Double transverse spin asymmetry in the $p^\uparrow \bar{p}^\uparrow$ Drell-Yan process from Sivers functions

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We show that the transverse double spin asymmetry in the Drell-Yan process contributed only from the Sivers functions can be picked out by the weighting function $\frac{Q_T}{M^2}(\cos(\phi - \phi_{S_1})\cos(\phi - \phi_{S_2}) + 3\sin(\phi - \phi_{S_1})\sin(\phi - \phi_{S_2}))$. The asymmetry is proportional to the product of two Sivers functions from each hadron $f_{1T}^{\perp(1)} \times f_{1T}^{\perp(1)}$. Using two sets of Sivers functions extracted from the semi-inclusive deeply elastic scattering data at HERMES, we estimate this asymmetry in the $p^\uparrow \bar{p}^\uparrow$ Drell-Yan process which is possible to be performed in high energy storage ring at GSI. The prediction of double spin asymmetry in the Drell-Yan process contributed by the function $g_{1T}(x, \mathbf{k}_T^2)$, which can be extracted by the weighting function $\frac{Q_T}{M^2}(3\cos(\phi - \phi_{S_1})\cos(\phi - \phi_{S_2}) + \sin(\phi - \phi_{S_1})\sin(\phi - \phi_{S_2}))$, is also given at GSI.

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I. INTRODUCTION

The Sivers effect [1] was proposed originally to explain the large single spin asymmetries (SSA) observed in inclusive pion hadroproduction ($p^\uparrow p \to \pi X$) at Fermi National Accelerator Laboratory [2]. The effect can be quantitatively described by a $\mathbf{k}_T$-dependent distribution called the Sivers function [3,4] $f_{1T}^\perp(x, \mathbf{k}_T^2)$, which is the distribution of unpolarized partons in a transversely polarized proton. It arises from a nontrivial correlation between the nucleon transverse spin and the intrinsic transverse momenta in the nucleon. Despite its (naively) T-odd property [5], the Sivers function has been proven to be nonvanishing [6] due to its special gauge-link property [7–9].

Recently the SSA measured in semi-inclusive deeply inelastic scattering (SIDIS) processes with transversely polarized targets at HERMES [10–12] and COMPASS [13,14] has been shown to be interpreted by the Sivers effect. The asymmetry is identified by the angular dependence $\sin(\phi - \phi_S)$, where ϕ and ϕ_S denote, respectively, the azimuthal angles of the produced hadron and of the nucleon spin polarization, with respect to the lepton scattering plane. The coexistent Collins asymmetry [5], with an angular dependence $\sin(\phi + \phi_S)$, has also been measured in those experiments. The data on the Sivers SSA have been utilized by different groups [15–19] to extract the Sivers functions of the proton, especially those for the u and d quarks, on the basis of the generalized factorization [20,21]. Those sets of parametrizations of the Sivers functions are qualitatively in agreement [22] among themselves, and were applied to predict the Sivers SSA in various processes in the established or planned facilities, such as the SIDIS at JLab, and the Drell-Yan process at COMPASS, RHIC, and GSI.

In this paper, we will investigate the role of the Sivers function on the transverse double spin asymmetry (DSA) in the Drell-Yan process. The transverse DSA has been investigated [23] for many years and is believed to be able to unravel the transverse spin property of the nucleon [24], especially the transversity distribution $h_1(x)$ [25]. Various azimuthal asymmetries contributed by different $\mathbf{k}_T$-dependent distribution functions have been analyzed and given in Refs. [26,27]. As shown in Ref. [27], the Sivers function contributes to the DSA in the Drell-Yan process through the product $f_{1T}^\perp \times f_{1T}^\perp$. However, this DSA is mixed with the contribution from another $\mathbf{k}_T$-dependent distribution function $g_{1T}(x, \mathbf{k}_T^2)$. We will show that, through the appropriate weighting function $\frac{Q_T^2}{M^2} \times (\cos(\phi - \phi_{S_1})\cos(\phi - \phi_{S_2}) + 3\sin(\phi - \phi_{S_1})\sin(\phi - \phi_{S_2}))$, the asymmetry from the Sivers function can be isolated without mixing with the contribution from other functions. Using two sets of parametrizations [16,18] of the Sivers functions, we calculate the double spin asymmetry from the Sivers functions in the $p^\uparrow \bar{p}^\uparrow$ Drell-Yan process at GSI. An asymmetry around 1% is predicted. The asymmetries estimated from these two sets of Sivers functions are quantitatively different. Therefore measuring the DSA in the Drell-Yan process can provide new information on the Sivers functions, especially their sizes. The transverse DSA contributed by $g_{1T}(x, \mathbf{k}_T^2)$ through the product $g_{1T} \times g_{1T}$ can also be picked out by another weighting function. We estimate this asymmetry by adopting a g_{1T} coming from the combination of a Lorentz invariance relation presented in Refs. [28,29] and the Wandzura-Wilczek approximation [30].

II. EXTRACTING DSA CONTRIBUTED BY THE SIVERS FUNCTIONS

The importance of the transverse-momentum distributions of quarks for a full understanding of the structure of hadrons has been widely recognized in the last decade

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[4,29,31,32]. A comprehensive, leading-twist, tree-level analysis of the (spin-dependent) Drell-Yan process in terms of $\mathbf{k}_T$-dependent distributions has been given in Ref. [26]. The role of the T-odd $\mathbf{k}_T$-dependent distributions in this process has been presented in Ref. [27]. In the Collins-Soper frame [33] the leading order unpolarized differential cross section for the Drell-Yan process $h_1(P_1) + h_2(P_2) \to \gamma^*(q) + X \to l^+(l_1) + l^-(l_2) + X$ has the form [27]

$$\frac{d\sigma^{(0)}(h_1h_2 \to l\bar{l}X)}{d\Omega dx_1 dx_2 d^2\mathbf{q}_T} = \frac{\alpha_{\rm em}^2}{3Q^2}\sum_q e_q^2\Big\{A(y)\mathcal{F}[f_1^q f_1^{\bar{q}}] + B(y)\cos 2\phi\,\mathcal{F}\Big[(2\hat{\mathbf{h}}\cdot\mathbf{p}_T\hat{\mathbf{h}}\cdot\mathbf{k}_T - \mathbf{p}_T\cdot\mathbf{k}_T)\frac{h_1^{\perp q}h_1^{\perp\bar{q}}}{M_1M_2}\Big]\Big\} \tag{1}$$

where q denotes the quark flavors, the notation

$$\mathcal{F}[f_1 f_1] = \int d^2\mathbf{p}_\perp d^2\mathbf{k}_\perp \delta^2(\mathbf{p}_T + \mathbf{k}_T - \mathbf{q}_T) f_1(x_1, \mathbf{p}_T^2) f_1(x_2, \mathbf{k}_T^2) \tag{2}$$

shows the convolution of transverse momenta, $Q^2 = q^2$ is the invariance mass of the lepton pair, q_T is the transverse-momentum of the lepton pair, $\hat{\mathbf{h}} = \mathbf{q}_T/Q_T$, ϕ is the angle between the hadron plane and the lepton plane, and

$$A(y) = (\tfrac{1}{2} - y + y^2) = \tfrac{1}{4}(1 + \cos^2\theta), \tag{3}$$

$$B(y) = y(1 - y) = \frac{1}{4}\sin^2\theta, \tag{4}$$

in the c.m. frame of the lepton pair.

The function $h_1^\perp$ in the third line of (1) is the Boer-Mulders function [4], the chiral-odd partner of the Sivers function. This function has attracted a lot of interest [34–36] recently because it can account for the anomalous $\cos 2\phi$ asymmetries [37,38] observed in the unpolarized Drell-Yan process, as the second term of Eq. (1) has shown.

The leading order differential cross section for the double transversely polarized Drell-Yan process is [27]

$$\begin{aligned}\frac{d\sigma^{(2)}(h_1^\uparrow h_2^\uparrow \to l\bar{l}X)}{d\Omega dx_1 dx_2 d^2\mathbf{q}_T} = \frac{\alpha_{\rm em}^2}{3Q^2}\sum_q\Big\{\ldots &+ \frac{A_1(y)}{2}|\mathbf{S}_{1T}||\mathbf{S}_{2T}|\cos(2\phi - \phi_{S_1} - \phi_{S_2})\mathcal{F}\Big[\hat{\mathbf{h}}\cdot\mathbf{p}_T\hat{\mathbf{h}}\cdot\mathbf{k}_T\frac{f_{1T}^{\perp q}f_{1T}^{\perp\bar{q}} - g_{1T}^q g_{1T}^{\bar{q}}}{M_1M_2}\Big]\\ &- \frac{A_1(y)}{2}|\mathbf{S}_{1T}||\mathbf{S}_{2T}|\cos(\phi - \phi_{S_1})\cos(\phi - \phi_{S_2})\mathcal{F}\Big[\mathbf{p}_T\cdot\mathbf{k}_T\frac{f_{1T}^{\perp q}f_{1T}^{\perp\bar{q}}}{M_1M_2}\Big]\\ &- \frac{A_1(y)}{2}|\mathbf{S}_{1T}||\mathbf{S}_{2T}|\sin(\phi - \phi_{S_1})\sin(\phi - \phi_{S_2})\mathcal{F}\Big[\mathbf{p}_T\cdot\mathbf{k}_T\frac{g_{1T}^q g_{1T}^{\bar{q}}}{M_1M_2}\Big]\Big\}.\end{aligned} \tag{5}$$

The ... indicates the terms which will not contribute in our analysis below; ϕ_{S_1} and ϕ_{S_2} are the angles between S_{1T}, S_{2T} and the lepton plane, respectively.

As shown in (5), the Sivers function can contribute to the transverse DSA through the product $f_{1T}^\perp \times f_{1T}^\perp$. However, this asymmetry is mixed with the asymmetry to which it contributes another $\mathbf{k}_T$-dependent distribution $g_{1T}(x, \mathbf{k}_T^2)$. The main goal of this paper is to isolate the asymmetry contributed by the Sivers function. The starting point is the method introduced in Ref. [39], by which one integrates the differential cross section with a proper weighting function $W(Q_T, \phi, \phi_{S_1}, \phi_{S_2})$, as follows:

$$\langle W(Q_T, \phi, \phi_{S_1}, \phi_{S_2})\rangle = \int d\phi d\phi_{S_1} d\mathbf{q}_T^2 \frac{d\sigma(h_1h_2 \to l\bar{l}X)}{d\Omega dx_1 dx_2 d^2\mathbf{q}_T} \times W(Q_T, \phi, \phi_{S_1}, \phi_{S_2}). \tag{6}$$

With the above weighting procedure, one can pick up the terms in which one is interested. Besides this, one can deconvolute the transverse-momentum integration in a model-independent way.

The unpolarized angular independent cross section can be picked out by using the weighting function 1, from Eq. (1):

$$\left(\frac{A(y)\alpha_{\rm em}^2}{3Q^2}\right)^{-1}\cdot\langle 1\rangle_{UU} = 4\pi^2\sum_q e_q^2 f_1^q(x_1) f_1^{\bar{q}}(x_2). \tag{7}$$

We denote $W_C = \cos(\phi - \phi_{S_1})\cos(\phi - \phi_{S_2})$ and $W_S = \sin(\phi - \phi_{S_1})\sin(\phi - \phi_{S_2})$. Given the weighting function $\frac{Q_T^2}{M^2}W_C$ (assuming $M_1 = M_2 = M$, i.e. the colliding two hadrons are nucleons), we can obtain the following term from (5):

$$\begin{aligned}\left(\frac{A(y)\alpha_{\rm em}^2}{3Q^2}\right)^{-1}\cdot\left\langle\frac{Q_T^2}{M_p^2}W_C\right\rangle_{TT} &= \pi^2\sum_q e_q^2\{3[f_{1T}^{\perp(1)q}(x_1)f_{1T}^{\perp(1)\bar{q}}(x_2) - g_{1T}^{(1)q}(x_1)g_{1T}^{(1)\bar{q}}(x_2)] - 2f_{1T}^{\perp(1)q}(x_1)f_{1T}^{\perp(1)\bar{q}}(x_2)\}\\ &= \pi^2\sum_a e_a^2[f_{1T}^{\perp(1)q}(x_1)f_{1T}^{\perp(1)\bar{q}}(x_2) - 3g_{1T}^{(1)q}(x_1)g_{1T}^{(1)\bar{q}}(x_2)],\end{aligned} \tag{8}$$

where $f_{1T}^{\perp(1)}(x)$ and $g_{1T}^{(1)}(x)$ are the first $\mathbf{k}_T^2$ moments, defined

as

$$f_{1T}^{\perp(1)}(x) = \int d^2\mathbf{k}_T \frac{\mathbf{k}_T^2}{2M^2} f_{1T}^{\perp}(x, \mathbf{k}_T^2), \tag{9}$$

$$g_{1T}^{(1)}(x) = \int d^2\mathbf{k}_T \frac{\mathbf{k}_T^2}{2M^2} g_{1T}(x, \mathbf{k}_T^2). \tag{10}$$

The factor Q_T^2 introduced in the weighting function ensures that the transverse-momentum integration in (8) can be deconvoluted (for details, refer to the Appendix). Again, applying the weighting function $\frac{Q_T^2}{M^2} W_S$ on (5), we arrive at

$$\begin{aligned}
&\left(\frac{A(y)\alpha_{\rm em}^2}{3Q^2}\right)^{-1} \cdot \left\langle \frac{Q_T^2}{M_p^2} W_S \right\rangle_{TT} \\
&= -\pi^2 \sum_q e_q^2 \{3[f_{1T}^{\perp(1)q}(x_1) f_{1T}^{\perp(1)\bar{q}}(x_2) \\
&\quad - g_{1T}^{(1)q}(x_1) g_{1T}^{(1)\bar{q}}(x_2)] - 2 g_{1T}^{(1)q}(x_1) g_{1T}^{(1)\bar{q}}(x_2)\} \\
&= \pi^2 \sum_a e_a^2 [-3 f_{1T}^{\perp(1)q}(x_1) f_{1T}^{\perp(1)\bar{q}}(x_2) \\
&\quad + g_{1T}^{(1)q}(x_1) g_{1T}^{(1)\bar{q}}(x_2)].
\end{aligned} \tag{11}$$

Therefore, combining (8) and (11), we can extract the term contributing to the transverse DSA and coming only from the Sivers functions:

$$\begin{aligned}
&\left(\frac{A(y)\alpha_{\rm em}^2}{3Q^2}\right)^{-1} \cdot \left\langle \frac{Q_T^2}{M_p^2} (W_C + 3W_S) \right\rangle_{TT} \\
&= -8\pi^2 \sum_q e_q^2 f_{1T}^{\perp(1)q}(x_1) f_{1T}^{\perp(1)\bar{q}}(x_2),
\end{aligned} \tag{12}$$

with the weighting function $\frac{Q_T^2}{M^2}(W_C + 3W_S)$.

By taking the ratio between (7) and (13), we define the weighted double spin asymmetry as follows:

$$\begin{aligned}
A_{TT}^f &= \frac{\langle \frac{Q_T^2}{M^2}(W_C + 3W_S)\rangle_{TT}}{\langle 1 \rangle_{UU}} \\
&= -\frac{2\sum_q e_q^2 f_{1T}^{\perp(1)q}(x_1) f_{1T}^{\perp(1)\bar{q}}(x_2)}{\sum_q e_q^2 f_1^q(x_1) f_1^{\bar{q}}(x_2)}.
\end{aligned} \tag{13}$$

The above equation thus provides a possibility to study the Sivers function by measuring the transverse DSA in the Drell-Yan process.

Also, from (7), (8), and (11) we can get another type of DSA:

$$A_{TT}^g = \frac{\langle \frac{Q_T^2}{M^2}(3W_C + W_S)\rangle_{TT}}{\langle 1 \rangle_{UU}} = -2\frac{\sum_q e_q^2 g_{1T}^{(1)q}(x_1) g_{1T}^{(1)\bar{q}}(x_2)}{\sum_q e_q^2 f_1^q(x_1) f_1^{\bar{q}}(x_2)}, \tag{14}$$

which is contributed only by g_{1T}.

III. NUMERICAL RESULTS

In this section we will give numerical results on the DSA from the Sivers functions. We consider the transversely polarized proton antiproton Drell-Yan process, where the valence Sivers functions are involved, so that a larger asymmetry should be measured compared to the $p^\uparrow p^\uparrow$ Drell-Yan process. It is possible to perform the $p^\uparrow \bar{p}^\uparrow$ Drell-Yan process in the planned high energy storage ring (HESR) [40] at GSI. We study the transverse DSA at GSI from the Sivers functions, based on Eq. (13). To do this we need to know the input for the Sivers functions. Several groups [16,18,19] have parametrized the Sivers functions based on the data of SIDIS at HERMES [11,12], and partially based on COMPASS data [13]. The kinematics in GSI can be chosen as the c.m. energy $s = 45\ \mathrm{GeV}^2$. For the invariance mass square of the lepton pair, we choose $Q^2 = 2.5\ \mathrm{GeV}^2$, which is close to the scale at HERMES. Therefore these sets of Sivers functions extracted from the data of HERMES can be applied to predict the asymmetries at GSI in the kinematics regime we give above. We will adopt two sets of Sivers functions, which are the sets in Refs. [16,18], respectively. The Sivers functions in Ref. [19] cannot be applied here since in that paper $f_{1T}^{\perp(1/2)}(x)$ is given while we use $f_{1T}^{\perp(1)}(x)$ in our calculation.

To use these Sivers functions one should notice that T-odd distribution functions in the DIS and in the Drell-Yan process have a minus sign difference [7]. However, in the $p^\uparrow \bar{p}^\uparrow$ Drell-Yan process two Sivers functions appear in the product; therefore the sign difference does not matter here and the functions can be used directly.

In Ref. [16] the Sivers functions are parametrized as

$$-\frac{\mathbf{k}_T}{M} f_{1T}^{\perp,q}(x, \mathbf{k}_T^2) = N_q(x) f_1^q(x) g(\mathbf{k}_T^2) h(\mathbf{k}_T^2), \tag{15}$$

with

$$N_q(x) = N_q x^{a_q}(1-x)^{b_q} \frac{(a_q + b_q)^{(a_q+b_q)}}{a_q^{a_q} b_q^{b_q}}, \tag{16}$$

$$g(\mathbf{k}_T^2) = \frac{e^{-\mathbf{k}_T^2/\langle k_T^2\rangle}}{\pi\langle k_T^2\rangle}, \tag{17}$$

for $q = u, d$. For the function $h(\mathbf{k}_T^2)$ two options are considered:

$$\text{(a)}\quad h(\mathbf{k}_T^2) = \frac{2k_T M_0}{k_T^2 + M_0^2}, \qquad \text{(b)}\quad \sqrt{2e}\frac{p_T}{M'} e^{-k_T^2/M'}. \tag{18}$$

In our calculation we will adopt option (b) in Eq. (18) and choose the central value of their fit. This parametrization has taken advantage of the more precise data [12] at HERMES.

In Ref. [18] the authors give the set of Sivers functions for the u and d quarks as

$$xf_{1T}^{\perp(1),u}(x) = -xf_{1T}^{\perp(1),d} = -0.17x^{0.66}(1-x)^5, \quad (19)$$

extracted from the published HERMES data [11], and whose form is based on the limit of a large number of colors N_c.

For the unpolarized distribution we use the MRST2001 (LO set) parametrization [41]. In Fig. 1 we present the DSA from the Sivers functions at GSI, as a function of x_1. A sizable asymmetry is predicted. The asymmetry (solid line) based on the Sivers functions from Ref. [16] is much larger than the asymmetry (dashed line) based on the Sivers functions from Ref. [18]. As explained in Ref. [42], taking into account the more precise data [12] of HERMES, larger Sivers functions can be extracted compared to the parametrization in Eq. (19), which will lead to a larger asymmetry compared to the dashed curve in Fig. 1. Thus the difference between the asymmetries from the two sets of Sivers functions may be reduced. Depending on the accuracy of the experimental measurements on the transverse DSA at GSI, useful constraints on the Sivers functions could be obtained, but it might be hard to distinguish between different parametrizations without high precision measurements.

Finally, we will predict the DSA contributed by the function $g_{1T}(x, \mathbf{k}_T^2)$ at GSI. This function, describing longitudinal polarization of quarks in the transversely polarized target, also plays a role in the double polarized (longitudinal-transverse) SIDIS process [28,43]. A treatment on $g_{1T}(x, \mathbf{k}_T^2)$ is the so-called Lorentz invariance relation that connects the first $\mathbf{k}_T^2$ moment of $g_{1T}(x, \mathbf{k}_T^2)$ with the twist-three distribution function $g_2(x)$:

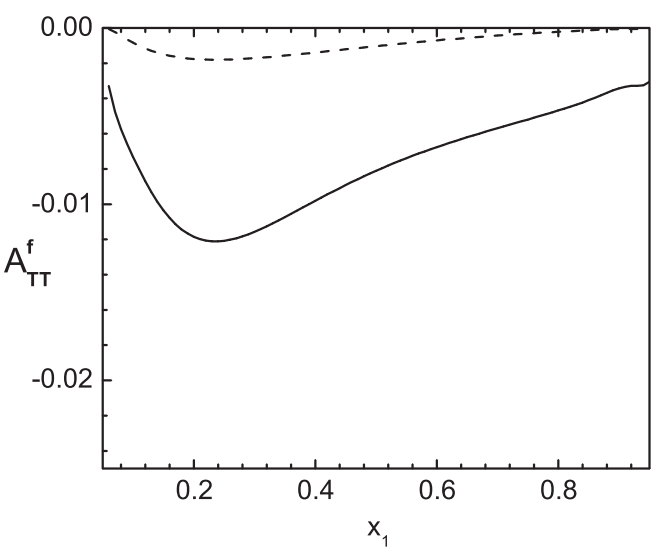

FIG. 1. The DSA in the proton antiproton Drell-Yan process at GSI coming only from the Sivers functions and calculated from Eq. (13). The kinematics are $s = 45$ GeV^2 and $Q^2 = 2.5$ GeV^2. The solid and dashed curves use the Sivers functions in Ref. [16] and in Ref. [18], respectively.

$$g_2^q(x) = \frac{d}{dx} g_{1T}^{q(1)}(x). \quad (20)$$

Using the Wandzura and Wilczek approximation for g_2^q,

$$g_2^q(x) \approx -g_1^q(x) + \int_x^1 dy \frac{g_1^q(x)}{y}, \quad (21)$$

the following relation was derived in Ref. [28]:

$$g_{1T}^{(1)q}(x) \approx x \int_x^1 dy \frac{g_1^q(x)}{y}. \quad (22)$$

For the polarized parton distribution we apply the GRSV2001 (standard scenario) parametrization [44], and for the unpolarized distribution we use the GRV98 LO parametrization [45], following the choice in Ref. [43]. In Fig. 2 we show the DSA contributed by $g_{1T}(x, \mathbf{k}_T^2)$ in the $p^\uparrow \bar{p}^\uparrow$ Drell-Yan process at GSI with $s = 45$ GeV^2 and $Q^2 = 2.5$ GeV^2. An asymmetry of 1% is predicted.

We end this section with some comments. In our calculation, especially in the case of Sivers DSA, we choose $Q^2 = 2.5$ GeV^2. This value is consistent with the averaged scale $\langle Q^2 \rangle$ in the HERMES experiment, from which the Sivers functions were extracted. Therefore, the parametrizations for Sivers functions in Refs. [16,18] can be applied here without further assumptions. Experimental measurements at GSI can also cover the continuous Drell-Yan masses 2–5 GeV which corresponds to Q^2 in the range 4–25 GeV^2. To estimate the asymmetries in this region one should use the fitted functions evolved to the relevant scale, which is not trivial for the $\mathbf{k}_T$-dependent distributions [46]. Therefore we assume that the ratios in Eqs. (13) and (14) scale with Q^2. In this region, the result is similar to the one which can be obtained at the fixed value of $Q^2 = 2.5$ GeV^2. Also there is the subtlety that the next to leading

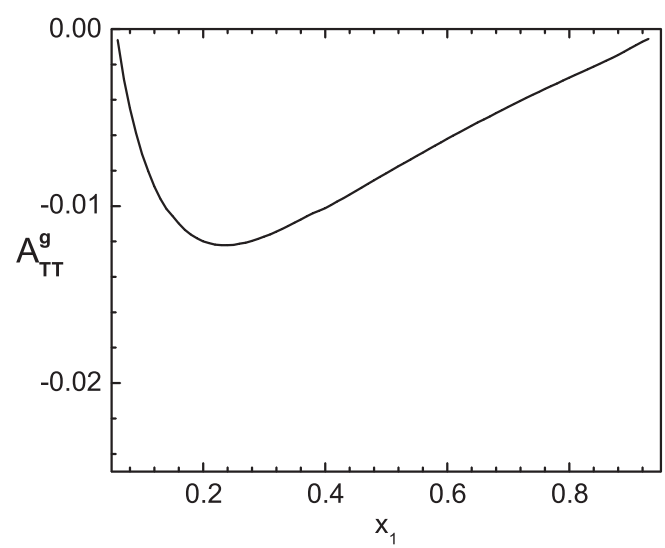

FIG. 2. The DSA in the proton antiproton Drell-Yan process at GSI coming from the function g_{1T}, defined in Eq. (14). The kinematics are $s = 45$ GeV^2 and $Q^2 = 2.5$ GeV^2.

order correction of the hard process could lead to the substantial K factor on the transversely polarized cross section. Since we calculate an asymmetry, which is essentially a ratio where the Q^2 dependences in the numerator and denominator tend to cancel each other, the effects of both the Q^2 dependence and the K factors do not introduce a strong influence on the resulting prediction coming from Eqs. (13) and (14).

IV. SUMMARY

We have performed an analysis of the transverse DSA in the Drell-Yan process contributed by the Sivers functions through the term $f_{1T}^{\perp} \times f_{1T}^{\perp}$. The asymmetry can be isolated through the appropriate weighting function $\frac{Q_T^2}{M^2} \times (\cos(\phi - \phi_{S_1})\cos(\phi - \phi_{S_2}) + 3\sin(\phi - \phi_{S_1}) \times \sin(\phi - \phi_{S_2}))$, without mixing with the contribution from other distribution functions. Using two sets of Sivers functions parametrizing the SSA data in the SIDIS process, we calculate the double spin asymmetry in the $p^{\uparrow}\bar{p}^{\uparrow}$ Drell-Yan process from the Sivers functions at GSI. An asymmetry around 1% is predicted. The asymmetries estimated from these two sets of Sivers functions are quantitatively different. Therefore measurements of the DSA in the Drell-Yan process can provide new information on the Sivers functions, especially their sizes. The transverse DSA contributed by $g_{1T}(x, \mathbf{k}_T^2)$ through the product $g_{1T} \times g_{1T}$ in the Drell-Yan process can also be picked out by a weighting function. We estimate this asymmetry at GSI by adopting g_{1T} from the combination of the Lorentz invariance relation and the Wandzura-Wilczek approximation. The investigation on the double transversely polarized Drell-Yan process can thus shed light on the knowledge of $\mathbf{k}_T$-dependent distribution functions, including the Sivers functions.

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APPENDIX: MOMENTS

To derive (8) and (11) we have used the following transverse-momentum integrations:

$$\begin{aligned}\int d^2\mathbf{k}_T d^2\mathbf{p}_T \delta^2(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T)\frac{Q_T^2}{M^2}(\mathbf{k}_T\cdot\mathbf{p}_T) f(x_1, \mathbf{k}_T^2) f(x_2, \mathbf{p}_T^2) &= \frac{1}{M^2}\int d^2\mathbf{k}_T d^2\mathbf{p}_T (\mathbf{k}_T + \mathbf{p}_T)^2 \mathbf{k}_T\cdot\mathbf{p}_T f(x_1, \mathbf{k}_T^2) f(x_2, \mathbf{p}_T^2)\\ &= \frac{2}{M^2}\int d^2\mathbf{k}_T d^2\mathbf{p}_T (\mathbf{k}_T\cdot\mathbf{p}_T)^2 f(x_1, \mathbf{k}_T^2) f(x_2, \mathbf{p}_T^2)\\ &= \frac{2}{M^2}\int d^2\mathbf{k}_T d^2\mathbf{p}_T (\mathbf{k}_{T1}^2\mathbf{p}_{T1}^2 + \mathbf{k}_{T2}^2\mathbf{p}_{T2}^2) f(x_1, \mathbf{k}_T^2) f(x_2, \mathbf{p}_T^2)\\ &= 4M^2 f^{(1)}(x_1) f^{(1)}(x_2), \end{aligned} \tag{A1}$$

$$\begin{aligned}&\int d^2\mathbf{k}_T d^2\mathbf{p}_T \delta^2(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T)\frac{Q^2}{M^2}\hat{\mathbf{h}}\cdot\mathbf{k}_T\hat{\mathbf{h}}\cdot\mathbf{p}_T f(x_1, \mathbf{k}_T^2) f(x_2, \mathbf{p}_T^2)\\ &= \frac{1}{M^2}\int d^2\mathbf{k}_T d^2\mathbf{p}_T (\mathbf{k}_T + \mathbf{p}_T)\cdot\mathbf{k}_T(\mathbf{k}_T + \mathbf{p}_T)\cdot\mathbf{p}_T f(x_1, \mathbf{k}_T^2) f(x_2, \mathbf{p}_T^2)\\ &= \frac{1}{M^2}\int d^2\mathbf{k}_T d^2\mathbf{p}_T (\mathbf{p}_T^2\mathbf{k}_T^2 + (\mathbf{k}_T\cdot\mathbf{p}_T)^2) f(x_1, \mathbf{k}_T^2) f(x_2, \mathbf{p}_T^2) = 6M^2 f^{(1)}(x_1) f^{(1)}(x_2). \end{aligned} \tag{A2}$$

In the above integrals, the terms containing odd numbers of $\mathbf{k}_T^i$ or $\mathbf{p}_T^i$ vanish after being integrated over $\mathbf{k}_T$ or $\mathbf{p}_T$.

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