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New criterion for finite-time stability of linear discrete-time systems with time-varying delay ☆

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Abstract

This paper is concerned with the problem of finite-time stability analysis of linear discrete-time systems with time-varying delay. The time-varying delay has lower and upper bounds. By choosing a novel Lyapunov–Krasovskii-like functional, a new sufficient condition is derived to guarantee that the state of the system with time-varying delay does not exceed a given threshold during a fixed time interval. Then, the corresponding corollary is developed for the case of constant time delay. Numerical examples are provided to demonstrate the effectiveness and merits of the proposed method.
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1. Introduction

Numerous existing literatures mainly concern about the stability analysis and controller design for the dynamical systems over infinite time interval. From the practical point of view, our interests are focused on the behavior of the system over a prescribed time interval in some cases. For instance, in the presence of saturation or controlling the trajectory of a space vehicle from an initial point to a final one in a prescribed time interval. That is, the time interval is fixed, the state of the system does not exceed a certain bound during this time interval. It is called finite-time stability (FTS) [1] or short time stability. Some early results on FTS [2,3] lack the operative test

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conditions. The problem of FTS has been revisited using linear matrix inequality technique, which allows to find feasible conditions guaranteeing FTS. Ref. [4] investigated the finite-time control of linear systems subject to parametric uncertainties and disturbances. Ref. [5] studied the problem of finite-time stabilization via dynamic output feedback. Refs. [6] and [7] addressed the finite-time control of discrete-time linear systems. The problem of finite-time stabilization was developed for nonlinear systems in [8] and [9]. However, time delay is not considered in all the above results .

In some practical systems (such as chemical engineering processing, neural network, inferred grinding model, etc.), time delay is inevitable, and the delay is always time-varying. This inherent feature of the system always causes instability and leads to unsatisfactory performance. Therefore, the study for the stability and stabilization of systems with time-varying delay is of significance. Refs. [10–16] investigated the stability criteria for systems with time-varying delays. Refs. [17–20] developed the stabilization conditions for systems with time-varying delay.

However, most of the existing results concentrate on the asymptotical stability, exponential stability or other problems, not the finite-time stability. In [25], San Filippo and Dorato had given a longitudinal flight control example to illustrate the wide practical use of the theoretic results on finite time stability. Therefore, the research on finite time stability is of great practical significance, which is also the motivation of our study. The study of finite-time stability and stabilization for systems with time-delay has received a lot of attention in recent years [21–23], but the time-delay to be considered is constant, not time-varying. On the other hand, it should be emphasized that the definition of the finite-time stability in [26,27] is different from what we investigate. These two papers used the FTS defined in [28], which requires the state trajectory should converge to the equilibrium in a finite time interval. On the other hand, these results mainly focus on the continuous-time systems, while little consideration has been taken on the discrete-time systems with time-varying delay. Ref. [24] investigated the finite-time control for discrete-time systems with time-varying delay.

The main contribution of this paper is that a new finite-time stability criterion of linear discrete-time system with time-varying delay is presented. We select a novel Lyapunov–Krasovskii-like functional, and present a sufficient condition to guarantee that the state of the system does not exceed a certain bound during a prescribed time interval. For the final part of the Lyapunov–Krasovskii-like functional, we can select different values for the positive definite matrices in this paper. However, there are some structural restrictions for the Lyapunov matrix in [24]. Moreover, some free-weighting matrices [29] are introduced to handle the useful items in the derivation process. It is shown that we can obtain better performance than that in [24] by numerical examples.

The rest of the paper is organized as follows. In Section 2, the considered system is stated, and some preliminaries are provided. In Section 3, by selecting a novel Lyapunov–Krasovskii-like functional, a sufficient condition is presented to guarantee the finite-time stability of linear discrete-time systems with time-varying delay, which is the main result of this paper. Section 4 gives numerical examples to show the advantage of the developed results. Finally, in Section 5, some conclusions are drawn.

Notations. R^n and $R^{n\times m}$ denote the n-dimensional Euclidean space and the set of $n\times m$ real matrices. $\lambda_{max}(\cdot)$ and $\lambda_{min}(\cdot)$ are the maximum and the minimum eigenvalues, respectively. N^+ represents the set of positive integers. In addition, in symmetric block matrices, we use $*$ as an ellipsis for the term that is induced by symmetry. $diag\{\ldots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. The notation $P>0$ (≥ 0) means P is symmetric and positive definite (positive semi-definite). I and 0 represent identity matrix and zero matrix, respectively.

2. Problem formulation and preliminaries

Consider the following linear discrete-time systems with time-varying delay:

$$\begin{aligned} &x(k+1)=Ax(k)+Dx(k-d(k)) \\ &\quad x(k)\equiv\phi(k), \quad \forall k\in[-d_M,0] \end{aligned} \tag{1}$$

where $x(k)\in R^n$ is the state vector. The time delay is assumed to be time-varying and has lower and upper bounds such that $0<d_m\le d(k)\le d_M$. A, D are constant matrices with appropriate dimensions. Define that $y(k)=x(k+1)-x(k)$, which satisfies $y^T(k)y(k)\le\delta$, for $k\in[-d_M,-1]$.

To study the finite-time stability of the linear discrete-time system (1), the following definition is necessarily introduced.

Definition 1. The linear discrete time-delay system (1) is said to be finite-time stable (FTS) with respect to (c_1,c_2,R,N), where R is a positive definite matrix, and $N\in N^+$, if

$$x^T(k^*)Rx(k^*)\le c_1, \quad \forall k^*\in[-d_M,0] \Rightarrow x^T(k)Rx(k)\le c_2, \quad \forall\ k\in[1,N]$$

Remark 1. The definition of FTS for time delay systems is quite different from that for systems without time delay, which requires all the initial states $x(k)\equiv\phi(k)$, $\forall\ k\in[-d_M,0]$ satisfying $x^T(k)Rx(k)\le c_1$. Furthermore, it is of interest to minimize the trajectory bound c_2 (or maximize the finite-time interval N). The smaller the c_2 is (or the bigger the N is), the better performance the system has.

3. Finite-time stability analysis

In this section, we will develop the finite-time stability criterion for system (1).

Theorem 1. *System (1) is FTS with respect to (c_1, c_2, R, N), if for scalar $\gamma>1$, there exist symmetric positive definite matrices P, Q_1, Q_2, R_1, $R_2\in R^{n\times n}$, U, $S\in R^{2n\times 2n}$, and matrices L, M, $W\in R^{2n\times n}$ and scalars $\lambda_1>0$, $\lambda_2>0$, $\lambda_3>0$, $\lambda_4>0$, $\lambda_5>0$, $\lambda_6>0$, such that*

$$\lambda_1 I\le\tilde{P}\le\lambda_2 I \tag{2}$$

$$0<\tilde{Q}_1\le\lambda_3 I, \quad 0<\tilde{Q}_2\le\lambda_4 I \tag{3}$$

$$0<R_1\le\lambda_5 I, \quad 0<R_2\le\lambda_6 I \tag{4}$$

$$\begin{aligned} &\gamma^N c_1[\lambda_2+\gamma^{d_M-1}d_M\lambda_3+\gamma^{d_m-1}d_m\lambda_4]-c_2\lambda_1 \\ &\quad +\gamma^N\delta\left[\gamma^{d_M-1}\frac{d_M(d_M-1)-d_m(d_m-1)}{2}\lambda_5+\gamma^{d_m-1}\frac{d_m(d_m-1)}{2}\lambda_6\right]<0 \end{aligned} \tag{5}$$

$$\Theta_i>0, \quad i=1,2,3 \tag{6}$$

$$\Omega=\begin{bmatrix} \Omega_{11} & \Omega_{12} & M_1-W_1 & -L_1 \\ * & \Omega_{22} & M_2-W_2 & -L_2 \\ * & * & -\gamma^{d_m}Q_2 & 0 \\ * & * & * & -\gamma^{d_M}Q_1 \end{bmatrix}<0 \tag{7}$$

where

$$\tilde{P}=R^{-1/2}PR^{-1/2}, \quad \tilde{Q}_1=R^{-1/2}Q_1R^{-1/2}, \quad \tilde{Q}_2=R^{-1/2}Q_2R^{-1/2}$$

$$U = \begin{bmatrix} U_{11} & U_{12} \\ * & U_{22} \end{bmatrix}, \quad S = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}, \quad L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}, \quad M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}, \quad W = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}$$

$$\Theta_1 = \begin{bmatrix} U & L \\ * & \gamma^{d_m+1} R_1 \end{bmatrix}, \quad \Theta_2 = \begin{bmatrix} U & M \\ * & \gamma^{d_m+1} R_1 \end{bmatrix}, \quad \Theta_3 = \begin{bmatrix} S & W \\ * & \gamma R_2 \end{bmatrix}$$

$$\Omega_{11} = A^T PA - \gamma P + Q_1 + Q_2 + (A-I)^T O_1 (A-I) + W_1^T + W_1 + (d_M - d_m) U_{11} + d_m S_{11}$$

$$\Omega_{12} = A^T PD + (A-I)^T O_1 D + L_1 - M_1 + W_2^T + (d_M - d_m) U_{12} + d_m S_{12}$$

$$\Omega_{22} = D^T (P + O_1) D + L_2^T + L_2 - M_2^T - M_2 + (d_M - d_m) U_{22} + d_m S_{22}$$

$$O_1 = (d_M - d_m) R_1 + d_m R_2$$

Proof. By defining $y(k) = x(k+1) - x(k)$, we get $y(k) = (A-I)x(k) + Dx(k-d(k))$. Let us select the following Lyapunov–Krasovskii-like functional

$$V(k) = V_1(k) + V_2(k) + V_3(k)$$

where

$$V_1(k) = x^T(k) P x(k)$$

$$V_2(k) = \sum_{s=k-d_M}^{k-1} \gamma^{k-1-s} x^T(s) Q_1 x(s) + \sum_{s=k-d_m}^{k-1} \gamma^{k-1-s} x^T(s) Q_2 x(s)$$

$$V_3(k) = \sum_{s=-d_M}^{-d_m-1} \sum_{v=k+s}^{k-1} \gamma^{k-1-v} y^T(v) R_1 y(v) + \sum_{s=-d_m}^{-1} \sum_{v=k+s}^{k-1} \gamma^{k-1-v} y^T(v) R_2 y(v)$$

Denote

$$\xi^T(k) = [x^T(k) \;\; x^T(k-d(k))]$$

$$\zeta^T(k,s) = [x^T(k) \;\; x^T(k-d(k)) \;\; y^T(s)]$$

$$\eta^T(k) = [x^T(k) \;\; x^T(k-d(k)) \;\; x^T(k-d_m) \;\; x^T(k-d_M)]$$

Let $\Delta V(k)$ be the difference operator. By taking the difference of $V(k)$ and using $\gamma > 1$, we can obtain

$$\begin{aligned} \Delta V(k) - (\gamma-1)V(k) = {} & x^T(k)[A^T PA - \gamma P + Q_1 + Q_2 + (A-I)^T O_1 (A-I)]x(k) \\ & + 2x^T(k)[A^T PD + (A-I)^T O_1 D]x(k-d(k)) \\ & + x^T(k-d(k)) D^T (P + O_1) D x(k-d(k)) \\ & - \gamma^{d_M} x^T(k-d_M) Q_1 x(k-d_M) - \gamma^{d_m} x^T(k-d_m) Q_2 x(k-d_m) \\ & - \sum_{s=-d_M}^{-d_m-1} \gamma^{-s} y^T(k+s) R_1 y(k+s) - \sum_{s=-d_m}^{-1} \gamma^{-s} y^T(k+s) R_2 y(k+s) \end{aligned} \tag{8}$$

$$\begin{aligned} < {} & x^T(k)[A^T PA - \gamma P + Q_1 + Q_2 + (A-I)^T O_1 (A-I)]x(k) \\ & + 2x^T(k)[A^T PD + (A-I)^T O_1 D]x(k-d(k)) \end{aligned}$$

$$
\begin{aligned}
&+x^T(k-d(k))D^T(P+Q_1)Dx(k-d(k))\\
&-\gamma^{d_M}x^T(k-d_M)Q_1x(k-d_M)-\gamma^{d_m}x^T(k-d_m)Q_2x(k-d_m)\\
&-\gamma^{d_m+1}\sum_{s=k-d_M}^{k-d_m-1}y^T(s)R_1y(s)-\gamma\sum_{s=k-d_m}^{k-1}y^T(s)R_2y(s)\\
&+2\xi^T(k)L\left[x(k-d(k))-x(k-d_M)-\sum_{s=k-d_M}^{k-d(k)-1}y(s)\right]\\
&+2\xi^T(k)M\left[x(k-d_m)-x(k-d(k))-\sum_{s=k-d(k)}^{k-d_m-1}y(s)\right]\\
&+2\xi^T(k)W\left[x(k)-x(k-d_m)-\sum_{s=k-d_m}^{k-1}y(s)\right] \qquad (9)
\end{aligned}
$$

Since

$$d_m\xi^T(k)S\xi(k)-\sum_{s=k-d_m}^{k-1}\xi^T(k)S\xi(k)=0 \qquad (10)$$

$$(d_M-d_m)\xi^T(k)U\xi(k)-\sum_{s=k-d_M}^{k-d(k)-1}\xi^T(k)U\xi(k)-\sum_{s=k-d(k)}^{k-d_m-1}\xi^T(k)U\xi(k)=0 \qquad (11)$$

we also have

$$\sum_{s=k-d_M}^{k-d_m-1}y^T(s)R_1y(s)=\sum_{s=k-d_M}^{k-d(k)-1}y^T(s)R_1y(s)+\sum_{s=k-d(k)}^{k-d_m-1}y^T(s)R_1y(s)$$

Adding the above Eqs. (10)–(11) to Eq. (9) and using Schur complement, we get

$$
\begin{aligned}
\Delta V(k)-(\gamma-1)V(k)<\eta^T(k)\Omega\eta(k)-\sum_{s=k-d_M}^{k-d(k)-1}\zeta^T(k,s)\Theta_1\zeta(k,s)\\
-\sum_{s=k-d(k)}^{k-d_m-1}\zeta^T(k,s)\Theta_2\zeta(k,s)-\sum_{s=k-d_m}^{k-1}\zeta^T(k,s)\Theta_3\zeta(k,s) \qquad (12)
\end{aligned}
$$

It is easy to see that Eqs. (6) and (7) ensure

$$\Delta V(k)<(\gamma-1)V(k) \qquad (13)$$

Note that condition (13) can be rewritten as

$$V(k)-V(k-1)<(\gamma-1)V(k-1)$$

$$V(k)<\gamma V(k-1)$$

By iteration, it follows that:

$$V(k)<\gamma^kV(0)$$

On the other hand, as $x(k)\equiv\phi(k)$, $\forall\ k\in[-d_M,0]$, then

$$V(0)=x^T(0)Px(0)+\sum_{s=-d_M}^{-1}\gamma^{-1-s}x^T(s)Q_1x(s)+\sum_{s=-d_m}^{-1}\gamma^{-1-s}x^T(s)Q_2x(s)$$

$$
\begin{aligned}
&+\sum_{s=-d_M}^{-d_m-1}\sum_{v=s}^{-1}\gamma^{-1-v}y^T(v)R_1y(v)+\sum_{s=-d_m}^{-1}\sum_{v=s}^{-1}\gamma^{-1-v}y^T(v)R_2y(v)\\
&\le\lambda_{max}(\tilde{P})x^T(0)Rx(0)\\
&\quad+\gamma^{d_M-1}\lambda_{max}(\tilde{Q}_1)\sum_{s=-d_M}^{-1}x^T(s)Rx(s)+\gamma^{d_m-1}\lambda_{max}(\tilde{Q}_2)\sum_{s=-d_m}^{-1}x^T(s)Rx(s)\\
&\quad+\gamma^{d_M-1}\lambda_{max}(R_1)\sum_{s=-d_M}^{d_m-1}\sum_{v=s}^{-1}y^T(v)y(v)+\gamma^{d_m-1}\lambda_{max}(R_2)\sum_{s=-d_m}^{-1}\sum_{v=s}^{-1}y^T(v)y(v)\\
&\le[\lambda_{max}(\tilde{P})+\gamma^{d_M-1}d_M\lambda_{max}(\tilde{Q}_1)+\gamma^{d_m-1}d_m\lambda_{max}(\tilde{Q}_2)]c_1\\
&\quad+\left[\gamma^{d_M-1}\lambda_{max}(R_1)\frac{d_M(d_M-1)-d_m(d_m-1)}{2}+\gamma^{d_m-1}\lambda_{max}(R_2)\frac{d_m(d_m-1)}{2}\right]\delta
\end{aligned}
$$

Note that

$$V(k)\ge\lambda_{min}(\tilde{P})x^T(k)Rx(k)$$

By Eqs. (2)–(5) and $\phi^T(k)R\phi(k)\le c_1$, we get

$$x^T(k)Rx(k)\le\gamma^k\frac{\Xi}{\lambda_{min}(\tilde{P})}\le c_2 \tag{14}$$

where

$$
\begin{aligned}
\Xi=&[\lambda_{max}(\tilde{P})+\gamma^{d_M-1}d_M\lambda_{max}(\tilde{Q}_1)+\gamma^{d_m-1}d_m\lambda_{max}(\tilde{Q}_2)]c_1\\
&+\left[\gamma^{d_M-1}\lambda_{max}(R_1)\frac{d_M(d_M-1)-d_m(d_m-1)}{2}+\gamma^{d_m-1}\lambda_{max}(R_2)\frac{d_m(d_m-1)}{2}\right]\delta
\end{aligned}
$$

Now, we can conclude that conditions (2)–(7) guarantee system (1) with time-varying delay is finite-time stable.

This completes the proof of Theorem 1. □

Remark 2. In the proof of Theorem 1, we choose a new Lyapunov–Krasovskii-like functional involving variable ratios $\gamma^{k-1-s}(\gamma^{k-1-v})$, which is different from that in [24]. By doing so, no inequality enlargement is required to obtain $\Delta V(k)<(\gamma-1)V(k)$. In Ref. [24], however, a traditional Lyapunov–Krasovskii candidate was constructed. As a result, ΔV is enlarged by $\Delta V(k)<(\gamma-1)x^T(k)Px(k)=(\gamma-1)V_1(k)<(\gamma-1)V(k)$ (see Eq. (17) in Ref. [24]). It demonstrates that our method contains more information of the system states and yields less conservatism.

Remark 3. If we ignore $-\gamma^{d_m+1}\sum_{s=k-d_M}^{k-d_m-1}y^T(s)R_1y(s)$ and $-\gamma\sum_{s=k-d_m}^{k-1}y^T(s)R_2y(s)$, the final two negative terms of Eq. (8), conservatism is inevitable. Here we introduce Eqs. (10) and (11) as well as the free-weighting matrices U, S, L, M, W in Theorem 1 to avoid such a treatment.

Remark 4. In Theorem 1, for the given N, γ, d_M and d_m, if c_1 is fixed, c_2 can be viewed as an optimization parameter. We can give the following optimization algorithm to get the minimal value of c_2:

$$\min_{\substack{P,Q_1,Q_2,R_1,R_2,U,S,\\ L,M,W,\lambda_i(i=1,\ldots,6)}} c_2$$

$$\text{s.t.} \quad \textit{Inequalities } (2)\text{–}(7) \tag{15}$$

Now, we consider the case where the time delay is constant.

Corollary 1. *System (1) with constant time delay d is FTS with respect to (c_1, c_2, R, N), if for scalar $\gamma > 1$, there exist symmetric positive definite matrices P, Q_1, $R_1 \in R^{n\times n}$, $S \in R^{2n\times 2n}$, and matrices $W \in R^{2n\times n}$ and scalars $\sigma_1 > 0$, $\sigma_2 > 0$, $\sigma_3 > 0$, $\sigma_4 > 0$, such that*

$$\sigma_1 I \le \tilde{P} \le \sigma_2 I \tag{16}$$

$$0 < \tilde{Q}_1 \le \sigma_3 I \tag{17}$$

$$0 < R_1 \le \sigma_4 I \tag{18}$$

$$\gamma^N c_1 \sigma_2 + \gamma^{N+d-1} c_1 d\sigma_3 + \frac{d(d-1)}{2}\gamma^{N+d-1} c_1 \delta\sigma_4 - c_2\sigma_1 < 0 \tag{19}$$

$$\Theta > 0 \tag{20}$$

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ * & \Omega_{22} \end{bmatrix} < 0 \tag{21}$$

where

$$\tilde{P} = R^{-1/2} P R^{-1/2}, \quad \tilde{Q}_1 = R^{-1/2} Q_1 R^{-1/2}$$

$$S = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}, \quad W = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}, \quad \Theta = \begin{bmatrix} S & W \\ * & \gamma R_1 \end{bmatrix}$$

$$\Omega_{11} = A^T P A - \gamma P + Q_1 + (A-I)^T d R_1 (A-I) + W_1^T + W_1 + dS_{11}$$

$$\Omega_{12} = A^T P D + (A-I)^T d R_1 D - W_1 + W_2^T + dS_{12}$$

$$\Omega_{22} = D^T (P + dR_1) D - \gamma^d Q_1 - W_2^T - W_2 + dS_{22}$$

Proof. In this case, we select $V(k)$ as follows:

$$V(k) = x^T(k) P x(k) + \sum_{s=k-d}^{k-1} \gamma^{k-1-s} x^T(s) Q_1 x(s) + \sum_{s=-d}^{-1} \sum_{v=k+s}^{k-1} \gamma^{k-1-v} y^T(v) R_1 y(v)$$

The remaining part of the proof is similar to that of Theorem 1, thus omitted. □

Remark 5. In Corollary 1, for the given N, γ and d, if c_1 is fixed, c_2 can be viewed as an optimization parameter. Similarly, we can get the minimal value of c_2 by the following:

$$\min_{\substack{P, Q_1, R_1, R_2, S, \\ W, \sigma_i (i=1,\ldots,4)}} c_2$$

$$\text{s.t.} \quad \textit{Inequalities } (16)\text{–}(21) \tag{22}$$

4. Numerical examples

In this section, numerical examples are provided to show the effectiveness of the method developed in this paper.

Example 1. Consider the system (1) with the following data:

$$A = \begin{bmatrix} 0.8 & 0 \\ 0.08 & 0.9 \end{bmatrix}, \quad D = \begin{bmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad N = 10, \quad \gamma = 1.2, \quad d_M = 5, \quad d_m = 2, \quad \delta = 1$$

For $c_1 = 1$ and $c_2 = 36$, a feasible solution exists for Theorem 1. The computed results are listed below

$$P = \begin{bmatrix} 496.8662 & -0.6007 \\ -0.6007 & 495.2389 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 36.6533 & 13.0610 \\ 13.0610 & 6.8791 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 34.6372 & -8.1141 \\ -8.1141 & 2.5149 \end{bmatrix}$$

$$R_1 = \begin{bmatrix} 51.9934 & 18.1162 \\ 18.1162 & 14.7738 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 70.3504 & 46.5769 \\ 46.5769 & 39.8650 \end{bmatrix}$$

Fig. 1 depicts several state trajectories starting from different initial points, and all the initial points are in the inner ellipsoid $x^T(k)Rx(k) = c_1$. We can see that the state trajectories will not exceed the

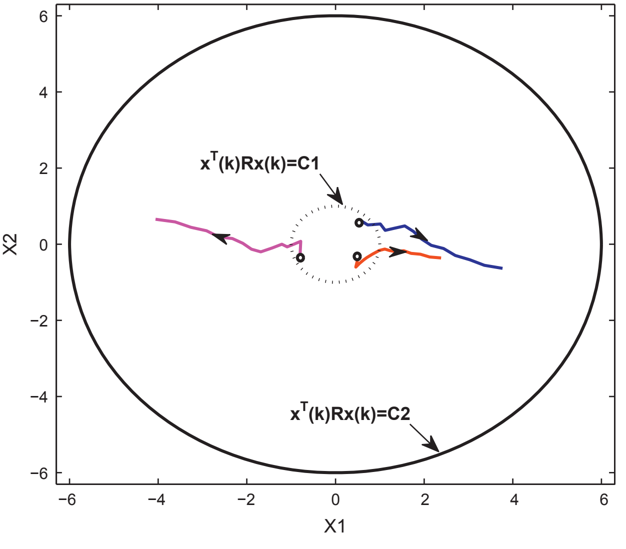

Fig. 1. State trajectories of system for Example 1.

outer bounding ellipsoid $x^T(k)Rx(k)=c_2$ in the fixed time interval N for Example 1. By the given Definition 1, we can say that system (1) with time-varying delay is finite-time stable.

Example 2. Here we show the advantage of Theorem 1 and Corollary 1 in two aspects.

Case (1). Consider system (1) with parameters as follows:

$$A=\begin{bmatrix}0.6 & 0\\ 0.35 & 0.7\end{bmatrix},\quad D=\begin{bmatrix}0.1 & 0\\ 0.2 & 0.1\end{bmatrix}$$

$$R=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad N=50,\quad \gamma=1.0011,\quad d_m=2,\quad \delta=1.1$$

For $c_1=2.1$, $c_2=50$, using Theorem 1 in this paper, we can get the maximum value of the finite-time interval $d_M=8$, which is bigger than $d_M=6$ computed in [24]. The bigger the N is, the better performance the system has. This demonstrates the superiority of our method.

Case (2). Consider system (1) with constant time delay and the following data:

$$A=\begin{bmatrix}0.6 & 0\\ 0.35 & 0.7\end{bmatrix},\quad D=\begin{bmatrix}0.2 & 0.25\\ 0.25 & 0.15\end{bmatrix}$$

We consider Corollary 1 to perform the optimization (22) over c_2 using the algorithm sketch below, with the aid of the Matlab LMI Toolbox:

Step (1) Choose some given fixed values for c_1, N, γ, R, d and δ.

Step (2) Decide an initial value for c_2.

Step (3) Solve the LMIs (16)–(21).

Step (4) If the problem is unfeasible, then the initial value for c_2 is need to be increased. Otherwise, we decrease c_2 until we get its minimum value.

For $c_1=2.1$, $N=10$, $\gamma=1.7258$, $R=I_2$, $d=2$, $\delta=1.1$, we can get the minimal value of the trajectory bound $c_2=674.62$, which is much smaller than $c_2=2425.04$ computed in [24].

Remark 6. The computational complexity of the above algorithm can be estimated from the number of scalar decision variables ν and the number κ of LMI rows. Based on the interior point methods used by LMI Control Toolbox, the complexity of the above algorithm can be estimated as being proportional to $\nu^3\kappa$ [30]. In Case (2) of Example 2, $\nu=6.5n^2+1.5n+4$ and $\kappa=8n+1$ (or for optimization problem (15), $\nu=14.5n^2+2.5n+6$ and $\kappa=18n+1$), where $n=2$ is the dimension of state variable.

Remark 7. Fig. 2 depicts several state trajectories starting from the inner ellipsoid $x^T(k)Rx(k)=c_1$ will not exceed the outer bounding ellipsoid $x^T(k)Rx(k)=c_2$ during a fixed time interval. The solid ellipsoid in Fig. 2 is plotted using the computed minimum value $c_2=674.62$, and the dashed one is $c_2=2425.04$ computed in [24]. The smaller the c_2 is, the better performance the system has. We can directly see that our method has obvious superiority.

Remark 8. Fig. 3 plots the system states during fixed time interval $N=10$ for the trajectory in the first quadrant in Fig. 2. We can see that the system states, which start from the inner bound $\sqrt{c_1}$, do not exceed the outer bound $\sqrt{c_2}$. The system, which is not asymptotically stable, may still be finite time stable.

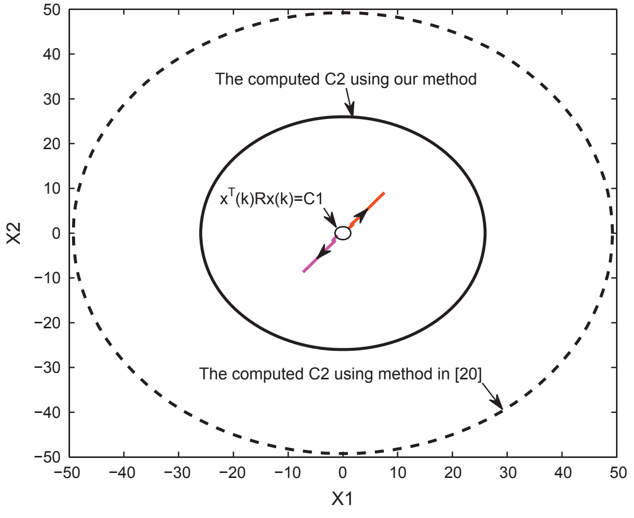

Fig. 2. Computed c_2 by different methods for Case 2 of Example 2.

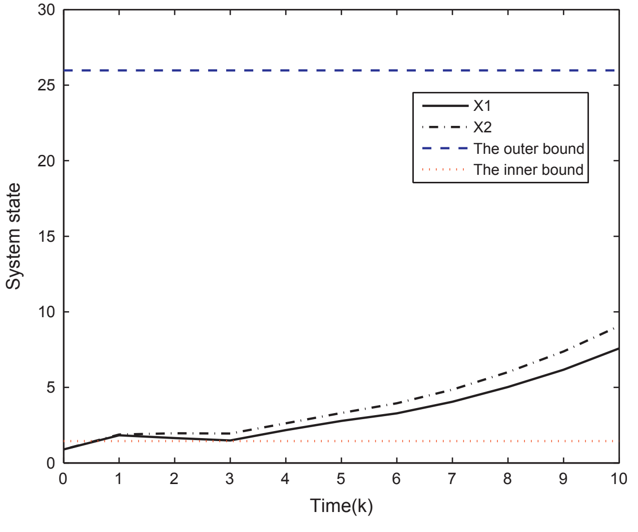

Fig. 3. System states for Case 2 of Example 2.

5. Conclusions

In this paper, a new criterion has been established to ensure the finite-time stability of the discrete-time system with time-varying delay. Based on the above result, we get the corresponding corollary for the case of constant time-delay. It is shown that all these results are given in terms of matrix inequalities. Finally, we have presented two numerical examples to illustrate the effectiveness and the merits of the proposed method.

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