A Minimum Value Based Threshold Setting Strategy for Frequency Domain Interference Excision

Shuai Wang, Jian-Ping An, *Member, IEEE*, Ai-hua Wang, and Xiang-yuan Bu

***Abstract*—We present a robust threshold setting strategy with modest computational complexity for frequency domain interference excision in direct sequence spread-spectrum (DSSS) communication systems. The proposed strategy calculates the threshold by multiplying the minimum value of the averaged squared magnitude with a predefined scaling factor. An analytical framework for choosing the scaling factor is developed based on the principle of constant false-alarm rate (CFAR). Numerical results indicate that the new strategy outperforms existing ones in a wide range of partial-band jamming scenarios.**

***Index Terms*—DSSS, FFT, interference excision, threshold.**

I. Introduction

It is well known the immunity of a DSSS communication system against partial-band jammer (PBJ) can be significantly enhanced by a threshold-based excisor working in the frequency domain [1]–[8]. As the power ratio of the PBJ to the wide-band DSSS signal and noise is generally unknown, it is advisable to have the threshold adapted to some statistical characteristics of the magnitude spectrum. A commonly used method is to set the threshold T based on the arithmetic mean of the bins' amplitude, for example: $T = \theta \cdot \mu$, where μ is the mean value and θ is a fixed scaling factor [1]. However, when the jammer is powerful enough or has a large bandwidth, μ will be biased away from the noise floor and leave the threshold considerably higher than needed. Similar problems also exist with variations of this strategy such as $T = \mu + \theta \cdot \delta$, where δ represents the standard deviation [2]. To get improved robustness, one may substitute μ with the median value $\bar{\mu}$ and form the threshold by $T = \theta \cdot \bar{\mu}$[1]. But the calculation of $\bar{\mu}$ is cumbersome as it involves a computationally intensive sorting procedure. Another competitive method is the so-called consecutive mean excision algorithm (CME) and its forward version, which is known as FCME [3], [4]. Due to the diagnostic concept on which they are based, CME and FCME may provide even better performance than that of the median value based strategy at highly reduced complexity. The shortcoming is that they both work in an iterative fashion and the convergent process is usually quite time-consuming.

Sometimes even the threshold has been set correctly there might be problems [5]. Since the length of FFT is often quite limited to keep the excision algorithm computationally acceptable, the magnitude of a single Fourier coefficient record is not always an accurate depiction for the power spectral density of the received signal. As a result, some corrupted bins may temporarily fall below the threshold, while a few clean ones may falsely yield to threshold crossings, both degrading the effectiveness of the excisor. To cope with these problems, a novel scheme named LAD (Localization Algorithm based on Double-thresholding) has been proposed recently in [5] as a double-threshold extension of FCME. Unfortunately, LAD cannot offer enough improvement on the bit error rate (BER), according to the simulation results provided therein.

In this letter we present a new threshold setting strategy that operates in two steps. First, the squared magnitudes for several adjacent coefficient records are averaged to form a reliable spectrum estimate, and then the threshold is calculated by multiplying the minimum value of the averaged squared magnitude with a predefined scaling factor. We prefer the minimum value over other statistics because it is extremely insensitive to the PBJ and can be computed efficiently by a sequential comparison procedure. An analytical framework to determine the scaling factor is developed based on the CFAR principle, and exact relationship between the false alarm rate (FAR) and the scaling factor is derived using the theory of *order statistics* (OS) [9].

It is worthwhile mentioning that the original idea of using order statistics (including the minimum value) in threshold setting strategy should be traced back to the OS-CFAR techniques proposed for radar target detection [10]. However, an important difference is that the radar target detector typically tests only one cell at a time, with a threshold calculated by the leading and lagging referenced cells within the sliding window [10]; But in the context of frequency domain interference excision, once the threshold for the current coefficient record is obtained, it will be applied to all the cells (or bins) of interest in that record [1]–[8]. To accommodate this difference we have to introduce a new definition of FAR other than the one used in [10]. More discussion can be found in Section III-C.

II. System Model

The received signal, denoted by $r(n)$, is modeled as a complex baseband sequence comprising three additive components, namely the desired DSSS signal $s(n)$, the narrow-band jammer $x(n)$, and the white Gaussian noise $g(n)$, which for each n has independent and identically distributed in-phase and quadrature parts of zero mean and variance σ^2. Prior to the FFT transform, the input sample stream $r(n)$ is segmented into frames of length N and each of these frames is multiplied with a tapered window function $w(n)$ to mitigate frequency domain sidelobes. Sometimes the dual-path overlap-and-add approach is adopted to alleviate the SNR loss caused by windowing (see [3, Fig.2] and references therein). Since the two parallel paths share the same structure and work independently, the following discussion actually applies to them both.

Manuscript received September 17, 2009; revised October 13, 2009. First published October 30, 2009; current version published March 31, 2010. This work was supported by the National High Technology Research and Development Program (863 Program) of China under Grant 2009AA01Z232. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Min Dong.

The authors are with the School of Information Science & Electronics, Beijing Institute of Technology, Beijing, China (e-mail: wah@bit.edu.cn).

Digital Object Identifier 10.1109/LSP.2009.2035414

III. Threshold Setting Strategy

A. The Averaged Squared Magnitude and Its Calculation

Passing through an N-point FFT transform, the windowed data frames $r_{\mathrm{w}}^{(0)}(n), r_{\mathrm{w}}^{(1)}(n), \ldots r_{\mathrm{w}}^{(m)}(n), \ldots$ yield their Fourier coefficient records $R_{\mathrm{w}}^{(0)}(k), R_{\mathrm{w}}^{(1)}(k), \ldots R_{\mathrm{w}}^{(m)}(k), \ldots$. Here we denote the squared magnitude of the mth coefficient record by $A_{\mathrm{w}}^{(m)}(k)$, i.e., $A_{\mathrm{w}}^{(m)}(k) = |R_{\mathrm{w}}^{(m)}(k)|^2$, and the mth averaged squared magnitude $\overline{A}_{\mathrm{w}}^{(m)}(k)$ is defined as:

$$\overline{A}_{\mathrm{w}}^{(m)}(k) = \frac{1}{L}\sum_{l=0}^{L-1} A_{\mathrm{w}}^{(m-l)}(k), \quad k = 0, 1, \ldots, N-1. \quad (1)$$

Since $\overline{A}_{\mathrm{w}}^{(m)}(k)$ is much more stable than $A_{\mathrm{w}}^{(m)}(k)$ as a spectrum estimate, the reliability of the excisor will be improved if the decision "to zeroize or not" at each frequency can be made by a threshold test applied to $\overline{A}_{\mathrm{w}}^{(m)}(k)$[6], [7]. To reduce the computational overhead posed by calculating $\overline{A}_{\mathrm{w}}^{(m)}(k)$ straightly, let's rewrite (1) into a sequential form:

$$\overline{A}_{\mathrm{w}}^{(m)}(k) = \overline{A}_{\mathrm{w}}^{(m-1)}(k) + \frac{1}{L}\left[A_{\mathrm{w}}^{(m)}(k) - A_{\mathrm{w}}^{(m-L)}(k)\right]. \quad (2)$$

Once the current $A_{\mathrm{w}}^{(m)}(k)$ has been obtained, we can easily update $\overline{A}_{\mathrm{w}}^{(m-1)}(k)$ to $\overline{A}_{\mathrm{w}}^{(m)}(k)$ by Formula (2). Note that the value of L should not be too large, or the excisor will be rendered inefficient in tracking nonstationary interferences [6], [7].

B. Minimum Value Based Threshold Setting Strategy

As stated earlier, the desired DSSS signal $s(n)$ is considered as a baseband sequence, so if we suppose there are $2M+1$ bins within the band of the desired signal ($2M+1 < N$), then the indices of these bins can be assumed as follows:

$$k \in \Psi \triangleq \left[0, 1, \ldots, M\right] \cup \left[N-M, N-M+1, \ldots, N-1\right]. \quad (3)$$

Our excision algorithm, which takes advantage of the averaged squared magnitude $\overline{A}_{\mathrm{w}}^{(m)}(k)$, is formulated by [1]

$$\tilde{R}_{\mathrm{w}}^{(m)}(k) = \begin{cases} \begin{cases} R_{\mathrm{w}}^{(m)}(k), & \overline{A}_{\mathrm{w}}^{(m)}(k) \le T \\ 0, & \overline{A}_{\mathrm{w}}^{(m)}(k) > T \end{cases}, & k \in \Psi. \quad (4a) \\ R_{\mathrm{w}}^{(m)}(k), & k \notin \Psi. \quad (4b) \end{cases}$$

in which $\tilde{R}_{\mathrm{w}}^{(m)}(k)$ stands for the output of the excisor, and T represents the interference detecting threshold:

$$T = \theta \cdot \xi \quad (5)$$

where $\xi = \mathrm{Min}_{\{k\in\Psi\}}\left[\overline{A}_{\mathrm{w}}^{(m)}(k)\right]$, and θ is the scaling factor.

We choose the minimum value because it has two unique merits which are indispensable to the design of robust and low-complexity threshold setting strategy. First, the minimum value as a quantile is less susceptible to the PBJ than statistical moments such as arithmetic mean or standard deviation [10]. Second, it only takes $2M$ comparisons to find the minimum value among $2M+1$ candidates, in contrast to other quantiles such as the median value of which the computational complexity is on the order of $M\log(M)$ at best [11]. Also it should be noted that unlike CME's family members, the proposed strategy does not involve any iterative operations. This is a significant advantage as we are considering online techniques for which processing speed is highly demanded.

C. Scaling Factor θ

Here we will present a CFAR framework by which the scaling factor θ can be found in an analytical way. As mentioned before, the proposed strategy is different from traditional OS-CFAR techniques in that all the $2M+1$ bins of interest are tested by the same threshold. So if the system is jammer-free, a false alarm will happen when *anyone* of these bins crosses the threshold, which implies the following definition of FAR shall make sense:

$$P_{\mathrm{FA}} = \Pr\left\{\bigcup_{k\in\Psi}\left[\overline{A}_{\mathrm{w}}^{(m)}(k) > T\right]\right\} = \Pr\left\{\eta > \theta\cdot\xi\right\} \quad (6)$$

where $\eta = \mathrm{Max}_{\{k\in\Psi\}}\left[\overline{A}_{\mathrm{w}}^{(m)}(k)\right]$. Clearly, the above defined FAR is distinct from its counterpart in OS-CFAR [10, eq. (11)], as only one tested cell is considered therein. Furthermore, it should not be confused with the *clean sample rejection rate* (CSRR) mentioned in [3]. To be specific, the FAR defined here is the overall probability of wrong excision, while CSRR reflects the *percentage* of mistakenly detected bins in a statistical sense. In [3], [4] a CSRR based framework has been used to determine the scaling factors for CME and FCME, actually it can also be applied to mean or median value based strategies. However, the same framework may not be readily adopted in our problem as it relies on an unbiased estimate for the noise floor but the minimum value, unlike the mean or median value, is not.

Now our aim is to find out a proper value for θ so that P_{FA} can be kept at a satisfactorily low level, say 0.01 or smaller. If there is no jammer, the received waveform is composed by the desired signal and AWGN. Here we further assume that noise is the dominating component, i.e., the SNR at the input of the receiver is low[2], then the following equation holds:

$$A_{\mathrm{w}}^{(m)}(k) \approx \left|\sum_{n=0}^{N-1} g^{(m)}(n)\mathrm{w}(n)e^{-j(2\pi/N)kn}\right|^2 = \left|G_{\mathrm{w}}^{(m)}(k)\right|^2 \quad (7)$$

where $g^{(m)}(n)$ represents the mth noise segment. As described in Section II, the noise samples $g^{(m)}(n)$ are complex Gaussian-distributed I.I.Ds with zero mean and variance σ^2, which implies $A_{\mathrm{w}}^{(m)}(k)/\sigma^2 E_{\mathrm{w}}$ and $L\cdot\overline{A}_{\mathrm{w}}^{(m)}(k)/\sigma^2 E_{\mathrm{w}}$ shall follow central chi-square distribution with 2 and $2L$ degrees of freedom, respectively ($E_{\mathrm{w}} = \sum_{n=0}^{N-1}|\mathrm{w}(n)|^2$). Therefore, the distributional properties of $\overline{A}_{\mathrm{w}}^{(m)}(k)$ for each k can be formulated as

$$\text{CDF}: \quad F(x) = \kappa\Big(\frac{L}{\sigma^2 E_{\mathrm{w}}}x\Big) = \int_0^{(L/\sigma^2 E_{\mathrm{w}})x}\kappa(t)dt \quad (8a)$$

$$\text{PDF}: \quad f(x) = \frac{L}{\sigma^2 E_{\mathrm{w}}}\kappa\Big(\frac{L}{\sigma^2 E_{\mathrm{w}}}x\Big) \quad (8b)$$

where $\kappa(x)$ and $\kappa(x)$ represent the CDF and PDF of central chi-square distribution with $2L$ degrees of freedom. Invoking the theory of order statistics [9, eq. (2.2.1)], the joint PDF of η and ξ can be derived easily:

$$h(\eta,\xi) = \begin{cases} 2M(2M+1)f(\eta)f(\xi)\left[F(\eta) - F(\xi)\right]^{2M-1}, \\ \qquad\qquad \eta > \xi > 0 \\ 0, \qquad\qquad \text{elsewhere.} \end{cases} \quad (9)$$

[1]As PBJs are detrimental to the DSSS system only when they fall into the band of interest, bins not belonging to Ψ are all left unchanged in (4b).

[2]This assumption is reasonable enough as typical DSSS systems are able to (sometimes even expected to) work in a low-SNR environment.

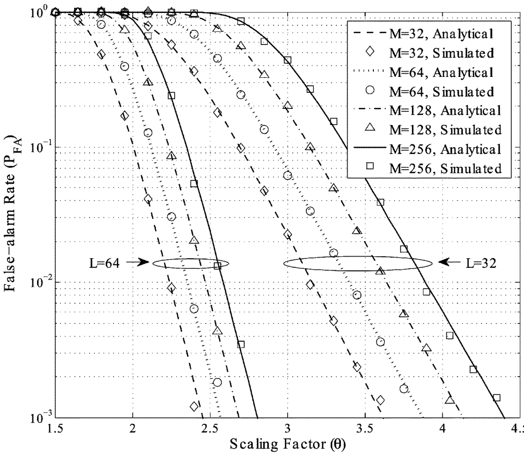

Fig. 1. Relationship between $P_{\rm FA}$ and θ over a few combinations of L and M. Simulations are conducted with SNR assumed to be -10 dB.

Then a more explicit expression for $P_{\rm FA}$ can be found in (10), shown at the bottom of the next page. According to (8a), $F(0) = 0$. Substituting (8) into (10) and replacing $(L/\sigma^2 E_\omega)\eta$ with a new variable y, finally we have

$$P_{\rm FA} = 1-(2M+1)\int_0^{+\infty} \kappa(y)\left[\kappa(y) - \kappa\left(\frac{y}{\theta}\right)\right]^{2M} dy. \quad (11)$$

Equation (11) clearly shows that $P_{\rm FA}$ is a function of L, M, and θ and it is independent of the noise variance σ^2. Via numerical integration, we were able to calculate the value of $P_{\rm FA}$ and illustrate it in Fig. 1. It is observed that the analytical results agree well with that of Monte-Carlo simulations if the SNR is low enough (-10 dB). We have also determined several possible values for θ, as listed in Table I. Note that the proposed (11) holds for arbitrary window functions, as we didn't impose any restriction on $w(n)$ throughout the analysis.

To finish this section, we point out that in statistics the ratio of η to ξ is often termed as *extremal quotient*, of which the CDF has been derived for Gamma distributed samples by Izenman [12, eq. (18)] in 1976. Since chi-square distribution is a special case of Gamma distribution, Izenman's result, which is more general mathematically, can be used as an alternative to evaluate (6). However, as for chi-square distribution, the proposed (11) is a perfect equivalent to [12, eq. (18)] which is simpler and thus more tractable for numerical computations.

IV. Numerical Results

The performance of the proposed strategy is evaluated and compared with that of others through computer simulations in which the QPSK-DSSS signal is generated by a 255-chip m sequence, and the PBJ is either modeled as a single tone or another PN signal with a chip rate lower than that of the desired signal. Without loss of generality, we consider $N = 4096$ and $M = 256$ for all the algorithms simulated[3], with a Hanning window adopted for the 50% overlap-and-add approach, resulting in overall distortion loss equal to 0 dB [8]. Regarding the proposed strategy, the scaling factor is set to 2.654 ($P_{\rm FA} = 0.005$), and the squared magnitude is averaged for 64 times ($L = 64$). The two scaling factors of LAD are set to 6.901 ($\rm CSRR = 0.001$) and 2.995 ($\rm CSRR = 0.05$), as recommended in [5][4]. For all the other single-threshold algorithms, the scaling factors are set to make CSRR= 0.005. In the following description, we will employ a new symbol α to denote the percentage of signal bandwidth corrupted.

TABLE I
SCALING FACTORS (θ) CORRESPONDING TO DIFFERENT LEVELS OF FAR ($P_{\rm FA}$)

	$M = 64$, $L = 32$	$M = 256$, $L = 32$	$M = 64$, $L = 64$	$M = 256$, $L = 64$
$P_{\rm FA} = 1.0 \times 10^{-2}$	$\theta = 3.399$	$\theta = 3.892$	$\theta = 2.355$	$\theta = 2.586$
$P_{\rm FA} = 0.5 \times 10^{-2}$	$\theta = 3.543$	$\theta = 4.046$	$\theta = 2.421$	$\theta = 2.654$
$P_{\rm FA} = 1.0 \times 10^{-3}$	$\theta = 3.870$	$\theta = 4.392$	$\theta = 2.568$	$\theta = 2.805$

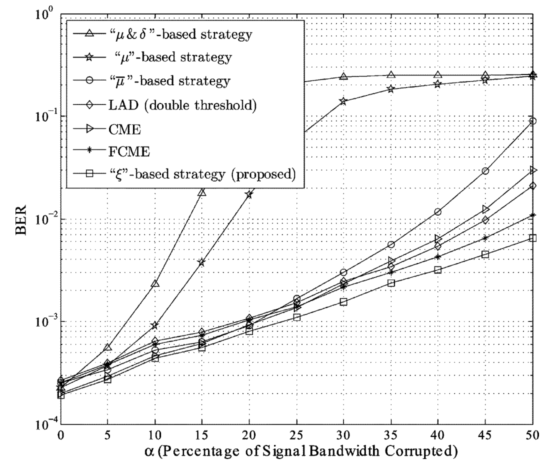

Fig. 2. BER versus α. $E_b/N_0 = 8$ dB, $\rm JSR = 30$ dB. Simulations repeated for 5×10^5 QPSK symbols. $\alpha = 0$ denotes sinusoid jammer.

In Fig. 2, the impact of jammer's bandwidth on various algorithms is illustrated. We can see the proposed strategy exhibits the finest robustness and gives significant advantage over other noniterative schemes, due to the insensitivity of minimum value to the PBJ. It is observed that iterative schemes such as FCME also perform well, especially when $\alpha \le 30\%$.

In Fig. 3, the BER performances of different strategies are plotted against JSR. It is interesting that most strategies do not perform so well when the JSR is between 10 dB to 25 dB. As

[3]Which means oversampling technique is used, i.e., the desired DSSS signal is sampled at a frequency eight times of its bandwidth.

[4]To achieve the same CSRR, the scaling factors used here are actually different from that of [5], as in our simulations the bins' squared magnitude is adopted but in [5] the authors considered their magnitude.

$$P_{\rm FA} = \Pr\{\eta > \theta\cdot\xi\} = \int_0^{+\infty} d\eta \int_0^{\eta/\theta} h(\eta,\xi)\, d\xi$$
$$= (2M+1)\int_0^{+\infty} f(\eta)\left\{\left[F(\eta) - F(0)\right]^{2M} - \left[F(\eta) - F\left(\frac{\eta}{\theta}\right)\right]^{2M}\right\} d\eta. \quad (10)$$

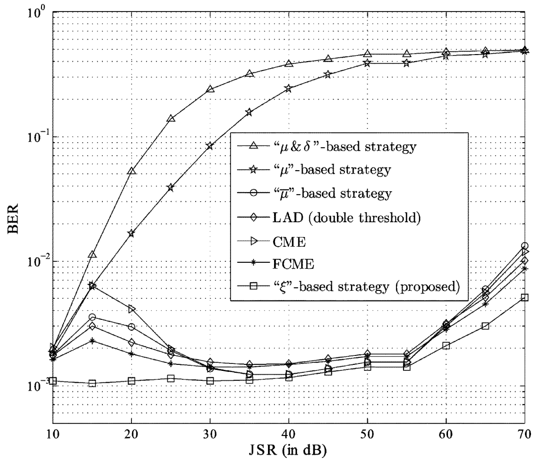

Fig. 3. BER versus JSR. $E_b/N_0 = 8$ dB, $\alpha = 25\%$. Simulations repeated for 5×10^5 QPSK symbols.

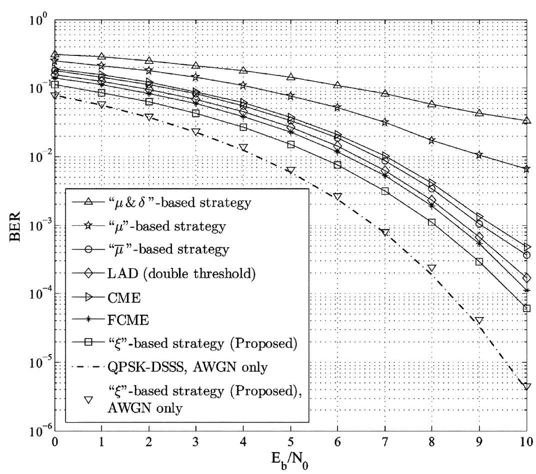

Fig. 4. BER versus E_b/N_0. $\alpha = 25\%$, $\mathrm{JSR} = 20$ dB (if not otherwise noted). Simulations repeated for 5×10^7 QPSK symbols.

mentioned in the Introduction, the reason is that in a medium interfered situation the corrupted bins do not always exceed the threshold, as the FFT is calculated on a limited observation interval. We can see that the proposed strategy which takes advantage of the averaged squared magnitude is hardly affected by this phenomenon. When the JSR goes to 60 dB or higher, the sidelobes of the jammer must be excised or the residual power left therein will still be harmful to the system. Again by virtue of averaging, the proposed strategy ensures reliable excision of the sidelobes and outperforms other strategies. However, performance degradation is inevitable as the percentage of excised bins substantially exceeds α.

In Fig. 4 we compare the BER performance of the proposed strategy with that of others at different E_b/N_0. The superiority of the proposed strategy is proved again. It is also observed that additional distortion loss caused by the proposed strategy is almost negligible if the system is jammer-free, as long as P_{FA} is kept low enough (here we have $P_{\mathrm{FA}} = 0.5 \times 10^{-2}$).

Finally, we note that according to our simulation results (not shown here for space reasons), the performance gap between other robust algorithms (such as the median value based strategy or CME's family members) and the proposed strategy diminishes evidently if equal amount of averaging is applied, especially when the JSR is medium or extremely high. We have also observed that the two mean value based strategies benefit far less from averaging.

V. Conclusions

A low-complexity threshold setting strategy based on the minimum value of the averaged squared magnitude has been presented for frequency domain interference excision and validated by simulations conducted under different PBJ scenarios. Note that the performance enhancement of the proposed strategy partially comes at the cost of reduced adaptability to nonstationary interferences, depending on the number of coefficient records involved in calculating $\overline{A}_{\mathrm{w}}^{(m)}(k)$. For future research direction, the transplant of this strategy into other transform domains where some wideband nonstationary jammers are well localized [13], [14] could be studied. Another potential limitation is that the calculation of $\overline{A}_{\mathrm{w}}^{(m)}(k)$ calls for additional memory to keep $\left\{A_{\mathrm{w}}^{(l)}(k) \middle| m-1 \le l \le m-L\right\}$. In practice, this problem can be somewhat alleviated by storing the bins belonging to Ψ only and abandoning the others.

References

[1] A. Pouttu, J. Juntti, and T. Kumpumäki, "Adaptive transform domain interference suppression in a hybrid DS/FH-system," in *Proc. IEEE Int. Symp. Spread Spectrum Techniques and Applications*, Sun City, South Africa, Sep. 2–4, 1998, vol. 1, pp. 351–355.

[2] P. T. Capozza, B. J. Holland, T. M. Hopkinson, and R. L. Landrau, "A single-chip narrow-band frequency-domain excisor for a Global Positioning System (GPS) receiver," *IEEE J. Solid-State Circuits*, vol. 35, pp. 401–411, Mar. 2000.

[3] H. Saarnisaari, "Consecutive mean excision algorithms in narrowband or short time interference mitigation," in *Proc. Position Location and Navigation Symp.*, Monterey, CA, Apr. 26–29, 2004, vol. 1, pp. 447–454.

[4] J. Vartiainen, J. J. Lehtomäki, H. Saarnisaari, and P. Henttu, "Estimation of signal detection threshold by CME algorithms," in *Proc. IEEE Vehicular Technology Conf.*, Milan, Italy, May 17–19, 2004, vol. 3, pp. 1654–1658.

[5] J. Vartiainen, J. J. Lehtomäki, and H. Saarnisaari, "Double-threshold based narrowband signal extraction," in *Proc. IEEE Vehicular Technology Conf.*, Stockholm, Sweden, May/Jun. 30–1, 2005, vol. 2, pp. 1288–1292.

[6] R. F. Guertin, R. C. Johnson, and D. A. Wamsley, "Monte Carlo analysis of matched-filter performance after narrowband interference suppression," in *Proc. IEEE Military Communications Conf.*, Monterey, CA, Sep./Oct. 30–3, 1990, vol. 3, pp. 1161–1166.

[7] J. Vartiainen, J. J. Lehtomäki, and H. Saarnisaari, "Detection of DS-CDMA signals under narrowband interference," in *Proc. XXVIII Convention on Radio Science and IV Finnish Wireless Communications Workshop (URSI/FWCW'03)*, Oulu, Finland, Oct. 16–17, 2003.

[8] J. A. Young and J. S. Lehnert, "Performance metrics for windows used in real-time DFT-based multiple-tone frequency excision," *IEEE Trans. Signal Process.*, vol. 47, no. 3, pp. 800–812, Mar. 1999.

[9] H. A. David, *Order Statistics*. New York: Wiley, 1970.

[10] H. Rohling, "Radar CFAR thresholding in clutter and multiple target situations," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 19, pp. 608–621, Jul. 1983.

[11] W. Press, W. Vetterling, S. Teukolsky, and B. Flannery, *Numerical Recipes in C*, 2nd ed. New York: Cambridge Univ. Press, 1992.

[12] A. J. Izenman, "On the extremal quotient from a Gamma sample," *Biometrika*, vol. 63, no. 1, pp. 185–190, Apr. 1976.

[13] H. Shen and A. Papandreou-Suppappola, "Wideband time-varying interference suppression using matched signal transforms," *IEEE Trans. Signal Process.*, vol. 53, no. 7, pp. 2607–2612, Jul. 2005.

[14] S. Aromaa, P. Henttu, and M. Juntti, "Transform-selective interference suppression algorithm for spread-spectrum communications," *IEEE Signal Process. Lett.*, vol. 12, pp. 49–51, Jan. 2005.