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Digital IIR filter design using multi-objective optimization evolutionary algorithm

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ABSTRACT

The research of applying evolutionary algorithms (EAs) to digital infinite-impulse response (IIR) filter design has gained much attention in recent years. Previously, most works treated digital IIR filter design as a single objective optimization problem of minimizing the magnitude response error with supplementary conditions. While the lack of considering the linear phase response error and the order may result in the loss of control on the structural flexibility, the distortion of output, and the dependency on pre-knowledge. The aim of this paper is to develop proper IIR filter designing method that (1) can provide relatively more complete optimal solutions with equal consideration of magnitude response, linear phase response and the order of structure; (2) can simultaneously optimize the structure and coefficients of digital IIR filter to obtain relatively better linear phase response and lower order, besides the good magnitude response. To achieve these targets, the digital IIR filter design problem is treated as a multi-objective optimization problem in this paper. A new local search operator enhanced multi-objective evolutionary algorithm (LS-MOEA) is specifically proposed for such kind of multi-objective optimization problems. To evaluate the effectiveness and efficiency of LS-MOEA, we experimentally compare it with classical methods and previously effective EAs for digital IIR filter design on four typical IIR filter design cases. Experimental results show that the proposed method can effectively improve the linear phase response of the designed filter, and can obtain filter of lower order. Besides, it achieves these by relatively much lower computational cost than compared EAs.

1. Introduction

Considered as an important and very hard task in digital signal processing, digital IIR filter design has attracted much attention in EA community [3,10,12,13,22–25,30]. In the previous works, some methods have been proposed to tackle such a hard task. The bilinear transformation approach is one of the early techniques [3] and has been widely adopted. Via this approach, the digital filter is transformed to the corresponding analog low-pass (LP) filter, and then, the well-known LP filter design methods, such as Butterworth, Chebyshev Type I, and Chebyshev Type II, are used to accomplish the design of analog LP filter. Finally, the analog LP filter is transformed back to the digital filter using bilinear transformation [17]. However, this procedure needs too much pre-knowledge and shows poor performance in most cases [24]. This stimulated the research on more effective optimization approaches with less pre-knowledge and higher accuracy [22,30].

Since the seminal work published in [10], a number of evolutionary algorithms (EAs) have been developed for digital IIR filter design. The major advantages of these EAs over other methods can be summarized as [2,15,20,26]: (1) pre-knowledge of the problems is not necessary for EAs, while the highly nonlinear characteristic must be approximated firstly for transformation approach and other mathematical optimization approaches; (2) EAs usually work with a population of candidate solutions and can handle the constraints adaptively under the strategy set beforehand in a single run. The current research of applying EAs to design digital IIR filter mainly focuses on the following two highly concerned issues:

- **Developing specific operators**: Similar to the other application areas, developing more effective GA operators has attracted wide attention for digital IIR filter design. The motivation is to strengthen the exploration ability, due to the fact that the fitness landscape of such a problem contains too many local optima. The important digital IIR filter optimization approaches include: hybrid genetic algorithm [12], hierarchical genetic algorithm (HGA) [22], Taguchi strategy enhanced GA (HTGA) [24] and GA including simulated annealing (SA) [13]. Among these approaches, only HGA considers optimizing the structure of digital IIR filter, which is also regarded as minimizing the order.

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- **Multi-algorithm framework**: In order to syncretize all qualitative aspects, a novel approach proposed recently is to implement different EA techniques to deal with different sub-problems [30]. In detail, the main idea of cooperative co-evolutionary genetic algorithm (CCGA) is inherited to divide and optimize the control genes (control genes define the structure of the filter) and coefficient genes separately. In the stage of optimizing the control genes, NSGA-II [7] is implemented to balance the relationship among magnitude response error, linear phase response error and order. In the stage of optimizing coefficient genes, the multi-objective situation is handled by a simple weight sum approach, and the simulated annealing (SA) is used. Clearly, the MOEA is only a part of this complex framework.

Based on the above literature review, it can be observed that most existing works treat digital IIR design as a single objective problem with some supplementary conditions, while the multiple criteria design has not attracted sufficient attention. The single objective methods inevitably cause the loss of consideration of the secondary or tertiary objectives, which are usually related to the linear phase response error and order of IIR filter [30]. These factors are also very important in digital IIR filter design. For example in HTGA [24] and HGA [22], the linear phase response error is not considered, which may result in large distortion. [14] indicates that IIR filters designed in such single objective optimization way can only be implemented in the applications, where phase responses are not very important. Besides, for single objective optimization algorithms, the users have to set weights to combine several criterions into one single optimization objective. Especially, the setting of weights always requires strong background of both designing digital IIR filter and optimization algorithm. Furthermore, a lot of previous works [12,13,24] do not take the optimization of the filter structure into account, which means that the order of IIR filter must be determined beforehand. It is well understood that higher order of IIR filter is, more complex the structure is and more expensive the cost is. In this case, EAs have not shown their full flexibility and potentiality in the tasks of designing optimal IIR filters.

With the equal consideration of all objectives, the essential goal of MOEA is to provide a set of complete Pareto optimal solutions to the decision maker. Therefore, for a static multi-objective optimization problem, a natural and effective choice is to implement MOEA due to its mature technique and proven excellent performance [27,32]. However, it is a little bit surprising that none of the existing works is found to try to handle digital IIR filter design totally by MOEAs, in defiance of the fact that the digital IIR filter design always needs to optimize more than one objective [30]. In this work, we try to investigate reasonable way of designing suitable digital IIR filter design algorithm from the perspective of MOEA. A new local search enhanced MOEA (LS-MOEA) is proposed specifically for digital IIR filter design. To fully evaluate the effectiveness of LS-MOEA, it is applied on four typical filter design cases, and its performance is compared comprehensively with those of the following algorithms: (1) several classical digital IIR filter design algorithms, to show the advantage of EA based algorithms; (2) the previous most effective algorithms based on single objective EAs, i.e. HGA and CCGA, to show the advantages of the usage of MOEA; and (3) the classical NSGA-II, to validate the advantages of the specifically designed operators for multi-objective digital IIR filter design. The experimental results evidently show that LS-MOEA is distinctly suitable for designing good digital IIR filter.

The remainder of this paper is structured as follows: In Section 2, the digital IIR filter design problem is introduced first. Then, the state-of-the-art works on MOEAs are briefly reviewed. In Section 3, the new algorithm LS-MOEA is proposed. Section 4 presents experimental evaluation of LS-MOEA on designing four types of digital IIR filters, including low-pass (LP), high-pass (HP), band-pass (BP) and band-stop (BS) filters. The previous effective algorithms, such as HGA and CCGA, and classical approaches are utilized to provide comparisons. In Section 5, a brief conclusion is given and the future work is outlined.

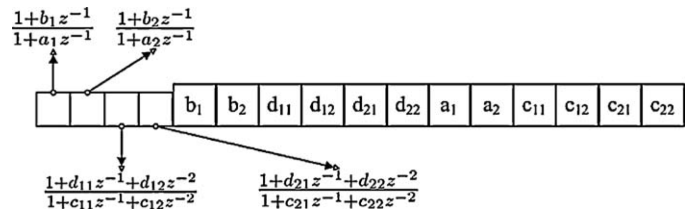

Fig. 1. Coding of digital IIR filter design.

2. Digital IIR filter optimization

The cascade form of an infinite-impulse response (IIR) filter can be described as follows [10,21,22]:

$$H(z) = K\prod_{k=1}^{n}\frac{1+b_kz^{-1}}{1+a_kz^{-1}}\prod_{i=1}^{m}\frac{1+d_{i1}z^{-1}+d_{i2}z^{-2}}{1+c_{i1}z^{-1}+c_{i2}z^{-2}} \tag{1}$$

where K is the gain, a_k and b_k for $k = 1, 2, \ldots, n$ are the first-order coefficients, and c_{i1}, c_{i2}, d_{i1} and d_{i2} for $i = 1, 2, \ldots, m$ are the second-order coefficients. In this paper, the optimization task is defined as searching an optimal structure with lowest order, minimal magnitude response error and minimal linear phase response error.

2.1. Coding scheme

The coding is important to bridge optimization algorithms to digital IIR filter design. As is claimed in [22] and [30], the coding is the essence of HGA. In principle, such a coded method represents a given digital IIR filter as a chromosome that contains control genes and coefficient genes. The control genes determine the structure of the filter, and the coefficient genes define the value of the coefficients in each block. Compared with HGA, the major difference of the coded method in [30] is that the control genes and the coefficient genes are separately considered to be corresponding species. The species can be flexibly combined to be the chromosome. In this paper, the classical coding method in [22] is inherited. Consider a digital IIR filter that can be expressed as Eq. (1) with $n = 2$ and $m = 2$. The graphical representation of the coding is shown in Fig. 1. The first four genes are control genes and the others are coefficient genes. The control genes are in binary bit form, which determine the activation of corresponding block by setting “1” for active block. The coefficient genes are in real number form. Note that coefficient genes for the inactive block are useless in calculation of the chromosome. It is apparent that such a coded method inevitably causes redundant genes. However, the MOEA is able to make use of the redundant genes to effectively maintain the diversity.

2.2. Formulation of digital IIR filter design

The digital IIR filter design problem can be formulated as a multi-objective optimization form as follows [30]:

$$\begin{array}{ll} \text{minimize} & f = \{f_1(x), f_2(x), f_3(x)\} \\ \text{subject to} & x \in X, \\ \text{Objective1} & f_1(x) \sim \text{magnitude response error}, \\ \text{Objective2} & f_2(x) \sim \text{linear phase response error}, \\ \text{Objective3} & f_3(x) \sim \text{order} \end{array} \tag{2}$$

where $X \subset (D^M \cup R^N)$ denotes the feasible decision space with M binary control variables and N continuous coefficient variables; $x = \{x_1, x_2, \ldots, x_{M+N}\} \in (D^M \cup R^N)$ is the decision variable vector; $f : X \rightarrow R^3$ stands for 3 objective functions for mapping from $M+N$ dimensional variable space to 3 dimensional objective value $f = \{f_1(x), f_2(x), f_3(x)\}$.

Definition: (*Pareto dominate and Pareto front*) x_1 'Pareto dominate' (is better than) x_2 is true when the following two conditions hold: (1) $f_j(x_1) \preccurlyeq f_j(x_2), j = 1, 2, 3$ which means that all objective values of x_1 are not worse than x_2. (2) $\exists j \in 1, 2, 3$, st. $f_j(x_1) < f_j(x_2)$, which means that at least one objective of x_1 is better than x_2. Based on the 'Pareto dominate' concept, the 'Pareto front' S_P is defined as a set of solutions that are not dominated by all the other feasible solutions.

The ultimate goal of MOEA is to achieve a set of solutions spread uniformly along the Pareto optimal front. Then, the decision maker can select the appropriate one which satisfies the tolerant conditions.

2.2.1. Magnitude response error

The magnitude response error can be simulated as the difference to the boundary of design requirement. It can be calculated as follows [22,30]:

$$H_p(\omega) = \begin{cases} 1 - \delta_1 - |H(e^{j\omega})|, & |H(e^{j\omega})| < 1 - \delta_1 \\ 0, & |H(e^{j\omega})| \geq 1 - \delta_1, \end{cases} \tag{3}$$

where ω is in the passband and the $H_p(\omega)$ is the passband magnitude response error at ω.

$$H_s(\omega) = \begin{cases} |H(e^{j\omega})| - \delta_2, & |H(e^{j\omega})| \geq \delta_2 \\ 0, & |H(e^{j\omega})| \leq \delta_2, \end{cases} \tag{4}$$

where ω is in the stopband and the $H_s(\omega)$ is the stopband magnitude response error at ω.

$$\min f_1 = \frac{1}{P_n}\sum_{i=1}^{P_n} H_p(\omega_i) + \frac{1}{S_n}\sum_{j=1}^{S_n} H_s(\omega_j) \tag{5}$$

where P_n and S_n are the sampling frequency in the passband and stopband respectively. In this case, the best fitness $f_1 = 0$ represents that the magnitude response of the solution is within $[1 - \delta_1, 1]$ in the passband and $[0, \delta_2]$ in the stopband.

2.2.2. Linear phase response error

Both passband and transition band are considered when calculating linear phase response error [30]. The sampling points are evenly distributed in the passband and transition band. Then the first order difference of phase sequences $\{\theta_1, \theta_2, \ldots, \theta_{P_n+T_n}\}$ can be calculated as follows:

$$\Delta\text{Phase} = \{\Delta\theta_1, \Delta\theta_2, \ldots, \Delta\theta_{P_n+T_n-1}\}, \tag{6}$$

where $\Delta\theta_i = \theta_{i+1} - \theta_i$; P_n and T_n are the sampling frequency in the passband and transition band respectively. Then, the Linear Phase response error is equal to the variance of the ΔPhase sequence:

$$\min f_2 = var\{\Delta\theta_i | \theta_i \in \text{passband} \cup \text{transition band}\} \tag{7}$$

where *var* is the operator to calculate the variance value. The best fitness $f_2 = 0$ represents that the elements of ΔPhase sequence are exactly the same.

Table 1
Procedure of NSGA-II

Algorithm: NSGA-II
Step 0 Randomly initialize a population $P(0)$. Set the iteration counter $t = 0$.
Step 1 Crossover [5].
Step 2 Mutation [5].
Step 3 Update population $P(t)$ using non-dominated sorting rank [7,5].
Step 4 Update population $P(t)$ using crowding distance strategy [7,5].
Step 5 If termination criterion is not satisfied, set $t = t + 1$ and go to Step 1, else report $P(t)$.

2.2.3. Order

The order is only determined by the control genes. It can be calculated as follows [22,30]:

$$\min f_3 = \sum_{i=1}^{n} p_i + 2\sum_{j=1}^{m} q_j \tag{8}$$

where n and m are the numbers of first order blocks and second order blocks respectively; p_i and q_j are the ith and jth control genes for the corresponding first order blocks and second order blocks respectively. The $f_3 = 0$ is not the best fitness, because it stands for an all-pass filter. Therefore, the feasible region of order is the integers within $[1, n + 2m]$.

2.3. Related works on multi-objective optimization evolutionary algorithm

Historically, the attempt of making use of the population based evolutionary approaches for multi-objective optimization problems goes back as far as 1985 [19]. Two books [4] and [6] summarize the early works on MOEAs. In the recent years, MOEAs have been successfully applied to many engineering applications, such as electric power dispatch problem [1], design of power distribution systems [18], bioinformatics and computational biology [11]. Generally speaking, the widely adopted MOEAs include non-dominated sorted based genetic algorithm II (NSGA-II) [7], strength Pareto evolutionary algorithm II (SPEA-II) [31] and Pareto develop based selection algorithm (PAES) [16]. For many multi-objective tasks, these approaches have shown excellent performances.

Due to the arising of new challenges, several effective MOEAs with improved strategies have been proposed, such as ϵ-MOEA [8], hypervolume based MOEA [9], regularity model-based multi-objective estimation of distribution algorithm (RM-MEDA) [32], and, but not least fast hypervolume based MOEA (FH-MOEA) [27]. Either more effective new offspring creating mechanism or elite maintenance with stronger selection pressure was adopted in these MOEAs. However, there are different problems of these algorithms when applying to digital IIR filter design, which can be summarized as follows: the computational costs of SPEA-II, hypervolume based MOEA and RM-MEDA are too expensive when dealing with the three objective problems; much pre-knowledge of the fitness landscape and bounds is needed for ϵ-MOEA; FH-MOEA seems not so effective for the problems with more than two objectives; the diversity maintenance of NSGA-II is not so effective.

Due to its high efficiency and low computational cost, the original NSGA-II algorithm proposed in [7] is selected to be the basic MOEA in this paper. It implements a fast non-dominated sorting approach to effectively conduct the selection strategy. The procedure of NSGA-II is shown in Table 1. The genetic operators, including crossover and mutation, are exactly the same as that used in single objective GAs. Without loss of diversity, step 4 is used to truncate the fixed number of individuals to form the population for the next generation. In detail, the individuals that locate at the sparse region are preserved while the ones that locate at the dense area are abandoned.

3. Algorithm

The goal of digital IIR filter designing is to find the optimal solution that satisfies the following requirements: (1) the magnitude response error $f_1 = 0$; (2) the phase response is as linear as possible; and (3) the order is as low as possible. The application of MOEAs to IIR filter design problem will generate a number of non-dominated solutions, called Pareto optimal solutions. To facilitate the satisfaction of requirement 2 above, a specific local search operator is proposed to enhance the search of higher quality solutions of phase response f_2 around the central point with $f_1 = 0$ and lowest order. Such a local search operator is the essence of LS-MOEA. Other components are mainly inherited from NSGA-II.

In principle, LS-MOEA tries to congregate multiple strategies to enhance various abilities of MOEA to meet the specific requirements of multi-objective IIR digital filter design, which can be summarized as follows:

- As is discussed in the above section, the diversity maintenance is the major demerit of NSGA-II. To make up it, the original NSGA-II is developed to be an improved-NSGA-II (INSGA-II) [28] by incorporating an extensive archive and an improved non-dominated selection strategy [32]. These two strategies can remarkably improve the diversity property compared with the original NSGA-II [28].
- To make the algorithm be specific for digital IIR filter design, a local search operator is introduced to improve the quality of the eligible solution. The main idea is to put large amount of attention around the previous best solution. It is beneficial to find superior solution in phase response error.
- The cooperation of diversity maintenance strategies and local search strategy can be expressed as follows: the extensive archive and improved non-dominated selection strategy help in keeping a good diversity of various filter structures (orders); when the solutions with $f_1 = 0$ appear, local search operator is launched to improve the linearity of phase response.

The details of LS-MOEA are shown in Table 2. At each generation, the proposed algorithm maintains: a population of *NP* solutions (i.e. points in decision space) with their objective values, and an external archive to record the non-dominated solutions found previously. In step 4, an improved selection operator removes solutions one at a time, which has the most crowding density. After a fixed number of generations, LS-MOEA detects whether there is some solutions with $f_1 = 0$ in the archive. If the above situation is satisfied, the local search is applied to the one with lowest order to generate 5 new individuals. A Gaussian based local search operator defined as Eq. (9) is used to implement the local search:

$$\text{for} \quad i = (m+n+1)\text{to}(5\cdot m + 3\cdot n)$$
$$v_i = \begin{cases} x_i + N(0,\sigma), & \text{if } r \text{ and } < 0.1 \\ x_i, & \text{otherwise,} \end{cases} \qquad (9)$$

where m and n come from Eq. (1); v is the new offspring solution generated by the selected solution x from the archive; genes $(m+n+1)$ to $(5\cdot m+3\cdot n)$ are the coefficient genes; *rand* is a uniformly generated number within [0, 1]; σ is a positive constant. The individual x will be replaced by v, if v dominate x. The Gaussian local search operator defined as Eq. (9) has been testified by many researches to be a good strategy to enhance the ability of elaborate search [29]. The Gaussian local search operator is adopted with probability 0.1. By performing such Gaussian local search operator, the selected archive solution can search for superior solutions around its current position.

4. Experimental study

To provide experimental evidence to study how LS-MOEA improves the performance of traditional digital IIR filter design approaches and how the incorporated strategies improve the performance of NSGA-II, we compare LS-MOEA with classical methods, single-objective EAs and NSGA-II on a widely used test suit [22,24,30], including four types of filters: (1) low-pass (LP); (2)

Table 2
Procedure of LS-MOEA

Algorithm: LS-MOEA
Step 0 Randomly initialize a population $P(0)$. The non-dominated solutions of $P(0)$ are copied to an archive population $A(0)$ one by one. Set the iteration counter $t = 0$.
Step 1 Crossover: Apply two-point crossover operator [5] to the control genes; apply the simulated binary crossover operator [5] to coefficient genes.
Step 2 Mutation: Apply bit-flip mutation operator [5] to the control genes; apply polynomial mutation operator [5] to the coefficient genes.
Step 3) Update population $P(t)$ using non-dominated sorting rank and crowding distance strategy [5,7].
Step 4 Update the archive with the new generated population using improved non-dominated selection.
Step 5 Apply local search operator to the selected individual.
Step 6 If termination criterion is not satisfied, set $t = t + 1$ and go to Step 1, else report $A(t)$.

Table 3
Design criteria.

Types	Order	ω_p	ω_s
LP	11	$[0\ 0.2\pi]$	$[0.3\pi\pi]$
HP	11	$[0.8\pi\pi]$	$[0\ 0.7\pi]$
BP	11	$[0.4\pi\ 0.6\pi]$	$[0\ 0.25\pi]\cup[0.75\pi\pi]$
BS	11	$[0\ 0.25\pi]\cup[0.75\pi\pi]$	$[0.4\pi\ 0.6\pi]$

Other uniform requirements are: (1) the phase response is linear in the pass band and transition band; (2) $\delta_1 = 0.1088$; (3) $\delta_2 = 0.17783$.

Table 4
Parameters settings of LS-MOEA.

Genes	Parameters	Settings
	Population	Population size 100, archive size 100
	Selection	Binary tournament selection
	Crossover rate	0.9
	Mutation	0.1
Control genes	Crossover	Two-point crossover
	Mutation	Bit-flip mutation
Coefficient genes	Crossover	Simulated binary crossover
	Mutation	Polynomial mutation

$G_{LS} = 5$ stands for the minimum number of generations between two local search operators. The termination condition for LS-MOEA is that the first objective function value $f_1 = 0$, and the other two objective values are less than the given values.

Table 5
The results obtained by LS-MOEA

LP :	$H(z) = 0.1828\,\frac{1+0.2094z^{-1}}{1-0.4384z^{-1}}\,\frac{1-0.8939z^{-1}+0.9316z^{-2}}{1-1.2338z^{-1}+0.6429z^{-2}}$
HP :	$H(z) = 0.2020\,\frac{1-0.5189z^{-1}}{1+0.3087z^{-1}}\,\frac{1+0.9351z^{-1}+0.9295z^{-2}}{1-1.1694z^{-1}+0.6127z^{-2}}$
BP :	$H(z) = 0.2192\,\frac{1-1.6642z^{-1}+0.9998z^{-2}}{1+0.5927z^{-1}+0.5120z^{-2}}\,\frac{1+1.6536z^{-1}+0.9949z^{-2}}{1-0.6168z^{-1}+0.5134z^{-2}}$
BS :	$H(z) = 0.4758\,\frac{1+0.3083z^{-1}+0.8687z^{-2}}{1+0.7223z^{-1}+0.4658z^{-2}}\,\frac{1-0.3112z^{-1}+0.8910z^{-2}}{1-0.7238z^{-1}+0.4758z^{-2}}$

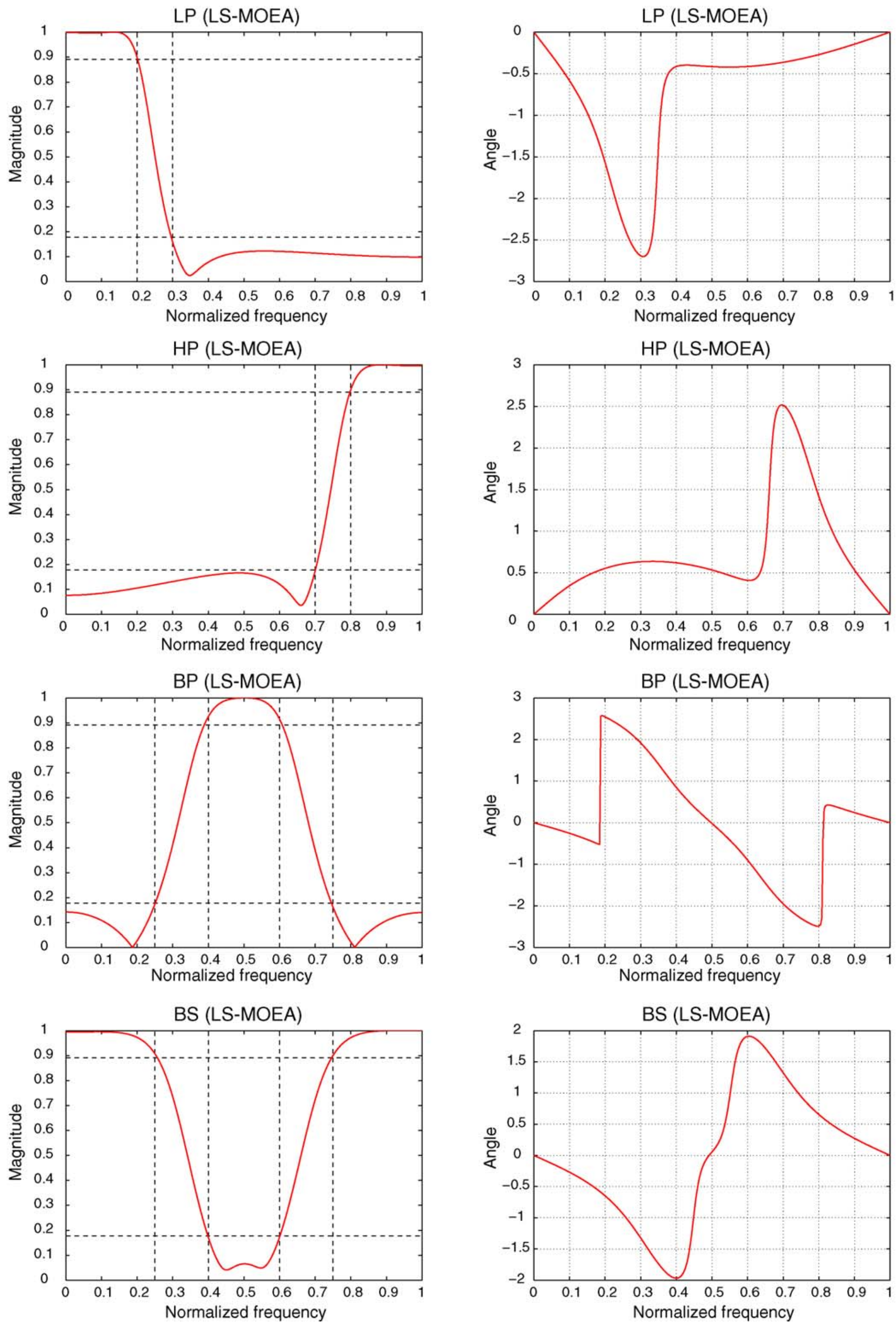

Fig. 2. Filter responses of four filter types (LS-MOEA).

high-pass (HP); (3) band-pass (BP); and (4) band-pass (BS). The fundamental structure of $H(z)$ is expressed as:

$$H(z) = K\prod_{k=1}^{3}\frac{1+b_k z^{-1}}{1+a_k z^{-1}}\prod_{i=1}^{4}\frac{1+d_{i1}z^{-1}+d_{i2}z^{-2}}{1+c_{i1}z^{-1}+c_{i2}z^{-2}}, \tag{10}$$

where there are $n=3$ first order blocks and $m=4$ second order blocks for Eq. (1). In the chromosome, there are 7 control genes and 22 coefficient genes. The parameter K is determined by unify-

Table 6
Lowest filter order due to various design schemes.

Filter	BWTH	CHBY1	CHBY2	ELTC	LS-MOEA
LP	6	4	4	3	3
HP	6	4	4	3	3
BP	12	8	8	6	4
BS	12	8	8	6	4

Table 7
Filter performance comparison on four types of digital IIR filters.

	Fitness evaluation size needed	Lowest filter order	Pass band magnitude response performance	Stop band magnitude response performance	Phase response error
LP filter					
HGA	–	3	$0.8862 \le \lvert H(e_{i\omega})\rvert \le 1$	$\lvert H(e_{i\omega})\rvert \le 0.1800$	1.6485E−04
CCGA	142480	3	$0.9034 \le \lvert H(e_{i\omega})\rvert \le 1$	$\lvert H(e_{i\omega})\rvert \le 0.1669$	1.4749E−04
NSGA-II	8900	3	$0.9117 \le \lvert H(e_{i\omega})\rvert \le 1$	$\lvert H(e_{i\omega})\rvert \le 0.1719$	1.2662E−04
LS-MOEA	**4900**	3	$0.9083 \le \lvert H(e_{i\omega})\rvert \le 1$	$\lvert H(e_{i\omega})\rvert \le 0.1586$	**1.0959E−04**
HP filter					
HGA	–	3	$0.9221 \le \lvert H(e_{i\omega})\rvert \le 1$	$\lvert H(e_{i\omega})\rvert \le 0.1819$	1.1212E−04
CCGA	341640	3	$0.9044 \le \lvert H(e_{i\omega})\rvert \le 1$	$\lvert H(e_{i\omega})\rvert \le 0.1749$	9.7746E−05
NSGA-II	148000	3	$0.8960 \le \lvert H(e_{i\omega})\rvert \le 1$	$\lvert H(e_{i\omega})\rvert \le 0.1769$	**9.1419E−05**
LS-MOEA	**42385**	3	$0.9004 \le \lvert H(e_{i\omega})\rvert \le 1$	$\lvert H(e_{i\omega})\rvert \le 0.1746$	9.6150E−05
BP filter					
HGA	–	6	$0.8956 \le \lvert H(e_{i\omega})\rvert \le 1$	$\lvert H(e_{i\omega})\rvert \le 0.1772$	1.1222E−04
CCGA	778960	4	$0.8920 \le \lvert H(e_{i\omega})\rvert \le 1$	$\lvert H(e_{i\omega})\rvert \le 0.1654$	8.1751E−05
NSGA-II	26500	4	$0.9100 \le \lvert H(e_{i\omega})\rvert \le 1$	$\lvert H(e_{i\omega})\rvert \le 0.1771$	3.6503E−04
LS-MOEA	**9995**	4	$0.9285 \le \lvert H(e_{i\omega})\rvert \le 1$	$\lvert H(e_{i\omega})\rvert \le 0.1734$	**6.0371E−05**
BS filter					
HGA	–	4	$0.8920 \le \lvert H(e_{i\omega})\rvert \le 1$	$\lvert H(e_{i\omega})\rvert \le 0.1726$	2.7074E−04
CCGA	775320	4	$0.8966 \le \lvert H(e_{i\omega})\rvert \le 1$	$\lvert H(e_{i\omega})\rvert \le 0.1733$	1.6198E−04
NSGA-II	307700	4	$0.8917 \le \lvert H(e_{i\omega})\rvert \le 1$	$\lvert H(e_{i\omega})\rvert \le 0.1770$	1.5190E−04
LS-MOEA	**123505**	4	$0.8967 \le \lvert H(e_{i\omega})\rvert \le 1$	$\lvert H(e_{i\omega})\rvert \le 0.1725$	**1.5084E−04**

ing the magnitude response of the digital IIR filter. Therefore, the feasible order of the designed digital IIR filter is [1 11].

The stable requirement of digital IIR filter was summarized in [21]. It can be expressed as follows:

$$-1 < a_{j2} < 1, \tag{11}$$

$$-1 - a_{j2} < a_{j1} < 1 + a_{j2}. \tag{12}$$

where Eq. (11) represents the stable requirement for first order blocks and Eq. (12) is for the second order blocks. The parameters for four types of digital IIR filters are shown in Table 3. The parameters settings for LS-MOEA are listed in Table 4. For fair comparison with HGA, CCGA and NSGA-II, the same 20 independent runs are performed by LS-MOEA. LS-MOEA is terminated when $f_1 = 0$, and the other two objectives are less than the given values. The functional structures obtained by LS-MOEA are shown in Table 5. Fig 2 depicts the filter performances of LS-MOEA.

(1) Comparison with classical methods: To show the advantage of LS-MOEA over the classical methods, the comparison of the lowest order between LS-MOEA and the classical methods, such as BWTH, CHBY1, CHBY2, and ELTC, is shown in Table 6. The results of these classical approaches are from the uses of the MATLAB toolbox [22]. It is observed that LS-MOEA always provides the lowest order for all cases. Especially, unlike the other methods, it is interesting to see that the orders of BP and BS obtained by LS-MOEA are less than twice of those of LP and HP.

(2) Comparison with single objective EAs (HGA and CCGA): As is easy to be appreciated in Table 7, the passband and the stopband magnitude response requirements are completely met by all algorithms. When looking into the lowest order, CCGA and LS-MOEA are comparable, and outperform HGA. However, LS-MOEA can accomplish the search procedure with significant lower computational cost, compared with CCGA. The best results for different aspects are marked in bold face in Table 7. In more detail, the optimization speed of LS-MOEA is improved up to 81.63%, 249.18%, 165.13% and 149.14% for respective types of filters, which is concluded on the aspect of FES needed. Especially, LS-MOEA provides the top performances of the phase response quality for all types of digital IIR filters among the three considered algorithms.

(3) Comparison with NSGA-II: In Table 7, it is interesting to see that besides LS-MOEA, NSGA-II can also obtain the filters, which completely meet all requirements, within remarkably lower computational costs than HGA and CCGA. Therefore, the effectiveness and efficiency of MOEAs on digital IIR filter is verified again. However, the advantage of LS-MOEA on computational cost is also very remarkable, even compared with NSGA-II. This is due to the implementation of local search in LS-MOEA, which specially focuses on the more promising search region. Furthermore, the qualities of the filters obtained by LS-MOEA are better than those obtained by NSGA-II, except the aspects of pass band magnitude response and phase respond error on HP filter.

In summary of the above three comparisons, it can be concluded that LS-MOEA is beneficial to effectively design digital IIR filters of better responses and lower order with lower computational cost.

5. Conclusion

In the previous studies, the single objective algorithms were the usual choices to design IIR filters [22,24,30]. In this paper, LS-MOEA is proposed to design digital IIR filter with three optimization objectives, minimizing magnitude response error, phase response error and order. Particularly, a local search operator is included with the pursuit of improving quality of phase response error. Furthermore, LS-MOEA is developed to optimize the structure and coefficients setting of filters simultaneously. In the experimental study, LS-MOEA gained better results than the compared state-of-the-art algorithms on four digital IIR filter design cases with remarkably lower computational cost. Due to the promising performance of LS-MOEA, MOEA based approaches deserve more attention in the area of digital IIR filter design.

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