Civil Engineering and Environmental Systems
Vol. 27, No. 1, March 2010, 33–52

ISIP: capacity planning for flood management systems under uncertainty

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(*Received 7 August 2006*)

An inexact two-stage stochastic integer programming (ISIP) model is developed for capacity planning of flood diversion under uncertainty. It incorporates the concepts of two-stage stochastic programming and chance-constrained programming within an interval-parameter integer programming framework. ISIP can facilitate dynamic analysis of capacity-expansion planning when uncertainties are presented in terms of probabilistic distributions and discrete intervals. Moreover, it can be used for examining various policy scenarios associated with different levels of economic penalties when the promised targets are violated. The developed method is applied to a case study of flood-diversion planning under uncertainty. Reasonable solutions are generated for binary and continuous variables. They provide the desired capacity-expansion schemes and flood-diversion patterns, which are related to a variety of trade-offs between system cost and constraint-violation risk. Decisions with a lower-risk level imply a higher system cost and an increased reliability in satisfying the system constraints; conversely, a desire for reducing the system cost could result in an increased risk of violating the system constraints.

Keywords: capacity planning; flood diversion; integer programming; interval optimisation; policy analysis; stochastic; uncertainty

1. Introduction

Flooding is the leading cause of losses from natural phenomena. Roughly a half of fatalities from natural hazards and one-third of economic losses were attributed to flooding (Munich 2000). Over the past decades, the frequency and intensity of floods have increased due to deteriorated ecosystems, decreased vegetation cover, reduced stream capacity, varied runoff pattern, and changed climate condition. Flood damages are becoming more severe. For example, the flood of 1997 in the Red River forced about 28,000 Manitobans to move away from their homes and caused an estimated loss of $400 million (Huang 2005). Losses can hardly be avoided when major floods occur; however, floodplains can be used to divert flows and reduce losses. Therefore, capacity planning for floodplain-management systems to meet the overall flood-diversion demand continues to challenge decision makers.

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ISSN 1028-6608 print/ISSN 1029-0249 online

DOI: 10.1080/10286600802284373
http://www.informaworld.com

Capacity planning for floodplains can be achieved through the mixed-integer linear programming (MILP) method, where integer variables can be typically used to indicate whether or not particular expansion options are to be undertaken. Previously, several studies of flood management by means of MILP were reported (van Dantzig 1956, Kumar *et al.* 1979, Windsor 1981, Randall 1997, Srinivasan *et al.* 1999, Needham *et al.* 2000). However, in real-world problems, many system parameters are highly uncertain and their interrelationships can be extremely complicated (Babaeyan-Koopaei *et al.* 2003, Byun *et al.* 2003). For example, in a flood management system, spatial and temporal variations in such components as flood flows, floodplain capacities, and water-diversion policies can exist. Also, costs for flood diversion and capacity expansion have many uncertainties. These complexities may be increased by interactions between the uncertain parameters and the associated economic implications. These difficulties place the planning problem beyond the conventional integer programming methods. A more robust approach is thus desired.

Several optimisation techniques such as interval, fuzzy, and stochastic programming were incorporated within the MILP framework to tackle uncertainties in integer optimisation problems. These led to methods of interval-parameter integer programming (IIP), fuzzy integer programming (FIP), and stochastic integer programming (SIP) (Glover 1976, Ignizio and Daniels 1983, Zimmermann and Pollatschek 1984, Teghem and Kunsch 1986a,b, Huang *et al.* 1995a,b, 2001, Chang *et al.*1997, Chanas and Kuchta 1998, Watkins and McKinney 1998, Vanderpooten 2003, Li *et al.* 2008). IIP allows uncertainties expressed as interval numbers to be directly communicated into the integer optimisation process and the resulting solution, such that multiple decision alternatives can be generated through the interpretation of the solutions (Huang *et al.* 1995a). In IIP, uncertain parameters are expressed as intervals with known lower and upper bounds but unknown membership or distribution functions. However, the IIP method may become infeasible when the model's right-hand side parameters are highly uncertain; this limits its practical use. Huang *et al.* (1995b) proposed an interval fuzzy integer programming (IFIP) method to deal with uncertainties presented as intervals and fuzzy sets. However, IFIP had difficulties reflecting uncertainties expressed as random variables and analysing economic consequences.

The two-stage stochastic programming (TSP) is effective for problems in which an analysis of policy scenarios is desired and the related data are random in nature with recourse. In the past decades, two-stage stochastic integer programming (TSIP) methods were explored and applied to capacity-expansion problems (Bean *et al.* 1992, Berman and Ganz 1994, Berman *et al.* 1994, Klein-Haneveld *et al.* 1996, Carøe and Tind 1998, Carøe and Schultz 1999, Ahmed 2000, Lund 2002, Ahmed *et al.* 2004, Li *et al.* 2006a, b). For example, Eppen *et al.* (1989) described a TSIP model for capacity planning in an automobile manufacturing plant. Berman *et al.* (1994) developed a two-stage stochastic capacity-expansion model for service industries, where the Lagrangian relaxation-based solution methodology was used. Klein-Haneveld *et al.* (1996) proposed solution schemes for TSIP problems with simple integer recourse, based on the construction of a convex hull for the second-stage value function. Schultz *et al.* (1998) proposed a finite scheme for TSPs with discrete distributions and integer second-stage variables, where only integer values of the right-hand side parameters for the second-stage decisions were considered. Ahmed *et al.* (2004) developed a finite branch-and-bound algorithm for TSPs with discrete distributions, where mixed-integer first-stage and pure-integer second-stage variables were involved. Albornoz *et al.* (2004) proposed a TSIP model for planning the expansion of a thermal power plant, where uncertainties related to the future availability of the thermal plant were reflected through analysing a finite group of scenarios. Li *et al.* (2006b) developed an interval-fuzzy two-stage stochastic MILP model for dealing with multiple uncertainties in municipal solid waste management systems. However, there were few reports on the applications of TSIP to flood management. Lund (2002) developed a two-stage integer linear programming model for floodplain planning, where the objective was to minimise the sum of expected annual damages and annualised expected flood response costs. The developed method could deal with uncertainties (flood flows) expressed as probability distributions; however, it had difficulties in dealing with independent uncertainties of the model's left-hand sides and cost coefficients (in the objective function) and accounting for the risks of violating uncertain system constraints.

A real-world flood management system may involve multiple complexities, such as (i) uncertainties that exist as intervals and/or random variables, (ii) dynamics in capacity-expansion planning, and (iii) variations in policy scenarios that are associated with different levels of economic penalties when the promised targets are violated. Therefore, this study is to develop an inexact two-stage stochastic mixed-integer programming (ISIP) method for capacity planning of flood management systems. The developed ISIP will be able to address uncertainties presented as probability distributions and interval values and to reflect the risk of violating system constraints under uncertainty. Moreover, fixed-charge cost function will be used to reflect the economies of scale (EOS) in the capacity-expansion costs. Then, the developed method will be applied to a case study of flood management, and the modelling results will support decisions of flood diversion and capacity expansion.

2. Model development

2.1. *Definitions for interval-parameter programming*

Prior to formulating the ISIP model for flood management and planning, we first introduce and review several ancillary definitions used in the earlier interval-parameter or grey system studies (Huang *et al.* 1992, 1995) that will be implemented through a series of model transformations to assist with computational efforts.

DEFINITION 1 *Let* x *denote a closed and bounded set of real numbers. An interval number with known lower and upper bounds but unknown distribution can be defined as follows*:

$$x^{\pm} = [x^{-}, x^{+}] = \{t \in x | x^{-} \leq t \leq x^{+}\} \tag{1}$$

where x^{-} *and* x^{+} *represent the lower and upper bounds of* $x^{\pm}$, *respectively. When* $x^{-} = x^{+}$, $x^{\pm}$ *becomes a deterministic number*, *i.e.* $x^{\pm} = x^{-} = x^{+}$.

DEFINITION 2 *For* $x^{\pm}$, *the following relationships hold*:

$$x^{\pm} \geq 0 \quad \textit{if } x^{-} \geq 0 \textit{ and } x^{+} \geq 0 \tag{2a}$$

$$x^{\pm} \leq 0 \quad \textit{if } x^{-} \leq 0 \textit{ and } x^{+} \leq 0 \tag{2b}$$

DEFINITION 3 *For* $x^{\pm}$ *and* $y^{\pm}$, *their order relations are*:

$$x^{\pm} \leq y^{\pm} \quad \textit{if } x^{-} \leq y^{-} \textit{ and } x^{+} \leq y^{+} \tag{3a}$$

$$x^{\pm} < y^{\pm} \quad \textit{if } x^{-} < y^{-} \textit{ and } x^{+} < y^{+} \tag{3b}$$

DEFINITION 4 *For* $x^{\pm}$, *Sign* $(x^{\pm})$ *is defined as follows*:

$$\text{Sign}\ (x^{\pm}) = \begin{matrix} 1 & \textit{if } x^{\pm} \geq 0 \\ -1 & \textit{if } x^{\pm} < 0 \end{matrix} \tag{4}$$

DEFINITION 5 *For $x^{\pm}$, its absolute value $|x|^{\pm}$ is defined as follows*:

$$|x|^{\pm} = \begin{cases} x^{\pm} & if\, x^{\pm} \geq 0 \\ -x^{\pm} & if\, x^{\pm} < 0 \end{cases} \tag{5a}$$

where

$$|x|^{-} = \begin{cases} x^{-} & if\, x^{\pm} \geq 0 \\ -x^{+} & if\, x^{\pm} < 0 \end{cases} \tag{5b}$$

and

$$|x|^{+} = \begin{cases} x^{+} & if\, x^{\pm} \geq 0 \\ -x^{-} & if\, x^{\pm} < 0 \end{cases} \tag{5c}$$

DEFINITION 6 *Let $\boldsymbol{R}^{\pm}$ denote a set of interval numbers. An interval vector $\boldsymbol{X}^{\pm}$ is a tuple of interval numbers, and an interval matrix $\boldsymbol{Y}^{\pm}$ has its elements being interval numbers*:

$$\boldsymbol{X}^{\pm} = \{x_i^{\pm} = [x_i^{-},\ x_i^{+}] | \forall i\}, \quad \boldsymbol{X}^{\pm} \in \{\boldsymbol{R}^{\pm}\}^{1\times n} \tag{6a}$$

$$\boldsymbol{Y}^{\pm} = \{y_{ij}^{\pm} = [y_{ij}^{-}, y_{ij}^{+}] | \forall i, j\}, \quad \boldsymbol{Y}^{\pm} \in \{\boldsymbol{R}^{\pm}\}^{m\times n} \tag{6b}$$

DEFINITION 7 *The lower/upper bounds of interval vector $\boldsymbol{X}^{\pm}$ and interval matrix $\boldsymbol{Y}^{\pm}$ are defined as follows*:

$$\boldsymbol{X}^{-} = \{x_i^{-}, | \forall i\} \tag{7a}$$

$$\boldsymbol{X}^{+} = \{x_i^{+}, | \forall i\} \tag{7b}$$

$$\boldsymbol{Y}^{-} = \{y_{ij}^{-}, | \forall i, j\} \tag{7c}$$

$$\boldsymbol{Y}^{+} = \{y_{ij}^{+}, | \forall i, j\} \tag{7d}$$

DEFINITION 8 *For an interval vectors* (*or matrix*), *we have*:

$$\boldsymbol{X}^{\pm} \geq 0, \quad if\, x_{ij}^{\pm} \geq 0,\ \ \forall i, j,\ \ \boldsymbol{X}^{\pm} \in \{\boldsymbol{R}^{\pm}\}^{m\times n}, \quad m \geq 1, \tag{8a}$$

$$\boldsymbol{X}^{\pm} \leq 0, \quad if\, x_{ij}^{\pm} \leq 0,\ \ \forall i, j,\ \ \boldsymbol{X}^{\pm} \in \{\boldsymbol{R}^{\pm}\}^{m\times n}, \quad m \geq 1. \tag{8b}$$

DEFINITION 9 *Let $* \in \{+, -, \times, \div\}$ be a binary operation on interval numbers. For interval numbers $x^{\pm}$ and $y^{\pm}$, we have*:

$$x^{\pm} * y^{\pm} = [\min\{x * y\}, \max\{x * y\}], \quad x^{-} \leq x \leq x^{+},\ \ y^{-} \leq y \leq y^{+}. \tag{9a}$$

In case of division, it is assumed that $y^{\pm}$ does not contain a zero. Hence, we have:

$$x^{\pm} + y^{\pm} = [x^{-} + y^{-},\ x^{+} + y^{+}], \tag{9b}$$

$$x^{\pm} - y^{\pm} = [x^{-} - y^{+},\ x^{+} - y^{-}], \tag{9c}$$

$$x^{\pm} \times y^{\pm} = [\min\{x \times y\},\ \max\{x \times y\}], \tag{9d}$$

$$x^{\pm} \div y^{\pm} = [\min\{x \div y\},\ \max\{x \div y\}]. \tag{9e}$$

Definition 10 *An IIP model can be formulated as follows*:

$$\text{Min} f^{\pm} = \boldsymbol{C}^{\pm}\boldsymbol{X}^{\pm}, \tag{10a}$$

subject to:

$$\boldsymbol{A}^{\pm}\boldsymbol{X}^{\pm} \leq \boldsymbol{B}^{\pm}, \tag{10b}$$

$$\boldsymbol{X}^{\pm} \geq 0, \tag{10c}$$

$$x_j^{\pm} = \textit{interval decision variables}, \ x_j^{\pm} \in \boldsymbol{X}^{\pm} \tag{10d}$$

where $\boldsymbol{A}^{\pm} \in \{\boldsymbol{R}^{\pm}\}^{m\times n}$, $\boldsymbol{B}^{\pm} \in \{\boldsymbol{R}^{\pm}\}^{m\times 1}$, $\boldsymbol{C}^{\pm} \in \{\boldsymbol{R}^{\pm}\}^{1\times n}$, $\boldsymbol{X}^{\pm} \in \{\boldsymbol{R}^{\pm}\}^{n\times 1}$, and $\boldsymbol{R}^{\pm}$ denotes a set of interval numbers. In model (10), the decision variables ($\boldsymbol{X}^{\pm}$) can be sorted into two categories: continuous and binary. An interactive solution algorithm has been developed to solve the above problem through analyses of the interrelationships between the parameters and the variables and between the objective function and the constraints. According to Huang *et al.* (1992, 1995), the solution for model (10) can be obtained through a two-step method, where a submodel corresponding to f^- (when the objective function is to be minimised) can be first formulated as follows (assume that $b_i^{\pm} > 0$, and $f^{\pm} > 0$):

$$\text{Min} f^{-} = \sum_{j=1}^{k_1} c_j^- x_j^- + \sum_{j=k_1+1}^{n} c_j^- x_j^+ \tag{11a}$$

subject to:

$$\sum_{j=1}^{k_1} |a_{ij}|^+ \ \text{Sign}(a_{ij}^+) x_j^- + \sum_{j=k_1+1}^{n} |a_{ij}|^- \ \text{Sign}(a_{ij}^-) x_j^+ \leq b_i^+, \forall i, \tag{11b}$$

$$x_j^{\pm} \geq 0, \forall j \tag{11c}$$

where $x_j^{\pm} (j = 1, 2, \ldots, k_1)$ are interval variables with positive coefficients in the objective function; $x_j^{\pm} (j = k_1 + 1, k_1 + 2, \ldots, n)$ are interval variables with negative coefficients. Solutions of $x_{j\text{opt}}^- (j = 1, 2, \ldots, k_1)$, $x_{j\text{opt}}^+ (j = k_1 + 1, k_1 + 2, \ldots, n)$, and f_{opt}^- can be obtained from submodel (11). Thus, the submodel corresponding to f^+ can be formulated as follows:

$$\text{Min} f^{+} = \sum_{j=1}^{k_1} c_j^+ x_j^+ + \sum_{j=k_1+1}^{n} c_j^+ x_j^- \tag{12a}$$

subject to:

$$\sum_{j=1}^{k_1} |a_{ij}|^- \ \text{Sign}(a_{ij}^-) x_j^+ + \sum_{j=k_1+1}^{n} |a_{ij}|^+ \ \text{Sign}(a_{ij}^+) x_j^- \leq b_i^-, \quad \forall i, \tag{12b}$$

$$x_j^+ \geq x_{j\,\text{opt}}^-, \quad j = 1, 2, \cdots, \ k_1 \tag{12c}$$

$$0 \leq x_j^- \leq x_{j\text{opt}}^+, \quad j = k_1 + 1, k_1 + 2, \cdots, n \tag{12d}$$

Solutions of $x_{j\text{opt}}^+ (j = 1, 2, \ldots, k_1)$, $x_{j\text{opt}}^- (j = k_1 + 1, k_1 + 2, \ldots, n)$, and f_{opt}^+ can be obtained from submodel (12). Then, through integration of the solutions of submodels (11) and (12), we can obtain interval solutions for model (10) as follows:

$$x_{j\text{opt}}^{\pm} = [x_{j\text{opt}}^{-}, x_{j\text{opt}}^{+}], \quad \forall j \tag{13a}$$

$$f_{\text{opt}}^{\pm} = [f_{\text{opt}}^{-}, f_{\text{opt}}^{+}] \tag{13b}$$

The IIP can directly handle uncertainties presented as interval numbers. However, it has difficulties in reflecting uncertainties expressed as probabilistic distributions; moreover, it is lack of linkage to economic consequences of violated policies pre-regulated by the authorities. When uncertainties of the model's right-hand sides are expressed as random variables and decisions need to be made periodically over time, the problem can be formulated as a TSP model. In TSP, a decision is first undertaken before random uncertainties are disclosed; then, after the random events have happened and their values are disclosed, a recourse decision can be made to minimise penalties that may appear due to any infeasibility (Birge and Louveaux 1988, 1997, Huang and Loucks 2000). However, the conventional TSP is associated with the following difficulties: (i) it can hardly deal with independent uncertainties of the model's left-hand sides and cost coefficients (in the objective function); (ii) it requires probabilistic specifications for uncertain parameters while, in many practical problems, the quality of information that can be obtained is mostly not satisfactory enough to be presented as probabilities; (iii) when multiple right-hand side parameters are expressed as probability distributions and are formulated into a TSP model, interactions among these uncertainties may lead to serious complexities, particularly for large-scale real-world problems. Therefore, one potential approach for tackling uncertainties presented as multiple formats is to incorporate the techniques of IIP and TSP within a general optimisation framework; this can lead to an ISIP method.

2.2. *ISIP for flood-diversion planning*

Consider a watershed system where flood volumes need to be diverted from a river to multiple diversion regions (i.e. flood-retention zones) over a flooding season. The river has a limited water-conveyance capacity and may overflow during flooding events. Associated with the local flood-management policies, a flood-warning water level (of the river) has been pre-formulated, and several projected flood-diversion regions have been assigned. If water in the river exceeds the pre-regulated warning level, the water will be allocated to the flood-retention zones. The flood-management system should contain both a specification of allowable levels of flood diversions and a scheme for efficiently using the diversion capacities. Especially, policies for diverting flood under limited diversion capacities are critical for minimising flows to densely populated communities. Decision makers desire sound flood-diversion and capacity-expansion schemes with both minimised cost and maximised safety.

Moreover, many uncertainties exist in the study system. The random characteristics of various natural processes and stream conditions, the errors in acquiring the modelling parameters, and the imprecision of the system objective and the related constraints are all possible sources of the uncertainties (Li *et al.* 2008). For example, stream flows may be presented as random variables with their values under different probability levels being available as discrete intervals, leading to dual uncertainties; randomness may also exist in flood-retention capacities, which can be expressed as a minimum requirement on the probability of satisfying the flood-diversion demands; the economic data for flood-diversion cost and floodplain expansion expense may be available as interval values (i.e. an interval value can be defined as a number with known lower and upper bounds but with unknown distribution information). Based on the above considerations, IIP is introduced into the TSP framework to communicate multiple uncertainties into the optimisation process. Thus, an

ISIP model for flood management can be formulated as follows:

$$\mathrm{Min}\, f^{\pm} = \sum_{i=1}^{u} C_i^{\pm} W_i^{\pm} + E\left[\sum_{i=1}^{u} [C_i^{\pm} T_{i\mathrm{FL}}^{\pm} + D_i^{\pm} S_{i\mathrm{FL}}^{\pm}]\right] + \sum_{i=1}^{u}\sum_{m=1}^{w} \left\{\left[A_i^{\pm} + (B_i^{\pm} \Delta R_{im}^{\pm})^{\delta_i}\right] y_i^{\pm}\right\} \tag{14a}$$

subject to:

$$\Pr\left\{[W_i^{\pm} + S_{i\mathrm{FL}}^{\pm}] \le R_{i\,\max}^{\pm}, \forall i\right\} \ge 1 - q \tag{14b}$$

(Existing capacity constraints)

$$\Pr\left\{\sum_{i=1}^{u}\left[W_i^{\pm} + S_{i\mathrm{FL}}^{\pm} + T_{i\mathrm{FL}}^{\pm}\right] \le \sum_{i=1}^{u}(R_{i\,\max}^{\pm} + \sum_{m=1}^{w} \Delta R_{im}^{\pm} y_{im}^{\pm}), \forall i\right\} \ge 1 - q \tag{14c}$$

$$T_{i\mathrm{FL}}^{\pm} \le \sum_{m=1}^{w} \Delta R_{im}^{\pm} y_{im}^{\pm}, \quad \forall i \tag{14d}$$

(Expanded capacity constraints)

$$\sum_{i=1}^{u} [W_i^{\pm} + S_{i\mathrm{FL}}^{\pm} + T_{i\mathrm{FL}}^{\pm}] \ge \tilde{\mathrm{FL}}^{\pm} \tag{14e}$$

(Flood-availability constraints)

$$\begin{array}{ll} y_{im}^{\pm} = 1, & \text{if capacity expansion is undertaken} \\ \quad\;\; = 0, & \text{if otherwise} \end{array}, \quad \forall i, m \tag{14f}$$

$$\sum_{m=1}^{w} y_{im}^{\pm} \le 1, \quad \forall i \tag{14g}$$

(Floodplain expansion constraints)

$$W_i^{\pm} \ge 0, \quad \forall i \tag{14h}$$

$$S_{i\mathrm{FL}}^{\pm} \ge 0, \quad \forall i \tag{14i}$$

$$T_{i\mathrm{FL}}^{\pm} \ge 0, \quad \forall i \tag{14j}$$

(Non-negative constraints)

where $f^{\pm}$ is expected net system cost; i denotes index of water-diversion region, and $i = 1, 2, \ldots, u$; $A_i^{\pm}$ is fixed-charge expansion cost in diversion region i (\$$10^6$); $B_i^{\pm}$ is variable expansion cost (\$/m^3); δ_i denotes the EOS exponent of region i; $C_i^{\pm}$ represents regular cost to region i per unit of allowable water diverted (\$/m^3) (the first-stage cost parameter); $D_i^{\pm}$ denotes penalty to region i per unit of surplus water diverted (\$/m^3) (the second-stage cost parameter); $E[\cdot]$ means expected value of a random variable; $\tilde{\mathrm{FL}}^{\pm}$ symbolises random variable equal to the total of available flood flow; q is a joint probability of violating constraints of flood-retention capacities, and $q \in [0, 1]$; $R_{i\,\max}^{\pm}$ denotes maximum level of the existing diversion capacity in region i (m^3); $\Delta R_{im}^{\pm}$ is the level of expansion option m for region i (m^3); $S_{i\mathrm{FL}}^{\pm}$ represents amount of surplus flood diverted to region i in reference to $W_i^{\pm}$ when the flood flow is $\tilde{\mathrm{FL}}^{\pm}$ (m^3) (the second-stage decision variable); $T_{i\mathrm{FL}}^{\pm}$ denotes the amount of increased allowance for region i when its diversion capacity is expanded under flood flow of $\tilde{\mathrm{FL}}^{\pm}$ (m^3) (the second-stage decision variable); $W_i^{\pm}$ stands for allowable amount of diversion to region i (m^3) (the first-stage decision variable); $y_{im}^{\pm}$ is binary variable for determining which region expansion option needs to be undertaken.

Model (12) includes continuous and binary decision variables. The continuous variables represent flood flows and the binary ones are for capacity planning decisions. The goal is to achieve optimal capacity-expansion schemes and relevant flood-diversion patterns with a minimised system cost and a maximised system safety. In model (12), the fixed-charge cost function is used to reflect the EOS in the capacity-expansion costs (Thuesen *et al.* 1977); moreover, the constraints of uncertain flood-retention capacities are enforced to be satisfied at a joint probability of at least q, and thus an increased robustness in controlling the system risk can be accomplished (Miller and Wager 1965, Zhang *et al.* 2002, Lejeune and Prekopa 2005). However, the above model is generally nonlinear. It will be equivalently formulated as a linear model, based on an assumption of discrete distribution for each random parameter. According to Huang and Loucks (2000), the distribution of flood flows ($\tilde{\mathrm{FL}}^{\pm}$) can be converted into an equivalent set of discrete values by letting each $\tilde{\mathrm{FL}}^{\pm}$ take value $\mathrm{FL}_j^{\pm}$ with probability $p_j (j = 1, 2, \ldots, v)$. Thus, we have:

$$E\left[\sum_{i=1}^{u}\left[C_i^{\pm}T_{i\mathrm{FL}}^{\pm} + D_i^{\pm}S_{i\mathrm{FL}}^{\pm}\right]\right] = \sum_{i=1}^{u}\sum_{j=1}^{v} p_j(C_i^{\pm}T_{ij}^{\pm} + D_i^{\pm}S_{ij}^{\pm}), \quad \forall i, j \tag{15a}$$

Secondly, by letting the random variables take a set of individual probabilistic constraints, the nonlinear joint probabilistic constraints (JPC) problem can be converted into a linear one. According to Lejeune and Prekopa (2005), Equations (14b) and (14c) can be converted into:

$$\left[W_i^{\pm} + S_{i\mathrm{FL}}^{\pm}\right] \le (R_{i\max}^{\pm})^{q_i}, \quad \forall i \tag{15b}$$

$$\sum_{i=1}^{u}\left[W_i^{\pm} + S_{i\mathrm{FL}}^{\pm} + T_{i\mathrm{FL}}^{\pm}\right] \le \sum_{i=1}^{u}\left[\left(R_{i\max}^{\pm}\right)^{q_i} + \sum_{m=1}^{w}\Delta R_{im}^{\pm}y_{im}^{\pm}\right], \quad \forall i \tag{15c}$$

$$\sum_{i=1}^{u} q_i \le q \tag{15d}$$

where q_i is the admissible probability of violating the capacity of floodplain i. Consequently, the above nonlinear ISIP model can be converted into a linear one. Moreover, in model (14), the flood-diversion targets ($W_i^{\pm}$) are expressed as interval numbers; however, as the first-stage decision variables, they should be identified before the related flood flows (i.e. random variables) are known (Huang and Loucks 2000, Dupaèová 2002). In this study, it is proposed introducing z_i as decision variables to identify an optimised set of target values. This optimised set will correspond to the lowest possible system cost under the uncertain flood-diversion targets. Thus, according to Huang and Loucks (2000), model (14) can be transformed into two deterministic submodels based on an interactive algorithm when $W_i^{\pm}$ are known. The transformation process is based on an interactive algorithm, which is different from normal best/worst case analysis. The resulting solution presents as intervals for the objective function value and decision variables, which can be easily interpreted for generating decision alternatives. Since the objective is to minimise the system cost, f^- submodel is first desired. Thus, we have:

$$\begin{aligned}\mathrm{Min}\, f^- = &\sum_{i=1}^{u} C_i^-(W_i^- + \Delta W_i z_i) + \sum_{i=1}^{u}\sum_{j=1}^{v} p_j(C_i^- T_{ij}^- + D_i^- S_{ij}^-)\\ &+ \sum_{i=1}^{u}\sum_{m=1}^{w}\left\{\left[A_i^- + (B_i^-\Delta R_{im}^-)^{\delta_i}\right] y_{im}^-\right\}\end{aligned} \tag{16a}$$

subject to:

$$(W_i^- + \Delta W_i z_i) + S_{ij}^- \le (R_{i\,\max}^+)^{q_i}, \quad \forall i, j \tag{16b}$$

$$\sum_{i=1}^{u} (W_i^- + \Delta W_i z_i + S_{ij}^- + T_{ij}^-) \le \sum_{i=1}^{u} \left[(R_{i\,\max}^+)^{q_i} + \sum_{m=1}^{w} \Delta R_{im}^- y_{im}^- \right], \quad \forall j \tag{16c}$$

$$T_{ij}^- \le \sum_{m=1}^{w} \Delta R_{im}^- y_{im}^-, \quad \forall i, j \tag{16d}$$

$$\sum_{i=1}^{u} (W_i^- + \Delta W_i z_i + S_{ij}^- + T_{ij}^-) \ge \mathrm{FL}_j^-, \quad \forall j \tag{16e}$$

$$W_i^- + \Delta W_i z_i \ge 0, \quad \forall i \tag{16f}$$

$$S_{ij}^- \ge 0, \quad \forall i, j \tag{16g}$$

$$T_{ij}^- \ge 0, \quad \forall i, j \tag{16h}$$

$$y_{im}^- \begin{array}{l} = 1, \quad \text{if capacity expansion is undertaken} \\ = 0, \quad \text{if otherwise} \end{array}, \quad \forall i, m \tag{16i}$$

$$\sum_{m=1}^{w} y_{im}^- \le 1, \quad \forall i \tag{16j}$$

$$\sum_{i=1}^{u} q_i \le q \tag{16k}$$

$$0 \le z_i \le 1, \quad \forall i \tag{16l}$$

where $\Delta W_i = W_i^+ - W_i^-$ and $z_i \in [0, 1]$. In submodel (16), S_{ij}^-, T_{ij}^-, and z_i are continuous variables and y_{im}^- are binary variables. Solution for f^- provides the lower-bound system cost under uncertain inputs of water-allocation targets. Let $S_{ij\mathrm{opt}}^-$, $T_{ij\mathrm{opt}}^-$, $z_{i\,\mathrm{opt}}$, and $y_{im\mathrm{opt}}^-$ be solutions of submodel (16). Then, the second submodel corresponding to f^+ under the optimised water-allocation target (i.e. $W_{i\,\mathrm{opt}}^\pm = W_i^- + \Delta W_i z_{i\,\mathrm{opt}}$) can be formulated as follows:

$$\begin{aligned} \mathrm{Min}\, f^+ = {} & \sum_{i=1}^{u} C_i^+ (W_i^- + \Delta W_i z_{i\,\mathrm{opt}}) + \sum_{i=1}^{u} \sum_{j=1}^{v} p_j (C_i^+ T_{ij}^+ + D_i^+ S_{ij}^+) \\ & + \sum_{i=1}^{u} \sum_{m=1}^{w} \left\{ \left[A_i^+ + (B_i^+ \Delta R_{im}^+)^{\delta_i} \right] y_{im}^+ \right\} \end{aligned} \tag{17a}$$

subject to:

$$(W_i^- + \Delta W_i z_{i\,\mathrm{opt}}) + S_{ij}^+ \le (R_{i\,\max}^-)^{q_i}, \quad \forall i, j \tag{17b}$$

$$\sum_{i=1}^{u} (W_i^- + \Delta W_i z_{i\,\mathrm{opt}} + S_{ij}^+ + T_{ij}^+) \le \sum_{i=1}^{u} [(R_{i\,\max}^-)^{q_i} + \sum_{m=1}^{w} \Delta R_{im}^+ y_{im}^+], \quad \forall j \tag{17c}$$

$$T_{ij}^+ \le \sum_{m=1}^{w} \Delta R_{im}^+ y_{im}^+, \quad \forall i, j \tag{17d}$$

$$\sum_{i=1}^{u}(W_i^- + \Delta W_i z_{i\,\mathrm{opt}} + S_{ij}^+ + T_{ij}^+) \geq \mathrm{FL}_j^+, \quad \forall j \tag{17e}$$

$$y_{im}^+ \begin{cases} = 1, & \text{if capacity expansion is undertaken} \\ = 0, & \text{if otherwise} \end{cases}, \quad \forall i, m \tag{17f}$$

$$\sum_{m=1}^{w} y_{im}^+ \leq 1, \quad \forall i \tag{17g}$$

$$\sum_{i=1}^{u} q_i \leq q \tag{17h}$$

$$S_{ij}^+ \geq S_{ij\mathrm{opt}}^- \geq 0, \quad \forall i, j \tag{17i}$$

$$T_{ij}^+ \geq T_{ij\mathrm{opt}}^- \geq 0, \quad \forall i, j \tag{17j}$$

$$y_{im}^+ \geq y_{im\mathrm{opt}}^-, \quad \forall i, m \tag{17k}$$

where S_{ij}^+, T_{ij}^+, and y_{im}^+ are decision variables. Let $S_{ij\mathrm{opt}}^+$, $T_{ij\mathrm{opt}}^+$, and $y_{im\mathrm{opt}}^+$ be the solutions of submodel (17). Thus, we can obtain the interval solution as follows:

$$T_{ij\mathrm{opt}}^{\pm} = [T_{ij\mathrm{opt}}^-, T_{ij\mathrm{opt}}^+], \quad \forall i, j \tag{18a}$$

$$S_{ij\mathrm{opt}}^{\pm} = [S_{ij\mathrm{opt}}^-, S_{ij\mathrm{opt}}^+], \quad \forall i, j \tag{18b}$$

$$y_{im\mathrm{opt}}^{\pm} = [y_{im\mathrm{opt}}^-, y_{im\mathrm{opt}}^+], \quad \forall i, m \tag{18c}$$

$$f_{\mathrm{opt}}^{\pm} = [f_{\mathrm{opt}}^-, f_{\mathrm{opt}}^+] \tag{18d}$$

The optimised flood-diversion patterns under varied flow levels are:

$$N_{ij\mathrm{opt}}^{\pm} = W_{i\,\mathrm{opt}}^{\pm} + T_{ij\mathrm{opt}}^{\pm}, \quad \forall i, j \tag{18e}$$

$$A_{ij\mathrm{opt}}^{\pm} = W_{i\,\mathrm{opt}}^{\pm} + T_{ij\mathrm{opt}}^{\pm} + S_{ij\mathrm{opt}}^{\pm}, \quad \forall i, j \tag{18f}$$

where $N_{ij\mathrm{opt}}^{\pm}$ is the sum of primal and incremental allowable flood-diversion levels; $A_{ij\mathrm{opt}}^{\pm}$ is the total diversion flow including primal allowance, incremental quota, and excess flow. The main advantage of the two-stage programming is its capability of incorporating multiple policies of flood management within the modelling framework. If the models were simply constructed with $A_{ij}^{\pm}$ (instead of $W_i^{\pm}$, $S_{ij}^{\pm}$, and $T_{ij}^{\pm}$) being decision variables, then the related flood-management policies and their implications would not have been reflected. Moreover, the ISIP method can facilitate dynamic analyses of capacity-expansion planning when uncertainties are expressed as probability distributions and interval values. Violations for capacity constraints are allowed under a range of significance levels, which are related to trade-offs between the system cost and the constraint-violation risk. Thus, the method can also support the assessment of reliability for satisfying the system constraints under uncertainty.

3. Case study

The study watershed contains three regions that are available to serve flood-diversion needs (Figure 1). An allowable flood-diversion level is pre-regulated according to the existing capacity. If this allowance is exceeded, it will mean a surplus diversion associated with economic penalties and/or capacity-expansion costs. The penalties may be expressed in terms of raised costs for

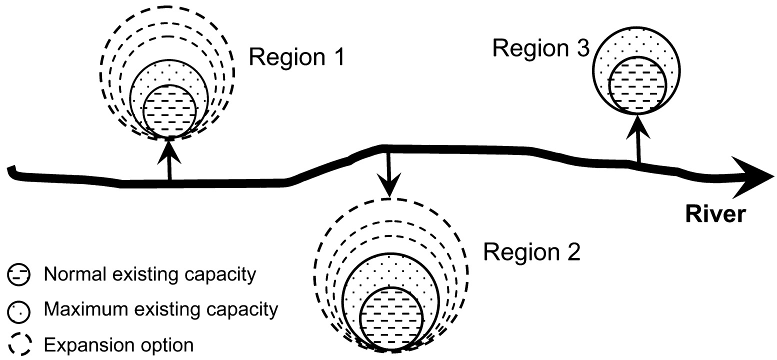

Figure 1. Schematic of flood diversion to assigned regions.

Table 1. Descriptions of flood-flow availability.

Stream flow level	Probability (p_j)	Flow volume ($q_j^{\pm}$) (10^6 m^3)
Low (j = 1)	0.10	[5.0, 8.0]
Low-medium (j = 2)	0.20	[8.0, 12.0]
Medium (j = 3)	0.40	[12.0, 16.5]
Medium-high (j = 4)	0.20	[16.5, 22.0]
High (j = 5)	0.10	[22.0, 28.0]

Table 2. Capacity-expansion options, allowable diversion flows, and the associated costs

	$i = 1$	$i = 2$	$i = 3$
Capacity-expansion option (10^6 m^3):			
$\Delta R_{i1}^{\pm}$ (option 1)	[3, 4]	[5, 7]	0
$\Delta R_{i2}^{\pm}$ (option 2)	[4, 5]	[6, 8]	0
$\Delta R_{i3}^{\pm}$ (option 3)	[5, 6]	[7, 9]	0
Capital cost of expansion:			
Fixed expansion cost, $A_i^{\pm}$ (\$$10^6$)	[8, 10]	[12, 15]	0
Variable expansion cost, $B_i^{\pm}$(\$/m^3)	[90, 100]	[110, 120]	0
Scale-economy index, δ_i	0.98	0.94	0
Allowable flood-diversion target $W_i^{\pm}$ (10^6 m^3)	[2.0, 3.0]	[3.0, 4.5]	[2.5, 3.5]
Regular cost for allowable flood diversion $C_i^{\pm}$ (\$/m^3)	[80, 100]	[90, 110]	[100, 130]
Penalty for excess flood diversion $D_i^{\pm}$ (\$/m^3)	[200, 250]	[150, 180]	[180, 210]

flood diversion and/or destruction of land-based infrastructure. The capacity expansions will help increase the allowable flood-diversion levels and thus reduce the penalties. Therefore, the total diverted flow will be a sum of the primal allowance, the incremental quota, and the probabilistic excess flow. Table 1 provides the flood flows as well as the associated probabilities of occurrences. Table 2 presents capacity-expansion options, allowable flood allocation, and the related costs. Based on the local flood-management policy, regions 1 and 2 can be expanded once by any of the three options. Besides, the existing capacities of floodplains 1, 2, and 3 are [4.0, 5.0] $\times 10^6$, [5.2, 6.0] $\times 10^6$ m^3, and [3.4, 4.4] $\times 10^6$ m^3, respectively. The problems under consideration include: (a) how to identify desired capacity-expansion schemes, (b) how to effectively divert flood flows to suitable regions, (c) how to achieve a minimised system cost under uncertainty,

Table 3. Solution of ISIP model for binary variables under different q levels.

Expansion option	Symbol		Solution of capacity expansion					
			$q = 0.05$		$q = 0.10$		$q = 0.20$	
	Region 1	Region 2	Region 1	Region 2	Region 1	Region 2	Region 1	Region 2
1	$Y_{11}^{\pm}$	$Y_{21}^{\pm}$	0	0	[0, 1]	0	[0, 1]	[1, 1]
2	$Y_{12}^{\pm}$	$Y_{22}^{\pm}$	[1, 1]	0	0	[1, 1]	0	0
3	$Y_{13}^{\pm}$	$Y_{23}^{\pm}$	0	[1, 1]	0	0	0	0

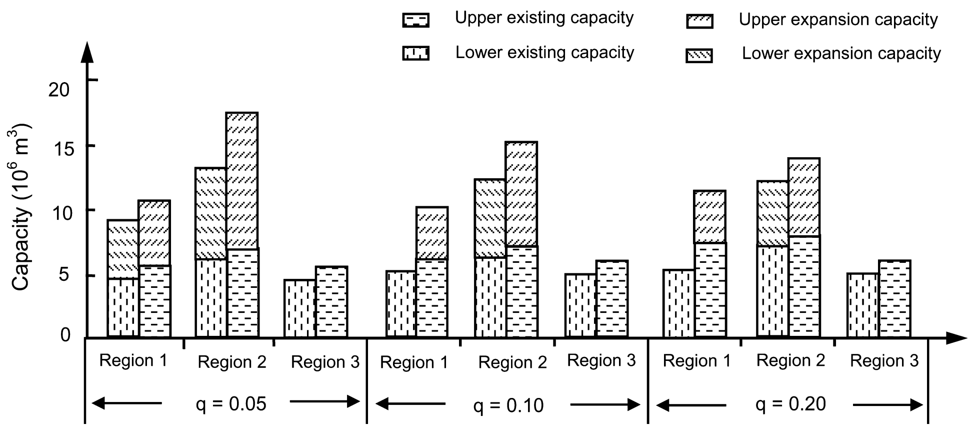

Figure 2. The original and expanded capacities under different q levels.

and (d) how to incorporate flood management policies within the modelling system. Therefore, the developed ISIP will be used for dealing with the planning problem.

Table 3 presents the solutions of binary variables from the ISIP model. In ISIP, the capacity constraints of the three flood-retention zones are considered to be satisfied at a set of joint probability level (q). The results indicate that different q levels can lead to varied capacity-expansion schemes. For example, region 1 would be expanded with an increment of [4, 5] $\times 10^6$ m^3 when $q = 0.05$; however, when $q = 0.10$ and 0.20, there would be two expansion options corresponding to f^- and f^+. When the decision scheme tends towards f^- under advantageous system conditions, this region would not be expanded; conversely, when the scheme tends towards f^+ under more demanding conditions, it would be expanded with a capacity of 4×10^6 m^3. For region 2, its expansion capacities would be [7, 9] $\times 10^6$ m^3 when $q = 0.05$, [6, 8] $\times 10^6$ m^3 when $q = 0.10$, and [5, 7] $\times 10^6$ m^3 when $q = 0.20$. Consequently, the total expanded capacities would be [11, 14] $\times 10^6$, [6, 12] $\times 10^6$, and [5, 11] $\times 10^6$ m^3 when q levels are 0.05, 0.10, and 0.20, respectively. Figure 2 presents the original and expanded capacities available for flood diversion under different q levels. An increased q level means a raised risk of constraint violation and, at the same time, it leads to a decreased strictness for the capacity constraints and thus corresponds to a lower capacity-expansion amounts, and vice versa. The capital costs for capacity expansion under $q = 0.05$, 0.10, and 0.20 would be \$[488.3, 670.0] $\times 10^6$, \$[207.2, 571.7] $\times 10^6$, and \$[176.4, 539.0] $\times 10^6$, respectively. In general, planning with a higher risk of violating the constraints could lead to a lower expansion cost. In comparison, planning for a lower risk would result in a higher cost.

In this study, expansions of regions 1 and 2 would lead to increased allowable flows to the two regions (i.e. $T_{ij}^{\pm}$); at the same time, the increased allowable flows could reflect the utilisation conditions of expanded capacities under different flood flows and q levels. An increase in the allowable flow would help to reduce excess flow and thus increase system reliability. For example, the results of $T^{\pm}_{11\text{opt}} = T^{\pm}_{21\text{opt}} = 0$ means that under significance levels of $q = 0.05$, 0.10, and 0.20, the expanded capacities of regions 1 and 2 would not be needed when the flow level is low associated with a probability of 10%. Under a low-medium flow level, the results of $T^{\pm}_{12\text{opt}} = [0.0, 3.5] \times 10^6\,\text{m}^3$ ($q = 0.05$), $T^{\pm}_{12\text{opt}} = [0.0, 2.0] \times 10^6\,\text{m}^3$ ($q = 0.10$), and $T^{\pm}_{12\text{opt}} = [0.0, 2.0] \times 10^6\,\text{m}^3$ ($q = 0.20$) indicate that region 1 would need to be expanded when the flow approaches its upper bound (i.e. FL^+_j). However, under a high flow level, regions 1 and 2 would both be expanded to satisfy the diversion need. The allowable flows to region 3 would keep being constant since this zone would not be expanded.

Tables 4–6 present the continuous variable solutions including those for the first- and second-stage variables. In the case of flooding events, the allowable flood flows are first diverted to the three regions with regular costs; if the water level in the river still exceeds the pre-regulated warning criterion, the probabilistic excess flow (in reference to the allowable water-diversion target) should continue to be diverted, leading to excess flow and/or increased allowance. The results of $z_{1\text{opt}} = 1$, $z_{2\text{opt}} = 0$, and $z_{3\text{opt}} = 0$ indicate that the allowable diversion levels to regions 1–3 would be 3.0×10^6, 3.0×10^6, and $2.5 \times 10^6\,\text{m}^3$ when $q = 0.05$. However, when $q = 0.10$ and 0.20, the allowable diversion levels to regions 1–3 would be $3.0 \times 10^6\,\text{m}^3$ ($z_{1\text{opt}} = 1$), $4.5 \times 10^6\,\text{m}^3$ ($z_{2\text{opt}} = 1$), and $2.5 \times 10^6\,\text{m}^3$ ($z_{3\text{opt}} = 0$), respectively. Generally, when $W^{\pm}_i$ reach their lower bounds (i.e. when $z_i = 0$), low system cost may be obtained if the flood-flow level is low; however, a higher penalty may have to be paid when the flood-flow level is high. Conversely, when $W^{\pm}_i$ approach their upper bounds (i.e. when $z_i = 1$), we may have a high system cost under a low flood-flow level but, at the same time, a lower risk of violating the promised targets (and thus lower penalty) when the flood-flow level is high.

For the three regions under $q = 0.05$, 0.10, and 0.20, there would be excess flows when the flood flows are medium-high to high, but no excess flow under low to medium flow levels. For example, under medium-high flow level, there would be excess flows of $[0.0, 1.0] \times 10^6\,\text{m}^3$ when

Table 4. Solution of the ISIP model for continuous variables under $q = 0.05$.

			Flood-diversion pattern ($10^6\,\text{m}^3$)				
Region	Flood-flow level	Probability (%)	Allowable diversion, $W^{\pm}_{i\,\text{opt}}$	Incremental diversion after expansion, $T^{\pm}_{ij\text{opt}}$	Regular diversion, $N^{\pm}_{ij\text{opt}}$	Excess diversion, $S^{\pm}_{ij\text{opt}}$	Optimised diversion, $A^{\pm}_{ij\text{opt}}$
1	Low	10	3.0	0	3.0	0	3.0
2	Low	10	3.0	0	3.0	0	3.0
3	Low	10	2.5	0	2.5	0	2.5
1	Low-medium	20	3.0	[0.0, 3.5]	[3.0, 6.5]	0	[3.0, 6.5]
2	Low-medium	20	3.0	0	3.0	0	3.0
3	Low-medium	20	2.5	0	2.5	0	2.5
1	Medium	40	3.0	[3.5, 5.0]	[6.5, 8.0]	0	[6.5, 8.0]
2	Medium	40	3.0	[0.0, 3.0]	[3.0, 6.0]	0	[3.0, 6.0]
3	Medium	40	2.5	0	2.5	0	2.5
1	Medium-high	20	3.0	5.0	3.0	0	3.0
2	Medium-high	20	3.0	[3.0, 8.5]	[6.0, 11.5]	0	[6.0, 11.5]
3	Medium-high	20	2.5	0	2.5	0	2.5
1	High	10	3.0	5.0	8.0	[0.0, 0.6]	[8.0, 8.6]
2	High	10	3.0	[8.5, 9.0]	[11.5, 12.0]	[0.0, 3.0]	[11.5, 15.0]
3	High	10	2.5	0	2.5	[0.0, 1.9]	[2.5, 4.4]

Decision variables: $z_{1\text{opt}} = 1$, $z_{2\text{opt}} = 0$, and $z_{3\text{opt}} = 0$
Net system cost ($\$10^6$): $f^{\pm}_{\text{opt}} = [1610.8, 2571.9]$

Table 5. Solution of the ISIP model for continuous variables under $q = 0.10$.

Region	Flood-flow level	Probability (%)	Flood-diversion pattern (10^6 m^3) Allowable diversion, $W^{\pm}_{i\,\mathrm{opt}}$	Incremental diversion after expansion, $T^{\pm}_{ij\mathrm{opt}}$	Regular diversion, $N^{\pm}_{ij\mathrm{opt}}$	Excess diversion, $S^{\pm}_{ij\mathrm{opt}}$	Optimised diversion, $A^{\pm}_{ij\mathrm{opt}}$
1	Low	10	3.0	0	3.0	0	3.0
2	Low	10	4.5	0	4.5	0	4.5
3	Low	10	2.5	0	2.5	0	2.5
1	Low-medium	20	3.0	[0.0, 2.0]	[3.0, 5.0]	0	[3.0, 5.0]
2	Low-medium	20	4.5	0	4.5	0	4.5
3	Low-medium	20	2.5	0	2.5	0	2.5
1	Medium	40	3.0	[0.0, 4.0]	[3.0, 7.0]	0	[3.0, 7.0]
2	Medium	40	4.5	[2.0, 2.5]	[6.5, 7.0]	0	[6.5, 7.0]
3	Medium	40	2.5	0	2.5	0	2.5
1	Medium-high	20	3.0	[0.0, 4.0]	[3.0, 7.0]	0	[3.0, 7.0]
2	Medium-high	20	4.5	[6.5, 8.0]	[11.0, 12.5]	0	[11.0, 12.5]
3	Medium-high	20	2.5	0	2.5	0	2.5
1	High	10	3.0	[0.0, 4.0]	[3.0, 7.0]	[0.0, 2.0]	[3.0, 9.0]
2	High	10	4.5	8.0	12.5	1.7	14.2
3	High	10	2.5	0	2.5	2.3	4.8

Decision variables: $z_{1\mathrm{opt}} = 1$, $z_{2\mathrm{opt}} = 1$ and $z_{3\mathrm{opt}} = 0$
Net system cost (\$$10^6$): $f^{\pm}_{\mathrm{opt}} = [1430.1, 2514.6]$

Table 6. Solution of the ISIP model for continuous variables under $q = 0.20$.

Region	Flood-flow level	Probability (%)	Flood-diversion pattern (10^6 m^3) Allowable diversion, $W^{\pm}_{i\,\mathrm{opt}}$	Incremental diversion after expansion, $T^{\pm}_{ij\mathrm{opt}}$	Regular diversion, $N^{\pm}_{ij\mathrm{opt}}$	Excess diversion, $S^{\pm}_{ij\mathrm{opt}}$	Optimised diversion, $A^{\pm}_{ij\mathrm{opt}}$
1	Low	10	3.0	0	3.0	0	3.0
2	Low	10	4.5	0	4.5	0	4.5
3	Low	10	2.5	0	2.5	0	2.5
1	Low-medium	20	3.0	[0.0, 2.0]	[3.0, 5.0]	0	[3.0, 5.0]
2	Low-medium	20	4.5	0	4.5	0	4.5
3	Low-medium	20	2.5	0	2.5	0	2.5
1	Medium	40	3.0	[0.0, 4.0]	[3.0, 7.0]	0	[3.0, 7.0]
2	Medium	40	4.5	[2.0, 2.5]	[6.5, 7.0]	0	[6.5, 7.0]
3	Medium	40	2.5	0	2.5	0	2.5
1	Medium-high	20	3.0	[0.0, 4.0]	[3.0, 7.0]	0	[3.0, 7.0]
2	Medium-high	20	4.5	[6.5, 7.0]	[11.0, 11.5]	[0.0, 1.0]	[11.0, 12.5]
3	Medium-high	20	2.5	0	2.5	0	2.5
1	High	10	3.0	[0.0, 4.0]	[3.0, 7.0]	[0.1, 2.1]	[3.1, 9.1]
2	High	10	4.5	7.0	11.5	2.5	14.0
3	High	10	2.5	0	2.5	2.4	4.9

Decision variables: $z_{1\mathrm{opt}} = 1$, $z_{2\mathrm{opt}} = 1$, and $z_{3\mathrm{opt}} = 0$
Net system cost (\$$10^6$): $f^{\pm}_{\mathrm{opt}} = [1406.6, 2503.9]$

$q = 0.20$; under high flow level, the total excess flows would be [0.0, 5.5] $\times 10^6$, [4.0, 6.0] $\times 10^6$, and [5.0, 7.0] $\times 10^6$ m^3 when $q = 0.05$, 0.10, and 0.20, respectively. This is because the expanded diversion capacities would generate increased allowances and thus reduce excess flows.

Correspondingly, the penalties for excess flood diversion would be \$[0.0, 108.9] $\times 10^6$, \$[66.9, 128.9] $\times 10^6$, and \$[83.2, 183.9] $\times 10^6$ when $q = 0.05$, 0.10, and 0.20, respectively, indicating an increasing tendency. Figures 3 and 4 present the optimised diversion patterns for regions 1

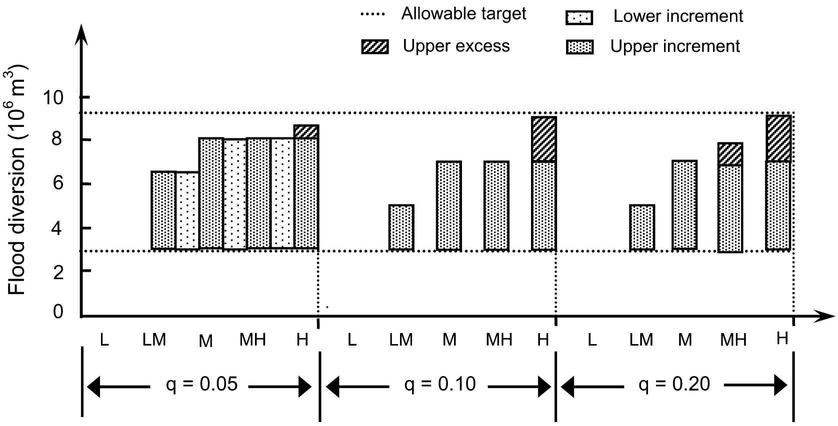

Figure 3. Optimised diversion patterns of region 1 under different q levels.

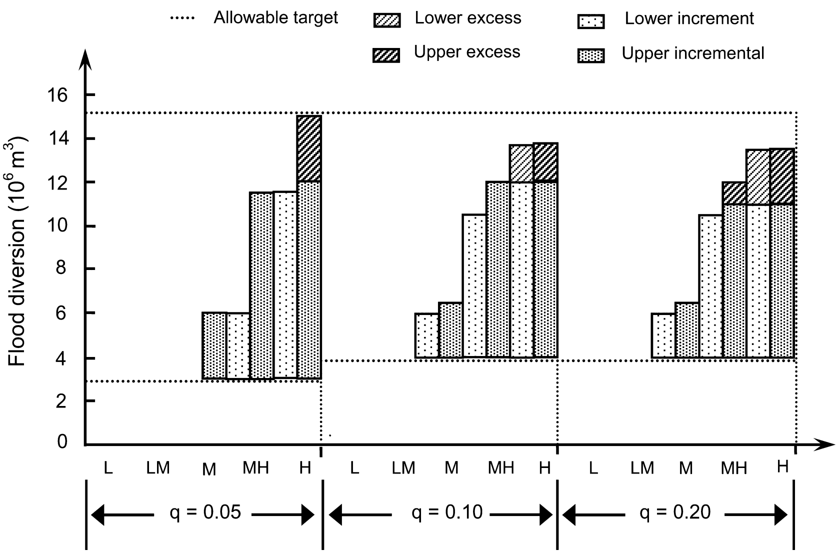

Figure 4. Optimised diversion patterns of region 2 under different q levels.

and 2 under different q levels. It is indicated that (i) the allowable and excess flows would both increase when the q level is raised; (ii) the expanded capacities would result in increased diversion allowances and reduced excess flows; (iii) more incremental allowances would be assigned to region 2 where a larger expansion capacity is planned.

Variations in the q levels also reveal the decision makers' preferences regarding the trade-off between system cost and constraint-violation risk. The objective function values are expressed as intervals under different q levels. The solutions of system cost ($f^{\pm}_{\text{opt}}$) are \$[1610.8, 2571.9] $\times 10^6$, \$[1430.1, 2514.6] $\times 10^6$, and \$[1406.6, 2503.9] $\times 10^6$ under $q = 0.05, 0.10$, and 0.20, respectively (listed in Tables 4–6). Figure 5 shows the variations of system cost with q level. Moreover, the solutions of system cost would also vary with individual probability of each reservoir-capacity constraint. Figure 6 provides the solutions for system cost under several scenarios (that correspond to different joint and individual-probability levels). For example, when joint probability equals 0.20, the system costs would be \$[1408.1, 2505.4] $\times 10^6$, \$[1411.6, 2507.4] $\times 10^6$, \$[1418.9,

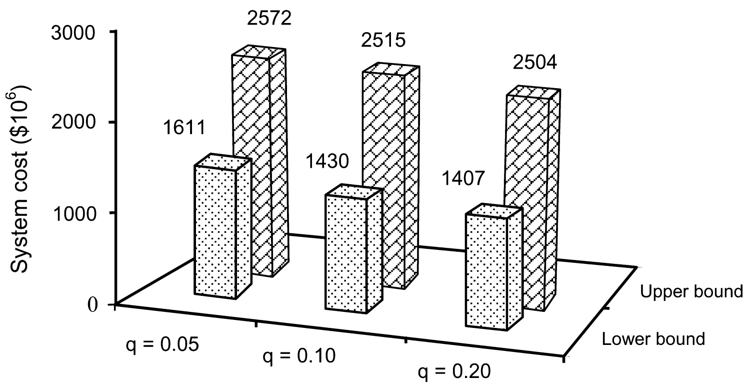

Figure 5. System costs under different q levels.

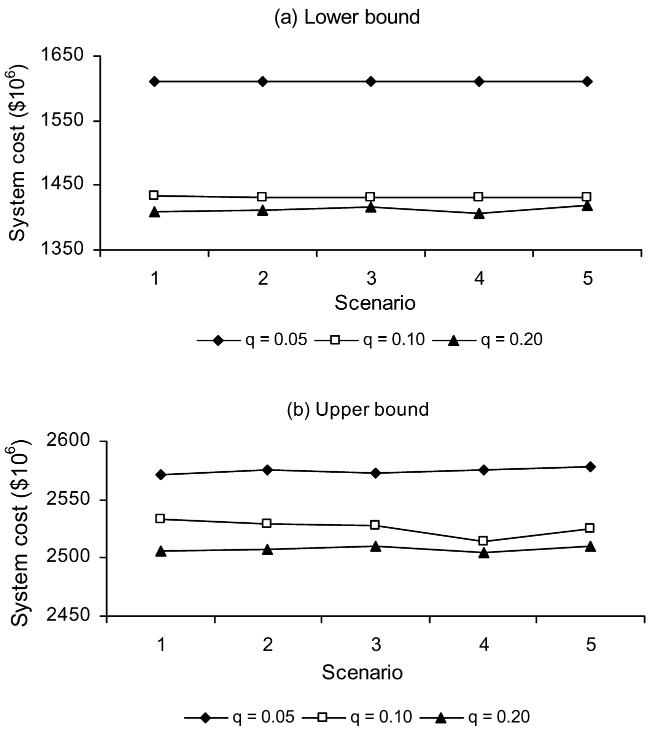

Figure 6. System costs under several scenarios.

2509.6] $\times 10^6$, \$[1406.6, 2503.9] $\times 10^6$, and \$[1427.7, 2510.6] $\times 10^6$ under different individual probability levels. In general, a lower q level (i.e. lower constraint-violation risk) would result in a higher cost; conversely, a higher q would sacrifice the system safety in order to reduce the cost.

4. Discussion

Solutions of the ISIP model provide desired flood-diversion patterns with minimised cost and maximised safety. The complexity is associated with the pre-regulation of the allowable flood-flow levels (i.e. the first-stage variable $W_i^{\pm}$) before the random flows are disclosed. Variations in the allowable flood-diversion levels could lead to multiple scenarios corresponding to different policies for managing the flood under uncertainty. When all $W_i^{\pm}$ reach their lower bounds (W_i^-), the system cost would be \$[1600.8, 2594.9] $\times 10^6$, \$[1418.6, 2557.6] $\times 10^6$, and \$[1407.1, 2546.9] $\times 10^6$ under $q = 0.05$, 0.10, and 0.20, respectively. Under this scenario, the decision makers are optimistic regarding the future conditions, and thus pre-regulate a low flood-diversion target for each region. This scenario would lead to a plan with both higher excess flows and higher penalties; the corresponding penalties would be \$[7.5, 151.9] $\times 10^6$ when $q = 0.05$, \$[144.4, 270.9] $\times 10^6$ when $q = 0.10$, and \$[190.7, 325.9] $\times 10^6$ when $q = 0.20$. Conversely, the solution when all $W_i^{\pm}$ reach their upper bounds represents a conservative consideration. It projects a high diversion target for each region. This scenario is associated with both lower excess flows and lower penalties; the penalties would be \$[0.0, 60.9] $\times 10^6$, \$[48.9, 107.9] $\times 10^6$, and \$[65.2, 126.9] $\times 10^6$ when $q = 0.05$, 0.10, and 0.20, respectively. Under this scenario, the system cost would be \$[1698.3, 2603.9] $\times 10^6$, \$[1458.1, 2537.6] $\times 10^6$, and \$[1434.6, 2512.9] $\times 10^6$ when $q = 0.05$, 0.10, and 0.20, respectively. Generally, a policy corresponding to a lower diversion target (W_i^-) may lead to a lower cost (f^-) under advantageous conditions; however, the system may be subjected to a higher risk of penalties under demanding conditions, and *vice versa*.

In the study problem, only one time period is considered. However, when the planning problem includes multiple periods with a sequential structure, the developed ISIP will have difficulties in reflecting dynamic variations of system conditions. Moreover, in ISIP, the capacity-expansion scheme for the entire planning horizon is determined at the first stage, while recourse actions to correct any infeasibility are taken at the second stage. These recourse actions can be interpreted as outsourcing additional capacities (Ahmed *et al.* 2003, Li *et al.* 2006c). Consequently, to deal with such a dynamic feature, a number of multi-stage stochastic programming (MSP) methods with recourse were developed as extensions of dynamic stochastic optimisation techniques. The multi-stage models improved upon TSP by permitting revised decisions in each time stage based on the real-time information of the uncertainties. The uncertainties and dynamics in MSP were often modelled through a multi-layer scenario tree. The solution approaches for MSP included nested benders decomposition and progressive hedging (Birge 1985, Rockafellar and Wets 1991). Several researchers also dealt with capacity-expansion issues under stochastic conditions through developing multi-stage stochastic integer programming methods (Chen *et al.* 2002, Ahmed *et al.* 2003, Lulli and Sen 2004). Therefore, one potential extension of this research will be about the reflection of the system dynamics within a multi-stage context in order to enhance the robustness of the developed method.

In fact, the objective of the developed ISIP is to minimise the expected system cost, which includes regular costs for diverting allowable flood flows, penalties for diverting excess flows, and capital costs for expanding flood-retention zones. ISIP is a single-objective model, and different from the conventional best–worst analysis. Based on an interactive algorithm, ISIP can be transformed into two deterministic submodels, where the second submodel is formulated based on the solutions of the first one (Huang *et al.* 1992). The resulting solutions present as intervals for the objective function value and decision variables, and can be interpreted for generating decision alternatives. In comparison, although multi-objective programming (MOP) methods can be used for decision analysis under multiple objectives subjected to a set of constraints, they may result in multiple decision alternatives. In general, in an MOP problem, an optimal solution or a Pareto set has to be determined. Determination of an optimal solution can be done through utility theory or other weighting methods; however, difficulties lie in the selection of correct utility functions and the quantification of trade-offs among multiple objectives (Sankararao and Gupta 2007). Moreover, for MOP problems, it can be problematic to directly analyse interrelationships among objectives through a combined function. A slight perturbation in the related parameters could result in significantly different solutions. An alternative to the MOP technique is to determine a Pareto set of solutions. The Pareto solutions are often preferred to the optimal solution since the final decisions are seldom based on the optima due to the existence of complex trade-offs among multiple conflicting factors. However, as the number of objectives increases, it can become increasingly overwhelming to identify desired compromises based on the Pareto set (Taboada *et al.* 2007). Challenges also exist in identifying such a Pareto set because an MOP problem often involves many subjective considerations and trade-off relationships.

5. Conclusions

An ISIP method has been developed for capacity planning of flood diversion under uncertainty. It incorporates techniques of TSP and chance-constrained programming within an IIP framework. The developed ISIP can facilitate dynamic analysis of capacity-expansion planning when uncertainties are presented in terms of probabilistic distributions and interval values. Moreover, it can support analyses for a variety of policy scenarios that are associated with multiple levels of economic penalties and expansion costs.

The developed method has been applied to a case study of flood management. Violations of capacity constraints are allowed under a range of joint-probability levels, which are related to trade-offs between system cost and constraint-violation risk. The results indicate that reasonable solutions have been generated for binary and continuous variables. The solutions provide desired capacity-expansion schemes and flood-diversion patterns with minimised system cost and maximised system safety. Decisions with a lower risk level would imply a higher system cost and an increased reliability in satisfying the system constraints; conversely, a desire for reducing the system cost could result in an increased risk of violating the system constraints. The results suggest that the developed ISIP method is applicable to the study case and can be extended to other problems that involve policy analysis, capacity planning, and risk assessment.

Acknowledgements

This research has been supported by the Natural Science Foundation of China (50849002 and 50675074) the Major State Basic Research Development Program of China (2003CB415201, 2005CB724200, 2006CB403307), and the Natural Science and Engineering Research Council of Canada. The authors are grateful to the editors and the anonymous reviewers for their insightful comments and suggestions.

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