Optics Communications 283 (2010) 4985–4992

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Iterative Wiener deconvolution based method for phase retrieval from noisy intensities

Shiling Zheng, Bindang Xue *

School of Astronautics, Beihang University, Beijing, 100191, China

ARTICLE INFO

Article history:
Received 28 May 2010
Received in revised form 29 July 2010
Accepted 3 August 2010

Keywords:
Phase retrieval
Iterative angular spectrum (IAS)
Wiener filter
Iterative Wiener deconvolution (IWD)

ABSTRACT

To retrieve the phase from the noisy measured intensities in the diffraction planes, an iterative Wiener deconvolution based method is proposed. With the same iterative scheme as the iterative angular spectrum method (IAS), the propagation of the optical wave function between the input plane and the diffraction planes is calculated by Wiener deconvolution in this method. The angular spectrum convolution kernel used in the iterative angular spectrum method is incorporated into the Wiener filter. The simulation experiments show that the proposed method can reduce the impact of the noise on the retrieved phase and performed better than the pre-denoising method. Furthermore, the proposed method exhibits great advantage compared to IAS for retrieving the complicated phase distribution from two measured intensities.

1. Introduction

Phase retrieval plays an important role in a diverse range of fields including electron microscopy, wavefront sensing, astronomy, X-ray imaging etc. It involves estimating the phase distribution from the intensity measurements produced by the field of interest. Many algorithms have been developed for phase retrieval from the diffraction intensity measurements. These algorithms can generally be divided into two classes: the non-iterative algorithms [1,2] and the iterative algorithms [3–12]. The Gerchberg–Saxton–Fienup (GSF) type algorithms [3,4] and the conjugate gradient methods [10,11] are the most widely used methods among the alternative iterative algorithms. The iterative angular spectrum (IAS) method, which calculates the propagation between two planes by the angular spectrum method [4], is a variant of the GSF type algorithms. The majority of the aforementioned algorithms are concerned with retrieving the phase from the noise-free measured intensities. However, almost every kind of data contains some degree of noise. The non-iterative algorithm is very sensitive to noise [6] and the iterative algorithms may be divergent or become stagnant before they reach the right solution if the measured intensities are corrupted by noise.

An approach for retrieving the phase from noisy data is to first remove the noise through image denoising techniques [13], and then retrieve the phase from the clean data by the general algorithms mentioned earlier. However, as the main difficulty in solving the phase retrieval problem is that we do not have enough information to determine the phase, the denoising operation may eliminate the important information from the original measurements necessary for retrieving the phase in the next step. A potentially more effective approach is to restrain the noise while retrieving the phase. In this study, we introduce such a method for retrieving the phase from the noisy data with additive noise. The proposed method is an iterative Wiener deconvolution (IWD) based method. In the IAS method, the angular spectrum method is used to calculate the propagation of the complex optical wave function between the desired plane (the plane in which we determine to retrieve the phase) and the diffraction plane (the plane in which the intensity measurements have been obtained), and the computed light intensities are replaced by the measured intensities. We interpreted this process form the view of deconvolution. The propagation calculated by the angular spectrum method can be considered to be inverse filtering. As Wiener deconvolution is better for restoring the original signal from the blurred and noisy data, we adopt it instead of the angular spectrum method to compute the propagation of the optical wave function. Thus, we get an iterative method which exploits Wiener deconvolution to calculate the forward propagation and backward propagation between the diffraction planes and corrects the phase iteratively. The properties of the proposed method are analyzed and numerical simulation experiments are conducted.

2. Description of algorithm

2.1. Wiener deconvolution

Wiener deconvolution is concerned with reconstruction of the original image from the known image corrupted by some kind of

* Corresponding author.
E-mail address: xuebd@buaa.edu.cn (B. Xue).

0030-4018/$ – see front matter
doi:10.1016/j.optcom.2010.08.010

noise. It applies the Wiener filter to the noise problems inherent in deconvolution, attempting to achieve trade-off between the suppression of noise and the reconstruction accuracy of high frequency components of the original image.

Given a linear imaging system:

$$f(x,y) = h(x,y) \otimes i(x,y) + w(x,y), \tag{1}$$

where $\otimes$ denotes convolution, $f(x,y)$ is the observed image at spatial coordinates (x,y), $i(x,y)$ is some unknown input image, $h(x,y)$ is the known impulse response of a linear space-invariant imaging system, $w(x,y)$ is zero-mean, additive Gaussian noise, independent of the input signal $i(x,y)$.

We perform the deconvolution on the corrupted signal $f(x,y)$ to recover the original image $i(x,y)$. Having restricted ourselves to a linear solution, our objective is to design a filter $g(x,y)$ so that we can estimate $i(x,y)$ in the following way:

$$\hat{i}(x,y) = g(x,y) \otimes f(x,y), \tag{2}$$

where $\hat{i}(x,y)$ is an estimate of $i(x,y)$ that minimizes the mean square error (MSE).

Assume that $i(x,y)$ is at least wide-sense stationary, then the Wiener deconvolution filter provides such a $g(x,y)$, which can be described conveniently in the frequency domain:

$$G\left(f_x,f_y\right) = \frac{H^*\left(f_x,f_y\right)P_f\left(f_x,f_y\right)}{\left|H\left(f_x,f_y\right)\right|^2 P_f\left(f_x,f_y\right) + P_w\left(f_x,f_y\right)}, \tag{3}$$

where, $G(f_x,f_y)$ and $H(f_x,f_y)$ are the Fourier transforms of $g(x,y)$ and $h(x,y)$ at spatial frequency (f_x,f_y). $P_f(f_x,f_y)$ and $P_w(f_x,f_y)$ are the corresponding mean power spectral densities of the input signal $f(x,y)$ and the noise $w(x,y)$, the superscript denotes complex conjugation.

The filtering operation is carried out in the frequency domain as follows:

$$\hat{I}\left(f_x,f_y\right) = G\left(f_x,f_y\right)F\left(f_x,f_y\right), \tag{4}$$

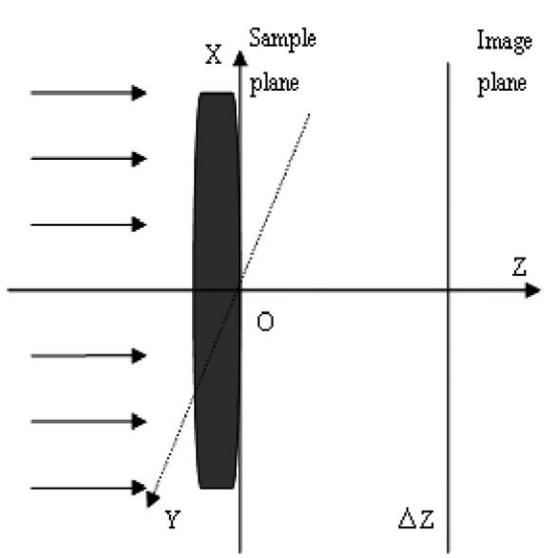

Fig. 1. Schematics of optical phase retrieval (originated from [9]).

where $\hat{I}(f_x,f_y)$ is the Fourier transform of $\hat{i}(x,y)$. Then by performing an inverse Fourier transform on $\hat{I}(f_x,f_y)$ we can obtain $\hat{i}(x,y)$.

2.2. Iterative angular spectrum phase retrieval method: a view from deconvolution

The IAS method is a modification of the Gerchberg–Saxton algorithm, which is mainly used to retrieve the phase from the measured intensities in the diffraction planes. Assuming an optical plane wave propagating along the Z axis is incident on a sample perpendicular to the Z axis (see Fig. 1). Consider that the transverse (x,y) plane where the plane wave just exits from the sample is in the $z=0$ plane, and the parallel plane which has a distance Δz from $z=0$ plane in $z=z_{\Delta z}$ plane. The complex-amplitude in these planes is composed by the amplitude and the phase, which can be expressed as:

$$U(r_\perp,z) = I^{1/2}(r_\perp,z)\exp[i\phi(r_\perp,z)], \tag{5}$$

where $r_\perp=(x,y)$ is a vector in the x–y plane and perpendicular to the Zaxis. $I(r_\perp,z)$ is the intensity. The complex-amplitude $U(r_\perp,z)$ can be decomposed into the angular spectrum $A(\alpha/\lambda,\beta/\lambda,z)$, which is defined by [14]:

$$A(\alpha/\lambda,\beta/\lambda,z) = \iint U(r_\perp,z)\exp[-j2\pi(\alpha x/\lambda + \beta y/\lambda)]dxdy, \tag{6}$$

where λ is the incident wavelength, and α,β are the direction cosines. According to the Helmholtz equation, the propagation of the angular spectrum from the $z=0$ plane to $z=z_{\Delta z}$ plane is written as [14]:

$$A(\alpha/\lambda,\beta/\lambda,z_{\Delta z}) = A(\alpha/\lambda,\beta/\lambda,z_0)\exp\left[j2\pi\left(1-\alpha^2-\beta^2\right)^{1/2}\Delta z/\lambda\right]. \tag{7}$$

In fact, the angular spectrum can be more conveniently used as the function of the spatial frequency (f_x,f_y). Thus, Eqs. (6) and (7) can be rewritten as [14]:

$$A\left(f_x,f_y,z\right) = F\{U(r_\perp,z)\} = \iint U(r_\perp,z)\exp\left[-j2\pi\left(f_x x + f_y y\right)\right]dxdy, \tag{8}$$

$$A\left(f_x,f_y,z_{\Delta z}\right) = A\left(f_x,f_y,z_0\right)H\left(f_x,f_y\right), \tag{9}$$

$$H\left(f_x,f_y\right) = \begin{cases} \exp\left(j2\pi\Delta z/\lambda\sqrt{1-(\lambda f_x)^2-\left(\lambda f_y\right)^2}\right) & \sqrt{f_x^2+f_y^2}<1/\lambda \\ 0 & \text{others} \end{cases}. \tag{10}$$

The direction cosines and the spatial frequency are related by

$$\alpha = \lambda f_x \qquad \beta = \lambda f_y. \tag{11}$$

Therefore, the propagation of the plane wave can be considered as a band-limited linear spatially invariant filter [14]. The backward propagation is:

$$A\left(f_x,f_y,z_0\right) = A\left(f_x,f_y,z_{\Delta z}\right)H_-\left(f_x,f_y\right), \tag{12}$$

$$H_-\left(f_x,f_y\right) = \begin{cases} \exp\left(-j2\pi\Delta z/\lambda\sqrt{1-(\lambda f_x)^2-\left(\lambda f_y\right)^2}\right) & \sqrt{f_x^2+f_y^2}<1/\lambda \\ 0 & \text{others} \end{cases}. \tag{13}$$

If we do the inverse Fourier transform to Eq. (9) with Eq. (10), then we can obtain:

$$U(r_\perp, z_{\Delta z}) = U(r_\perp, z_0) \otimes h(x, y), \tag{14}$$

$$h(x, y) = F^{-1}\left\{H\left(f_x, f_y\right)\right\}. \tag{15}$$

Similarly, from Eqs. (12) and (13), we get

$$U(r_\perp, z_0) = U(r_\perp, z_{\Delta z}) \otimes h_-(x, y), \tag{16}$$

$$h_-(x, y) = F^{-1}\left\{H_-\left(f_x, f_y\right)\right\}, \tag{17}$$

where the symbols H_- and h_- denote backward propagator convolution operators. From Eqs. (14)–(17), we can see that if we consider $U(r_\perp,z_{\Delta z})$ that is generated by the forward propagation as the degraded signal of $U(r_\perp,z_0)$, then the backward propagation can be considered as estimating $U(r_\perp,z_0)$ from the degraded signal $U(r_\perp,z_{\Delta z})$. Eq. (17) shows that the process of obtaining $U(r_\perp,z_0)$ is just like direct inverse filtering $U(r_\perp,z_{\Delta z})$ (The specific expression of the convolution kernel h and h_- are not necessary because the proposed method is mainly performed in the frequency domain).

Assume $I_m(r_\perp,z_0)$ and $I_m(r_\perp,z_{\Delta z})$ respectively denote the measured intensities in the $z=0$ plane and the $z=z_{\Delta z}$ plane. The IAS method is a process of propagating the complex wavefront between the two planes by the angular spectrum method and replacing the calculated intensities by the measured ones iteratively. The algorithm can be described as follows:

Step1: Let $n=0$. Construct the complex wave $U_n(r_\perp,z_0)$ in the $z=0$ plane with an initial phase estimate and the measured intensity $I_m(r_\perp,z_0)$ by Eq. (5).
Step2: Propagate $U_n(r_\perp,z_0)$ to the $z=z_{\Delta z}$ plane by Eq. (9), and do the 2D inverse Fourier transform to obtain $U_n'(r_\perp,z_{\Delta z})$.
Step3: Replace the amplitude of $U_n'(r_\perp,z_{\Delta z})$ with $I_m^{1/2}(r_\perp,z_{\Delta z})$.
Step4: Propagate the corrected wavefront back to the $z=0$ plane by Eq. (12), and do the 2D inverse Fourier transform to get $U_n'(r_\perp,z_0)$.
Step5: Replace the amplitude of $U_n'(r_\perp,z_0)$ with $I_m^{1/2}(r_\perp,z_0)$, yielding the refreshed wavefront $U_n(r_\perp,z_0)$.
Step6: Let $n=n+1$. Go to Step2 if the calculation accuracy or the maximum iteration number is not reached.

Here the calculation accuracy is estimated by the sum-squared error (SSE), defined by:

$$SSE = \sum_{x,y}\left[I_m(r_\perp, z_0) - |U_n'(r_\perp, z_0)|^2\right]^2 \Big/ \sum_{x,y} I_m^2(r_\perp, z_0). \tag{18}$$

From the given description, we can see that the IAS method is a formal resemblance to the iterative inverse filtering procedure. Moreover, we could expect that the noise contained in the intensity measurements is propagated with the complex wave.

3. Iterative Wiener deconvolution based method for phase retrieval

Consider noisy $U(r_\perp,z_{\Delta z})$ as the degraded signal of $U(r_\perp,z_0)$, and assume that the complex field of the monochromatic light and the noise propagating with the phase are at least wide-sense stationary. Then the propagation relationship between $U(r_\perp,z_0)$ and $U(r_\perp,z_{\Delta z})$ can be calculated by Wiener deconvolution, which is expressed as:

$$F\{U(r_\perp, z_{\Delta z})\} = F\{U(r_\perp, z_0)\}H\left(f_x, f_y\right) + w\left(f_x, f_y, z_{\Delta z}\right), \tag{19}$$

where $w(f_x,f_y,z_{\Delta z})=F\{w(x,y,z_{\Delta z})\}$ denotes the noise in the $z=z_{\Delta z}$ plane. Note that $w(x,y,z_{\Delta z})$ is assumed zero-mean, additive complex Gaussian noise, independent of $U(r_\perp,z_0)$. Similarly, we could obtain:

$$F\{U(r_\perp, z_0)\} = F\{U(r_\perp, z_{\Delta z})\}H_-\left(f_x, f_y\right) + w\left(f_x, f_y, z_0\right). \tag{20}$$

Then the main process of the IWD based method is replacing Eqs. (9)–(13) in the IAS method with the following equations:

$$F\{U(r_\perp, z_0)\} = F\{U(r_\perp, z_{\Delta z})\}G_-\left(f_x, f_y\right), \tag{21}$$

$$G_-\left(f_x, f_y\right) = \frac{H^*\left(f_x, f_y\right)P_{U_{true}(r_\perp, z_0)}\left(f_x, f_y\right)}{\left|H\left(f_x, f_y\right)\right|^2 P_{U_{true}(r_\perp, z_0)}\left(f_x, f_y\right) + P_w\left(f_x, f_y, z_{\Delta z}\right)}. \tag{22}$$

$$F\{U(r_\perp, z_{\Delta z})\} = F\{U(r_\perp, z_0)\}G\left(f_x, f_y\right), \tag{23}$$

$$G\left(f_x, f_y\right) = \frac{H_-^*\left(f_x, f_y\right)P_{U_{true}(r_\perp, z_{\Delta z})}\left(f_x, f_y\right)}{\left|H_-\left(f_x, f_y\right)\right|^2 P_{U_{true}(r_\perp, z_{\Delta z})}\left(f_x, f_y\right) + P_w\left(f_x, f_y, z_0\right)}. \tag{24}$$

Since the power spectrums of the true signal and the noise are two very important parameters for Wiener deconvolution, they should be estimated first. In this study, the power spectrum of the true complex-amplitude is estimated by the following method, which is:

$$P_{U_{true}(r_\perp, z_{\Delta z})}\left(f_x, f_y\right) = \frac{P_{U(r_\perp, z_0)}\left(f_x, f_y\right) - P_{w(r_\perp, z_0)}\left(f_x, f_y\right)}{\left|H_-\left(f_x, f_y\right)\right|^2}, \tag{25}$$

$$P_{U_{true}(r_\perp, z_0)}\left(f_x, f_y\right) = \frac{P_{U(r_\perp, z_{\Delta z})}\left(f_x, f_y\right) - P_{w(r_\perp, z_{\Delta z})}\left(f_x, f_y\right)}{\left|H\left(f_x, f_y\right)\right|^2}, \tag{26}$$

where $P_{U(r_\perp,z_0)}$ and $P_{U(r_\perp,z_{\Delta z})}$ are the power spectrums of the degraded signals, $P_{w(r_\perp,z_0)}$ and $P_{w(r_\perp,z_{\Delta z})}$ are the power spectrums of the assumed zero-mean, additive complex white Gaussian noise. $P_{U(r_\perp,z_0)}$ and $P_{U(r_\perp,z_{\Delta z})}$ are estimated with the conventional periodogram method [13]:

$$P_{U(r_\perp, z_0)}\left(f_x, f_y\right) = |F\{U(r_\perp, z_0)\}|^2,\ P_{U(r_\perp, z_{\Delta z})}\left(f_x, f_y\right) = |F\{U(r_\perp, z_{\Delta z})\}|^2. \tag{27}$$

$P_{w(r_\perp,z_0)}$ and $P_{w(r_\perp,z_{\Delta z})}$ are estimated from the average of the variances measured in a set of image block[15]. In the method, the variance of a local neighborhood of each sampling point is firstly calculated. Then the mean of the variance is taken as the noise variance of that sampling point.

According to the given description, the IWD based phase retrieval method can be summarized as follows:

Step1: Let $n=0$. Give an initial guess of the phase distribution and use $I_m(r_\perp,z_0)$ to create the complex-amplitude $U_n(r_\perp,z_0)$ in the $z=0$ plane by Eq. (5).
Step2: Propagate $U_n(r_\perp,z_0)$ to the $z=z_{\Delta z}$ plane by Eq. (23), and the 2D inverse Fourier transform is performed on the calculated result to obtain $U_n'(r_\perp,z_{\Delta z})$.
Step3: Replace the amplitude of $U_n'(r_\perp,z_{\Delta z})$ by $I_m^{1/2}(r_\perp,z_{\Delta z})$.
Step4: Propagate the corrected complex wavefront to the $z=0$ plane by Eq. (21), and do the 2D inverse Fourier transform to get $U_n'(r_\perp,z_0)$.
Step5: Replace the amplitude of $U_n'(r_\perp,z_0)$ by $I_m^{1/2}(r_\perp,z_0)$, get $U_n(r_\perp,z_0)$.
Step6: Let $n=n+1$. Go to Step2 until the termination criteria are met.

4. Simulations

In this section, simulations are conducted to test the three methods: the IAS method, the IWD based method and the method in which the noises were removed in the measurements by Wiener filter first and subsequently the IAS method was used, denoted by W-IAS. The simulations are conducted on the PIV3.4Ghz 1 GB RAM PC. The well-known Lena image is chosen as the phase profile (see Fig. 2). The radiation wavelength is chosen to be 550 nm. Note that in the angular spectrum method the light field outside a finite area is neglected. This assumption may cause some high frequency components introduced by diffraction lost and so will reduce the light propagation accuracy. The accuracy decreases as the propagation distance increases. Therefore, the angular spectrum method for light propagation is generally limited to $z \le L^2/\lambda N$, where L is the physical size, $N \times N$ is the number of sample points. In this study, the propagation distance should satisfy $z \le 0.0102m$. Hence the dimensions of the image are 256×256 pixels with an assigned physical size of $1.2 \times 10^{-3} \times 1.2 \times 10^{-3}$ m^2. The phase is bounded between 0 and π, which is corresponding to the thin samples or small aberrations in the optical systems. The intensity distribution in the input plane is uniform and its intensity value is 1, which corresponds to the pure phase object. We should note that the Lena image is not a stationary process and stationary processes rarely exist in the practical application. Despite this, we will show in our experiments that the proposed method is still effective and easy to implement. The initial phases for all the experiments are set to be uniform zero phases. Neighborhoods of size 25×25 is used to estimated the local sampling point block variances. The relative root-mean-square error (RMSE) is employed to quantify the accuracy of the retrieved phase:

$$RMSE = \sqrt{\sum_{x,y} [\theta_1(x,y) - \theta_0(x,y)]^2 \Big/ \sum_{x,y} \theta_0^2(x,y)}, \tag{28}$$

where, $\theta_1(x,y)$ and $\theta_0(x,y)$ respectively denote the simulated phase distribution and the calculated phase distribution at pixel (x,y).

4.1. Phase retrieval from three intensity measurements

In the first tests, we used the measured intensities in the $z=0$ plane and $z=\pm 0.01$ m planes to retrieve the phase in the $z=0$ plane. The diffraction intensities without noise are shown in Fig. 3.

Fig. 2. Simulated phase distributions in the $z=0$ plane.

Initially, the IAS and IWD methods were tested by retrieving the phase from noise-free intensities. In this situation, since the Wiener deconvolution reduced to the inverse filtering, the proposed method reduced to the IAS method. Thus, the retrieved phases are almost the same. When the maximum iteration number was set to 100, the retrieved phases are shown in Fig. 4. The RMSEs of the retrieved phases are around 0.95%.

Then we test the following three methods: the IAS method, the W-IAS method and the IWD based method. The noisy data, shown in Fig. 5, are assumed to be corrupted by the additive zero-mean Gaussian white noise with a signal-to-noise ratio (SNR) of 25 dB and the corresponding retrieved results are shown in Fig. 6(a), (b) and (c). These results show that IWD and W-IAS obtained better retrieved phase profiles. Since IWD needs to estimate the power spectrum of the noise and the true signal, we implemented the three methods respectively with 55, 55, and 50 iterations for different levels of the noisy measurements to execute them approximately at the same time. Each method for every level of the noisy intensities was carried out for 30 times and the averages of the corresponding RMSE and the consumed time were calculated. The results are presented in Table 1. As shown, IWD and W-IAS achieved higher

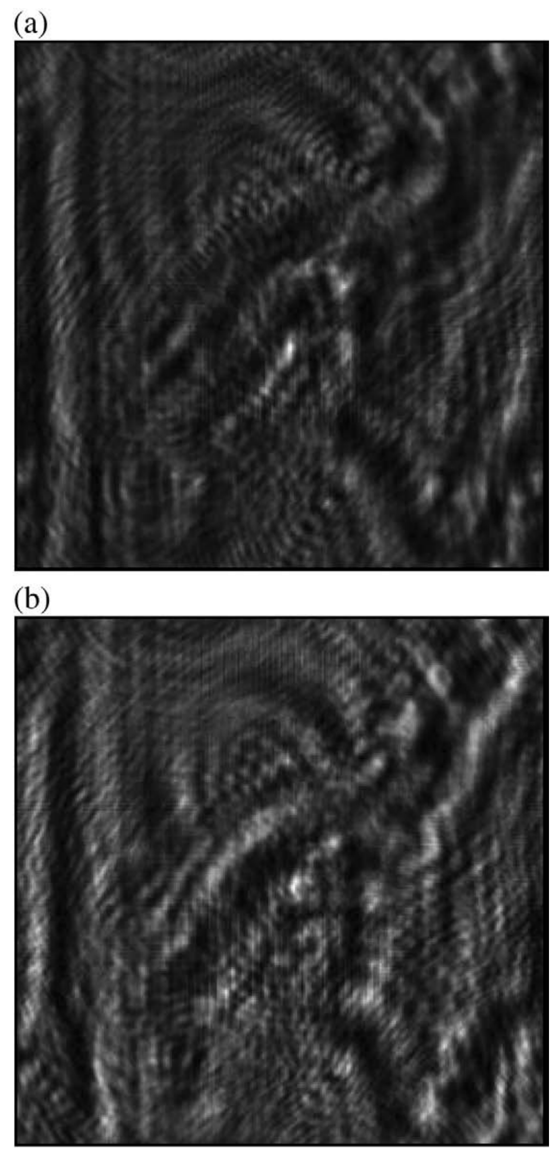

Fig. 3. Noise-free intensity measurements in two diffraction planes: (a) $z=0.01$ m, (b) $z=-0.01$ m.

accuracy than IAS. Particularly, our method performed best for retrieving the phase from measured intensities with the SNR of 30 dB–45 dB.

4.2. Phase retrieval from two intensity measurements

As is known, the iterative methods from two intensities can often get trapped in a local minimum and thus cannot obtain an acceptable solution [5]. In the second set of numerical tests, two diffraction intensities were used to retrieve the phase distribution in the $z = 0$ plane.

Firstly, we applied the IAS method and the IWD based method to retrieve the phase in the $z = 0$ plane from the noise-free measured intensities in the $z = 0$ plane and $z = 0.01$ m plane. The maximum iteration numbers of the two methods were respectively set to 90 times and 80 times. The retrieved phases were shown in Fig. 7. As shown, the result of the IAS contained much noise, whereas IWD showed significant improvement.

Fig. 4. Retrieved phases from noise-free intensity measurements in the $z = 0$ plane and $z = \pm 0.01$ m planes by (a) IAS method and (b) IWD based method.

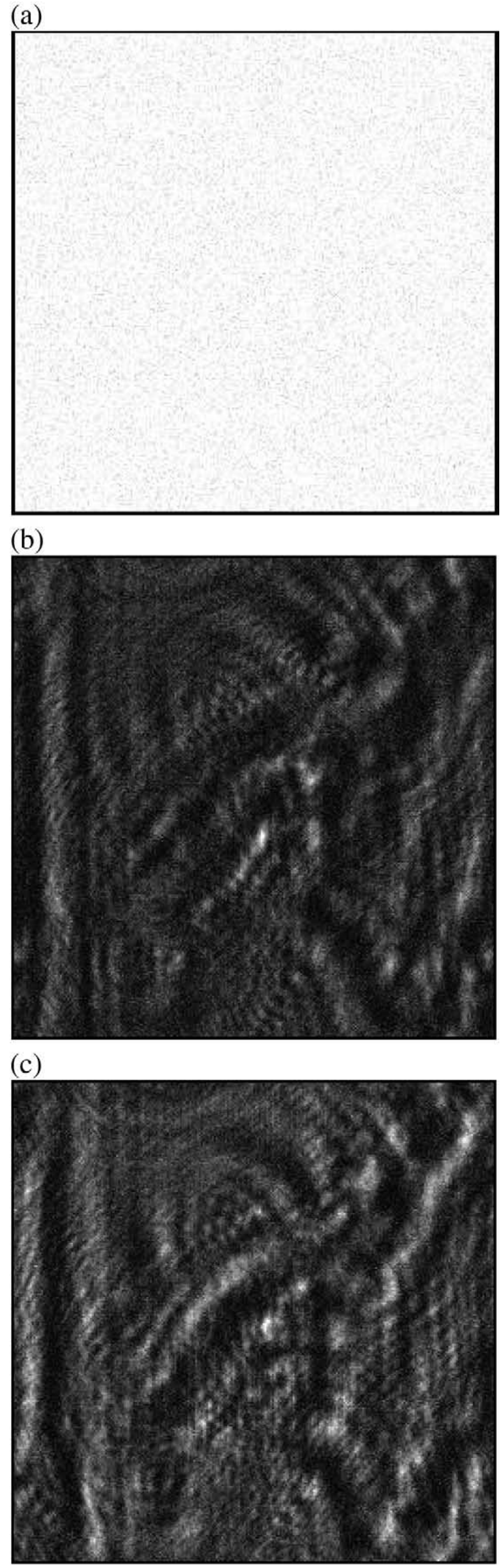

Fig. 5. Noisy intensity measurements with SNR of 25 dB in three diffraction planes: (a) $z = 0$, (b) $z = 0.01$ m, (c) $z = -0.01$ m.

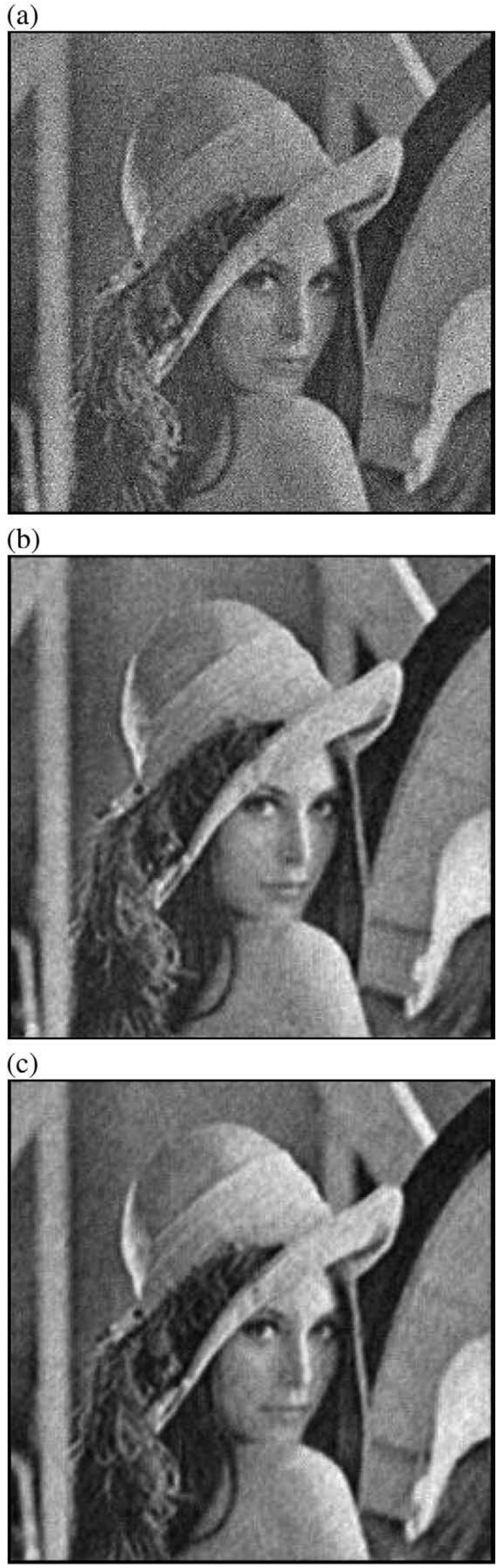

Fig. 6. Retrieved phases from noisy intensity measurements with SNR of 25 dB in the $z=0$ plane and $z=\pm 0.01$ m planes by (a) IAS method, (b) W-IAS method and (c) IWD based method.

Table 1
RMSE of three methods for phase retrieval by three intensity measurements under different SNR conditions.

SNR/dB	Methods	RMSE (%)	Time(s)
45	IAS	5.69	57.44
	W-IAS	5.74	57.52
	IWD	4.16	57.16
40	IAS	5.87	57.36
	W-IAS	5.72	57.44
	IWD	4.26	57.69
35	IAS	7.45	57.77
	W-IAS	5.95	57.85
	IWD	4.57	57.19
30	IAS	10.82	57.71
	W-IAS	6.53	57.79
	IWD	6.33	57.79
25	IAS	21.41	57.74
	W-IAS	8.36	57.83
	IWD	13.13	57.41

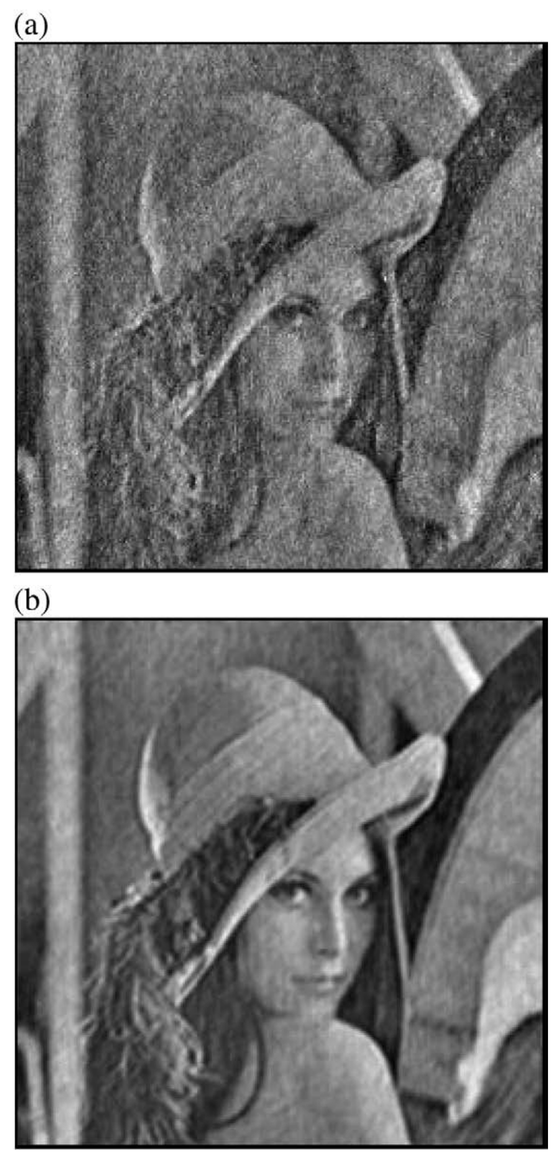

Fig. 7. Retrieved phases from noise-free intensity measurements in the $z=0$ plane and $z=0.01$ m plane by (a) IAS method and (b) IWD based method.

Secondly, the experiment was performed under different SNR conditions. The retrieved phases from the measured intensities with the SNR of 35 dB (see Fig. 8) by the IAS, W-IAS and IWD methods were respectively shown in Fig. 9. Visually, W-IAS and IWD achieved better results. The RMSE and the consumed time for the three methods were presented in Table 2. As shown, the three methods consumed almost the same time when the high SNR intensities were used. The largest difference was only 2 s, which was approximately 4% of the average execution time for each method. However, IWD improves the reconstruction accuracy by about 30% compared to IAS.

From the simulations we can conclude that both the IWD based method and the W-IAS method can reduce the noise in the retrieved phase, thereby achieve substantially higher accurate phase reconstruction from noisy intensities. However, IWD performed better for retrieving the phase from three measured intensities with the SNR of 30 dB–45 dB. With the 45 dB SNR, the W-IAS has a higher RMSE than IAS. This may be in accordance to our discussion that pre-processing method will reduce some useful information for phase retrieval. Especially, the proposed method could retrieve the phase from two measured intensities to an acceptable extent and should be an effective method for phase retrieval.

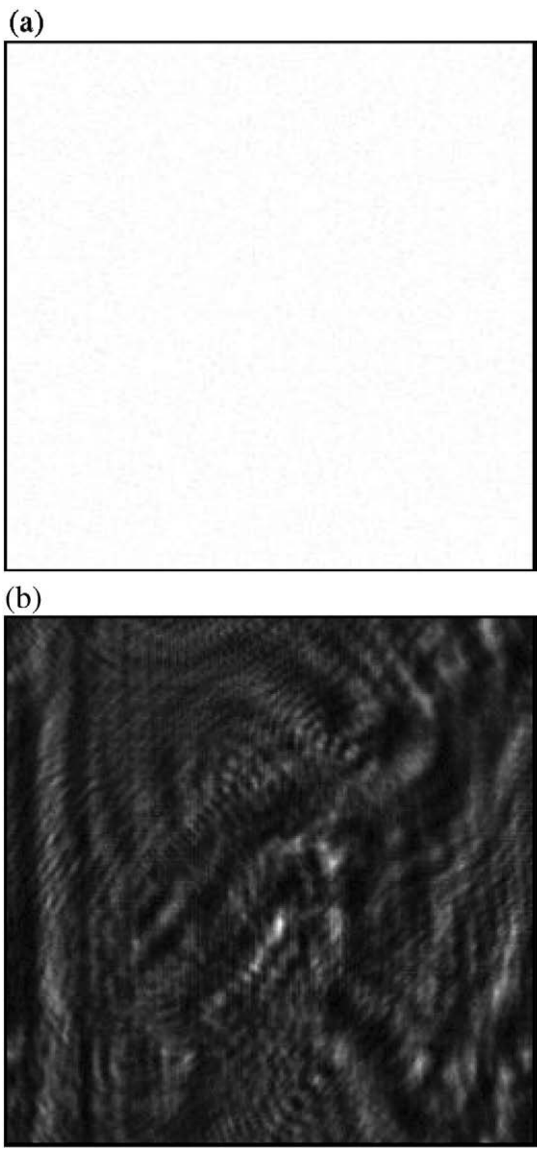

Fig. 8. Noisy intensity measurements with SNR of 35 dB in two diffraction planes: (a) $z=0$, (b) $z=0.01$ m.

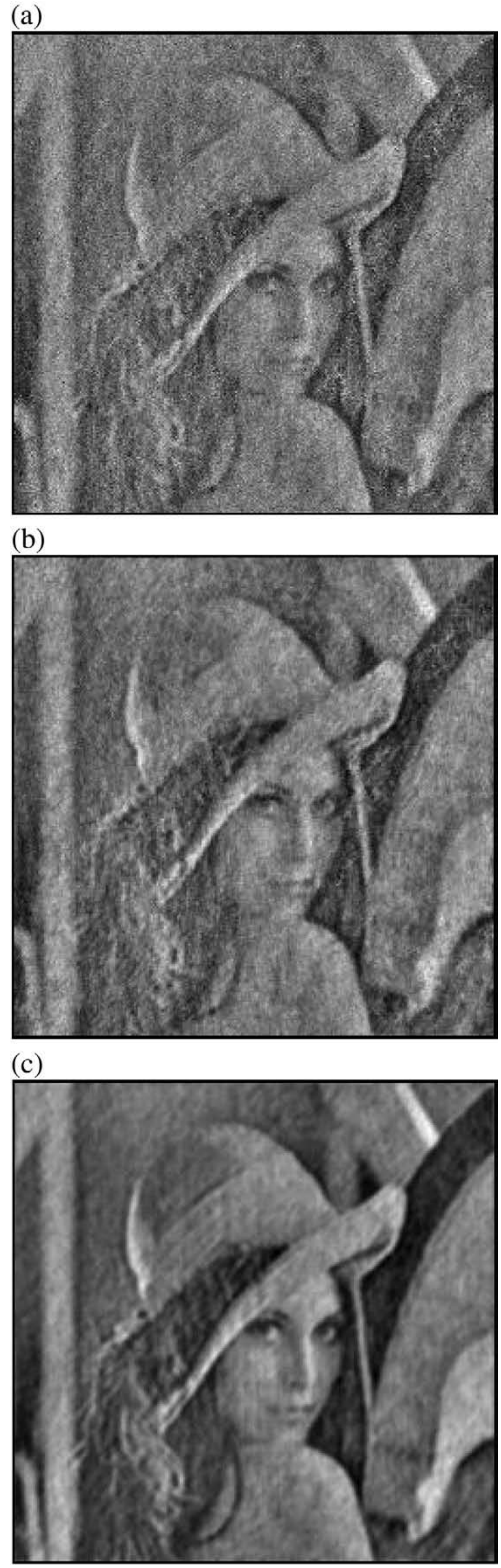

Fig. 9. Retrieved phases from noisy intensity measurements with SNR of 35 dB in the $z=0$ plane and $z=0.01$ m plane by (a) IAS method, (b) W-IAS method and (c) IWD based method.

Table 2
RMSE of three methods for phase retrieval by two intensity measurements under different SNR conditions.

SNR/db	Methods	RMSE (%)	Time(s)
70	IAS	22.24	47.1614
	W-IAS	23.02	47.2133
	IWD	16.69	48.9025
60	IAS	29.38	47.3488
	W-IAS	26.56	47.4007
	IWD	17.44	49.2102
55	IAS	27.40	47.2113
	W-IAS	23.59	47.2633
	IWD	16.73	49.0037
45	IAS	26.42	47.2056
	W-IAS	23.00	47.2575
	IWD	16.45	48.7822
35	IAS	27.65	47.4101
	W-IAS	23.27	47.4602
	IWD	16.57	49.1735
25	IAS	34.05	51.0721
	W-IAS	28.03	51.1240
	IWD	27.21	52.9013

5. Discussions and conclusions

By explaining the IAS method from the point-of-view of deconvolution, we put forward the IWD based method for retrieving the phase from noisy intensity measurements. The IWD based phase retrieval method utilizes Wiener deconvolution to minimize the mean square error of the estimated and true complex wave field in each of the diffraction planes. Although the method was derived under the stationary assumption, we tested it with the non-stationary process. The simulations showed that the method could reduce the impact of the noise on the retrieved phase and improve the retrieved accuracy compared to the IAS method and the pre-denoising method.

Obviously, the accuracy of the proposed method depends on the estimation precision of the power spectrums of the true signal and the noise. If these two parameters are not good estimates, the algorithm can easily fall into a local minimum and stop converging. As confirmed by simulations, the method used in this study is very effective for retrieving the phase from the noisy data. The quality of the phase images retrieved from three measured intensities is improved significantly and the accuracy is increased by more than 50% under the poor SNR conditions compared to the IAS method. Moreover, the simulations show that the proposed method exhibits good characteristics for retrieving the phase from two measured intensities, no matter whether the intensities are corrupted by noise or not.

The proposed method has the same iterative framework with the IAS method, and so the behavior of the method is similar to the latter. When the noise-free intensity measurements are used, the IWD based method is reduced to the IAS method. Therefore, the proposed method inherits some shortcomings of the IAS method, such as the convergence stagnation problem. Nevertheless, the IWD based phase retrieval method is better than other phase retrieval methods in dealing with the noisy data. Therefore, it is competitive among the methods for retrieving the phase. However, although the pre-denoising method, i.e., W-IAS introduced in this study, performs less well than IWD method, but much better than the IAS method. This means that proper pre-denoising of the measured intensities is also effective for phase retrieval.

Despite the effectiveness of the algorithm in the simulation, further work should be done in the future. First, we developed the theory from the stationary statistics. This condition generally will not be satisfied in the practical application, so how to design an optimal filter for the non-stationary phase retrieval should be explored. Second, we have assumed that the zero-mean, complex additive complex Gaussian noise is propagating with the complex wave. A more realistic and general noise model should be applied to designing an optimal filter for phase retrieval.

Acknowledgements

This work was partially supported by a grant from the National Natural Science Foundation of China (No. 60502018) and a grant from the Aeronautical Science Foundation of China. (No.20080151009).

References

[1] M.R. Teague, J. Opt. Soc. Am. 73 (1983) 1434.
[2] T.E. Gureyev, K.A. Nugent, J. Opt. Soc. Am. A 13 (1996) 1670.
[3] G.W. Gerchberg, W.O. Saxon, Optik (Stuttgart) 35 (1972) 237.
[4] D.L. Misell, J. Phys. D Appl. Phys. 6 (1973) L6.
[5] J.R. Fienup, Appl. Opt. 21 (1982) 2758.
[6] L.J. Allen, M.P. Oxley, Opt. Commun. 199 (2001) 65.
[7] R. Vincent, Ultramicroscopy 90 (2002) 135.
[8] Laurent Bruel, SPIE 4932 (2003) 590.
[9] Heinz H. Bauschke, Patrick L. Combettes, D. Russell Luke, J. Opt. Soc. Am. A 20 (2003) 1025.
[10] Gregory R. Brady, James R. Fienup, Opt. Express 14 (2006) 474.
[11] Fa. Zeng, Qiaofeng Tan, Yingbai Yan, Guofan Jin, Appl. Opt. 46 (2007) 6872.
[12] A.M. Maiden, J.M. Rodenburg, Ultramicroscopy 109 (2009) 1256.
[13] J.S. Lim, Two-dimensional Signal and Image Processing, Prentice-Hall, 1990.
[14] J.W. Goodman, Introduction to Fourier Optics, Roberts and Co. Publishers, New York, 2004, p. 55.
[15] S.I. Olsen, CVGIP: Graphical Model Image Process. 55 (1993) 319.