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# New Predictive Control Scheme for Networked Control Systems

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**Abstract** This paper is concerned with the design of networked control systems with random network-induced delay and data dropout. It presents a new control scheme, which is termed networked predictive control with optimal estimation. Based on Multirate Kalman Filtering, the measured data which are out of sequence or delayed can be used to improve the precision of estimation. The control prediction generator provides a set of future control predictions to make the closed-loop system achieve the desired control performance and the compensator removes the effects of the network transmission with time delay and data dropout. Simulation results are presented to illustrate the effectiveness of the control strategy via comparing with control schemes without any compensation for the network.

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## 1 Introduction

With the development of network technology, an increasing number of network technologies have been applied to traditional control systems and the networked control and data fusion technology has become a popular topic which has been extensively studied under various assumptions and scenarios (for example, see [6, 10, 14, 20, 21, 26]). There has been a growing interest in the design of controllers based on the networks, such as traffic, communication, aviation and spaceflight (see [1, 4, 22, 24]). Particularly, the rapid rising of Internet makes Internet based control systems accomplish remote monitoring and adjustment over long distance. This makes the control systems benefit from the ways of retrieving data and reacting to plant fluctuations from anywhere around the world at any time (for example, see [2, 12, 13, 19]). In a networked control systems (NCS), the plant, controller, sensor, actuator and reference command are connected through a network. As the structure of networked control systems is different from that of tradition control systems, there exist various specific problems in networked control systems, for example, network-induced time delay, loss of data packets, out of sequence, bandwidth constraints, network security and safety [23].

Several methodologies have been reported in literature to handle with the problems mentioned above in networked systems. Among these papers, two basic control strategies are applied when time delay and packet dropping happens, they are zero-input scheme by which the actuator output is set to zero when the current control packet is delayed or lost, and hold-input scheme which implies the previous control input is used again when the current control packet delays or drops. The further research is proposed in [15] by directly comparing the two control methods. In [25], the stability problem of closed-loop NCS in the presence of network delays and data packet drops has been addressed under an assumption that the network-induced delay is less than the sampling period. To reduce network traffic load, a sampled-data NCS scheme combining the model-based control methods has been presented and some necessary and sufficient conditions for global exponential stability of the closed-loop systems via state/output feedback, without/with network delays have been established in [11]. Since $H_\infty$ control can make a well established connection between the performance index being optimized and performance requirements encountered in practical situations, a method is proposed in [16] where the packet dropouts and channel delays are modeled as Markov Chains with the usual assumption that all the transition probabilities are completely accessible. The work of [9] presents a novel control technique combining modified Model Predictive Control (MPC) and modified Smith Predictor to guarantee the stability of the networked control systems. Especially, the key point in this paper is that the future control sequence is used to compensate for the forward communication time delay and predictor is responsible for compensating the time delay in the backward channel. Paper [17] has proposed the method to optimize the estimation of the states and to compensate for the time delays which occur in the channel from the sensor to the controller. In [8], a novel networked predictive control method is proposed to deal with NCSs with random time delay existing in both feedback and forward channels. The dynamics of the plant is explicitly used to derive a sequence of forward control predictions, which are sent to the actuator simultaneously and the actuator chooses the appropriate one to compensate for the delays.

Although much research work has been done in networked control systems, however, many of those results simply treat the NCS as a traditional controlled object and a controller, which are connected with each other over unreliable networks. The dynamics of networks feature, like time delays and data dropouts, is modeled separately. In order to solve the problem, Markovian jump system can be used to model the random time delay and data dropout associated with the controlled object. Moreover, most work has also ignored another very important feature of networked control systems. This feature is that the communication networks can transmit a packet of data at the same time, which does not appear in traditional control systems. Therefore, in this paper, we make full use of this networks feature and propose a new networked predictive control scheme with the optimal estimation, which overcomes the effects caused by network time delay and data dropout modeled as Markov chain. In the controller, multirate Kalman filter, which is modified from the method presented in the [17], is proposed to overcome the time delay and data dropout in the feedback channel. A buffer is set to store the data coming from the sensor and the length of the buffer can be set to store the data in this step and before. Based on multirate Kalman Filtering, the signal from the sensor is optimized to estimate the state and the generator produces a series control signals by using the estimate of the state. Then, the actuator will receive a series control signals from the controller through network and the actuator will chose the best one from the series control signals to control the plant.

The paper is organized as follows. Section 2 presents a novel networked predictive control scheme and a optimal estimation method for the state estimate such that the closed-loop system is asymptotically stable. The linear inverted pendulum of a networked control system is modeled with random time delay and packet dropouts in Sect. 3. Numerical simulations are presented in Sect. 4. Some conclusion remarks are given in Sect. 5.

## 2 Networked Predictive Control for Systems

To overcome unknown network transmission with time delay, data dropout and out of sequence, a multirate Kalman filter and a networked predictive control scheme are proposed. The multirate Kalman filter is applied to estimate the state based on the measurements with time delay, data dropout and out of sequence in the feedback channel. The networked predictive control mainly consists of a control prediction generator and a network delay and data dropout compensator. The control prediction generator is designed to generate a set of future control predictions based on the optimal state estimate at each time instant. The network time delay and data dropout compensator is used to compensate for the unknown random networked delay and data dropout. This networked predictive control system (NPCS) structure is shown in Fig. 1. This paper mainly concerns about the random transmission delay, data dropout and out of sequence existing in both feedback and forward channels in NCSs.

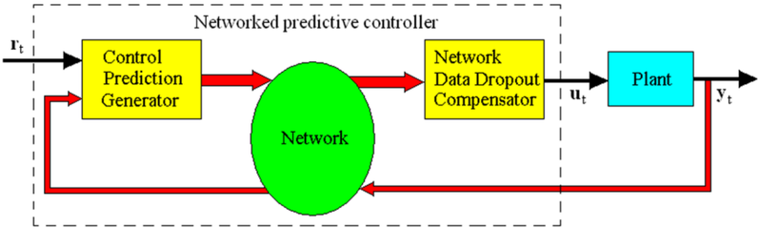



**Fig. 1** The networked predictive control system

### 2.1 System Model

Consider a MIMO discrete time system described in the state-space form

$$\begin{aligned} x_{t+1} &= Ax_t + Bu_t + w_t \\ y_t &= Cx_t + v_t \end{aligned} \tag{1}$$

where $x_t \in R^n$, $u_t \in R^m$, and $y_t \in R^l$ are the system state, input, and output vectors, respectively. The noise process $\{w_t\}$ and $\{v_t\}$ are white, zero-mean, uncorrelated, and have known covariance matrices $Q_1$ and $R_1$. $A$, $B$, $C$ are matrices of appropriate dimensions. For the simplicity of stability analysis, it is assumed that the reference input of the system is zero and the following assumptions are made.

The arrival of the observation at time as a binary random variable $\gamma_t$, with probability distribution $P(\gamma_t = 1) = \lambda_t, \lambda_t \in (0, 1)$, and with $\gamma_t$ independent of $\gamma_s$ if $t \neq s$.

Assumption 1: The pair, $(A, B)$, is completely controllable, and the pair, $(A, C)$ is completely observable.

Assumption 2: The number of consecutive data dropouts must be less than $N_1$ (a positive integer).

Assumption 3: The upper bound of the network delay is not greater than $N$ (a positive integer).

*Remark 1* In a real NCS, if the data packet does not arrive at a destination in a certain transmission time, it means this data packet is lost, based on the commonly used network protocols. From the physical point of view, it is natural to assume that only a finite number of consecutive data dropouts can be tolerated in order to avoid that the NCS becomes open-loop. Thus, the number of consecutive data dropouts should be less than a finite number $N_1$.

Some notations are defined as follows:

$$\begin{aligned} Y_t &= [y_0, \dots, y_t]^T \\ \Gamma_t &= [\gamma_0, \dots, \gamma_t]^T \\ \hat{x}_{t|t} &= E[x_t | Y_t, \Gamma_t] \\ P_{t|t} &= E\big[(x_t - \hat{x}_{t|t})(x_t - \hat{x}_{t|t})^T | Y_t, \Gamma_t\big] \end{aligned} \tag{2}$$

$$
\begin{aligned}
\hat{x}_{t+1|t} &= E[x_{t+1}|Y_t, \Gamma_t] \\
P_{t+1|t} &= E\big[(x_{t+1} - \hat{x}_{t+1|t})(x_{t+1} - \hat{x}_{t+1|t})^T|Y_t, \Gamma_t\big] \\
\hat{y}_{t+1|t} &= E[y_{t+1}|Y_t, \Gamma_t]
\end{aligned}
$$

### 2.2 Network-Induced Delay and Data Dropout

As we know, more and more control systems use the network to transmit control signal. With the network used in the control system, there are various factors introduced as a result of the addition of the communication network, such as time delay, data dropout and time disorder which have to be considered for ensuring the desired performance of the NCS.

In this paper, we assume that the measurement data $y_t$ from the sensor is sent across a network with delay and data dropout to the controller. Time delays and data dropout occur in a networked control system due to the addition of a network. This delay can destabilize a system designed or can degrade the system performance. Network delay can be further subdivided into sensor-to-controller delay, controller-to-actuator delay, and the computational delay in the controller. In this paper, the sensor-to-controller delay and controller-to-actuator delay will be considered and we assume that the time of the data computing in the controller is very small. So there will not consider the computational delay.

The problem of network time delay over a network has been proposed in [3, 5], and some types of model had already established. In this paper, a simple approach of random time delay that we set up the Markov model, for its accurate and sample. To construct a Markov chain in discrete time, the time delay is a stochastic process with an upper bound $N$ ($N$ is a positive integer) which is multiples of the sampling period of the system.

### 2.3 The Predictive Control Scheme to Compensate for Time Delay and Data Dropout in the Forward Channel

The Kalman Filter (**KF**) based state observer is designed as

$$
\begin{aligned}
\hat{x}_{t|t-1} &= A\hat{x}_{t-1|t-1} + Bu_{t-1} \\
\hat{x}_{t|t} &= \hat{x}_{t|t-1} + \gamma_t K_t(y_t - C\hat{x}_{t|t-1}) \\
P_{t|t} &= P_{t|t-1} - \gamma_t K_t C P_{t|t-1} \\
K_t &= P_{t|t-1}C^T\big(CP_{t|t-1}C^T + R\big)^{-1}
\end{aligned}
\tag{3}
$$

where $\hat{x}_{t|t-1} \in R^n$ and $u_{t-1} \in R^m$ are the one-step ahead state prediction and the input of the observer at time $t-1$ and $\gamma_t$ indicates the signal is received or not by controller at time $t$. If the $\gamma_t = 1$, it means the signal received by the controller at time $t$, otherwise the $\gamma_t = 0$ means the signal is not received by the controller at time $t$. $P_{t|t-1}$ is the solution of the following modified Riccati equation [18]:

$$\begin{aligned} P_{t|t-1} &= AP_{t-1|t-2}A^T \\ &\quad + Q - \gamma_t AP_{t-1|t-2}C^T\left(CP_{t-1|t-2}C^T + R\right)^{-1}CP_{t-1|t-2}A^T \end{aligned} \tag{4}$$

The state predictions at time $t$ are constructed as

$$\begin{aligned} \hat{x}_{t+1|t} &= A\hat{x}_{t|t-1} + Bu_t + \gamma_t AK_t(y_t - C\hat{x}_{t|t-1}) \\ \hat{x}_{t+2|t} &= A\hat{x}_{t+1|t} + Bu_{t+1|t} \\ &\vdots \\ \hat{x}_{t+N|t} &= A\hat{x}_{t+N-1|t} + Bu_{t+N-1|t} \end{aligned} \tag{5}$$

Assume that the controller is of the form

$$u_t = u_{t|t} = L_t\hat{x}_{t|t} \tag{6}$$

where $L_t \in R^{m\times n}$ is the state feedback control matrix to be determined using modern control theory. Then, the control predictions are generated by

$$u_{t+k|t} = L_t\hat{x}_{t+k|t}, \quad \text{for } k = 0, 1, 2, \ldots, N \tag{7}$$

Thus, it follows from (5) that

$$\hat{x}_{t+k|t} = (A + BL_t)^{k-1}\hat{x}_{t+1|t} \tag{8}$$

Based on (3), (5) and (7), it can be shown that

$$\begin{aligned} \hat{x}_{t+k|t} &= (A + BL_t)^{k-1}\left[A\hat{x}_{t|t-1} + Bu_t + \gamma_t AK_t(y_t - Cx_{t|t-1})\right] \\ &= (A + BL_t)^{k-1}\left[\left(A + BL_t - \gamma_t(AK_t - BL_tK_t)C\right)\hat{x}_{t|t-1} \right. \\ &\quad \left. + \gamma_t(AK_t + BL_tK_t)Cx_t\right] \end{aligned} \tag{9}$$

and

$$\begin{aligned} u_{t+k|t} &= L_t\hat{x}_{t+k|t} \\ &= L_t(A + BL_t)^{k-1}\left[\left(A + BL_t - \gamma_t(AK_t - BL_tK_t)C\right)\hat{x}_{t|t-1} \right. \\ &\quad \left. + \gamma_t(AK_t + BL_tK_t)Cx_t\right] \end{aligned} \tag{10}$$

In order to compensate for the network transmission delay and data loss, a network delay compensator is proposed. A very important characteristic of the network is that it can transmit a set of data at the same time. Thus, it is assumed that predictive control sequence at time $t$ is packed and sent to the plant side through a network. The network delay compensator chooses the latest control value from the control prediction sequences available on the plant side. For example, if the following predictive control sequences are received on the plant side:

$$\left[u_{t|t}^T, u_{t+1|t}^T, \ldots, u_{t+N-1|t}^T, \ldots, u_{t+N|t}^T\right]^T \tag{11}$$

On the side of actuator, $u_{t|t}$ is used as input to the actuator. If new control package containing control predictive sequences are delayed or dropped at the next step is dropped, one step control prediction $u_{t+1|t}$ will be used as control input to the actuator. Then, if new packages of control predictive sequences are received, the first control input will be used as the input to the actuator. Otherwise, $u_{t+2|t}$ will be used as input to the actuator, ... and so on. Based on the assumption, it is reasonable that there are at least $N$ time consecutive time delay or data dropout. The stochastic stability criteria of the closed-loop networked predictive control systems can be analytically derived based on the methods in previous work [8]. In fact, using the networked predictive control scheme presented in this section, the control performance of the closed-loop system with delay and data dropout is very similar to that of the closed-loop system without delay and data dropout.

### 2.4 Multirate Kalman Filter to Compensate for Time Delay and Data Dropout in the Feedback Channel

In this subsection, we present a multirate Kalman filter, which is modified from the method proposed in [17], to overcome the time delay and data dropout in the feedback channel. It is called Multirate Kalman Filter in this paper since it works faster than sensor and actuator. A buffer is set to store the received data from the sensor. Figure 2 shows the structure of the network system with a buffer in the controller.

We assume that the length of the buffer is set to be $D$, so the controller discards the data, which are delayed by $D$ times or more, from the sensor. For example, if $y_{t-D}$ is not received by the controller before $t$, then even if $y_{t-D}$ arrives at $t$ or at a later time, it will be discarded by the controller. In Sect. 2.3, the form of the Kalman Filter is shown in (3). Let $\gamma_t$ be the indicator function for $y_t$ at time $k, t \le k$, i.e., $\gamma_t = 1$ if $y_t$ arrives at $k$ and $\gamma_t = 0$ otherwise. Depending on whether $y_t$ is received or not, i.e., $\gamma_t = 1$ or 0. $(\hat{x}_{t|t}, P_{t|t})$ is known to be computed by Kalman Filter (**KF**) through (3). We write the $(\hat{x}_{t|t}, P_{t|t})$ in compact form as follows.

$$(\hat{x}_{t|t}, P_{t|t}) = \mathbf{KF}(\hat{x}_{t-1|t-1}, P_{t-1|t-1}, \gamma_t, y_t, u_{t-1}) \tag{12}$$

which represents (3).

A multirate Kalman filter method is presented in this part to estimate the state with time delay in the feedback channel. As $y_{t-i}$ may arrive at time $t$ due to the delays or out of sequence introduced by the network, we can improve the control quality by recalculating $\hat{x}_{t-i|t-i}$ utilizing the new available measurement $y_{t-i}$. Once $\hat{x}_{t-i|t-i}$ is updated, we can update $\hat{x}_{t-i+1|t-i+1}$ in a similar fashion. The following summarizes the estimation process.

Let $y_{t-i}, i \in [0, D-1]$ be the oldest measurement received by the estimator at time $t$. Then $\hat{x}_{t|t}$ is computed by $i+1$ **KF**s as

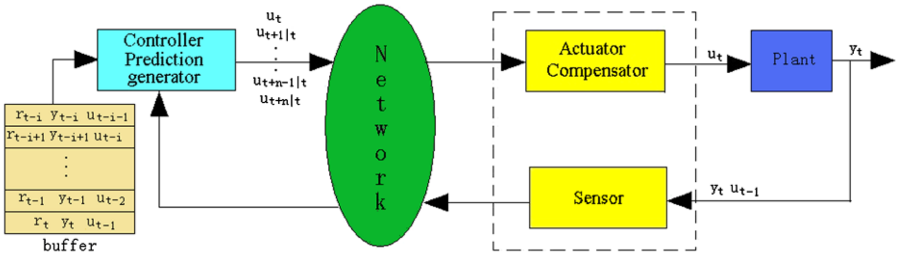



**Fig. 2** The structure of the networked control system

$$
\begin{aligned}
(\hat{x}_{t-i|t-i}, P_{t-i|t-i}) &= \mathbf{KF}(\hat{x}_{t-i-1|t-i-1}, P_{t-i-1|t-i-1}, 1, y_{t-i}, u_{t-i-1}) \\
(\hat{x}_{t-i+1|t-i+1}, P_{t-i+1|t-i+1}) &= \mathbf{KF}(\hat{x}_{t-i|t-i}, P_{t-i|t-i}, \gamma_{t-i+1}, y_{t-i+1}, u_{t-i}) \\
&\vdots \\
(\hat{x}_{t-1|t-1}, P_{t-1|t-1}) &= \mathbf{KF}(\hat{x}_{t-2|t-2}, P_{t-2|t-2}, \gamma_{t-1}, y_{t-1}, u_{t-2}) \\
(\hat{x}_{t|t}, P_{t|t}) &= \mathbf{KF}(\hat{x}_{t-1|t-1}, P_{t-1|t-1}, \gamma_t, y_t, u_{t-1})
\end{aligned}
\tag{13}
$$

With the new $\hat{x}_{t|t}$, the controller can give out the control signal $u_{t|t}$ and the predictive control sequences $u_{t+1|t}, \ldots, u_{t+N|t}$ and send it to the actuator through network. In the controller, if the controller receive the new data $y_{t-i}$ at time $t$ and $i \in [0, D-1]$, the controller will store the new data in the corresponding cell of the buffer and set the corresponding $\gamma_{t-i}$ to 1. Then the controller will find the oldest measurement stored in the buffer with $\gamma_{t-i} = 1$ and calculate the $\hat{x}_{t|t}$ according to (13). Finally, the controller can calculate the predictive state sequences $x_{t+1|t}, x_{t+2|t}, \ldots, x_{t+N|t}$ according to (5) and get the predictive signal sequences $u_{t|t}, u_{t+1|t}, \ldots, u_{t+N|t}$ which are sent to the actuator to control the plant. After the actuator receives the predictive control sequences, the compensator will choose the best control signal from the sequences according to the law which have been proposed in Sect. 2.3. Using this method, we can compensate for the time delay and data dropout, which happen in the both forward and feedback channels, and optimize the performance of the system.

*Remark 2* Some researchers adopt an algorithm, which discards all measurements data packets out of sequence, for example, see [7]. The observer estimates the state based on the intermittent measurements. This method is time-efficient, low-computational-burden, but it does not take full advantage of all the measurements received by the observer, since parts of them is abandoned duo to the network-induced time delay. In this paper, the received measurements out of sequence is reused to update the estimate of state, if the their delay times do not exceed the predefined buffer length $D$. The proposed method maintains the advantage of time-efficiency, moreover, improves the utilization ratio of the networks resource. However, it indeed needs much more computational capacity than the previous algorithm, then increases the cost of hardware.

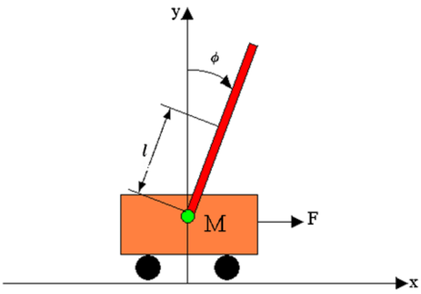



**Fig. 3** Structure of the inverted pendulum

**Table 1** Parameters used in the linear inverted pendulum model

| Symbol | Value | Meaning |
|---|---|---|
| $m$ | 0.109 kg | Mass of the pendulum rod |
| $M$ | 1.096 kg | Mass of the cart |
| $l$ | 0.25 m | Length from the pendulum's axis to its centroid |
| $b$ | 0.1 N/m/s | The friction coefficient of the cart |
| $I$ | 0.0034 kg m m | Inertia of the rod |
| $g$ | 9.81 $\mathrm{m/s^2}$ | Gravitational constant |

## 3 Choose the Model of System Structure

In this section, in order to show the effectiveness of the proposed method, we adopt an inverted pendulum and its linearized model. The network-induced delay and data dropout is modeled by Markov Chain.

### 3.1 Inverted Pendulum Model

Consider a single inverted pendulum, it contains two parts, a vertical level and a cart. The structure of the inverted pendulum is shown in Fig. 3. Let $m$ denote the pendulum's mass and $l$ denote the length from the pendulum's axis to its centroid, while the cart's mass is $M$. The pendulum's angle is denoted by $\phi$, the cart's position is denoted by $x$, and the inverted pendulum's angular velocity and the cart's velocity derivatives are denoted by $\dot{\phi}$ and $\dot{x}$. The parameters used in this experiment are shown at Table 1.

We can obtain the nonlinear dynamics equations (3.1) of the inverted pendulum, through the mechanical analysis of the inverted pendulum.

(1) Nonlinear Model

$$\begin{aligned} &(M+m)\ddot{x}+b\dot{x}+ml\ddot{\phi}\cos\phi-ml\dot{\phi}^2\sin\phi=F \\ &-\left(I+ml^2\right)\ddot{\phi}+mgl\sin\phi=ml\ddot{x}\cos\phi \end{aligned} \tag{14}$$

where $F$ means the force, which can drive the cart moving to keep the pendulum rod in the vertical upward.

(2) Linear Model

By linearizing the nonlinear dynamics equations, using the $\cos\phi = -1, \sin\phi = \phi, (\frac{d\phi}{dt})^2 = 0$ and $u$ instead of $F$, we can get the following:

$$\begin{aligned} -\big(I + ml^2\big)\ddot{\phi} + mgl\phi &= ml\ddot{x} \\ (M + m)\ddot{x} + b\dot{x} + ml\ddot{\phi} &= u \end{aligned} \tag{15}$$

with $X^T = [x, \dot{x}, \phi, \dot{\phi}]$, $u = -\ddot{x}$ and $I = \frac{1}{3}ml^2$, the state-space model is

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{3g}{4l} & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ -1 \\ 0 \\ \frac{3}{4l} \end{bmatrix} u \tag{16}$$

$$y = \begin{bmatrix} x \\ \phi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u \tag{17}$$

By using the plant parameters of Table 1, thus, we can calculate the system parameter matrices $A$ and $B$ as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 29.4 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 3 \end{bmatrix} \tag{18}$$

For the purpose of controller design, we next discretize the system dynamics, with the sample time 0.01 s, to obtain the form:

$$X_{k+1} = A_d X_k + B_d u_k = \begin{bmatrix} 1 & 0.01 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1.0015 & 0.01 \\ 0 & 0 & 0.2941 & 1.0015 \end{bmatrix} X_k + \begin{bmatrix} -0.0001 \\ -0.01 \\ 0.0002 \\ 0.03 \end{bmatrix} u_k \tag{19}$$

### 3.2 Random Networked Time Delay Model

As described in Sect. 2.2, the networks with time delay and data dropout are simulated by the Markov model. We set the every state of the Markov chain to represent the different random time delay and data dropout. With the state jump in the Markov chain, the time delay of the network is variable. In this example, we assume a 5 states Markov chain with the $N = 4$, which means that the most large step of time delay is 4 in the both forward and feedback channel. Thus the largest step of time delay in the control loop is 8 step and the state transition probability matrix of the forward and feedback channels are shown in the following:

$$\begin{aligned} P_{\text{forward}} &= \begin{bmatrix} 0.3 & 0.2 & 0.2 & 0.2 & 0.1 \\ 0.2 & 0.3 & 0.2 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.4 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.2 & 0.3 & 0.2 \\ 0.1 & 0.1 & 0.2 & 0.2 & 0.4 \end{bmatrix} \\ P_{\text{feedback}} &= \begin{bmatrix} 0.3 & 0.2 & 0.2 & 0.2 & 0.1 \\ 0.2 & 0.3 & 0.2 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.4 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.2 & 0.3 & 0.2 \\ 0.1 & 0.1 & 0.2 & 0.2 & 0.4 \end{bmatrix} \end{aligned} \tag{20}$$

where the $P_{\text{forward}}$ is the state transition probability matrix of the forward channel and the $P_{\text{feedback}}$ is the state transition probability matrix of the feedback channel.

### 3.3 Structure of the Controller

According to the discrete LQR state feedback control, the linearized model (19) could be used to derive controller on the form

$$u = -LX \tag{21}$$

Design of a discrete LQR state feedback control law for state regulation is then obtained by minimizing the cost function:

$$J = \int_0^\infty \left(X^T QX + u^T Ru\right) dt \tag{22}$$

## 4 Simulation Results

In order to demonstrate the effectiveness of the proposed predictive control scheme (10) and the optimal estimation method (13) in the feedback channel, numerical simulations have been performed and presented comparing with the control method without any compensation in this section. The plant is chosen as linear inverted pendulum model (16) and the pendulum parameter values are selected in Table 1. This section gives the trajectory curves of the pendulum's angle $\phi$ and the cart's position $x$ when the pendulum's angle $\phi$ is stable from the initial state $\phi = 0.2$ ($X = [0\ 0\ 0.2\ 0]'$) with different control methods. The numerical simulations are discussed in three steps. We first consider the model of the linear inverted pendulum (16) and LQR state feedback control law (21) without network. Then, the LQR state feedback control law without any compensation will be considered to control the inverted pendulum with time delay and data dropout. Finally, the predictive control scheme (10) in the forward channel and the multirate Kalman filter (13) in the feedback channel are applied for NCSs to compensate for time delay and data dropout, and we will show that the predictive control schemes and the multirate Kalman filter are superior than control the

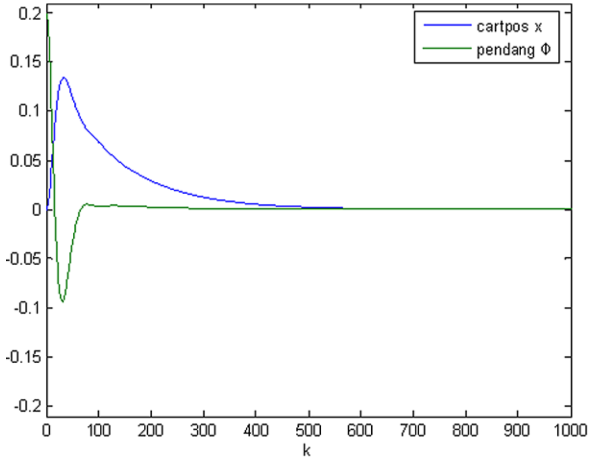



**Fig. 4** Control of the inverted pendulum without network

system without compensation through network, and the simulation results indicate that the predictive control strategy with compensation we proposed in this paper has the best performance when time delay and data dropout appear simultaneously in the network.

## 4.1 Control the Inverted Pendulum Without Network

Consider the model of the pendulum (19) and LQR state feedback control law (21) without data dropouts and noises. The parameters in cost function (22) are selected as

$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 500 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad R = 1 \tag{23}$$

which yields the feedback gain

$$L = [-31.6228 \quad -20.8404 \quad -77.0990 \quad -13.6080] \tag{24}$$

The trajectory of the pendulum's angle,which reaches at zero from the initial position and keeps stable, and the trajectory of the cart's position is depicted in Fig. 4, which indicates that, without networks, the pendulum stability can be guaranteed under the LQR state feedback control law.

From Fig. 4, we can see that the cart position and the pendulum angle reach stability before the $k = 600$ and the overshoot of the pendulum angle is smaller than 0.1. The curve of the cart position shows that the cart does not leave far away the center position $x = 0$.

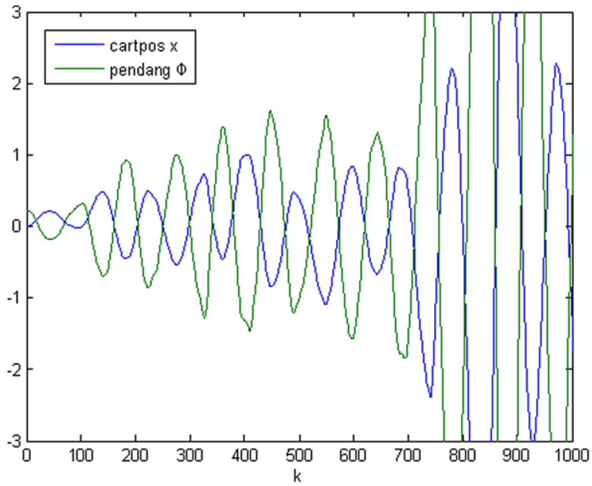



**Fig. 5** Control the inverted pendulum through network without compensation

## 4.2 Control of the Inverted Pendulum Through Network Without Compensation for the Time Delay and Data Dropout

In this subsection, we consider the NCSs with time delay and data dropouts. System performance of the control without any compensation is shown when the time delay and data dropout happen in both the forward and feedback channels.

The method without any compensation means that if the control signals over networks are delayed or discarded, the actuator gives out a zero signal to the plant. As Fig. 5 shown, the curves of the pendulum angle and cart position are divergence.

## 4.3 Predictive Control Scheme and Multirate Kalman Filter Applied for NCSs

It has been shown that the performance of control the pendulum with network is worse than it without network in preceding. Next, the predictive control and multirate Kalman filter will be added to the NCSs for improving the performance of the system.

We first consider that only the predictive control is applied in the NCSs. Figure 6 shows the performance of predictive control, which compensates for the time delay and data dropout. From the figure, we can find that the pendulum angle is well stabilized in the equilibrium position, when there exist time delay and data dropout in both forward and feedback channels. Compared with Fig. 5, the pendulum angle and the cart position are stabilized in the equilibrium position, although the cart position has more large overshoot than it is in Fig. 4 when the time delay and data dropout happen in both the forward and feedback channels with network. Once the predictive control is introduced to control system, the pendulum angle can be stabilized in the vertical position, but the performance of the system is not good and the cart position is far away from the center position. Next, we will introduce multirate Kalman filter to the controller to improve the performance of the system.

Figure 7 shows the performance of the proposed method. The performance of the system is indeed improved after applying both the predictive control and multirate Kalman filter. Compared Fig. 7 with Fig. 6, it shows that cart position, in Fig. 7, has smaller overshoot and settling time than that in Fig. 6.

**Fig. 6** Control of the inverted pendulum through network with predictive control

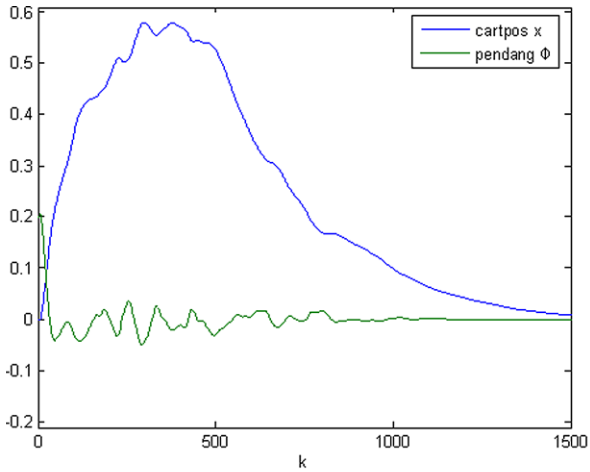
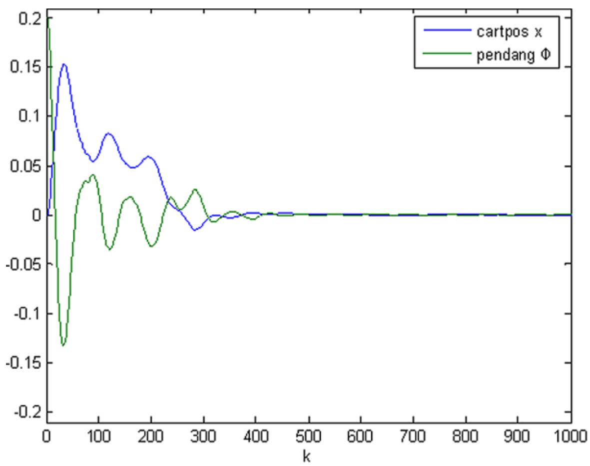



**Fig. 7** Control of the inverted pendulum through network with predictive control and optimal estimation




## 5 Conclusions

A new networked predictive control scheme and multirate Kalman filter have been proposed for MIMO networked distributed control systems with random network time delay and data dropout. Based on the network feature of transmitting a set of data each time, the proposed networked predictive controller consists of the control prediction generator and the network time delay and data loss compensator. The former uses the optimal estimate to compute a set of future control predictions for satisfying the system performance requirements. The latter compensates the random network transmission time delay and packet loss. Simulation results are presented to illustrate the effectiveness of the proposed predictive control and the multirate Kalman filter strategy via comparing with the control scheme without any compensation.

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