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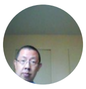

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Dynamics analysis of chaotic circuit with two memristors

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Based on Chua's chaotic oscillation circuit, a fifth-order chaotic circuit with two memristors is designed and its corresponding dimensionless mathematic model is established. By using conventional dynamical analysis methods, stability analysis of the equilibrium set of the circuit is performed, the distribution of stable and unstable regions corresponding to the memristor initial states is achieved, and the complex dynamical behaviors of the circuit depending on the circuit parameters and the memristor initial states are investigated. The theoretical analysis and numerical simulation results demonstrate that the proposed chaotic circuit with two memristors has an equilibrium set located on the plane constituted by the inner state variables of two memristors. The stability of the equilibrium set depends on both the circuit parameters and the initial states of the two memristors. Rich nonlinear dynamical phenomena, such as state transitions, transient hyperchaos and so on, are expected.

memristor, chaotic circuit, initial state, equilibrium point set, stability

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1 Introduction

Memristor, a missing circuit element proposed by Leon O Chua in 1971 [1], and realized by Stan Williams's group of HP Labs in 2008 [2], is a passive two-terminal electronic device described by nonlinear constitutive relation of charge and flux. Currently, the physical realization of various memristors and memristive systems [2–5], the modeling of basic memristor circuits [6–13], the designs and analyses of memsistor based application circuits [14–21], etc., have attracted much attention in the electrical and electronic engineering communities.

Due to the nonlinearity of memristor element [9], the memristor based circuits can easily generate chaotic signal, which enhances research interests in the design of the chaotic memristive circuits [14, 15, 17–21]. Itoh and Chua [17] derived several oscillators from Chua's oscillators by replacing Chua's diodes with memristors characterized by monotone-increasing and piecewise-linear function. Muthuswamy and Kokate [18] proposed memristor based chaotic circuits with the memductance mathematically defined as a discontinuous function. These studies revealed that the memristor based chaotic circuits could generate various chaotic attractors and exhibit rich nonlinear phenomena. However, the constitutive relations of the memristors in refs. [17, 18] were piecewise-linear and resulted in discontinuous characteristics of the corresponding memristance and memductance, which makes the physical realization of such non-smooth memristors more difficult. Muthuswamy [15] provided a practical implementation of a memristor based chaotic circuit, where the memristor was characterized by a cubic nonlinearity and realized by the off-the-shelf components such as resistor, capacitor, operational amplifier and analog multiplier. However, the more detailed dynamical

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behavior of the proposed memristor based chaotic circuit were not investigated in refs. [15, 17, 18]. It should be noted that many novel nonlinear physical phenomena in the memristive circuit appear because of the introduction of the memristors. Recently, we developed several smooth flux-controlled memristors characterized by cubic or piecewise-quadratic nonlinearities in refs. [19–21]. The dependence of the dynamics of the memristor based chaotic circuits on the initial state of memristor was reported [21], and special complicated dynamical behaviors of the memristor based chaotic circuit were investigated [20].

Different from general dynamical systems [22–24], the equilibrium points of the memristive system are an equilibrium point set located on the axis corresponding to the inner state variable of memristor and therefore, the stability of the memristive system also depends on the initial state of the memristor [21]. The trajectories of the system starting from different initial states asymptotically tend to stable sink or limit cycle or chaotic orbit or infinity [19–21]. In refs. [19–21], we reported the chaotic circuits with one memristor and found that the circuits had complex dynamical phenomena, such as transient chaos, intermittent chaos, state transition of system trajectory and global steady period oscillation with transient chaos. However, for the circuit with two memristors, there exists an equilibrium point set located in the plane constructed by the inner state variables of two memristors, which makes the qualitative stability analysis of the equilibrium point more complicated. In this paper, we propose a chaotic circuit with two memristors. Through mathematical modeling, we emphasise the distributions of stability regions of the equilibrium point set in the plane, and reveal and analyze the complex nonlinear dynamics of this chaotic circuit under the variation of its circuit parameters and the initial states of the two memristors.

2 Chaotic circuit with two memristors

The chaotic circuit with two passive two-terminal smooth flux-controlled memristors is shown in Figure 1. This circuit is evolved from Chua's chaotic circuit by replacing the Chua's diode with an active memristive circuit consisting of memristor M_1 and a resistor with negative resistance $-1/G$, and by inserting memristor M_2 between LC_2 resonance circuit and the output nonlinear filter circuit. The proposed circuit consists of five dynamic elements, including two memristors, two capacitors and one inductor. The corresponding state variables are ϕ_1, ϕ_2, v_3, v_4, and i_5 respectively, where ϕ_1 and ϕ_2 are inner state variables of the two memristors M_1 and M_2.

Applying Kirchhoff's circuit laws to the circuit in Figure 1, we obtain a set of five first-order differential equations, which define the relation among the five circuit variables:

$$
\begin{aligned}
&\frac{\mathrm{d}\phi_1}{\mathrm{d}t} = v_3,\\
&\frac{\mathrm{d}\phi_2}{\mathrm{d}t} = \frac{v_4 - v_3}{RW_2 + 1},\\
&\frac{\mathrm{d}v_3}{\mathrm{d}t} = \frac{1}{C_1}\left[\left(G - W_1\right)v_3 - \frac{W_2}{RW_2+1}\left(v_3 - v_4\right)\right],\\
&\frac{\mathrm{d}v_4}{\mathrm{d}t} = \frac{1}{C_2}\left[\frac{W_2}{RW_2+1}\left(v_3 - v_4\right) + i_5\right],\\
&\frac{\mathrm{d}i_5}{\mathrm{d}t} = -\frac{1}{L}v_4 - \frac{r}{L}i_5.
\end{aligned} \tag{1}
$$

By letting $x = \phi_1$, $y = \phi_2$, $z = v_3$, $u = v_4$, $v = i_5$, $a = 1/C_1$, $b = 1/L$, $c = r/L$, $d = G$, $e = R$, $C_2 = 1$, and defining the nonlinear functions $q(x)$ and $W(x)$ as [15, 20]

$$
\begin{aligned}
&q(\xi) = \xi + \xi^3,\\
&W(\xi) = \mathrm{d}q(\xi)/\mathrm{d}\xi = 1 + 3\xi^2,
\end{aligned} \tag{2}
$$

respectively, the state equations of (1) can be rewritten in dimensionless form as:

$$
\begin{aligned}
&\dot{x} = z,\\
&\dot{y} = (u - z)/(1 + eW_2),\\
&\dot{z} = az(d - W_1) - a(z - u)W_2/(1 + eW_2),\\
&\dot{u} = (z - u)W_2/(1 + eW_2) + v,\\
&\dot{v} = -bu - cv,
\end{aligned} \tag{3}
$$

where $W_1 = 1 + 3x^2$, $W_2 = 1 + 3y^2$. Thus, the proposed chaotic circuit with two memristors is a five-dimensional system, with its dynamics described by eq. (3). Based on which the corresponding theoretical analyses and numerical simulations can be conducted.

Let the dimensionless parameters a=8, b=10, c=0, d=2, and e=0.1. For initial conditions (0, 0, 0, 1×10^{-4}, 0), system (3) is chaotic and displays a 2-scroll chaotic attractor, as shown in Figure 2. By means of Wolf's method to calculate Lyapunov exponents, the results are L_1=0.6254, L_2=0, L_3=0, and L_4=–3.8697, and the Lyapunov dimension is d_L=2.1616. Therefore, observed from the phase portraits, the Lyapunov exponents and the Lyapunov dimension, we can see clearly that the fifth-order memristive circuit is in a chaotic state.

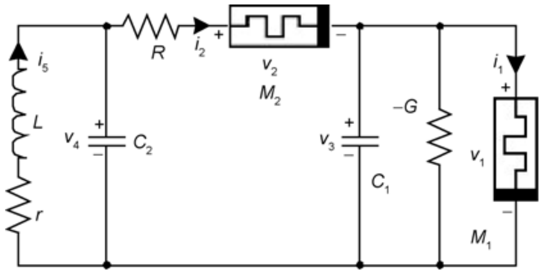

Figure 1 Chaotic circuit with two memristors.

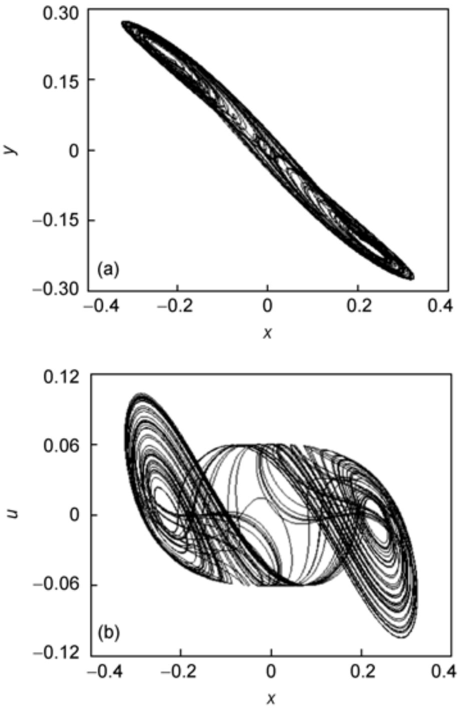

Figure 2 Attractor of the chaotic circuit with two memristors. (a) x–y; (b) x–u.

3 Equilibrium point set and stability analysis

To find the equilibrium points of system (3), let $\dot{x} = \dot{y} = \dot{z} = \dot{w} = 0$. It is obvious that the equilibrium state of system (3) is given by an equilibrium point set

$$E=\{(x, y, z, u, v)|\ z=u=v=0, x=c_1, y=c_2\}, \tag{4}$$

which corresponds to the x-y plane, *i.e.*, any point located in x-y plane is an equilibrium point. Here, c_1 and c_2 are real constants. When parameters a=8, b=10, c=0, and d=2 while parameters e, c_1 and c_2 are varying, the Jacobian matrix J_E of eq. (3) at the equilibrium set E is given by

$$J_E = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\rho & \rho & 0 \\ 0 & 0 & 8(1-\rho-3c_1^2-3\rho c_2^2) & 8\rho(1+3c_2^2) & 0 \\ 0 & 0 & \rho+3\rho c_2^2 & -\rho-3\rho c_2^2 & 1 \\ 0 & 0 & 0 & -10 & 0 \end{bmatrix}, \tag{5}$$

where $\rho = 1/(1+e+3ec_2^2)$.

The characteristic eq. (3) at the equilibrium set E is given by

$$\lambda^2(\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3) = 0, \tag{6}$$

where

$$\begin{aligned} a_1 &= 24c_1^2 + 27\rho c_2^2 + 9\rho - 8, \\ a_2 &= 8\rho(3c_2^2+1)(3c_1^2-1)+10, \\ a_3 &= 80(3c_1^2 + 3\rho c_2^2 + \rho - 1). \end{aligned}$$

Eq. (6) indicates that the Jacobian matrix J_E of eq. (3) has two zero values and three non-zero values. The coefficients of the cubic polynomial equation in brackets of eq. (6) are all non-zero. Then according to Routh-Hurwitz condition, except two zero eigenvalues, the real parts of the roots of eq. (6) are negative if and only if

$$H_k = \begin{vmatrix} a_1 & a_3 & 0 \\ 1 & a_2 & 0 \\ 0 & a_1 & a_3 \end{vmatrix} > 0, \tag{7}$$

where k=1, 2, 3, namely,

$$\begin{aligned} H_1 &= a_1 > 0, \\ H_2 &= a_1a_2 - a_3 > 0, \\ H_3 &= a_3(a_1a_2 - a_3) > 0. \end{aligned} \tag{8}$$

Let e=0.1, i.e., the circuit parameters are fixed, and the constants c_1 and c_2 are in the range of c_1=–0.8 ~ 0.8, c_2=–0.4 ~ 0.4 respectively. The constants c_1 and c_2 in the cubic polynomial equation need to be chosen to satisfy the conditions of eq. (8), which are distributed over the white regions consisting of the two stable regions in Figure 3. Note that in Figure 3, the white regions are stable regions while the gray regions are unstable regions. If the initial states of the two memristors satisfy the conditions given by eq. (8) and the initial states of other dynamic elements are zero, then the orbits of system (3) starting from the two stable regions is asymptotically stable. The real parts of the three non-zero eigenvalues of the equilibrium set E are negative. Otherwise, the initial states of the two memristors located in the gray regions do not satisfy the conditions of eq. (8), in which the equilibrium set E are unstable. Ignoring the effects of the two zero eigenvalues, the orbits of system (3) starting from the two unstable regions are unstable and asymptotically tend to a limit cycle or chaotic attractor or infinity.

For c_2=0, the corresponding unstable regions in c_1 coordinate axis is

$$|c_1| < 0.1741,\ 0.234 < |c_1| < 0.5206. \tag{9}$$

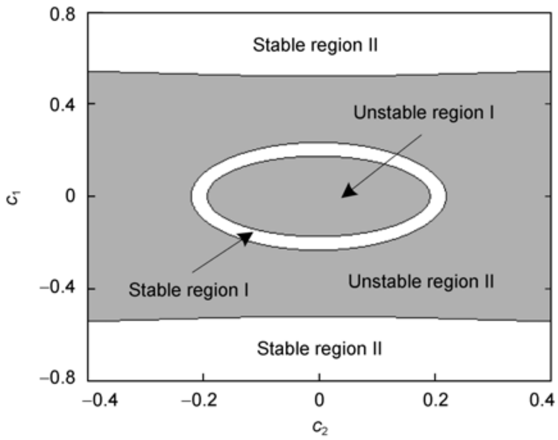

Figure 3 Stability region distributions of the three non-zero eigenvalues.

For c_1=0, the corresponding unstable regions in c_2 coordinate axis is

$$\left|c_2\right| < 0.1925,\ \left|c_2\right| > 0.2202. \tag{10}$$

It is remarkable that the stability of the chaotic circuit with the two memristors can not be simply determined by the three non-zero eigenvalues of the equilibrium set. In the following, the numerical simulation results indicate that the two zero eigenvalues also have influence on the dynamics of the chaotic circuit with the two memristors under some circuit parameters.

For the above parameters, parameters $x(0)$ and $y(0)$ in the initial states $[x(0), y(0), 0, 1\times10^{-4}, 0]$ are varying. When $y(0)$=0, the Lyapunov exponents with the variation of initial state $x(0)=c_1$ is shown in Figure 4(a); when $x(0)$=0, the Lyapunov exponents with the variation of initial state $y(0)=c_2$ is depicted in Figure 4(b). For clarity, only the first four Lyapunov exponents are presented.

Between the numerical simulation results in Figure 4 and the aforementioned theoretical analysis results, there exist differences in regions $0.12<c_1<0.23$ and $-0.19<c_2<-0.16$, in which system (3) is in stable sink. These differences are mainly evolved by the two zero eigenvalues, not by the three non-zero eigenvalues, of the equilibrium set E.

4 Dynamics dependent on circuit parameters

Similar to dynamical analysis of general chaotic circuit, by utilizing the conventional dynamical analysis tools such as bifurcation diagram, Lyapunov exponent spectra and so on, the dynamical behaviors of the chaotic circuit with two memristors shown in Figure 1 are studied under the variation of circuit parameters.

For the above circuit parameters and c_1=0, c_2=0, the circuit parameter e is a varying parameter, i.e., the resistance R is adjustable. When parameter e increases gradually, the Lyapunov exponent spectra and the corresponding bifurcation diagram of state variable x are shown in Figures 5(a) and (b), respectively. Note that the Poincaré section z=0 is selected to plot the bifurcation diagram. It can be seen that the stable and unstable regions described by the bifurcation diagram well coincide with those by the Lyapunov exponent spectra. Different from the general chaotic system, the maximum Lyapunov exponent of the chaotic circuit with two memristors in periodic region is a positive value. Observed from Figure 5(b), the routes to chaos of system (3) are that when resistance R decreases gradually, the orbit settles at sink first and then goes to unstable periodic and chaotic state through period-doubling bifurcation; afterwards, the orbit has a transition from chaotic to periodic behaviors through reverse period-doubling bifurcation and jumps to sink abruptly. Obviously, there are several periodic windows in the chaotic region. Table 1 lists the domains of circuit parameter e and the corresponding dynamics, with which system (3) has abundant and complex dynamical behaviors.

The projections on the y-z plane for some typical periodic

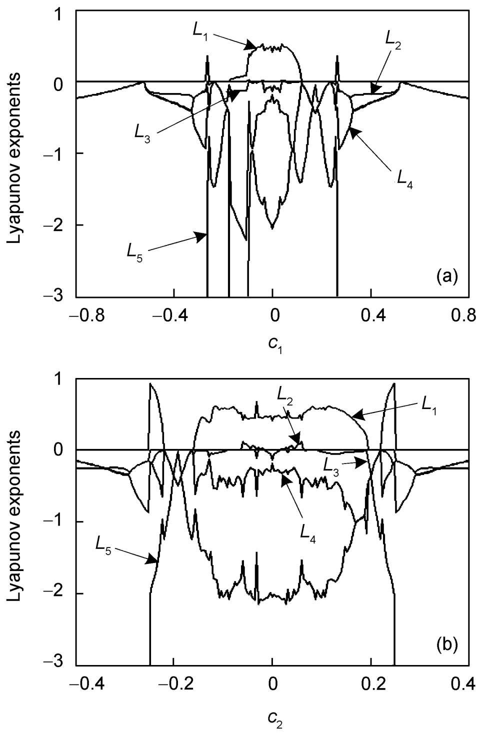

Figure 4 Lyapunov exponent spectra when the initial states vary. (a) c_1 varying; (b) c_2 varying.

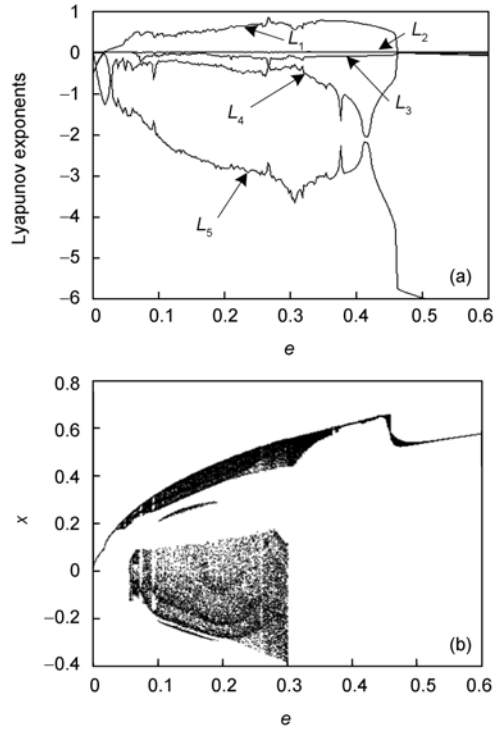

Figure 5 Dynamics dependent on circuit parameter e. (a) Lyapunov exponents; (b) bifurcation diagram.

Table 1 Domains of parameter e and the corresponding dynamics

For $0 \leqslant e \leqslant 0.019$, stable sink
For $0.019 < e < 0.043$, limit cycle
$e = 0.035, 0.04$, period-2, -4 bifurcation
For $0.043 \leqslant e < 0.358$, chaotic attractor
$0.047 < e \leqslant 0.051$, period-3 windows
Near $e = 0.06$, period-5 windows
$0.072 \leqslant e < 0.076$, period-3 windows
$0.091 < e < 0.096$, period-5 windows
Near $e = 0.26$, period-5 windows
For $0.358 \leqslant e \leqslant 0.46$, limit cycle
$e = 0.361, 0.387$, period-4, -2 bifurcation
For $0.46 < e \leqslant 0.6$, stable sink

orbits, chaotic orbits and stable sinks generated by system (3) are obtained from numerical simulations, as shown in Figure 6. It is marked that Figure 6(a) is period 1 (e=0.032), Figure 6(b) is period 2 (e=0.038), Figure 6(c) is period 5 (e=0.26), Figure 6(d) is 2-scroll chaotic attractor (e=0.28), Figure 6(e) is 1-scroll chaotic attractor (e=0.32), and Figure 6(f) is stable sink (e=0.47). Figures 6(a) and (b) depict the limit cycles with period 1 and period 2, which indicate that system (3) shows period-doubling bifurcation. Figure 6(c) illustrates a period-5 limit cycle within a periodic windows. Figures 6(d) and (e) exhibit the chaotic attractors with 2-scroll and 1-scroll in chaotic region. Figure 6(f) demonstrates that the orbit of system (3) asymptotically tends to a stable sink.

5 Dynamics dependent on initial states of memristors

The equilibrium points of the chaotic circuit with two memristors are the points located over the entire x-y plane and have the distributions of stable and unstable regions. The orbits of this circuit starting from different regions have different dynamics. In the following numerical simulation analysis, the circuit parameters a=8, b=0, c=0, d=2, e=0.1 and the initial states c_1 and c_2 are varying parameters.

5.1 State transitions

When $c_1 = 0.264$ and $c_2 = 0$, it can be observed from Figure 4(a) that the memristor based chaotic circuit has a positive Lyapunov exponent ($L_1 > 0$), which implies that this circuit should be in chaotic state. However, the numerical simulation results indicate that this circuit only generates transient and steady limit cycle with period 1. The evolutions of Lyapunov exponent spectra and the corresponding orbits with the time are shown in Figures 7(a) and (b). The result in Figure 7(a) illustrates that there exists a positive Lyapunov exponent after a long time interval, while the result in Figure 7(b) exhibits that accompanying with the evolutions of time series, the complicated state transition phenomenon occurs in the memristor based chaotic circuit.

As observed clearly from Figure 7(b), the circuit orbit settles at sink first and then gradually goes to a limit cycle with period 1 at t≈60 s. With further evolution, the dynamic amplitude of the limit cycle in z-axis expands slowly from [–0.22, 0.25] to [–0.31, 0.36] and jumps abruptly to [–4.92, 4.65] at t≈640 s.

5.2 Transient hyperchaos

As observed from Figure 4(b), in region $|c_2| \in (0.02, 0.06]$, the second Lyapunov exponent L_2>0, which implies that the hyperchaos phenomenon occurs in system (3).

When c_2=0.06 and c_1=0, the evolutions of Lyapunov exponent spectra and the corresponding orbits of the memristor based chaotic circuit with time are shown in Figures 8(a) and (b). From Figure 8(a), it can be known that with the evolutions of time series, the Lyapunov exponent spectra

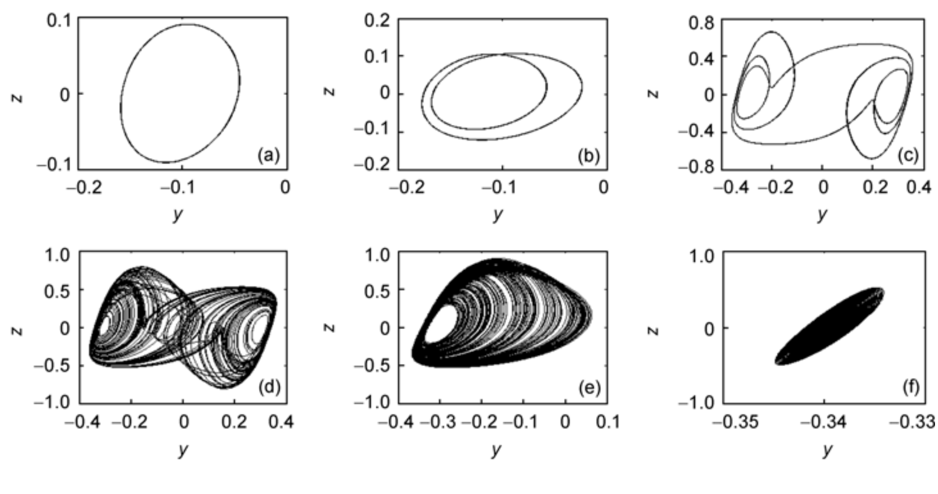

Figure 6 Phase portrait of system (3) with different circuit parameters.

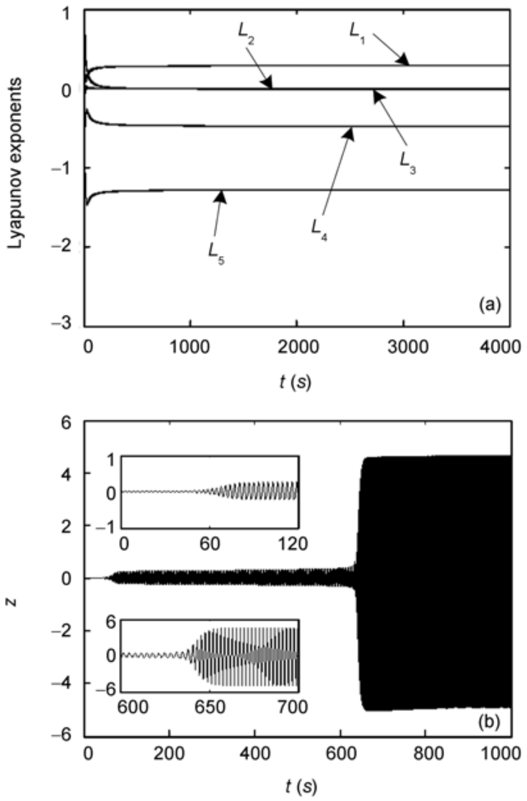

Figure 7 State transition phenomenon. (a) Lyapunov exponent evolution with time; (b) orbit evolution with time.

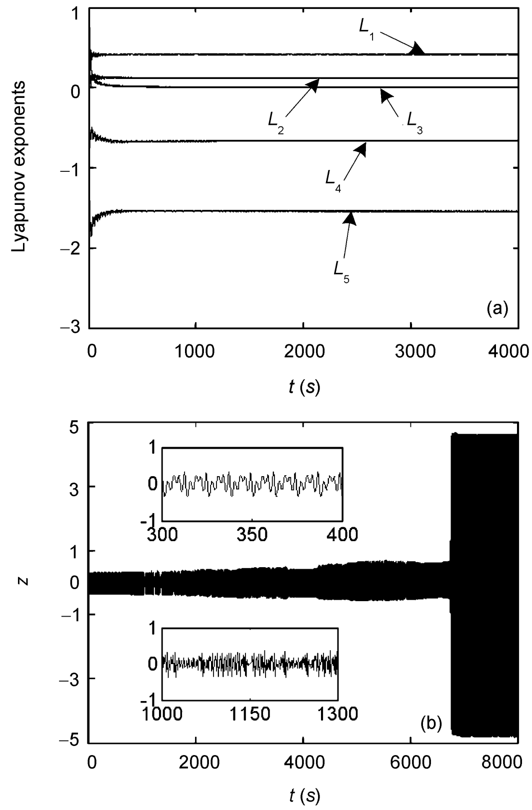

Figure 8 Transient hyperchaos phenomenon. (a) Lyapunov exponent evolution with time; (b) orbit evolution with time.

tends to steady values L_1=0.417, L_2=0.1204, L_3=0, L_4= –0.6639, and L_5=–1.5419. Therefore, the orbits of system (3) starting from the initial states extend in two directions simultaneously, which makes the dynamics of system (3) more complex. Figure 8(b) shows the time series of state variable z in system (3), where a longer transient transition state with different dynamical behaviors locates in the time interval $t \in [0 \text{ s}, 6750 \text{ s}]$, and a steady limit cycle with period 1 appears after $t > 6750$ seconds. The appearance of chaos on finite time scales is known as transient chaos [19, 22]. Since there are two positive Lyapunov exponents in the circuit, the complicated transient transition state shown in Figure 8(b) is referred to as the transient hyperchaos phenomenon.

Clearly, the time series in Figure 8(b) is completely different from the time series generated by other chaotic systems [23–26]. The orbit has a transition from an original limit cycle with period 4 to a 2-scroll hyperchaotic attractor after a span of time and then to a 1-scroll chaotic attractor, to a limit cycle with period 2 and further to a 1-scroll chaotic attractor and ultimately settles down at a limit cycle with period 1. The orbit of the memristor based chaotic circuit is continuous in the entire transition process with time. The projections on the y-z plane for some typical time intervals are shown in Figure 9. Note that Figure 9(a) is in the time interval [200 s, 600 s] (period 4), Figure 9(b) is [1000 s, 1400 s] (hyperchaos), Figure 9(c) is [3000 s, 3400 s] (chaos), Figure 9(d) is [3900 s, 4200 s] (period 2), Figure 6(e) is [6000 s, 6400 s] (chaos), and Figure 6(f) is [7600 s, 8000s] (period 1). Figures 9(a) to (e) reveal the complicated transient behaviors with period 1, period 2, hyperchaos and chaos, reflecting that complex dynamical behaviors on finite time scales occur in the memristor based chaotic circuit. Figure 9(f) represents the ultimately formed periodic orbit after a longer transient process.

5.3 Analysis of complex phenomenon

The occurrence of the above complex nonlinear phenomena is due to the existence of one or two positive Lyapunov exponents and the distributions of stable and unstable regions over the x-y plane in the memristor based chaotic circuit.

On the one hand, the orbit starting from the initial states in unstable regions expands continually in one or two directions because of the positive Lyapunov exponents, and contracts or folds continually in these directions because of the negative sum of all Lyapunov exponents, from which the chaotic or hyperchaotic attractors are emerged. On the other hand, there are two stable regions and two unstable regions over the x-y plane in the memristor based chaotic circuit,

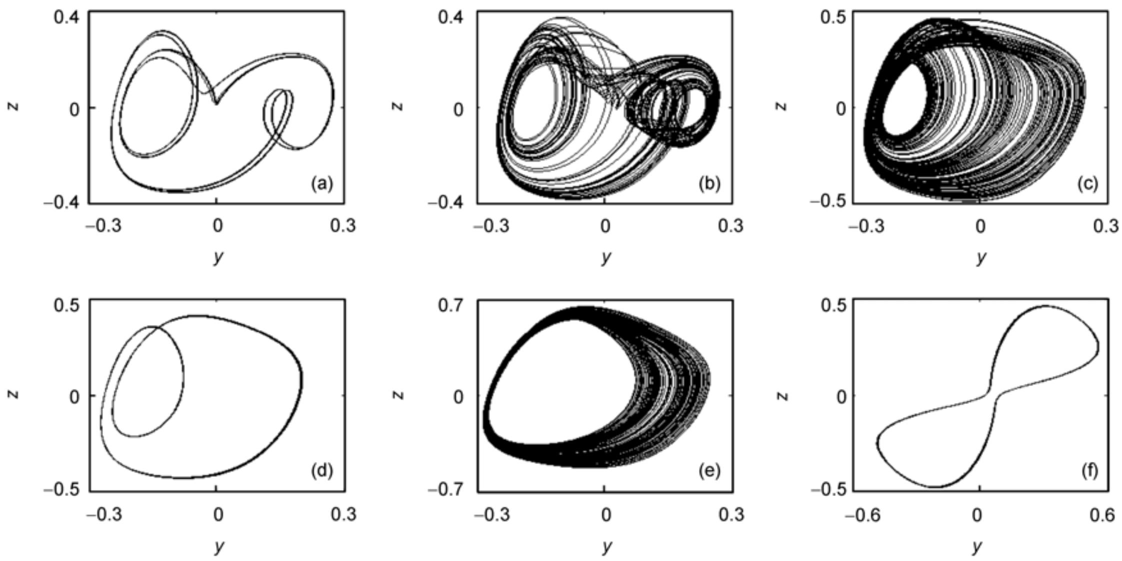

Figure 9 Phase portraits of eq. (3) in different time intervals.

and there exists a gap of stable region between two unstable regions. The orbit starting from the unstable region expands or contracts continually in the five-dimensional phase space, but its direction suffers certain degree interference when it passes through the gap of stable region, which alters the original dynamical property of the memristor based chaotic circuit and causes its dynamical behavior unpredictable. The trajectory flow with one positive Lyapunov exponent suffers interference only in one moving direction, while the trajectory flow with two positive Lyapunov exponents receives interference in two moving directions. For this reason, with the time evolution, the dynamical behaviors in the transient hyperchaos phenomenon are more complex than those in the state transition phenomenon.

6 Conclusions

The chaotic circuit with two memristors has an equilibrium point set located in the plane constructed by the inner state variables of the two memristors. Stable and unstable regions of three non-zero eigenvalues of the equilibrium point set exist in the plane of the inner state variables of the two memristors. Therefore, the chaotic circuit with two memristors is different from general chaotic systems, whose dynamical characteristics, accompanied with the variations of the circuit parameters, closely depend on the initial states of the two memristors in the circuit. Consequently, some distinctive nonlinear dynamical behaviors, such as the dynamical phenomena of state transitions, transient hyperchaos and so on, can be generated from the circuit.

In this paper, a new chaotic circuit with two memristors was derived from Chua's chaotic oscillators by introducing two flux-controlled memristors characterized by smooth cubic nonlinearities. By making use of the conventional dynamical analysis methods, the distributions of the stability regions of the equilibrium point set of the memristor based chaotic circuit in the plane were qualitatively analyzed. With the help of numerical simulation tool, the complex dynamical phenomena generated by the memristor based chaotic circuit under different circuit parameters and memristor initial states were investigated. The proposed qualitative analysis methods for the stabilities of the equilibrium point set can be adapted to study the stabilities of other special memristor based circuits or complex systems with equilibrium point sets. The revealed complicated physical phenomena can enhance the related contents in chaos dynamic theory. The research results in this paper have important theoretic significance and high engineering application value.

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