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Adaptive function project synchronization of Rössler hyperchaotic system with uncertain parameters

Luo Runzi

Department of Mathematics, Nanchang University, 330031, PR China

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Abstract

This Letter addresses the function project synchronization problem of two Rössler hyperchaotic in the presence of unknown system parameters. Based on Lyapunov stability theory an adaptive control law is proposed to make the states of two identical Rössler hyperchaotic systems asymptotically synchronized. Numerical simulations are presented to show the effectiveness of the proposed schemes.

Keywords: Function project synchronization; Adaptive control; Rössler hyperchaotic system; Lyapunov method

1. Introduction

Chaos synchronization is an important subject both theoretically and practically and has been intensively studied in the last three decades. Since Pecora and Carrol [1] introduced a method to synchronize two identical chaotic systems with different initial conditions, a variety of method and techniques have been proposed for the synchronization of chaotic systems such as drive-response synchronization [1], linear and nonlinear feedback synchronization [2,3], coupled synchronization [4], impulsive synchronization [5–7], adaptive synchronization [8,9], phase synchronization [10,11], generalized synchronization (GS) [12–14], etc. In [12], Li considered a new GS method, called modified projective synchronization (MPS), where the responses of the synchronized dynamical states synchronize up to a constant scaling matrix. Recently, Park [15,16] proposed an AMPS scheme, named adaptive modified projective synchronization, to acquire a general kind of proportional relationship between the drive and response systems with uncertain parameters. More recently, Yong Chen et al. [17] extended the modified projective synchronization and raised a new projective synchronization, called function projective synchronization, where the responses of the synchronized dynamical states synchronize up to a scaling function factor.

However, most of the existing synchronization methods mainly concern the synchronization of chaotic systems with low dimensional attractor, characterized by one positive Lyapunov exponent. Recently, owing to they possesses at least two positive Lyapunov exponents and has more complex behavior than chaotic system, hyperchaotic system are more suitable for some special engineering application such as secure communication and have received much attention, particularly the hyperchaotic Rössler attractors and its variation [18–20].

The object of this Letter is to study function project synchronization of two identical Rössler hyperchaotic system with unknown parameters. A novel parameters identification and synchronization method is derived for Rössler hyperchaotic system with all the system parameters unknown based upon adaptive control. By this method, one can achieve hyperchaotic synchronization and identify the unknown parameters simultaneously.

E-mail address: luo_rz@163.com.

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The organization of this Letter is as follows. In Section 2, the problem statement and master-slave synchronization scheme are presented for Rössler hyperchaotic. In Section 3, a numerical example is given to demonstrate the effectiveness of the proposed method. Finally some concluding remarks are given.

2. Function projective synchronization of the Rössler hyperchaotic system

The function projective synchronization [17] is illustrated like this. Consider the following chaotic systems:

$$\dot{U} = f(U), \tag{1}$$

$$\dot{V} = g\bigl(V, \xi(U, V)\bigr), \tag{2}$$

where $U = (u_1(t), u_2(t), \ldots, u_n(t))^T$, $V = (v_1(t), v_2(t), \ldots, v_n(t))^T$, and $\xi(U, V) = (\xi_1(U, V), \xi_2(U, V), \ldots, \xi_n(U, V))^T$, which is the controller to be determined later and satisfies $\xi(0, 0) = 0$, $g(V, \xi(0, 0)) = f(V)$. The letters U and V stand for the drive (or master) and response (or slave) systems, respectively. If there exists a function $Q(U) = (Q_1(u_1(t)), Q_2(u_2(t)), \ldots, Q_n(u_n(t)))^T$ satisfying $e = V - Q(U)U^T$, $\lim_{t\to\infty} \|e\| = 0$, then systems (1) and (2) achieve function projective synchronization, where $e = (e_1, e_2, \ldots, e_n)^T$, and we call Q a "scaling function factor".

Rössler hyperchaotic system was provided by Rössler in describing dynamics of some hypothetical chemical reaction and is a first example of hyperchaotic system with two positive Lyapunov exponents. The nonlinear differential equations that describe Rössler hyperchaotic system are

$$\begin{cases} \dot{x} = -y - z, \\ \dot{y} = x + ay + w, \\ \dot{z} = b + xz, \\ \dot{w} = -cz + dw, \end{cases} \tag{3}$$

where a, b, c, d are real constants and x, y, z, w are state variables. Rössler hyperchaotic system has a hyperchaotic attractor when $a = 0.25$, $b = 3$, $c = 0.5$, $d = 0.05$ [21].

We assume that we have two Rössler hyperchaotic systems where the master system with the subscript m drives the slave system having identical equations denoted by the subscript s. For the systems (3), the master (or drive) and slave (or response) systems are defined below, respectively,

$$\begin{cases} \dot{x}_m = -y_m - z_m, \\ \dot{y}_m = x_m + ay_m + w_m, \\ \dot{z}_m = b + x_m z_m, \\ \dot{w}_m = -cz_m + dw_m, \end{cases} \tag{4}$$

and

$$\begin{cases} \dot{x}_s = -y_s - z_s + u_1, \\ \dot{y}_s = x_s + a_1 y_s + w_s + u_2, \\ \dot{z}_s = b_1 + x_s z_s + u_3, \\ \dot{w}_s = -c_1 z_s + d_1 w_s + u_4, \end{cases} \tag{5}$$

where a_1, b_1, c_1 and d_1 are parameters of the slave system which needs to be estimated, and u_1, u_2, u_3 and u_4 are the nonlinear controller such that two hyperchaotic systems can be synchronized in the sense that

$$\begin{cases} \lim_{t\to\infty} \|x_s - f_1(x_m)x_m\| = 0, \\ \lim_{t\to\infty} \|y_s - f_2(y_m)y_m\| = 0, \\ \lim_{t\to\infty} \|z_s - f_3(z_m)z_m\| = 0, \\ \lim_{t\to\infty} \|w_s - f_4(w_m)w_m\| = 0. \end{cases} \tag{6}$$

In our synchronization scheme we assume $f_1(x_m) = \alpha_{11}x_m + \alpha_{12}$, $f_2(y_m) = \alpha_{21}y_m + \alpha_{22}$, $f_3(z_m) = \alpha_{31}z_m + \alpha_{32}$, $f_4(w_m) = \alpha_{41}w_m + \alpha_{42}$, where $\alpha_{i1}\alpha_{i2} \neq 0$ $(i = 1, 2, 3, 4)$.

By subtracting Eq. (4) from Eq. (5) we have

$$\begin{aligned} \dot{e}_1 &= \dot{x}_s - (\alpha_{11}\dot{x}_m)x_m - (\alpha_{11}x_m + \alpha_{12})\dot{x}_m \\ &= -y_s - z_s + u_1 - 2\alpha_{11}(-y_m - z_m)x_m - \alpha_{12}(-y_m - z_m), \\ \dot{e}_2 &= \dot{y}_s - (\alpha_{21}\dot{y}_m)y_m - (\alpha_{21}y_m + \alpha_{22})\dot{y}_m = \dot{y}_s - 2\alpha_{21}\dot{y}_m y_m - \alpha_{22}\dot{y}_m \\ &= x_s + a_1 y_s + w_s + u_2 - 2\alpha_{21}(x_m + ay_m + w_m)y_m - \alpha_{22}(x_m + ay_m + w_m), \end{aligned}$$

$$\begin{aligned}\dot e_3 &= \dot z_s - (\alpha_{31}\dot z_m)z_m - (\alpha_{31}z_m + \alpha_{32})\dot z_m = \dot z_s - 2\alpha_{31}\dot z_m z_m - \alpha_{32}\dot z_m\\ &= b_1 + x_s z_s + u_3 - 2\alpha_{31}(b + x_m z_m)z_m - \alpha_{32}(b + x_m z_m),\\ \dot e_4 &= \dot w_s - (\alpha_{41}\dot w_m)w_m - (\alpha_{41}w_m + \alpha_{42})\dot w_m = \dot w_s - 2\alpha_{41}\dot w_m w_m - \alpha_{42}\dot w_m\\ &= -c_1 z_s + d_1 w_s + u_4 - 2\alpha_{41}(-cz_m + dw_m)w_m - \alpha_{42}(-cz_m + dw_m),\end{aligned}$$

where

$$\begin{cases} e_1 = x_s - (\alpha_{11}x_m + \alpha_{12})x_m,\\ e_2 = y_s - (\alpha_{21}y_m + \alpha_{22})y_m,\\ e_3 = z_s - (\alpha_{31}z_m + \alpha_{32})z_m,\\ e_4 = w_s - (\alpha_{41}w_m + \alpha_{42})w_m. \end{cases} \tag{7}$$

Thus we have the error dynamical system between Eqs. (4) and (5)

$$\begin{cases} \dot e_1 = -y_s - z_s + u_1 - 2\alpha_{11}(-y_m - z_m)x_m - \alpha_{12}(-y_m - z_m),\\ \dot e_2 = x_s + a_1 y_s + w_s + u_2 - 2\alpha_{21}(x_m + ay_m + w_m)y_m - \alpha_{22}(x_m + ay_m + w_m),\\ \dot e_3 = b_1 + x_s z_s + u_3 - 2\alpha_{31}(b + x_m z_m)z_m - \alpha_{32}(b + x_m z_m),\\ \dot e_4 = -c_1 z_s + d_1 w_s + u_4 - 2\alpha_{41}(-cz_m + dw_m)w_m - \alpha_{42}(-cz_m + dw_m). \end{cases} \tag{8}$$

Our aim is to find control laws u_i $(i = 1, 2, 3)$ for stabilizing the error variables of system (8) at the origin. For this end, we propose the following control law and update rule for system (5):

$$\begin{cases} u_1 = y_s + z_s - 2\alpha_{11}(y_m + z_m)x_m - \alpha_{12}(y_m + z_m) - k_1 e_1,\\ u_2 = -x_s - w_s + 2\alpha_{21}(x_m + w_m)y_m + \alpha_{22}(x_m + w_m) - a_1 y_s + a_1(2\alpha_{21}y_m^2 + \alpha_{22}y_m) - k_2 e_2,\\ u_3 = -b_1 - x_s z_s + 2\alpha_{31}x_m z_m^2 + \alpha_{32}x_m z_m + b_1(2\alpha_{31}z_m + \alpha_{32}) - k_3 e_3,\\ u_4 = c_1 z_s - d_1 w_s - c_1(2\alpha_{41}z_m w_m + \alpha_{42}z_m) + d_1(2\alpha_{41}w_m^2 + \alpha_{42}w_m) - k_4 e_4, \end{cases} \tag{9}$$

and the update rule for four unknown parameters a_1, b_1, c_1, d_1

$$\begin{cases} \dot a_1 = -(2\alpha_{21}y_m^2 + \alpha_{22}y_m)e_2 - k_5(a_1 - a),\\ \dot b_1 = (2\alpha_{31}z_m + \alpha_{32})e_3 - k_6(b_1 - b),\\ \dot c_1 = (2\alpha_{41}z_m w_m + \alpha_{42}z_m)e_4 - k_7(c_1 - c),\\ \dot d_1 = (-2\alpha_{41}w_m^2 - \alpha_{42}w_m)e_4 - k_8(d_1 - d), \end{cases} \tag{10}$$

where $k_i > 0$ $(i = 1, \ldots, 8)$.

Thus, we can establish the following theorem.

Theorem. *For given nonzero scaling function factors f_i $(i = 1, 2, 3, 4)$, the function projective synchronization between drive systems (4) and response system (5) will occur by the control law (9) and the update rule (10) and satisfy* $\lim_{t\to+\infty}(a_1 - a) = \lim_{t\to+\infty}(b_1 - b) = \lim_{t\to+\infty}(c_1 - c) = \lim_{t\to+\infty}(d_1 - d) = 0$.

Proof. Choose the following Lyapunov function

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2\right), \tag{11}$$

where $e_a = a_1 - a$, $e_b = b_1 - b$, $e_c = c_1 - c$, $e_d = d_1 - d$. The time derivative of V along the trajectory of error system (8) is

$$\begin{aligned}\frac{dV}{dt} &= \dot e_1 e_1 + \dot e_2 e_2 + \dot e_3 e_3 + \dot e_4 e_4 + \dot e_a e_a + \dot e_b e_b + \dot e_c e_c + \dot e_d e_d\\ &= e_1\left[-y_s - z_s + u_1 - 2\alpha_{11}(-y_m - z_m)x_m - \alpha_{12}(-y_m - z_m)\right] + e_2\left[x_s + a_1 y_s + w_s + u_2\right.\\ &\quad \left. - 2\alpha_{21}(x_m + ay_m + w_m)y_m - \alpha_{22}(x_m + ay_m + w_m)\right] + e_3\left[b_1 + x_s z_s + u_3 - 2\alpha_{31}(b + x_m z_m)z_m - \alpha_{32}(b + x_m z_m)\right]\\ &\quad + e_4\left[-c_1 z_s + d_1 w_s + u_4 - 2\alpha_{41}(-cz_m + dw_m)w_m - \alpha_{42}(-cz_m + dw_m)\right]\\ &\quad + \dot a_1(a_1 - a) + \dot b_1(b_1 - b) + \dot c_1(c_1 - c) + \dot d_1(d_1 - d).\end{aligned} \tag{12}$$

By substituting the update rule (10) into Eq. (12), we have

$$\begin{aligned}\frac{dV}{dt} &= \left[-y_s - z_s - 2\alpha_{11}(-y_m - z_m)x_m - \alpha_{12}(-y_m - z_m)\right]e_1 + \left[x_s + w_s - 2\alpha_{21}(x_m + w_m)y_m\right.\\ &\quad \left. - \alpha_{22}(x_m + w_m) + a_1 y_s - a_1\left(2\alpha_{21}y_m^2 + \alpha_{22}y_m\right)\right]e_2 + \left[b_1 + x_s z_s - 2\alpha_{31}x_m z_m^2\right.\end{aligned}$$

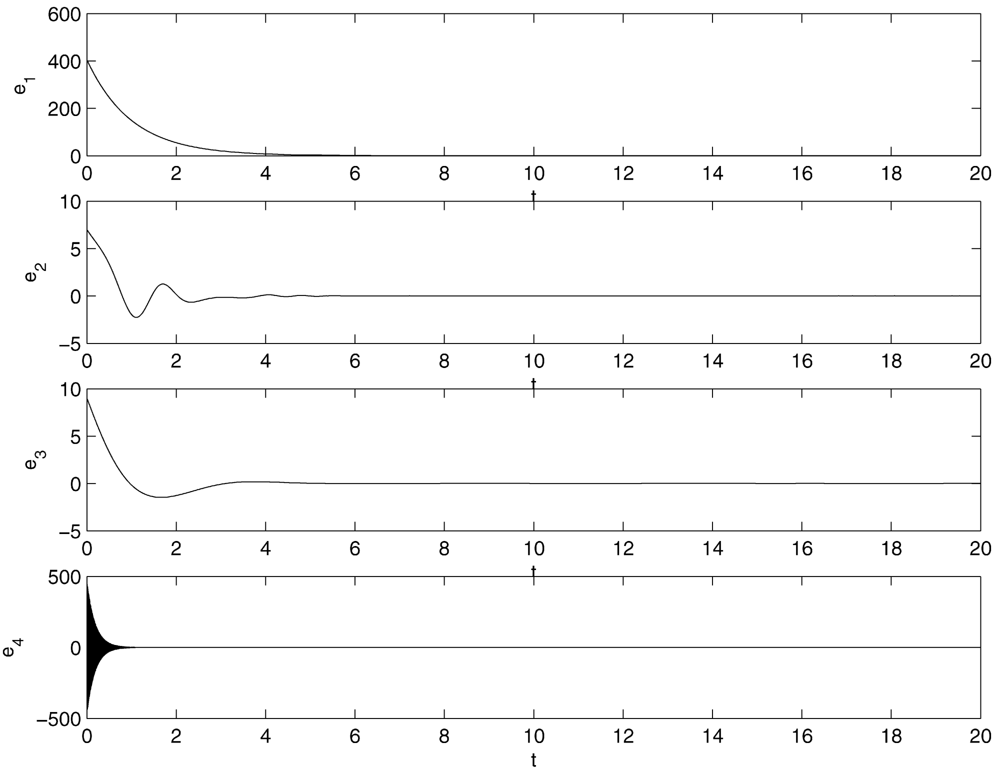

Fig. 1. Error signals between drive and response systems.

$$- \alpha_{32}x_m z_m + b_1(-2\alpha_{31}z_m - \alpha_{32})\big]e_3 + \big[-c_1 z_s + d_1 w_s + c_1(2\alpha_{41}z_m w_m + \alpha_{42}z_m) + d_1\big(-2\alpha_{41}w_m^2 - \alpha_{42}w_m\big)\big]e_4 + u_1e_1 + u_2e_2 + u_3e_3 + u_4e_4 - k_5(a_1-a)^2 - k_6(b_1-b)^2 - k_7(c_1-c)^2 - k_8(d_1-d)^2. \tag{13}$$

Utilizing the control input (9) gives that

$$\frac{dV}{dt} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 - k_5e_a^2 - k_6e_b^2 - k_7e_c^2 - k_8e_d^2 = -e^TKe, \tag{14}$$

where $e = (e_1, e_2, e_3, e_4, e_a, e_b, e_c, e_d)^T$, and $K = \mathrm{diag}(k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8)^T$.

Since $\dot{V} \leqslant 0$, we have $e_1, e_2, e_3, e_4, e_a, e_b, e_c, e_d \to 0$ as $t \to \infty$, i.e., $\lim_{t\to\infty} \|e\| = 0$. This completes the proof. □

Remark. When $f_1 = f_2 = f_3 = f_4 = 1$, $f_1 = f_2 = f_3 = f_4 = \alpha$, $f_4 = \alpha$, $f_1 = \alpha_1$, $f_2 = \alpha_2$, $f_3 = \alpha_3$, $f_4 = \alpha_4$, complete synchronization, project synchronization, modified project synchronization will appear, respectively.

3. Numerical simulation

Numerical simulations results are presented to demonstrate the effectiveness of the proposed synchronization methods. Fourth-order Runge–Kutta method is used to solve the systems of differential equations (4), (5) and (8). In addition, a time step of size 0.001 is employed. The parameters are chosen to be $a = 0.25$, $b = 3$, $c = 0.5$ and $d = 0.05$ in all simulations so that the Rössler hyperchaotic system exhibits a chaotic behavior if no control is applied. The initial states of the drive system are $x_m(0) = -20$, $y_m(0) = 0$, $z_m(0) = 0$ and $w_m(0) = 15$ and initial states of the response system are $x_s(0) = 5$, $y_s(0) = 7$, $z_s(0) = 9$ and $w_s(0) = 11$. Suppose the function factors are $f_1(x_m) = -x_m$, $f_2(y_m) = -1$, $f_3(z_m) = z_m + 1$ and $f_4(w_m) = 2w_m + 2$, then the error system has the initial values $e_1(0) = 405$, $e_2(0) = 7$, $e_3(0) = 9$ and $e_4(0) = -369$. Furthermore, the initial values of estimated parameters are chosen as $a_1(0) = b_1(0) = c_1(0) = d_1(0) = 0$ and the control gains are $(k_1, k_2, k_3, k_4) = (1, 1, 1, 1)$ and $(k_5, k_6, k_7, k_8) = (1, 1, 2, 10)$. Synchronization of systems (4) and (5) via adaptive control law (9) and (10) are shown in Figs. 1 and 2. Fig. 1 displays the synchronization errors between systems (2) and (3). Obviously, they tend to zero after a sufficiently long time. Fig. 2 indicates that the identified parameters $a_1(t)$, $b_1(t)$, $c_1(t)$, $d_1(t)$ approach the desired values: $a = 0.25$, $b = 3$, $c = 0.5$, $d = 0.05$ as $t \to \infty$.

4. Conclusions

This Letter has investigated the function project synchronization problem of two Rössler hyperchaotic system. A novel parameters identification and synchronization method have been proposed for Rössler hyperchaotic system with all the system parameters

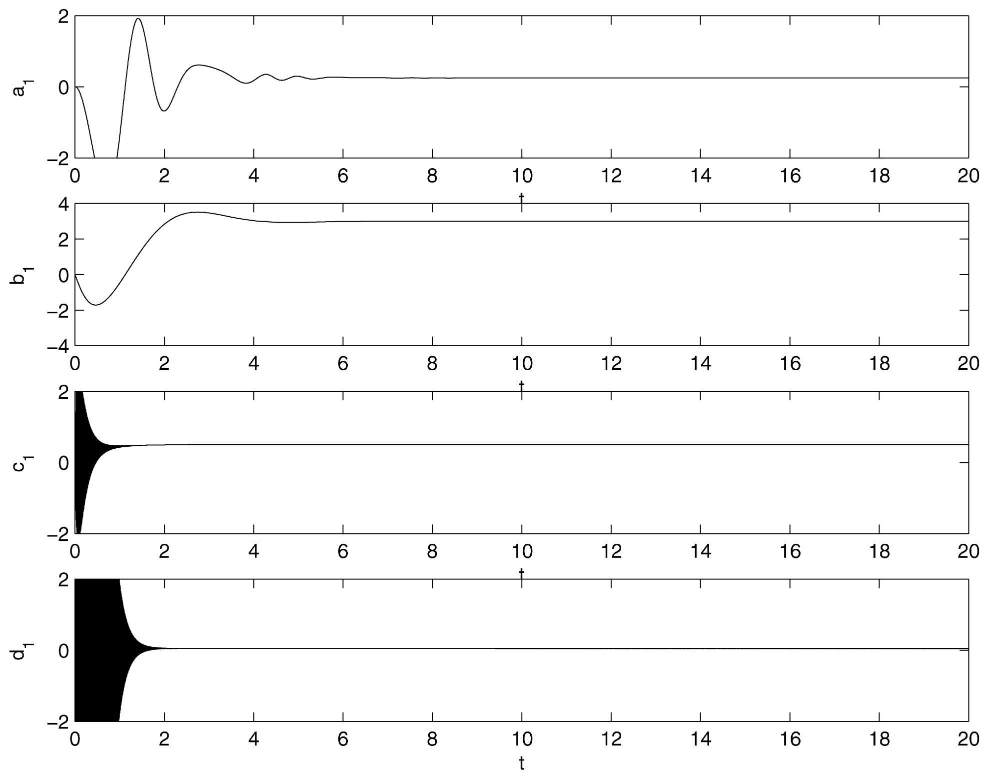

Fig. 2. Estimated values for unknown parameters.

unknown based upon adaptive control. By this method, one can achieve hyperchaotic synchronization and identify the unknown parameters simultaneously. Numerical simulation are used to verify the effectiveness of the proposed chaos synchronization scheme.

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