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Design of arbitrarily shaped concentrators based on conformally optical transformation of nonuniform rational *B*-spline surfaces

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We study the design of arbitrarily shaped electromagnetic (EM) concentrators and their potential applications. To obtain closed-form formulas of EM parameters for an arbitrarily shaped concentrator, we employ nonuniform rational *B*-spline (NURBS) to represent the geometrical boundary. Using the conformally optical transformation of NURBS surfaces, we propose the analytical design of arbitrarily shaped concentrators, which are composed of anisotropic and inhomogeneous metamaterials with closed-form constitutive tensors. The designed concentrators are numerically validated by full-wave simulations, which show perfectly directed EM behaviors. As one of the potential applications, we demonstrate a way to amplify plane waves using a rectangular concentrator, which is much more efficient and easier than the existing techniques. Using NURBS expands the generality of the transformation optics and could lead toward making a very general tool that would interface with commercial softwares such as 3D STUDIOMAX and MAYA. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2951485]

Since the concept of invisible cloak was proposed[1] and the first practical implementation was realized in the microwave regime using simplified medium parameters,[2] more and more researches have been investigated in this exciting topic.[3–6] More recently, other devices based on the method of optical transformation have been proposed, such as the electromagnetic (EM) rotator[7] and concentrator.[8] In this letter, we investigate the design and analysis of arbitrarily shaped EM concentrators using the coordinate transformation and the nonuniform rational *B*-spline,[9] (NURBS) in representation of the geometrical boundary. We also propose a way to amplify plane waves using a rectangular concentrator.

NURBS is one of the most common approaches to represent curves in geometric modeling. It is possible to split an arbitrary curve represented by NURBS into a sequence of rational Bézier curves. Usually, rational Bézier curves of the first and second degrees can describe all familiar sections: lines, circles, ellipses, parabolas, hyperbolas, etc. Hence we use *piecewise* rational Bézier curves to represent an arbitrarily geometric model. A first-degree Bézier curve consists two control vertices, and a second-degree curve consists of three control vertices and three weights. Generally, the Bézier curve is classified into a rational one and a polynomial one since the rational Bézier curve is reduced to a polynomial Bézier curve if the control weights are equal. The usage of NURBS will expand the generality of the transformation optics and could make a very general tool to interface with commercial softwares such as 3D STUDIOMAX and MAYA. Hence the transformation optics is possible to become industry.

We investigate two-dimensional (2D) arbitrarily cylindrical concentrators. In order to obtain closed-form formulas, we take a section of an arbitrary concentrator for consideration, which can be represented by a second-degree Bézier curve, as shown in Fig. 1. Here, the control points are $A(a_0,b_0)$, $C(a_1,b_1)$, $B(a_2,b_2)$ and the control weights are w_0,w_1, w_2 respectively. The parametric equations of the curve $\hat{AB}$ are expressed as

$$x(u)=\frac{w_0a_0(1-u)^2+2w_1a_1(1-u)u+w_2a_2u^2}{w_0(1-u)^2+2w_1(1-u)u+w_2u^2}, \quad (1)$$

$$y(u)=\frac{w_0b_0(1-u)^2+2w_1b_1(1-u)u+w_2b_2u^2}{w_0(1-u)^2+2w_1(1-u)u+w_2u^2}, \quad (2)$$

in which $0\leqslant u\leqslant 1$.

Choose an arbitrary point $H(x,y)$ in the original coordinate system (see Fig. 1), whose mapping point is $H'(x',y')$ in the transformed system (not shown). We denote the dis-

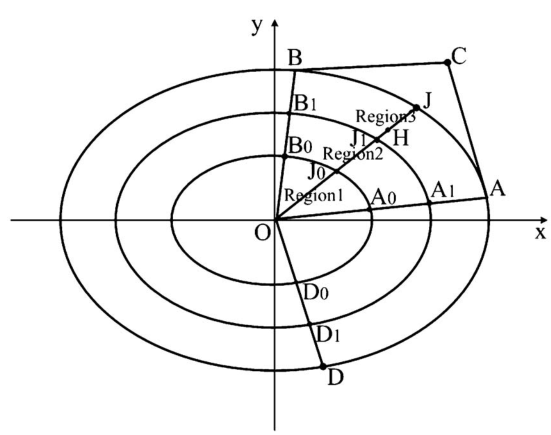

FIG. 1. The cross section of an arbitrarily shaped concentrator in the Cartesian coordinate system, in which H is an arbitrary point in the original space.

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tance OH as $r=\sqrt{x^2+y^2}$ and OH' as $r'=\sqrt{x'^2+y'^2}$. In order to make a conformal coordinate transformation, we let $|OJ_0|/|OJ_1|=|OA_0|/|OA_1|=k_1$, $|J_0J|/|J_1J|=|A_0A|/|A_1A|=k_2$, and $|J_0J_1|/|J_1J|=|A_0A_1|/|A_1A|=k_3$ for an arbitrary point J on the outer boundary, as shown in Fig. 1. Then the coordinate-transformation equations for a concentrator are expressed as

$$r'=\begin{cases} k_1 r & H \text{ is in region 1 or 2,} \\ k_2 r - k_3 R_3 & H \text{ is in region 3,} \end{cases} \tag{3}$$

in which R_3 is the distance of OJ. Obviously, H' is in region 1 if H is in region 1 or 2; and H' is in region 2 or 3 if H is in region 3. The transformation implies that the space is compressed into region 1 at the expense of an expansion of regions 2 and 3. We denote the slopes of OH, OA, and OB as t, t_1, and t_2, respectively. Hence the slopes of OH' and OJ are also t. Then we easily obtain

$$\frac{w_0b_0(1-u)^2+2w_1b_1(1-u)u+w_2b_2u^2}{w_0a_0(1-u)^2+2w_1a_1(1-u)u+w_2a_2u^2}=t. \tag{4}$$

Let $L=(w_0a_0-2w_1a_1+w_2a_2)t-(w_0a_0-2w_1a_1+w_2a_2)$, $M=(-2w_0a_0+2w_1a_1)t+2w_0b_0-2w_1b_1$, and $N=w_0a_0t-w_0b_0$, we obtain the solution as

$$u=(-M\pm\sqrt{M^2-4LN})/(2L), \tag{5}$$

in which the sign "+" or "−"is determined by the following two conditions: (1) $u=0$, if $t=t_0$ and (2) $u=1$, if $t=t_2$.

Hence the solution to Eq. (4) is determined and we get the coordinate at the point J, (x_J, y_J), from Eqs. (1) and (2), and furthermore the length of OJ, $R_3=R(x,y)$. We define operators $\mathbf{A}=k_3R(x,y)/r$ and $\mathbf{A}'=k_3R'(x',y')/r'$. Due to the transformation invariance, the unit vectors in the original and transformed spaces must be equal: $x'/r'=x/r$ and $y'/r'=y/r$. Hence the coordinate-transformation formulas are derived as

$$(x',y',z')=\begin{cases} (k_1x, k_1y, z) & \text{region 1 or 2,} \\ [(k_2+\mathbf{A})x, (k_2+\mathbf{A})y, z] & \text{region 3.} \end{cases} \tag{6}$$

With above equations, we easily obtain the Jacobian transformation matrix[3] as

$$\Lambda=\begin{cases} \begin{pmatrix} k_1 & 0 & 0 \\ 0 & k_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} & \text{region 1 or 2} \\ \begin{pmatrix} k_2+\mathbf{A}+x\mathbf{A}_x & x\mathbf{A}_y & 0 \\ y\mathbf{A}_x & k_2+\mathbf{A}+y\mathbf{A}_y & 0 \\ 0 & 0 & 1 \end{pmatrix} & \text{region 3} \end{cases}$$

in which $\mathbf{A}_x$ and $\mathbf{A}_y$ represent the derivatives of $\mathbf{A}$ with respect to x and y, respectively. The determinant of the Jacobian matrix has a closed-form expression

$$\det(\bar{\Lambda})=(k_2+\mathbf{A})^2+(k_2+\mathbf{A})(x\mathbf{A}_x+y\mathbf{A}_y). \tag{7}$$

By the metric invariance of Maxwell's equations, we then obtain the constitutive parameter tensors of the medium in the transformed space

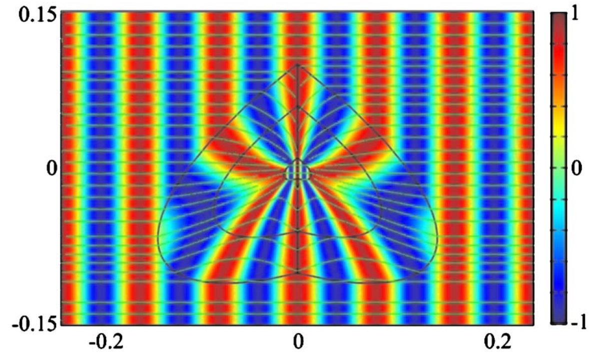

FIG. 2. (Color online) The electric-field distributions and power-flow lines in the computational domain for a heart-shaped concentrator. The incident plane waves propagate from the left to the right.

$$\overline{\varepsilon'}=\frac{\bar{\Lambda}\cdot\bar{\varepsilon}\cdot\bar{\Lambda}^T}{\det(\bar{\Lambda})}, \quad \overline{\mu'}=\frac{\bar{\Lambda}\cdot\bar{\mu}\cdot\bar{\Lambda}^T}{\det(\bar{\Lambda})}, \tag{8}$$

in which an $\exp(-i\omega t)$ time dependence has been assumed. In our study, we assume the original space to be free space: $\bar{\varepsilon}=\bar{\mathbf{I}}\varepsilon_0$ and $\bar{\mu}=\bar{\mathbf{I}}\mu_0$. Hence the relative permittivity and permeability tensors in the concentrator are simplified as

$$\overline{\varepsilon_r'}=\overline{\mu_r'}=\begin{cases} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/k_1^2 \end{pmatrix} & \text{region 1,} \\ \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix} & \text{regions 2 and 3,} \end{cases} \tag{9}$$

in which the expressions for ε_{xx}, ε_{xy}, ε_{yx}, ε_{yy}, and ε_{zz} can be easily derived from Eq. (8).

The above equations provide full design parameters to the permittivity and permeability tensors in the region OAB of the concentrator. The material parameters for other regions can be obtained similarly using the proposed method. Clearly, the concentrator is composed of inhomogeneous and anisotropic metamaterials.

In order to validate the design formulas, we make full-wave simulations of a heart-shaped concentrator and a rectangular concentrator using the commercial software, COMSOL Multiphysics, which is based on the method of finite element. When a transverse-electric (TE) polarized plane wave is incident upon a cylindrical concentrator, there exists only z components of electric fields. Hence the concentrator is reduced to a 2D problem. In the following examples, we set $k_1=1/6$ and $k_2=9/4$, and the working frequency is 4 GHz.

Figure 2 demonstrates the numerical result of the heart-shaped concentrator, when the TE wave propagates from the left to the right. The concentrator is divided into two parts: the left and the right. The outer boundary on the left part is described by the Bézier curve with control points $(0,-0.1)$, $(-0.4,-0.2)$, $(0,0.1)$ and control weights 2,1,2, respectively. Hence we obtain the parametric equations of such a part as

$$x=0.4u(u-1)/w, \quad y=(0.2u^2-0.1)/w, \tag{10}$$

in which $w=u^2-u+1$, and $u\in[0,1]$. The ratio among the principal radii of the three "hearts" is 1:6:10 and the unit of length is meter. We construct the coordinate transformation and compute the permittivity and permeability tensors of the concentrator as described earlier. In Fig. 2, the electric-field distributions and power-flow lines are demonstrated in the computational domain. Clearly, the field intensities are strongly enhanced in the inner region of the concentrator. In fact, the concentrator can focus plane waves incident from arbitrary directions. There is no reflected waves outside the concentrator due to the inherent impedance matching in the method of optical transformation.

In above analysis, the heart-shaped concentrator is represented by the second-order Bézier curves. Next we give a rectangular concentrator whose boundary is described by the first-order Bézier curves. The widths of three rectangles are 0.02, 0.12, and 0.2 m, respectively, and the length-width ratio is 2:1. We divide the rectangular concentrator into four parts. The boundary of the left part is represented by the rational Bézier curve with control points $(0.1,-0.2)$ and $(0.1,0.2)$ and the parametric equations are $x=0.1$ and $y=0.4u-0.2$ In this case, it is easier to compute the constitutive parameters as $\varepsilon_{xx}=1+1/(8x)$, $\varepsilon_{xy}=\varepsilon_{yx}=y/(8x^2)$, $\varepsilon_{yy}=(y^2+64x^4)/(64x^4+8x^3)$, and $\varepsilon_{zz}=2(8x+1)/(81x)$.

Similar to the above procedure, the constitutive tensors of other three parts are obtained. Figure 3 illustrates the full-wave simulation result of the rectangular concentrator when the TE waves are incident from the left to the right. We observe that the fraction of plane-wave extending in the middle rectangle is completely focused to the concentrator in the inner rectangle and the fields within the outer layer are expanded to the middle rectangle.

The above rectangular concentrator can be easily used in amplifying plane waves. A plane-wave amplifier, first proposed in Ref. 10, is an array of small power amplifiers that receive input signals via input antennas and radiate amplified output signals from an output antenna. The design of a monolithic plane-wave pseudomorphic high electron mobility transistor amplifier was presented in Ref. 11. Obviously, such a plane-wave amplifier is very expensive and complicated. If a power amplifier is placed at the center of the rectangular concentrator, the plane waves will be amplified when passing through the concentrator. Hence we can amplify plane waves easily via the rectangular concentrator and a *single power amplifier*.

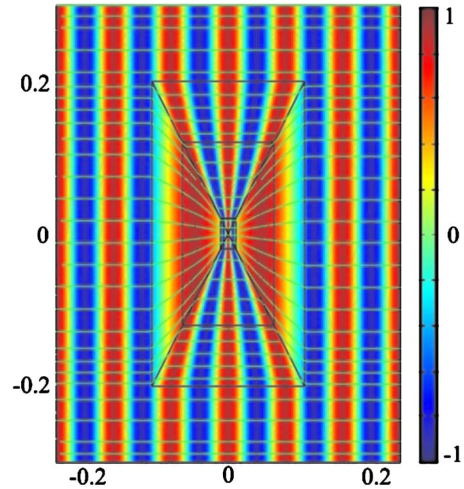

FIG. 3. (Color online) The electric-field distributions and power-flow lines in the computational domain for a rectangular concentrator. The plane waves are incident in the horizontal direction.

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[1] J. B. Pendry, D. Schurig, and D. R. Smith, Science **312**, 1780 (2006).
[2] D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, Science **314**, 977 (2006).
[3] D. Schurig, J. B. Pendry, and D. R. Smith, Opt. Express **14**, 9704 (2006).
[4] S. A. Cummer, B. I. Popa, D. Schurig, and D. R. Smith, Phys. Rev. E **74**, 036621 (2006).
[5] F. Zolla, S. Guenneau, A. Nicolet, and J. B. Pendry, Opt. Lett. **32**, 1069 (2007).
[6] W. Cai, U. K. Chettiar, A. V. Kildishev, V. M. Shalaev, and G. W. Milton, Appl. Phys. Lett. **91**, 111105 (2007).
[7] H. Chen and C. T. Chan, Appl. Phys. Lett. **90**, 241105 (2007).
[8] M. Rahm, D. Schurig, D. A. Roberts, S. A. Cummer, D. R. Smith, and J. B. Pendry, Phot. Nano.-Fund. Appl. **6**, 87 (2008).
[9] L. Piegl and W. Tiller, *The NURBS Book*, 2nd Ed. (Springer, New York, 1996).
[10] J. A. Higgins, E. A. Sovero, and W. J. Ho, IEEE Microw. Guid. Wave Lett. **5**, 10 (1995).
[11] E. A. Sovero, Y. Kwon, D. S. Deakin, A. L. Sailer, and J. A. Higgins, IEEE MTT-S Int. Microwave Symp. Dig. **2**, 1111 (1996).