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Dynamic Bayesian estimation of displacement parameters of continuous curve box based on Novozhilov theory *

ZHANG Jian (张剑)[1], YE Jian-shu (叶见曙)[1], ZHAO Xin-ming (赵新铭)[2]

(1. College of Transportation, Southeast University, Nanjing 210096, P. R. China;
2. Civil Engineering Institute, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, P. R. China)

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Abstract The finite strip controlling equation of pinned curve box was deduced on basis of Novozhilov theory and with flexibility method, and the problem of continuous curve box was resolved. Dynamic Bayesian error function of displacement parameters of continuous curve box was found. The corresponding formulas of dynamic Bayesian expectation and variance were derived. After the method of solving the automatic search of step length was put forward, the optimization estimation computing formulas were also obtained by adapting conjugate gradient method. Then the steps of dynamic Bayesian estimation were given in detail. Through analysis of a classic example, the criterion of judging the precision of the known information is gained as well as some other important conclusions about dynamic Bayesian stochastic estimation of displacement parameters of continuous curve box.

Key words displacement parameters, Bayesian estimation, Novozhilov theory, continuous curve box

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Introduction

Box cutting plane has been widely applied in practical engineering for its own merits such as excellent torsion property and slight weight. With the development of civil engineering and bridge engineering and with the advance of construction technology, curve box girder is used more and more widely[1]. As for continuous curve box, it is much complex to complete spatial analysis precisely[2−4]. However, finite curve strip method can satisfy the stability and precision conditions and at the same time the computing parameters which are necessary to be input are few, which makes finite curve strip method become the efficient computational method[5,6]. Though the parameters which are needed to be input are not many, displacement parameters must be input. If not, the structural analysis cannot be carried on. Whereas, it is not easy to grasp displacement parameters exactly. The displacement parameters are often determined by experiments, which cannot take the influence of stochastic factors into account and cannot accurately reflect the factual circumstance. Thus, if displacement parameters can be dynamically estimated on basis of displacement measuring values of different measuring times and spatial spots, it will help to evaluate and forecast performances of the curve box more precisely. In fact, parameter estimation theory has some applications in practical engineering. But the research results focus on parameter static estimation[7−9] and the estimation regularities

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Corresponding author ZHANG Jian, Doctor, E-mail: jianzhang78@126.com

are still needed to be studied. During the current research results, there is no result of dynamical estimation of displacement parameters of continuous curve box appearing.

Thus, this paper is to direct against continuous curve box structure. The combinative method based on finite curve strip method and flexibility method is studied. Dynamic Bayesian error function of displacement parameters of continuous curve box is found and the corresponding formulas of dynamic Bayesian expectation and variance are deduced. After the method of the automatic search of step length is put forward, the optimization estimation computing formulas are also obtained by adapting CG method. Some dynamic Bayesian estimation regularities of continuous curve box structure are gained.

1 Solution of continuous curve box based on finite curve strip-flexibility method

1.1 Novozhilov theory[5]

Novozhilov theory utilizes the different kinds of φ, called conic angle, to depict different relations between strain and displacement (Fig.1). If $\varphi = 0°$, the shell is a pole shell; if $\varphi = 90°$, it is a sector; if $0° < \varphi < 90°$, it is a conic shell whose top part is cut off.

The equation of depicting relation between strain and displacement based on Novozhilov theory is as follows:

$$\begin{cases} \varepsilon_x = \dfrac{\partial u}{\partial x}, \quad \varepsilon_\theta = \dfrac{1}{r}\dfrac{\partial v}{\partial \theta} + \dfrac{w\cos\varphi + u\sin\varphi}{r}, \quad \gamma_{x\theta} = \dfrac{1}{r}\dfrac{\partial u}{\partial \theta} + \dfrac{\partial v}{\partial x} - \dfrac{v\sin\varphi}{r}, \\ \boldsymbol{X}_x = -\dfrac{\partial^2 w}{\partial x^2}, \quad \boldsymbol{X}_\theta = -\dfrac{1}{r^2}\dfrac{\partial^2 w}{\partial \theta^2} + \dfrac{\cos\varphi}{r^2}\dfrac{\partial v}{\partial \theta} - \dfrac{\sin\varphi}{r}\dfrac{\partial w}{\partial x}, \\ \boldsymbol{X}_{x\theta} = 2\left(-\dfrac{1}{r}\dfrac{\partial^2 w}{\partial x\partial\theta} + \dfrac{\sin\varphi}{r^2}\dfrac{\partial w}{\partial \theta} + \dfrac{\cos\varphi}{r}\dfrac{\partial v}{\partial x} - \dfrac{\sin\varphi\cos\varphi}{r^2}v\right). \end{cases} \tag{1}$$

1.2 Finite curve strip controlling equation of pinned curve box

As for pinned curve box structure, displacement interposition function of curve strip element (Fig.2) is

$$f = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{m=1}^{r} \begin{Bmatrix} u_m \\ v_m \\ w_m \end{Bmatrix} = \sum_{m=1}^{r} [N_m]\{\delta_m\}, \tag{2}$$

where u_m, v_m, w_m are the mth items of displacement of x-, y-, z-directions, respectively. $[N_m]$ is the function of $Y_m = \sin K_m\theta$, where $K_m = \frac{m\pi}{\alpha}$ and α is the angle as shown in Fig.1. θ is the angular coordination and b is the width of the strip element.

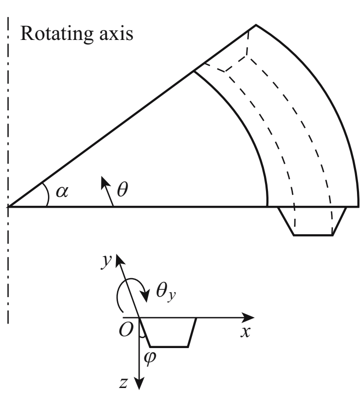

Fig.1 Curve box structure

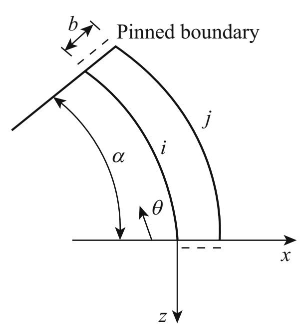

Fig.2 Curve strip element

Based on theory of elasticity, the stiff matrix of curve strip element can be expressed as follows:

$$\begin{aligned}\boldsymbol{K}_{\mathrm{e}} &= \int_V \boldsymbol{B}^{\mathrm{T}}\boldsymbol{D}\boldsymbol{B}dV \\ &= \int_V \begin{bmatrix}\boldsymbol{B}_1 & \boldsymbol{B}_2 & \cdots & \boldsymbol{B}_r\end{bmatrix}^{\mathrm{T}} \boldsymbol{D} \begin{bmatrix}\boldsymbol{B}_1 & \boldsymbol{B}_2 & \cdots & \boldsymbol{B}_r\end{bmatrix} dV \\ &= \int_V \begin{bmatrix}\boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{D}\boldsymbol{B}_1 & \boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{D}\boldsymbol{B}_2 & \cdots & \boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{D}\boldsymbol{B}_r \\ \boldsymbol{B}_2^{\mathrm{T}}\boldsymbol{D}\boldsymbol{B}_1 & \boldsymbol{B}_2^{\mathrm{T}}\boldsymbol{D}\boldsymbol{B}_2 & \cdots & \boldsymbol{B}_2^{\mathrm{T}}\boldsymbol{D}\boldsymbol{B}_r \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{B}_r^{\mathrm{T}}\boldsymbol{D}\boldsymbol{B}_1 & \boldsymbol{B}_r^{\mathrm{T}}\boldsymbol{D}\boldsymbol{B}_2 & \cdots & \boldsymbol{B}_r^{\mathrm{T}}\boldsymbol{D}\boldsymbol{B}_r\end{bmatrix} dV = \begin{bmatrix}\boldsymbol{K}_{\mathrm{e}11} & \boldsymbol{K}_{\mathrm{e}12} & \cdots & \boldsymbol{K}_{\mathrm{e}1r} \\ \boldsymbol{K}_{\mathrm{e}21} & \boldsymbol{K}_{\mathrm{e}22} & \cdots & \boldsymbol{K}_{\mathrm{e}2r} \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{K}_{\mathrm{e}r1} & \boldsymbol{K}_{\mathrm{e}r2} & \cdots & \boldsymbol{K}_{\mathrm{e}rr}\end{bmatrix}, \end{aligned} \tag{3}$$

where $\boldsymbol{D}$ is an elastic matrix and the strain matrix $\boldsymbol{B}$ can be derived by putting Eq.(2) into Novozhilov theory equation (1). In Eq.(3), we have

$$\boldsymbol{K}_{\mathrm{e}mn} = \int_V \boldsymbol{B}_m^{\mathrm{T}}\boldsymbol{D}\boldsymbol{B}_n dV. \tag{4}$$

By using the orthogonal property of the function $Y_m = \sin\left(\frac{m\pi}{\alpha}\theta\right)$, Eq.(3) can be written as follows:

$$\boldsymbol{K}_{\mathrm{e}} = \mathrm{diag}\begin{bmatrix}\boldsymbol{K}_{\mathrm{e}11} & \boldsymbol{K}_{\mathrm{e}22} & \cdots & \boldsymbol{K}_{\mathrm{e}rr}\end{bmatrix}. \tag{5}$$

The mth item of finite curve strip controlling equation of pinned curve box is

$$\boldsymbol{K}_{mm}\boldsymbol{U}_m = \boldsymbol{R}_m, \tag{6}$$

where $\boldsymbol{K}_{mm} = \sum\limits_{\mathrm{e}} \boldsymbol{T}^{\mathrm{T}}\boldsymbol{K}_{\mathrm{e}mm}\boldsymbol{T}$. $\boldsymbol{T}$ is a rotational matrix from total coordination system to local coordination system. $\boldsymbol{U}_m$ and $\boldsymbol{R}_m$ are the mth items of total displacement vector and total load vector, respectively. The pinned curve box displacement $\boldsymbol{U}$ is

$$\boldsymbol{U} = \sum_{m=1}^{r} \boldsymbol{U}_m. \tag{7}$$

1.3 Flexibility method of continuous curve box structure

For the reason that $Y_m = \sin K_m\theta$ in Eq.(2) is unable to depict the middle pedestals, it is necessary to use flexibility method to solve the problem of continuous curve box structure. The structure is diverted into an indeterminate structure with outer surplus forces. Liberating the restrictions of middle pedestals' restrictions, the structure is diverted into pinned curve box structure. Compatibility equation of structural deformation of the nodes at outer restrictions can be written as

$$\boldsymbol{F}\boldsymbol{R} + \boldsymbol{\Delta} = \boldsymbol{0}, \tag{8}$$

where $\boldsymbol{F}$ is the flexibility matrix of continuous curve box, $\boldsymbol{R}$ is the vector of outer surplus forces and $\boldsymbol{\Delta}$ is the displacement vector of the basic system (pinned curve box) which the loads are acting on. Then let $\boldsymbol{R}$ act on the basic system and the displacement field of continuous curve box can be computed by Eqs.(6) and (7).

2 Dynamic Bayesian error function of displacement parameters of continuous curve box

During the process of Bayesian estimation of displacement parameters of continuous curve box, Young's moduli of the top, the middle and the bottom boards are all viewed as random variables, which are written as the random vector $\boldsymbol{E} = [E_1 \quad E_2 \quad E_3]$ to carry on the poly-parameters estimation. It is supposed that displacement parameter $\boldsymbol{E}$ are fitted to Gaussian distribution and both means and variances of $\boldsymbol{E}$ are known. That is, the pre-testing distribution $f(\boldsymbol{E})$ is known. Bayesian parametric estimation is a theory that when the conditional distribution of the measuring displacements $f(\boldsymbol{U}^*|\boldsymbol{E})$ and the pre-testing distribution $f(\boldsymbol{E})$ are given, the statistical properties of displacement parameters are what we need only if the post-testing distribution $f(\boldsymbol{E}|\boldsymbol{U}^*)$ gets its maximum. From Bayesian theory, we have

$$f(\boldsymbol{E}|\boldsymbol{U}^*) = \frac{f(\boldsymbol{U}^*|\boldsymbol{E})f(\boldsymbol{E})}{f(\boldsymbol{U}^*)}, \tag{9}$$

where the expression of the pre-testing distribution $f(\boldsymbol{E})$ is

$$f(\boldsymbol{E}) = (2\pi)^{-\frac{m}{2}}|\boldsymbol{C}_{\boldsymbol{E}}|^{-1}\exp\left[-\frac{1}{2}(\boldsymbol{E}-\boldsymbol{E}_0)^{\mathrm{T}}\boldsymbol{C}_{\boldsymbol{E}}^{-1}(\boldsymbol{E}-\boldsymbol{E}_0)\right], \tag{10}$$

where m is the dimension of displacement parameter $\boldsymbol{E}$ of continuous curve box, and $\boldsymbol{C}_{\boldsymbol{E}}, \boldsymbol{E}_0$ are, respectively, the covariance matrix and expectation vector of displacement parameter $\boldsymbol{E}$.

In practical engineering, the displacements at the measuring nodes must be measured many times and the measuring displacement data $\boldsymbol{U}_i^*$ of each time are all samples of $\boldsymbol{U}^*$. If the routine Bayesian error function[9] is set up to estimate parameters, there is much work repeated. Thus the dynamic Bayesian error function of displacement parameters of continuous curve box is set up. The united density function of $\boldsymbol{U}_i^*$ is $\prod_{i=1}^{n} f(\boldsymbol{U}_i^*|\boldsymbol{E})$, where n is the times of measuring displacements. From maximum likelihood theory, we have

$$L(\boldsymbol{U}^*|\boldsymbol{E}) = f(\boldsymbol{U}^*|\boldsymbol{E}) = (2\pi)^{-\frac{mn}{2}}\prod_{i=1}^{n}|\boldsymbol{C}_{\boldsymbol{U}_i^*}|^{-1}\times\exp\left[-\frac{1}{2}\sum_{i=1}^{n}(\boldsymbol{U}_i^*-\boldsymbol{U}_i)^{\mathrm{T}}\boldsymbol{C}_{\boldsymbol{U}_i^*}^{-1}(\boldsymbol{U}_i^*-\boldsymbol{U}_i)\right]. \tag{11}$$

Putting Eq.(10) and Eq.(11) into Eq.(9), in order to make the post-testing distribution $f(\boldsymbol{E}|\boldsymbol{U}^*)$ get its maximum, we should study

$$J = \sum_{i=1}^{n}(\boldsymbol{U}_i^*-\boldsymbol{U}_i)^{\mathrm{T}}\boldsymbol{C}_{\boldsymbol{U}_i^*}^{-1}(\boldsymbol{U}_i^*-\boldsymbol{U}_i) + (\boldsymbol{E}-\boldsymbol{E}_0)^{\mathrm{T}}\boldsymbol{C}_{\boldsymbol{E}}^{-1}(\boldsymbol{E}-\boldsymbol{E}_0), \tag{12}$$

where J is called the Bayesian error function of displacement parameters of continuous curve box and because n is changeable, the function is called 'dynamic'. The partial differentiation of dynamic error function J to displacement parameter $\boldsymbol{E}$ is

$$\frac{\partial J}{\partial \boldsymbol{E}} = \sum_{i=1}^{n} 2\left(\frac{\partial \boldsymbol{U}_i}{\partial \boldsymbol{E}}\right)^{\mathrm{T}}\boldsymbol{C}_{\boldsymbol{U}_i^*}^{-1}(\boldsymbol{U}_i-\boldsymbol{U}_i^*) + 2\boldsymbol{C}_{\boldsymbol{E}}^{-1}(\boldsymbol{E}-\boldsymbol{E}_0), \tag{13}$$

where $\boldsymbol{U}_i = \boldsymbol{U}_i(\boldsymbol{E})$. Putting $\boldsymbol{U}_i(\boldsymbol{E})$ into the form of Taylor formula at the expectation point $\boldsymbol{E}$ and selecting only the zero order item and the one order item, we have

$$\boldsymbol{U}_i(\boldsymbol{E}) = \boldsymbol{U}_i(\bar{\boldsymbol{E}}) + \boldsymbol{S}_i(\bar{\boldsymbol{E}})(\boldsymbol{E}-\bar{\boldsymbol{E}}), \tag{14}$$

where the sensitivity matrix $\boldsymbol{S}_i(\bar{\boldsymbol{E}}) = \frac{\partial \boldsymbol{U}_i}{\partial \boldsymbol{E}}\big|_{\boldsymbol{E}=\bar{\boldsymbol{E}}}$. Putting Eq.(14) into Eq.(13), we have

$$\frac{\partial J}{\partial \boldsymbol{E}} = \sum_{i=1}^{n} 2\boldsymbol{S}_i^{\mathrm{T}}\boldsymbol{C}_{\boldsymbol{U}_i^*}^{-1}(\bar{\boldsymbol{U}}_i + \boldsymbol{S}_i\boldsymbol{E} - \boldsymbol{S}_i\bar{\boldsymbol{E}} - \boldsymbol{U}_i^*) + 2\boldsymbol{C}_{\boldsymbol{E}}^{-1}(\boldsymbol{E}-\boldsymbol{E}_0), \tag{15}$$

where $\bar{\boldsymbol{U}}_i = \boldsymbol{U}_i(\bar{\boldsymbol{E}})$. When the error function J reaches the minimum value, Eq.(15) equals zero. Then we have

$$\left[\sum_{i=1}^{n} \boldsymbol{S}_i^{\mathrm{T}} \boldsymbol{C}_{\boldsymbol{U}_i^*}^{-1} \boldsymbol{S}_i + \boldsymbol{C}_{\boldsymbol{E}}^{-1}\right] \boldsymbol{E} = \sum_{i=1}^{n} \boldsymbol{S}_i^{\mathrm{T}} \boldsymbol{C}_{\boldsymbol{U}_i^*}^{-1} (\boldsymbol{U}_i^* - \bar{\boldsymbol{U}}_i + \boldsymbol{S}_i \bar{\boldsymbol{E}}) + \boldsymbol{C}_{\boldsymbol{E}}^{-1} \boldsymbol{E}_0. \tag{16}$$

Supposing $\boldsymbol{H} = \sum\limits_{i=1}^{n} \boldsymbol{S}_i^{\mathrm{T}} \boldsymbol{C}_{\boldsymbol{U}_i^*}^{-1} \boldsymbol{S}_i + \boldsymbol{C}_{\boldsymbol{E}}^{-1}$ and $\boldsymbol{M} = \boldsymbol{H}^{-1}[\boldsymbol{S}_1^{\mathrm{T}} \boldsymbol{C}_{\boldsymbol{U}_1^*}^{-1}, \boldsymbol{S}_2^{\mathrm{T}} \boldsymbol{C}_{\boldsymbol{U}_2^*}^{-1}, \cdots, \boldsymbol{S}_n^{\mathrm{T}} \boldsymbol{C}_{\boldsymbol{U}_n^*}^{-1}]$, from Eq.(16) the estimation value $\hat{\boldsymbol{E}}$ of displacement parameter $\boldsymbol{E}$ of continuous curve box can be written as

$$\hat{\boldsymbol{E}} = (\boldsymbol{I} - \boldsymbol{M}\boldsymbol{S})\boldsymbol{E}_0 + \boldsymbol{M}\boldsymbol{U}^* - \boldsymbol{M}(\bar{\boldsymbol{U}} - \boldsymbol{S}\bar{\boldsymbol{E}}), \tag{17}$$

where $\boldsymbol{I}$ is a unit matrix and $\boldsymbol{U}^* = [\boldsymbol{U}_1^*, \boldsymbol{U}_2^*, \cdots, \boldsymbol{U}_n^*]^{\mathrm{T}}$, where $\boldsymbol{U}_i^*$ is the vector of measuring displacement data of the ith time. And $\bar{\boldsymbol{U}} = [\bar{\boldsymbol{U}}_1, \bar{\boldsymbol{U}}_2, \cdots, \bar{\boldsymbol{U}}_n]^{\mathrm{T}}$, where $\bar{\boldsymbol{U}}_i$ is the vector of computing displacement data of the ith time at the expectation point $\bar{\boldsymbol{E}}$. $\boldsymbol{S} = [\boldsymbol{S}_1, \boldsymbol{S}_2, \cdots, \boldsymbol{S}_n]^{\mathrm{T}}$, where $\boldsymbol{S}_i$ is the sensitivity matrix of measuring displacements of the ith time. Supposing the pre-known information $\boldsymbol{E}_0$ of displacement parameter $\boldsymbol{E}$ of continuous curve box has no relation with the measuring displacement data $\boldsymbol{U}^*$, from Eq.(17) the variance of $\hat{\boldsymbol{E}}$ can be written as follows:

$$\boldsymbol{C}_{\hat{\boldsymbol{E}}} = [\boldsymbol{I} - \boldsymbol{M}\boldsymbol{S}]\boldsymbol{C}_{\boldsymbol{E}}[\boldsymbol{I} - \boldsymbol{M}\boldsymbol{S}]^{\mathrm{T}} + \boldsymbol{M}\boldsymbol{C}_{\boldsymbol{U}^*}\boldsymbol{M}^{\mathrm{T}}, \tag{18}$$

where $\boldsymbol{C}_{\boldsymbol{U}^*} = \mathrm{diag}(\boldsymbol{C}_{\boldsymbol{U}_1^*}, \boldsymbol{C}_{\boldsymbol{U}_2^*}, \cdots, \boldsymbol{C}_{\boldsymbol{U}_n^*})$ and $\boldsymbol{C}_{\boldsymbol{U}_i^*}$ is the covariance matrix of measuring displacement data of the ith time. Using the non-singularity property of $\boldsymbol{C}_{\boldsymbol{E}}$ and $\boldsymbol{C}_{\boldsymbol{U}^*}$, Eq.(18) can be written as

$$\boldsymbol{C}_{\hat{\boldsymbol{E}}} = [\boldsymbol{C}_{\boldsymbol{E}}^{-1} + \boldsymbol{S}^{\mathrm{T}}\boldsymbol{C}_{\boldsymbol{U}^*}^{-1}\boldsymbol{S}]^{-1}. \tag{19}$$

3 Optimization method and dynamic Bayesian estimation steps of displacement parameters of continuous curve box

3.1 CG method

The available optimization methods can be roughly classified into two kinds: directly searching method (simplex method, complex method, et al) and gradient analyzing method (CG method, DFP method, et al). An important merit of gradient analyzing method is that when the parameters are estimated, the parameters' sensitivity is simultaneously gained, which makes the calculation of structural reliability easy and convenient. Among gradient analyzing methods, CG method is an efficient optimization method, which incessantly changes the spatial scale (matrix) to engender new searching orientations and optimize the error function efficiently.

3.2 Dynamic Bayesian estimation steps of displacement parameters of continuous curve box

Applying CG method, dynamic Bayesian estimation steps of displacement parameters of continuous curve box are as follows:

(i) Select the initial value $\boldsymbol{E}_0$ of displacement parameter $\boldsymbol{E}$ of continuous curve box and let the iterative variable $i = 0$.

(ii) From Eq.(13), compute both the gradient $\nabla J_i = \boldsymbol{G}_i$ of dynamic Bayesian error function J at $\boldsymbol{E}_i$ and $\|\boldsymbol{G}_i\|_2^2$.

(iii) If $\|\boldsymbol{G}_i\|_2^2 < \varepsilon_1$ or $\|\boldsymbol{E}_i - \boldsymbol{E}_{i-1}\|_1 < \varepsilon_2$, the displacement parameter $\boldsymbol{E}_i$ of continuous curve box satisfies the convergence criterion; otherwise, go on with the next step.

(iv) If $i = 0$, let $\boldsymbol{P}_i = -\boldsymbol{G}_i$; otherwise, let $\boldsymbol{P}_i = -\boldsymbol{G}_i + \frac{\|\boldsymbol{G}_i\|_2^2}{\|\boldsymbol{G}_{i-1}\|_2^2}\boldsymbol{P}_{i-1}$, where $\boldsymbol{P}_i$ is the conjugate gradient of $\boldsymbol{G}_i$.

(v) From Eq.(12), search the step length h of one dimension to make $J(\boldsymbol{E}_{i+1}) = \min\limits_{h} J(\boldsymbol{E}_i + h\boldsymbol{P}_i)$ and let $i = i + 1$.

(vi) If $i > m$, reset the initial value $\boldsymbol{E}_0 = \boldsymbol{E}_{i+1}$ and go back to step (i) to continue iterating; if $i \le m$, go back to step (ii) to continue optimization, where m is the dimension of displacement parameter $\boldsymbol{E}$ of continuous curve box (here m is equal to 3).

(vii) Compute the covariance $\boldsymbol{C}_{\hat{\boldsymbol{E}}}$ of displacement parameter $\boldsymbol{E}$ of continuous curve box by dynamic Bayesian variance formula (19).

3.3 One-dimensional searching of h

One-dimensional searching of h is necessary in the step (v) of dynamic Bayesian estimation steps of displacement parameters of continuous curve box, which is a complex problem in the domain of parameter estimation. In the researching results available, the method of one-dimensional searching is the gold section method on the whole[8,9], because this method can ensure the precision of computation and simplicity of making programs. But the range h lies in must be known. However, correctly estimating the range h lies in is not easy (usually by trying different ranges to determine the one h lies in) and especially as for the problem of poly-parameters' estimation, correctly estimating the range h lies in becomes much more difficult. This paper adopts the inner-outer searching method introduced in Ref.[10], which can automatically determine the range h lies in and carry on optimization without supposing the initial range h lies in. The method successfully solves the problem of automatically optimizing h.

4 Development of program and analysis of examples

The dynamic Bayesian estimation of displacement parameter $\boldsymbol{E} = [E_1\ \ E_2\ \ E_3]$ of continuous curve concrete box is studied in this paper (E_1, E_2 and E_3 are, respectively, the Young's moduli of the plane board, bottom board and middle board). The curve strip element and the number of the line are shown in Fig.3. The radius is $R = 300$ cm and the angle is $\alpha = 0.4$ rad and the length of the middle axis is $L = 120$ cm. Poisson's ratio μ is 0.17. The true values of displacement parameter $\boldsymbol{E}$ and the widths of the cutting plane are shown in Table 1. The variation coefficient is 0.1. The vertical loads $P_1 = 9$ kN and $P_2 = 8$ kN are applied to the curve box at the nodes on the cutting planes of $1/4L$ and $3/4L$ (Fig.4). In order to carry on dynamic Bayesian estimation of displacement parameters of continuous curve box, the program called INVCCB is developed.

Select the four points (Nos.2, 5, 10, 14) as displacement measuring points and measure the displacements of the four points for five times. The expectations and variances of the measuring displacements are shown in Table 2.

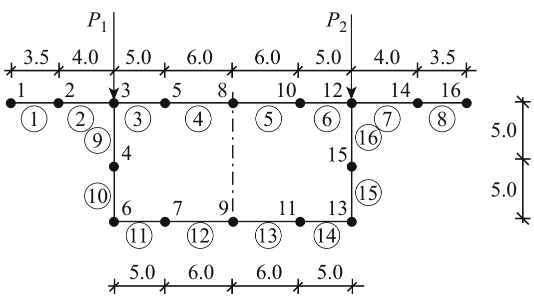

Fig.3 Curve strip element and the number of the line (Unit: cm)

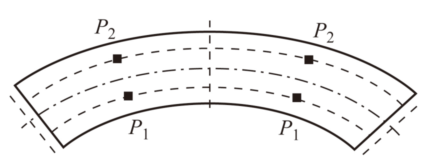

Fig.4 Applied loads of continuous curve box

Table 1 True values of displacement parameters and the widths of the cutting plane

Parameter	$E_1/(\mathrm{N\cdot cm^{-2}})$	$E_2/(\mathrm{N\cdot cm^{-2}})$	$E_3/(\mathrm{N\cdot cm^{-2}})$	d_1/cm	d_2/cm	d_3/cm
Value	2.8E+6	2.0E+6	2.4E+6	0.6	0.6	0.4

Table 2 Expectations and variances of the measuring displacements (Unit: cm)

Selected points	U_1	U_2	U_3	U_4	U_5	σ_{U_1}	σ_{U_2}	σ_{U_3}	σ_{U_4}	σ_{U_5}
2	0.949	0.946	0.943	0.951	0.953	0.158	0.157	0.158	0.153	0.154
5	0.884	0.883	0.887	0.886	0.881	0.144	0.146	0.141	0.143	0.145
10	0.642	0.648	0.641	0.642	0.647	0.094	0.095	0.091	0.096	0.094
14	0.582	0.580	0.584	0.586	0.585	0.080	0.079	0.076	0.082	0.084

4.1 Dynamic Bayesian estimation of displacement parameters of continuous curve box when the pre-known information is precise

Let the pre-known information $\boldsymbol{E}_0 = [2.9, 2.1, 2.5]$. Respectively select the initial values $\boldsymbol{E}_{1\text{ini}} = [1.5, 3.8, 1.2]$ and $\boldsymbol{E}_{2\text{ini}} = [5.0, 4.0, 1.0]$, which are put into the program of INVCCB together with the data in Table 2. Suppose the convergence criteria $\varepsilon_1 = 0.001, \varepsilon_2 = 0.001$ and the iterative results of displacement parameters are shown in Table 3, Figs.5 and 6.

Table 3 Results of dynamic Bayesian estimation of displacement parameters of continuous curve box when the pre-known information is precise (Unit: 10^6 N/cm^2)

Displacement parameters	E_1	E_2	E_3	E_1	E_2	E_3
Initial value	1.500	3.800	1.200	5.000	4.000	1.000
Final value	2.861	2.038	2.423	2.820	2.059	2.396
Iterative times	10	10	10	8	8	8
Relative error/%	2.179	1.900	0.958	0.714	2.950	0.167
Convergence criterion	ε_1	ε_1	ε_1	ε_2	ε_2	ε_2

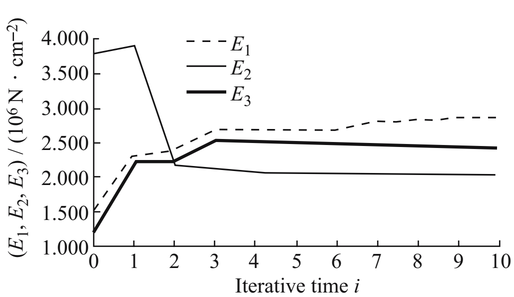

Fig.5 Iterative results when $\boldsymbol{E}_{1\text{ini}}$ is selected (Unit: 10^6 N/cm^2)

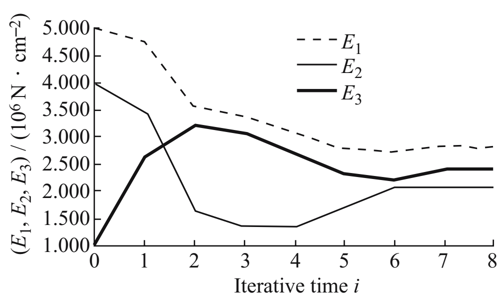

Fig.6 Iterative results when $\boldsymbol{E}_{2\text{ini}}$ is selected (Unit: 10^6 N/cm^2)

It is known from computing process that dynamic Bayesian estimation can consider displacement measuring data of five times at the same time, which is efficient in calculation. From Figs.5 and 6, it is known that when the pre-known information is precise, the iterative process of dynamic Bayesian estimation of displacement parameter $\boldsymbol{E}$ of continuous curve box is steady and whether the initial parameter values are far from the true parameter values or near to them, the iteration is convergent to the true parameter values.

As a research paper, displacement parameters of the top, the middle and the bottom boards of continuous curve box are all viewed as random variables. In fact, the middle board may consist of different materials, which makes the number of random variables added during estimation of displacement parameters. If the expectations of random variables lie in the same numerical scale, displacement parameters of curve box can be correctly estimated by using dynamic Bayesian method based on Novozhilov theory, which means that the convergence of displacement parameters is not affected by the number of random variables; if not, numerical scale of the parameter should be tackled on basis of sensitivity property of the parameter to the systematic responses[9].

4.2 Dynamic Bayesian estimation of displacement parameters of continuous curve box when the pre-known information is not precise

Let pre-known information $\boldsymbol{E}_0 = [2.0, 3.0, 1.5]$. In order to make comparison conveniently, still let parameter initial values $\boldsymbol{E}_{1\text{ini}} = [1.5, 3.8, 1.2]$ and $\boldsymbol{E}_{2\text{ini}} = [5.0, 4.0, 1.0]$. The iterative results of displacement parameters are shown in Table 4.

It can be found from Table 4 that when the pre-known information is not precise, the iterative process of displacement parameters is not convergent by the criterion of ε_1 or ε_2 when the iterative time equals 30 and the relative errors of parameters are far more than 5%. As to practical engineering, the true values of displacement parameters of continuous curve box are unknown. Without parameter true values, now the problem is how to judge whether the pre-known information given is precise or not. Otherwise, it will give wrong guide to practical engineering. Through research and analysis, it is found that when the pre-known information given is precise, the iterative process can be convergent on basis of the criterion of ε_1 or ε_2; when the information given is not precise, the iterative process can not be convergent even the iterative times are increased.

Table 4 Results of dynamic Bayesian estimation of displacement parameters of continuous curve box when the pre-known information is not precise (Unit: 10^6 N/cm^2)

Displacement parameters	E_1	E_2	E_3	E_1	E_2	E_3
Initial value	1.500	3.800	1.200	5.000	4.000	1.000
Final value	2.356	3.642	1.784	2.384	3.592	1.694
Iterative times	30	30	30	30	30	30
Relative error/%	15.857	82.100	25.667	14.857	79.600	29.417
Convergence criterion	-	-	-	-	-	-

5 Conclusions

Dynamic Bayesian estimation of displacement parameters of continuous curve box is studied in this paper. From theoretic research and analysis of examples, some conclusions are drawn as follows. Dynamic Bayesian estimation of displacement parameters of continuous curve box considers the stochastic property of both displacement parameters and the measuring displacement data, which is efficient in computation because the measuring displacement data of different times are considered simultaneously. In-out searching method is adapted, which makes the range h lies in determined automatically. There is no need to suppose the range h lies in as the gold section method does. The problem of automatic optimization of h during optimizing process is solved. When the pre-known information is precise, the iterative process of dynamic Bayesian estimation of displacement parameters of continuous curve box is steadily convergent to the true values and the convergence property of displacement parameters is not affected by the number of random variables; when the pre-known information is not precise, the iterative process of displacement parameters is divergent.

References

[1] Xiang Haifan. Advanced theory of bridge structure[M]. Beijing: The Peoples' Communications Press, 2001 (in Chinese).

[2] Xiao Rucheng. Analysis of bridge structure and program system[M]. Beijing: The Peoples' Communications Press, 2002 (in Chinese).

[3] Ghani R A, Hangang U. Thin-walled multicell box-girder finite element[J]. *Structure Engineering, ASCE*, 1991, **117**(10):2953–2971.

[4] Martti J M, Jahn P. Finite element analysis of box girders[J]. *Structure Division, ASCE*, 1980, (10):1343–1357.

[5] Zhao Zhenming, Chen Baochun. Analysis of pole system and box girder bridge and program designing[M]. Guangzhou: Huanan University of Science and Engineering Press, 1997 (in Chinese).

[6] Zhang Youqi. Finite strip analysis of box girder bridge[M]. Beijing: The Peoples' Communications Press, 1978 (in Chinese).

[7] Gioda. Back analysis procedure for the interpretation of field measurements in geomechanics[J]. *Numerical and Analytic Method in Geomechanics*, 1987, (11):555–583.

[8] Ohkami T, Ichikawa Y. A boundary element method for identifying orthotropic material parameters[J]. *Numerical and Analytic Method in Geomechanics*, 1991, (15):609–625.

[9] Shu Shanhang. Parameter stochastic back analysis method and object oriented programming design[D]. M Sc Dissertation. Nanjing: Hohai University, 2000 (in Chinese).

[10] Fan Mingyu, Zhang Yin. Basis of optimization technology[M]. Beijing: Tsinghua University Press, 1982 (in Chinese).