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Non-fragile Dynamic Output Feedback H_∞ Control for Discrete-Time Systems with FWL Consideration

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Abstract: The non-fragile dynamic output feedback H_∞ controller design problem is investigated. The controller to be designed is assumed to be with additive gain variations. A two-step procedure is proposed to develop sufficient conditions for the non-fragile H_∞ controller design by employing the structured vertex separator. A comparison between the proposed and the existing controller design methods is provided, and a numerical example is carried out to support the theoretical findings.

Keywords: Additive gain variations, FWL, Non-fragile H_∞ control, two-step procedure.

1. INTRODUCTION

In the usual design process, an assumption is often made that the controller can be implemented exactly. However, there will inevitably be some amount of uncertainty in the controller digital complement. In the course of controller implementation based on different design algorithms, it turns out that the controllers can be sensitive with respect to errors in the controller coefficients [1,2]. This brings a new issue at the stage of designing controllers: how to design a controller for a given plant such that the controller is insensitive to some amount of error with respect to its gains. This issue has received some attention from the control systems community, and some relevant results have appeared in the last decade to tackle the problem of designing controllers which are capable of tolerating some level of its gain variations [1,3,4].

All the above mentioned works are concerned with the non-fragile problem with the norm-bounded type of controller uncertainty. However, this kind of uncertainty cannot describe exactly the uncertain information, while the interval type of parameter uncertainty [5] can describe more exactly the uncertain information than the norm-bounded type. But, due to the fact that the vertices of the set of uncertain parameters grow exponentially with the number of uncertain parameters, which may result in numerical computational problem for systems with high dimensions, up to present, there are only our works [6] and [7] studied the non-fragile filters design problem with the consideration of the interval type of parameter uncertainty. However, there is no work on the non-fragile controller design problem with taking interval gain uncertainty into account. Moreover, similar to the case that the problem of designing a output-feedback controller for polytopic uncertain systems is known to be a non-convex optimization problem [8], the problem of designing full-order non-fragile dynamic output feedback H_∞ controllers with interval type of gain uncertainty is also a non-convex one. These problems motivate the work in this paper.

This paper is concerned with the problem of non-fragile dynamic output feedback H_∞ controller design for linear discrete-time systems with FWL consideration. The controller to be designed is assumed to be with interval additive gain variations which are due to the FWL effects when the controller is implemented. A two-step procedure is adopted to solve this non-convex problem. It will be very difficult to apply the result to systems with high dimensions. To overcome the difficulty, a notion of structured vertex separator is employed and is exploited to develop sufficient conditions for the non-fragile H_∞ controller design in terms of solutions to a set of LMIs. The structured vertex separator-based method can significantly reduce the number of the LMI constraints involved in the design conditions. It can be proved that the worst case of our proposed method is less conservativeness than the conventional method given in [2] in theory, which can be illustrated in Section 3.

2. PROBLEM STATEMENT

Consider a linear time-invariant (LTI) discrete-time system as

$$\begin{aligned} x(k+1)&=Ax(k)+B_1\omega(k)+B_2u(k),\\ z(k)&=C_1x(k)+D_{12}u(k),\\ y(k)&=C_2x(k)+D_{21}\omega(k), \end{aligned} \tag{1}$$

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where $x(k)\in R^n$ is the state, $u(k)\in R^q$ is the control input, $\omega(k)\in R^r$ is the disturbance input, $y(k)\in R^p$ is the measured output and $z(k)\in R^m$ is the regulated output, respectively, and $A, B_1, B_2, C_1, C_2, D_{12}$ and D_{21} are known constant matrices of appropriate dimensions. To formulate the control problem, we consider a controller with gain variations of the following form:

$$\begin{aligned} \xi(k+1) &= (A_k+\Delta A_k)\xi(k)+(B_k+\Delta B_k)y(k), \\ u(k) &= (C_k+\Delta C_k)\xi(k), \end{aligned} \tag{2}$$

where $\xi(k)\in R^n$ is the controller state, A_k, ΔB_k and C_k are controller gain matrices of appropriate dimensions to be designed. ΔA_k, ΔB_k and ΔC_k represent the additive gain variations of the following interval type:

$$\begin{aligned} \Delta A_k &= [\delta_{aij}]_{n\times n},\ |\delta_{aij}|\le \delta_a,\ i,j=1,\cdots,n, \\ \Delta B_k &= [\delta_{bij}]_{n\times n},\ |\delta_{bij}|\le \delta_a,\ i=1,\cdots,n,\ j=1,\cdots,p, \\ \Delta C_k &= [\delta_{cij}]_{q\times n},\ |\delta_{cij}|\le \delta_a,\ i=1,\cdots,q,\ j=1,\cdots,n. \end{aligned} \tag{3}$$

Let $e_k\in R^n$, $h_k\in R^p$ and $g_k\in R^q$ denote the column vectors in which the kth element equals 1 and the others equal 0. Then the gain variations of the form (3) can be described as :

$$\Delta A_k=\sum_{i=1}^{n}\sum_{j=1}^{p}\delta_{aij}e_ie_j^T,\quad \Delta B_k=\sum_{i=1}^{n}\sum_{j=1}^{p}\delta_{bij}e_ih_j^T,$$

$$\Delta C_k=\sum_{i=1}^{q}\sum_{j=1}^{n}\delta_{cij}g_ie_j^T.$$

Applying controller (2) to system (1), this yields:

$$\begin{aligned} x_e(k+1) &= A_ex_e(k)+B_e\omega(k), \\ z(k) &= C_ex_e(k), \end{aligned} \tag{4}$$

where $x_e(k)=[x(k)^T,\xi(k)^T]$, and

$$A_e=\begin{bmatrix} A & B_2(C_k+\Delta C_k) \\ (B_k+\Delta B_k)C_2 & A_k+\Delta A_k \end{bmatrix},$$

$$B_e=\begin{bmatrix} B_1 \\ (B_k+\Delta B_k)D_{21} \end{bmatrix},\quad C_e=\begin{bmatrix} C_1 & D_{12}(C_k+\Delta C_k) \end{bmatrix}.$$

This paper addresses the following problem:

Non-fragile H_∞ control problem with controller gain variations: Given a positive constant γ, find a dynamic output feedback controller of the form (2) with the gain variations (3) such that the resulting closed-loop system (4) is asymptotically stable and the H_∞ norm of the closed-loop transfer function from disturbance to the controlled output is strictly less than a prescribed positive scalar γ.

3. NON-FRAGILE H_∞ CONTROLLER DESIGN

In this section we will present a two-step procedure which can be used for solving the non-fragile H_∞ control problem, and a comparison is made between the new proposed method and the existing method.

3.1. Non-fragile H_∞ control with known gain C_k

In this subsection, we will give non-fragile H_∞ controller design methods under the assumption that the controller gain C_k is known, where the gain C_k will be designed in the next subsection. To facilitate the presentation, we denote

$$M_0(\Delta A_k,\Delta B_k,\Delta C_k) = \begin{bmatrix} \Xi_1 & \Xi_2 & 0 & \Xi_4 & S^TA & S^TB_1 \\ * & \Xi_3 & 0 & \Xi_5 & \Xi_6 & \Xi_7 \\ * & * & -I & \Xi_8 & C_1 & 0 \\ * & * & * & -\bar{P}_{11} & -\bar{P}_{12} & 0 \\ * & * & * & * & -\bar{P}_{22} & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

where

$$\begin{aligned}
\Xi_1 &= \bar{P}_{11}-S-S^T, \Xi_2=\bar{P}_{12}-S-S^T, \\
\Xi_3 &= \bar{P}_{22}-S-S^T+N+N^T, \\
\Xi_4 &= S^TA+S^TB_2(C_k+\Delta C_k), \\
\Xi_5 &= (S-N)^TA+F_BC_2+N^T\Delta B_kC_2+F_AN^T\Delta A_k \\
&\quad +(S-N)^TB_2(C_k+\Delta C_k), \\
\Xi_6 &= (S-N)^TA+F_BC_2+N^T\Delta B_kC_2, \\
\Xi_7 &= (S-N)^TB_1+F_BD_{21}+N^T\Delta B_kD_{21}, \\
\Xi_8 &= C_1+D_{12}(C_k+\Delta C_k).
\end{aligned}$$

Then the following theorem presents a sufficient condition for the problem of non-fragile H_∞ control.

Theorem 1: Consider system (1). Let scalar $\gamma>0$, $\delta_a>0$ and gain matrix C_k be given. If there exist matrices F_A, F_B, S, N, $\bar{P}_{12}$ and $\bar{P}_{11}>0$, $\bar{P}_{22}>0$, such that the following LMIs hold:

$$\begin{aligned} & M_0(\Delta A_k,\Delta B_k,\Delta C_k)<0, \delta_{aij},\delta_{bik},\delta_{clj}\in\{-\delta_a,\delta_a\}, \\ & \qquad i,j=1,\cdots,n;\ k=1,\cdots,p;\ l=1,\cdots,q, \end{aligned} \tag{5}$$

then controller (2) with additive uncertainty (3), C_k and

$$A_k=(N^T)^{-1}F_A, B_k=(N^T)^{-1}F_B \tag{6}$$

solves the non-fragile H_∞ control problem for system (1).

Proof: Due to the limit of space, the proof is omitted. The reader who is interested in the proof can connect with the authors.

For the non-fragile H_∞ controller design method, it should be noted that the number of LMIs involved in (5) is $2^{n^2+np+nq}$, which results in the difficulty of implementing the LMI constraints in computation. For example, when n = 6 and p = q = 1, the number of LMIs involved in (5) is 2^{48}, which already exceeds the capacity of the current LMI solver in Matlab. To overcome the difficulty the following method is developed.

Denote

$$F_{a1}=[f_{a11}\ \ f_{a12}\ \ \cdots\ \ f_{a1l_a}],$$

$$F_{a2}=[f_{a21}^T\ \ f_{a22}^T\ \ \cdots\ \ f_{a2l_a}^T]^T,$$

where $l_a = n^2 + np + nq$, and

$$f_{ak1} = [0_{1\times n} \quad (N^T e_i)^T \quad 0_{1\times q} \quad 0_{1\times n} \quad 0_{1\times n} \quad 0_{1\times r}]^T,$$
$$f_{ak2} = [0_{1\times n} \quad 0_{1\times n} \quad 0_{1\times q} \quad e_j^T \quad 0_{1\times n} \quad 0_{1\times r}],$$
$$\text{for } k = (i-1)n + j, i, j = 1, \cdots, n,$$
$$f_{a1k} = [0_{1\times n} \quad (N^T e_i)^T \quad 0_{1\times q} \quad 0_{1\times n} \quad 0_{1\times n} \quad 0_{1\times r}]^T,$$
$$f_{a2k} = [0_{1\times n} \quad 0_{1\times n} \quad 0_{1\times q} \quad h_j^T C_2 \quad h_j^T C_2 \quad h_j^T D_{21}],$$
$$\text{for } k = n^2 + (i-1)p + j, i = 1, \cdots, n, j = 1, \cdots, p,$$
$$f_{a1k} = [\Omega_1 \quad \Omega_2 \quad (D_{12} g_i)^T \quad 0_{1\times n} \quad 0_{1\times n} \quad 0_{1\times r}]^T,$$
$$f_{a2k} = [0_{1\times n} \quad 0_{1\times n} \quad 0_{1\times q} \quad e_j^T \quad 0_{1\times n} \quad 0_{1\times r}],$$
$$\text{for } k = n^2 + np + (i-1)n + j, i = 1, \cdots, q, j = 1, \cdots, n,$$

where $\Omega_1 = (S^T B_2 g_i)^T$, $\Omega_1 = [(S-N)^T B_2 g_i]^T$, and $0_{i\times j}$ represents zero matrix of i rows and j columns.

Let $k_0, k_1, \cdots, k_{sa}$ be integers satisfying $k_0 = 0 < k_1 < \cdots < k_{sa} = l_a$ and matrix θ have the following structure

$$\Theta = \begin{bmatrix} diag\left[\theta_{11}^1 \ \cdots \ \theta_{11}^{S_a}\right] & diag\left[\theta_{12}^1 \ \cdots \ \theta_{12}^{S_a}\right] \\ diag\left[\theta_{12}^1 \ \cdots \ \theta_{12}^{S_a}\right]^T & diag\left[\theta_{22}^1 \ \cdots \ \theta_{22}^{S_a}\right]^T \end{bmatrix}, \tag{8}$$

where θ_{11}^i, θ_{12}^i, and $\theta_{22}^i \in R^{(k_i - k_{i-1})\times(k_i - k_{i-1})}$, $i = 1, \cdots, S_a$. Then, we have

Theorem 2: Consider system (1). Let scalar $\gamma > 0$, $\delta_a > 0$ and gain matrix C_k be given. If there exist matrices F_A, F_B, S, N, $\bar{P}_{12}$, $\bar{P}_{11} > 0$, $\bar{P}_{22} > 0$, and symmetric matrix Θ with the structure described by (8) such that the following LMIs hold:

$$\begin{bmatrix} Q & F_{a1} \\ F_{a1}^T & 0 \end{bmatrix} + \begin{bmatrix} F_{a2} & 0 \\ 0 & I \end{bmatrix}^T \Theta \begin{bmatrix} F_{a2} & 0 \\ 0 & I \end{bmatrix} < 0, \tag{9}$$

$$\begin{bmatrix} I \\ diag\left[\delta_{k_{i-1+j}} \ \cdots \ \delta_{k_i}\right] \end{bmatrix}^T \begin{bmatrix} \theta_{11}^i & \theta_{12}^i \\ (\theta_{12}^i)^T & \theta_{22}^i \end{bmatrix} \times \begin{bmatrix} I \\ diag\left[\delta_{k_{i-1+j}} \ \cdots \ \delta_{k_i}\right] \end{bmatrix} \geq 0 \tag{10}$$

for all $\delta_{k_{i-1+j}} \in \{-\delta_a, \delta_a\} j = 1, \cdots, k_i - k_{i-1}$, $i = 1, \cdots, S_a$, where

$$Q = \begin{bmatrix} \Xi_1 & \Xi_2 & 0 & \Psi_1 & S^T A & S^T B_1 \\ * & \Xi_3 & 0 & \Psi_2 & \Psi_3 & \Psi_4 \\ * & * & -I & C_1 + D_{12} C_k & C_1 & 0 \\ * & * & * & -\bar{P}_{11} & -\bar{P}_{12} & 0 \\ * & * & * & * & -\bar{P}_{22} & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

with Ξ_1, Ξ_2, Ξ_3 defined by (6) and

$$\Psi_1 = S^T A + S^T B_2 C_k, \Psi_3 = (S-N)^T A + F_B C_2,$$
$$\Psi_2 = (S-N)^T A + F_B C_2 + F_A + (S-N)^T B_2 C_k,$$
$$\Psi_4 = (S-N)^T B_1 + F_B D_{21}.$$

Then controller (2) with additive uncertainty (3) and the controller gain parameters given by (6) solves the non-fragile H_∞ control problem for system (1).

Proof: It is similar to the proof of Theorem 7 in [6] and is omitted here.

3.2. Comparison with the existing design method

In this part, the result of a non-fragile H_∞ controller design with norm-bounded gain variations is introduced, and the comparison with our result is made.

Similar to [2] for non-fragile problem with norm-bounded uncertainty, the norm-bounded type of gain variations ΔA_k, ΔB_k and ΔC_k can be overbounded by the following norm-bounded uncertainty:

$$\Delta A_k = M_a F_1(t) E_a, \quad \Delta B_k = M_b F_2(t) E_b, \quad \Delta C_k = M_c F_3(t) E_c, \tag{11}$$

where

$$M_a = [M_{a1} \ \cdots \ M_{an^2}], \quad E_a = [E_{a1}^T \ \cdots \ E_{an^2}^T]^T,$$
$$M_b = [M_{b1} \ \cdots \ M_{bnp}], \quad E_b = [E_{b1}^T \ \cdots \ E_{bnp}^T]^T,$$
$$M_c = [M_{c1} \ \cdots \ M_{cnq}], \quad E_c = [E_{c1}^T \ \cdots \ E_{c1}^T]^T$$

with

$$M_{ak} = e_i, \quad E_{ak} = e_j^T,$$
$$\text{for } k = (i-1)n + j, i, j = 1, \cdots, n,$$
$$M_{bk} = e_i, \quad E_{bk} = h_j^T$$
$$\text{for } k = n^2 + (i-1)p + j, \ i = 1, \cdots, n, \ j = 1, \cdots, p,$$
$$M_{ck} = g_i, \quad E_{ck} = e_j^T$$
$$\text{for } k = n^2 + np + (i-1)n + j, \ i = 1, \cdots, q, j = 1, \cdots, n$$

and $F_i^T(t) F_i(t) \leq \delta_a^2 I$ for $i = 1, 2, 3$ represent the uncertain parameters, here δ_a is the same as before.

Noting that the problem of non-fragile dynamic output feedback H_∞ controller design with norm-bounded gain variations is also a non-convex problem, and similar to Theorem 2, when the controller gain C_k is known, it can be converted to a convex one.

To facilitate the presentation, we denote

$$\bar{F}_A = \bar{N} A_k, \quad \bar{F}_B = \bar{N} B_k,$$
$$M_{a1} = \begin{bmatrix} 0 & \bar{S} B_2 M_c & 0 \\ \bar{N} M_a & (\bar{S} - \bar{N}) B_2 M_c & \bar{N} M_b \\ 0 & D_{12} M_c & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$M_{a2}=\begin{bmatrix} 0 & 0 & 0 & E_a & 0 & 0 \\ 0 & 0 & 0 & E_c & 0 & 0 \\ 0 & 0 & 0 & E_bC_2 & E_bC_2 & E_bD_{21} \end{bmatrix}.$$

Assume that C_k is known, by using the method in [2], the non-fragile H_∞ controller design with norm-bounded gain variations is reduced to solve the following LMI:

$$\begin{bmatrix} \bar{Q} & M_{a1} & \delta_a\varepsilon M_{a2}^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix}<0 \tag{12}$$

with matrix variables $\bar{S}>0$, $\bar{N}<0$ and scalar $\varepsilon>0$, where

$$\bar{Q}=\begin{bmatrix} -\bar{S} & -\bar{S} & 0 & \bar{S}(A+B_2C_k) & \bar{S}A & \bar{S}B_1 \\ * & -\bar{S}+\bar{N} & 0 & Q_1 & Q_2 & Q_3 \\ * & * & -I & C_1+D_{12}C_k & C_1 & 0 \\ * & * & * & -\bar{S} & -\bar{S} & 0 \\ * & * & * & * & -\bar{S}+\bar{N} & 0 \\ & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

with $Q_1=(\bar{S}-\bar{N})(A+B_2C_k)+\bar{F}_A+\bar{F}_BC_2$, $Q_2=(\bar{S}-\bar{N})A+F_BC_2$, $Q_3=(\bar{S}-\bar{N})B_1+\bar{F}_BD_{21}$.

Then, we have

Lemma 1: Consider system (1), if condition (12) is feasible, then the controllers design condition given in Theorem 2 is feasible.

Proof: Due to the limit of space, the proof is omitted.

Remark 1: It needs be pointed out the existing non-fragile H_∞ controller design method with the norm-bounded gain variations is more conservative than the one given by Theorem 2 even for the worst case of the new proposed method, i.e., $S_a=l_a$.

3.3. Design an initial controller gain C_k

Consider the controller (2) with $\Delta A_k=0$ and $\Delta B_k=0$, which is described by

$$\begin{aligned} \dot{\xi}(k)&=A_k\xi(k)+B_ky(k),\\ u(k)&=(C_k+\Delta C_k)\xi(k), \end{aligned} \tag{13}$$

where ΔC_k is the same as in (11).

Combining controller (13) with system (1), we obtain :

$$\begin{aligned} \dot{x}_e(k)&=A_{edc}x_e(k)+B_{edc}\omega(k),\\ z(k)&=C_ex_e(k) \end{aligned} \tag{14}$$

with $A_{edc}=\begin{bmatrix} A & B_2(C_k+\Delta C_k) \\ B_kC_2 & A_k \end{bmatrix}$, $B_{edc}=\begin{bmatrix} B_1 \\ B_kD_{21} \end{bmatrix}$, and C_e is the same as the one in (4).

Then the following theorem gives a design method of the initial controller gain C_k.

Theorem 3: Consider system (1), $\gamma>0$, and $\delta_a>0$ are constants. If there exist matrices $\hat{A}$, $\hat{B}$, $\hat{C}$, $X>0$, $Y>0$, and a constant $\varepsilon_c>0$ such that

$$\begin{bmatrix} \Xi_{11} & 0 & \Xi_{12} & \Xi_{13} & \Xi_{14} & 0 \\ * & -I & \Xi_{15} & 0 & D_{12}M_c & 0 \\ * & * & \Xi_{11} & 0 & 0 & \Xi_{16} \\ * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & -\varepsilon_c I & 0 \\ * & * & * & * & * & -\varepsilon_c I \end{bmatrix}<0, \tag{15}$$

where

$$\Xi_{11}=\begin{bmatrix} -X & -I \\ * & -Y \end{bmatrix},\quad \Xi_{12}=\begin{bmatrix} AX+B_2\hat{C} & A \\ \hat{A} & YA+\hat{B}C_2 \end{bmatrix},$$

$$\Xi_{13}=\begin{bmatrix} B_1 \\ YB_1+\hat{B}D_{21} \end{bmatrix},\quad \Xi_{14}=\begin{bmatrix} B_2M_c \\ YB_2M_c \end{bmatrix},$$

$$\Xi_{15}=\begin{bmatrix} C_1X+D_{12}\hat{C} & C_1 \end{bmatrix},\quad \Xi_{16}=\begin{bmatrix} \varepsilon_c\delta_aXE_c^T \\ 0 \end{bmatrix}.$$

Then controller (13) with

$$\begin{aligned} A_k&=(X^{-1}-Y)^{-1}(\hat{A}-YAX-\hat{B}C_2X-YB_2\hat{C})X^{-1},\\ B_k&=(X^{-1}-Y)^{-1}B,\quad C_k=\hat{C}X^{-1} \end{aligned}$$

solves the non-fragile H_∞ control problem for system (1).

Proof: Due to the limit of space, the proof is omitted.

Remark 2: Theorem 3 shows that the non-fragile controller design problem with $\Delta A_k=0$, $\Delta B_k=0$ and ΔC_k in the norm-bounded form defined by (11) can be converted into a convex one depending a single parameter $\varepsilon_c>0$.

Combining the results in Subsection 3.1 and Theorem 3, a two-step procedure is summarized as follows:

Step 1: Minimize γ subject to $X>0$; $Y>0$ and LMI (15). Denote the optimal solutions as $X=X_{opt}$ and $\hat{C}=\hat{C}_{opt}$. Then, $C_{kopt}=\hat{C}_{opt}X_{opt}^{-1}$.

Step 2: Let $C_k=C_{kopt}$, minimize γ subject to F_A, A_B, N, S, $\bar{P}_{12}$, $\bar{P}_{11}>0$, $\bar{P}_{22}>0$, and LMIs (9), (10). Denote the optimal solutions as $N=N_{opt}$, $F_A=F_{Aopt}$, and $F_B=F_{Bopt}$. Then according to (6), we obtain $A_k=(N^T)^{-1}F_{Aopt}$, $B_k=(N^T)^{-1}F_{Bopt}$.

The resulting A_k, B_k and C_k will form the non-fragile dynamic output feedback H_∞ controller gains.

4. EXAMPLE

In the following, an example is given to illustrate the effectiveness of the proposed method.

Example: Consider a linear system of form (1) with

$$A=\begin{bmatrix} 0 & -1 & 0 \\ 0 & -1 & 0.5 \\ 0.5 & -1 & 1 \end{bmatrix},\quad B_1=\begin{bmatrix} -0.5 & 0 \\ -0.5 & 0 \\ -1 & 0 \end{bmatrix},\quad B_2=\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix},$$

$$C_1=\begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix},\quad D_{12}=\begin{bmatrix} 0 \\ 1 \end{bmatrix},\quad C_2=\begin{bmatrix} -1 & 1 & -2 \end{bmatrix},$$

$$D_{21}=[0 \quad 1].$$

Table 1. The performance index by design with δ_a=0.006.

	Conventional	Proposed (s_a=15)	Proposed (s_a=5)
γ	2.8645	2.5204	2.5099
Degradation compared with γ_{opt1}	32.48 %	16.57%	16.08 %

By the standard H_∞ controller design method, we obtain the optimal H_∞ performance index for the system as γ_{opt}=2.1622. On the other hand, assume that the designed controller is with form (13). Let S_a=0.05, by Theorem 3 with ε_c=155.9999, we obtain $C_{kini} = [0.2573 \;\; -0.2351 \;\; 0.3380]$.

Firstly, we design an H_∞ controller by the conventional method (condition (12)) with $C_k=C_{kini}$. Assume that the designed controller is with norm-bounded additive uncertainties described by (11), by applying condition (12) with δ_a=0.006 to design a non-fragile controller, the obtained H_∞ performance index of the obtained non-fragile controller is 2.8645.

Then, we design an H_∞ controller by the proposed method Theorem 2 with $C_k=C_{kini}$. Assume that the designed controller is with the additive uncertainties described by (3). By applying Theorem 2 with δ_a=0.006 and $k_i=i$, $i=1,\cdots,15$, i.e., S_a=15 as well as $k_i=3i$, $i=1,\cdots,5$ i.e., S_a=5 to design a non-fragile H_∞ controller, and the H_∞ performance indexes of the obtained non-fragile controllers are γ=2.5204 (S_a=15) and γ=2.5099 (S_a=5), respectively.

In the following, Table 1 shows the H_∞ performance indices achieved by the designs of the existing method (Condition (12)) and the proposed method (Theorem 2). The above table shows that the worst case (S_a=15) of the proposed method also is more effective than the conventional non-fragile H_∞ controller design method condition (12).

5. CONCLUSIONS

The problem of non-fragile dynamic output feed-back H_∞ controller design for linear discrete-time systems is studied. The controller to be designed is assumed to be with additive gain variations of interval type. The structured vertex separator is exploited to develop sufficient conditions for the non-fragile H_∞ controller design via a two-step procedure. A comparison between our method and the existing method for non-fragile H_∞ controller design is presented, and a numerical example is given to illustrate the design methods.

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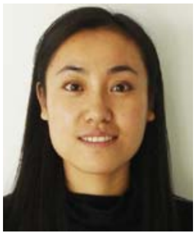

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