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A GDP fluctuation model based on interacting firms

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ABSTRACT

A distinctive feature of the market economies is the short-run fluctuations in output around the trend of long-run growth over time, and we regard this feature is internal to complex economic systems composed of interacting heterogeneous units. To explore such internal mechanisms of macroeconomic fluctuations, we present a multi-agent Keynesian theory-based model, which can provide a good approximation to the key empirical features of the western business cycles in the 20th Century, such as the structure of the autocorrelation function of overall output growth, correlations between the output growth of individual agents over time, the distribution of recessions, etc.

1. Introduction

Economic movements have always attracted the attention of economists. Generally, economists decompose it as two subfields: one is economic growth theory, which cares about the long-run trend; while the other is economic fluctuations theory (also often referred to as "business cycle theory"), which focuses on the short-run fluctuations deviating from the trend. Mainstream economists usually regard the economic fluctuations as nothing but just a series of random shocks which are exogenous to the economic system. But such an argument appears to be difficult to give a reasonable explanation to some crucial properties of economic fluctuations [1]. In fact, until now, economics lacks a satisfactory explanation of economic fluctuations. Recently, more and more econophysicists put their eyes on the complex economic system, and some scholars attempt to give a better explanation to the economic fluctuations phenomenon by injecting some physics methodology [2]. Here we follow these pioneers, especially to extend the business cycle model proposed by Ormerod [1], and present a multi-agent Keynesian theory-based model, which enables more key empirical features of the US business cycles in the 20th Century to be explained.

As a classical expression of the Keynesian theory on macroeconomic fluctuations, Keynes once wrote "By a cyclical movement we mean that as the system progresses in, e.g., the upward direction, the forces propelling it upwards at first gather force and have a cumulative effect on one another but gradually lose their strength until at a certain point they tend to be replaced by forces operating in the opposite direction; which in turn gather force for a time and accentuate one another, until they too, having reached their maximum development, wane and give place to their opposite" [3]. Inspired by this insightful idea, we propose our agent-based model as follows: The economic system is composed of N individual firms, namely agents. When firms set their output growth, they are influenced by two opposite forces: one is "force of inertia", which corresponds to the trend-chasing expectation; the other is "force of recovery", which corresponds to the reverse expectation. Technically, taking zero economic growth rate as a base point, when the detrended growth rate departs not too far away from zero, the trend-chasing force dominates the dynamics; whereas in the opposite situation, the reverse force dominates the movement. Here every firm determines its own output growth rate independently but obeys such a

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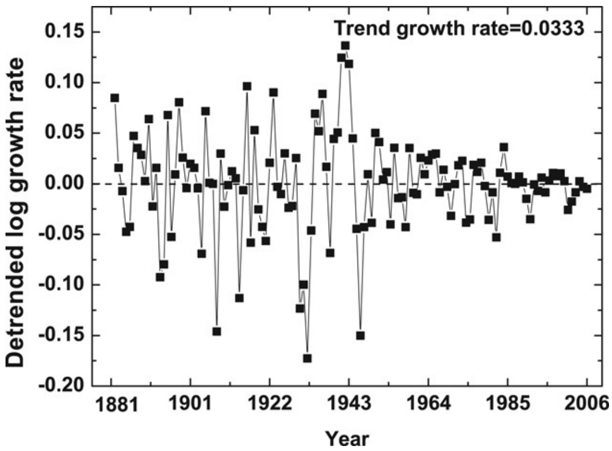

Fig. 1. Time series of real detrended log economic growth rates in the US 1881–2006.

Keynesian-style rule, so the firm's output decision is influenced both by a common fluctuation and its own uncertainty. The overall economic growth rate is just the weighted sum of each firm's output growth rate, and the weight reflects each firm's contribution to the overall output.

The model evolves on a step-by-step basis, and gives a time series of overall economic growth rates, which exhibits some key empirical features of the US business cycle in the 20th Century, such as the structure of the autocorrelation function of overall output growth, correlations between the output growth of individual agents over time, the distribution of recessions, etc.

Furthermore, our model behaves in a "conservation" way, which means the GDP growth rate can not go beyond a "reasonable" range. In this paper, we explain this conservation nature by the expectation dynamics. Behind the expectation dynamics, this flocking-type dynamic mechanism may emerge from more basic conservation principles, such as the conservation of agent and money, as Wright advocated [4]. But in this paper, instead of clarifying what are the underlying factors to dominate the expectation dynamics, we just take the expectation dynamics as our key assumption.

In what follows, Section 2 reviews the stylized facts on the macroeconomic fluctuations, Section 3 presents the theoretical model, and Section 4 displays the properties of the model compared to the stylized facts. Section 5 gives a brief conclusion.

2. Stylized facts on GDP fluctuations

Unlike in the financial markets, empirical studies about GDP (Gross Domestic Product) data meet two fundamental problems: First, GDP data are estimated, so they naturally contain potential estimate errors and are subject to future revision. Second, GDP data are scarce in the content of statistics. Without doubt, these problems bring difficulties to the study of empirical features of GDP fluctuations. So the stylized facts about GDP are revealed to be limited and debates on some features also do exist [5–7]. Furthermore, the problems also potentially affect the foundation of theoretical models, because the empirical criterion to evaluate a model is limited and doubtful. Here we just review some researches on the empirical features of GDP fluctuations as the following stylized facts. Fig. 1 shows the annual log growth rates of the US GDP [8] (detrended by the long-run average value), which gives an intuitionistic interpretation of Keynes's cited words.

(1) The structure feature of the time series of GDP growth rates

The autocorrelation function is used to explore the structure feature of GDP growth rates in the time domain. Ormerod underlined three points about this: low positive first-order autocorrelation (For the sampled data we used in Fig. 1, the first-order autocorrelation is 0.252.); lower, but in general statistically significant, negative autocorrelation at lags 3–5; other lags of the autocorrelation usually not significantly different from zero [1]. While from the frequency view, the power spectrum can give another exploration of the structure feature of GDP. As Ormerod also mentioned in Ref. [1], there is a weak concentration of the power spectrum at frequencies between 6 and 9 year.

(2) The statistical distribution of GDP growth rates

Lee, Amaral, Canning, Meyer and Stanley analyzed the fluctuations in GDP of 152 countries for the period 1950–1992 and found that the probability distribution of annual log growth rates is consistent, for a certain range of its absolute values, with an exponential decay [9,10].

(3) Correlations between the output growth rates of individual agents.

Ormerod mentioned that there exist fairly strong positive correlations between the output growth rates of individual agents over time [1].

(4) The distribution of firm size

Axtell [11] used data on the entire population of tax-paying firms in the US and showed the firm size follows a Zipf distribution. He also indicated that the result holds for data from multiple years and for various definitions of firm size.

Gaffeo et al. [12] analyzed the average size, employing different proxies, distribution of a pool of the G7 group's firms over the period 1987–2000 and found that the empirical distributions are all consistent with a power law but firm size distribution generally is not Zipf.

(5) The distribution of the duration and magnitude in recessions

Following Ormerod [5], here a recession is defined as a period in which real GDP growth rates are negative, the duration of a recession is measured by the number of continuative years in the recession and the magnitude of a recession is defined as the cumulative growth rate during the recession. Ormerod and Mounfield analyzed data from 17 capitalist economies between 1870 and 1994 and concluded that power law relationship gives a good description of the frequency of duration of recessions [5]. But Wright analyzed the same data and claimed that the data rather follow an exponential law [6]. Ausloos, Miśkiewicz and Sanglier examined quarterly GDP data for 21 countries over 14 years and summarized that the hypothesis stated by Ormerod and Mounfield seems to be fine for recession occurrences, though a theory is still needed [7]. Moreover, Ormerod analyzed the same data used in Ref. [5] and held that there is a bimodal distribution of magnitude of recessions, with an exponential fit for the bulk of the data, and a second peak describing a small number of very large recessions [13].

3. The model

The modeled economic system is populated by N individual firms. Each agent i separately sets its individual growth rate of output x_t^i in period t by the following rule:

$$x_t^i = \alpha y_{t-1} + \beta_t^i y_{t-1} + \varepsilon_t + \eta_t^i \quad i = 1, 2 \ldots N \tag{1}$$

where y_{t-1} is the overall growth rate of output of economy (GDP) at period $t-1$. Each firm can obtain the information from the public official estimates of the national GDP. Certainly, in reality, the growth of individual firms are causally linked via market exchanges that involve transfers of money [4]. However, our model only covers real economic variables, not including the money factor, so we can not introduce the exchange mechanism in our model directly. Yet it may contain a transmission mechanism from GDP to a firm's output decision via market exchanges as follows: the GDP, resulting from the overall sentiment of all agents, influences a firm's expectation on its market money income, then influences its output decision. ε_t is a random variable drawn separately for each period from a normal distribution $N(0, \sigma_\varepsilon)$, which reflects uncertainty stemming from macroeconomic shocks of the current state of economy. η_t^i is a random variable drawn separately for each agent in each period from a normal distribution $N(0, \sigma_\eta)$, which embodies uncertainty in the agent's decision-making process and implies that agents are heterogeneous, that is, η_t^i is regarded as an endogenetic idiosyncratic shock which firms separately experience. This is different from mainstream economic models based on a single representative agent. $\alpha \in [0, 1]$ is a general coefficient for all the agents, while β_t^i is an intrinsic parameter constituted by two variables as follows:

$$\beta_t^i = s_t^i b_i \tag{2}$$

where b_i is a random positive variable drawn separately for each agent from a uniform distribution $U(0, b_m)$, which implies another agent-heterogeneity; and b_m is the regulable boundary of the uniform distribution, which in a way reflects the force of agents adopting different expectations in their decision-making process. s_t^i is a symbol variable switching value $+1$ or -1 by the following probability:

$$s_t^i = \begin{cases} 1 & \text{with } p = 2/[1 + \exp(\lambda \mid y_{t-1} \mid)]; \\ -1 & \text{with } 1 - p \end{cases} \tag{3}$$

where λ is a positive parameter, and $s_t^i = +1$ implies the agent adopts the trend-chasing expectation while $s_t^i = -1$ implies the agent adopts the reverse expectation. The formula suggests that agents switch their expectations following the Keynesian-style rule: when economic growth rate departs not too far away from zero, they prefer to chase the overall trend; whereas in the opposite situation, they prefer to go against the stream.

Given the individual output growth rate x_t^i, each agent's output z_t^i can be calculated by the following formula:

$$\ln z_t^i = \ln z_{t-1}^i + x_t^i. \tag{4}$$

Here we just use firm's current output to measure firm size, and it also determines a firm's contribution to (or influence on) GDP. So we define the output weight of each firm i to the aggregate output, w_t^i, as

$$w_t^i = z_t^i / \Sigma z_t^i. \tag{5}$$

Then we can obtain the overall growth rate of economy y_t as follows:

$$y_t = \Sigma w_t^i x_t^i. \tag{6}$$

Hereunto, our dynamical model is closed.

Table 1
The model-generated structure of the autocorrelation function of GDP growth rates

Lag	1	2	3	4	5	6	7	8	9	10
ACF	0.23	0.084	−0.006	−0.100	−0.110	−0.073	0.008	−0.021	0.016	0.007

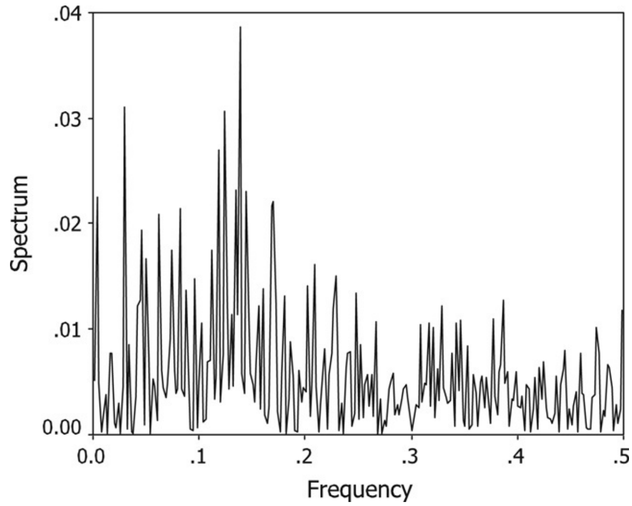

Fig. 2. The power spectrum of the model-generated GDP growth rates.

4. Results of the numerical simulations

Considering the stochastic nature of the theoretical model and without loss of generality for setting the initial values ($z_0^i = 1, x_0^i = 0, i = 1, 2 \ldots N$), as an example, we have run 100 simulations for a given set of value of parameters ($\alpha = 0.6$, $\sigma_\varepsilon = 0.048$, $\sigma_\eta = 0.088$, $b_m = 1.2$, $\lambda = 50$, $N = 1000$), and every simulation was carried out over 1000 periods. Now, we compare the simulated results with the key features stated in Section 2 one by one.

(1) Based on the data from 100 simulations, we can obtain the qualitative structure feature of the series of GDP growth rates both in the time domain using the autocorrelation function and in the frequency domain using the power spectrum as Ormerod described [1]. Across 100 simulations, a good quantitative approximation of the autocorrelation function is shown in Table 1: low positive autocorrelation at lag 1, lower negative autocorrelation at lag 3–5 among which figures at lag 4–5 are statistically significant, other lags of autocorrelations usually not significantly different from zero; while Fig. 2 plots a good quantitative approximation of the power spectrum which is concentrated at frequencies between about 6 and 9 year, exactly according with the empirical study.

(2) In every simulation, along with time going by, we obtain a time series of growth rates of GDP, then we can explore its statistical distribution. We once tried to fit the data with a Laplace distribution, but failed to get an acceptable result. This means our model does not reproduce the empirical findings [9,10].

(3) Our model also registers the output growth rates of individual agents in each period. By one simulation, we can get the time series of output growth rates of 1000 individual firms. Then we calculate the cross-correlations between them and take an average of them. After 100 simulations, we can obtain the mean correlation across 100 simulations is 0.218, which validates the fairly positive correlations between the output growth rates of individual agents over time.

(4) Firm size is defined as the output of a firm here. At the terminal time of each simulation, we can obtain the output of each firm. After 100 simulations, we gather the data together and find it is visually apparent that the distribution of firm size fits a power law very well (see Fig. 3). The estimate value for the power law exponent is −1.64, which means the distribution is not a Zipf. This does not accord with empirical findings by Axtell [11], but is consistent with the empirical conclusion from Gaffeo et al. [12].

(5) Based on the definitions in Section 2, we explore the model-generated data and find the distribution of duration of recessions follows an exponential law with exponent −0.23 (see Fig. 4(a)). This is consistent with Wright's empirical result [6]; while the distribution of magnitude of recessions is consistent with Ormerod's empirical finding [13] (see Fig. 4(b)): on the whole, the distribution follows an exponential law, but not very well. We find that when the magnitude of a recession exceeds 0.3, the data are not well fitted by a least-squares regression. As Ormerod said, there seems a bimodal distribution, with an exponential fit for most data and a second peak describing a small number of larger recessions.

By all appearances, it is inadequate to provide a single sample in the model's parameter space to exhibit the economic evolutionary behavior. Thus, here we try to explore the parameter space and parameter sensitivity by numerical simulations.

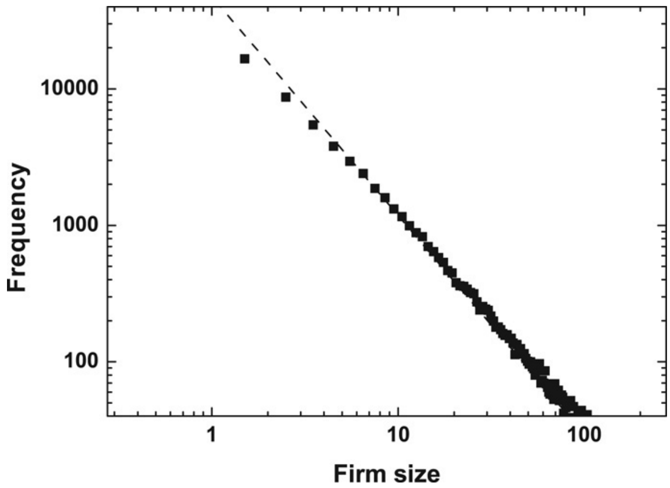

Fig. 3. The distribution of firm size (measured by output) at terminal time with a power law fit with exponent −1.64.

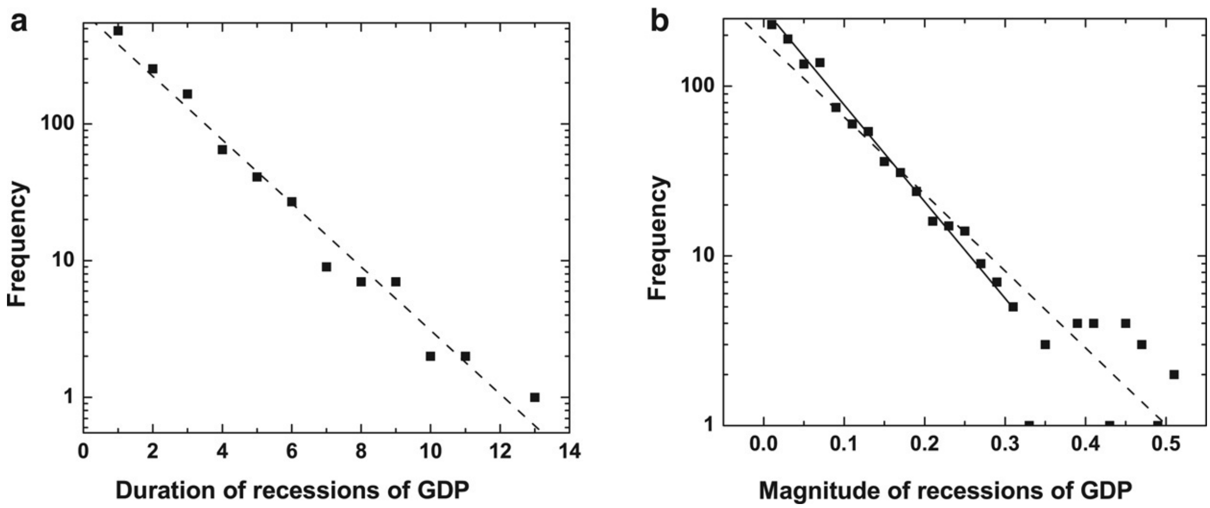

Fig. 4. In figure (a) is plotted the distribution of duration of recessions with an exponential fit with exponent −0.23. In figure (b) is plotted the distribution of magnitude of recessions. The dashed line indicates an exponential behavior with slope −4.53 fitting for all the points; while the solid line indicates an exponential behavior with slope −5.43 fitting for the points with their abscissa below 0.3.

With a view to the stochastic nature of the model and without loss of generality for setting the initial values ($z_0^i = 1, x_0^i = 0, i = 1, 2 \ldots N$), we have run 100 simulations each carried out over 500 periods, for any given set of values of the parameters. We find, with $\alpha \in [0.5, 0.65]$, $b_m \in [1.0, 1.4]$, $\lambda \in [60, 80]$ and $\sigma_\varepsilon, \sigma_\eta \in [0.03, 0.09]$, the model can reproduce all the qualitative stylized facts we have mentioned above and limit the range of fluctuations of GDP growth rates at $[-0.2, 0.2]$ and the correlations of firms at $[0.15, 0.55]$.

We also find (i) the number of agents in the system, N, appears to have no significant effects on the results of the model except that the range of fluctuations of GDP growth rates series becomes too big and the structure of the autocorrelation function of GDP growth rates seems to be unsteady when N is too small. (ii) α and b_m measure the influence that the last GDP growth rate brings to agents' present output decisions. The smaller α or the bigger b_m, the less obviously positive (or even negative) the first lag of the autocorrelation of GDP growth rates; while the bigger α or the smaller b_m, the more obviously positive the autocorrelation of GDP growth rates not only for the first lag, but also for longer lags. Moreover, the smaller b_m relative to α, the stronger the correlations between firms. (iii) λ represents the sensitivity of the agents' reactions to the exceptional GDP level. The smaller λ, the more obviously positive autocorrelation of GDP growth rates not only for the first lag, but also for longer lags, the stronger the correlations between firms, the longer duration and bigger magnitude in recessions of GDP. (iv) σ_ε and σ_η measure the diversity degree of the agents' decision making. Both parameters make very little difference to the results except some quantitative effects on the correlations between firms and the autocorrelation of GDP growth rates.

5. Conclusion

In this paper, we focus on an attractive feature of western market economies: the local short-run fluctuations in output around the overall slow but persistent growth trend over time. Based on Keynes's idea and inspired by some agent-based models, we offer a new parsimonious agent-based model to try to recur to some basic features of the short-run economic fluctuations. Our model creates an economy stage and places N diversified actors (firms), they put on a drama composed of their colorful uncertain individual economic decision process and their potential interactions. Through presenting the drama again and again, we can get an evolving economy system which is capable of replicating crucial properties of the US short-run economic fluctuations. We expect this work can make some contributions to the explanations about economic fluctuations and give a new insight to the complex phenomena.

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References

[1] P. Ormerod, Physica A 314 (2002) 774–785.
[2] L.A.N. Amaral, S.V. Buldyrev, S. Havlin, M.A. Salinger, H.E. Stanley, Phys. Rev. Lett. 80 (1998) 1385–1388.
[3] J.M. Keynes, The general theory of employment, in: Interest and Money, Macmillan Cambridge University Press, New York, 1936.
[4] I. Wright, Physica A 346 (2005) 589–620.
[5] P. Ormerod, C. Mounfield, Physica A 293 (2001) 573–582.
[6] I. Wright, Physica A 345 (2005) 608–610.
[7] M. Ausloos, J. Miśkiewicz, M. Sanglier, Physica A 339 (2004) 548–558.
[8] The annual US GDP data have been obtained from L.D. Johnston and S.H. Williamson, The Annual Real and Nominal GDP for the United States, 1790-Present. Economic History Services (2005) at, http://www.eh.net/hmit/gdp/.
[9] Y. Lee, L.A.N. Amaral, D. Canning, M. Meyer, H.E. Stanley, Phys. Rev. Lett. 81 (15) (1998) 3275–3278.
[10] D. Canning, L.A.N. Amaral, Y. Lee, M. Meyer, H.E. Stanley, Econom. Lett. 60 (1998) 335–341.
[11] R.L. Axtell, Science 293 (2001) 1818–1820.
[12] E. Gaffeo, M. Gallegati, A. Palestrini, Physica A 324 (2003) 117–123.
[13] P. Ormerod, Physica A 341 (2004) 556–568.