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ASSESSMENT OF DAMAGE AND LOSS OF SEISMICALLY EXCITED STRUCTURES BASED ON CONVEX ANALYSIS

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Probabilistic results drawn upon inadequate information are suspicious. The convex set theory, which requires much less information, is employed to model the uncertainties of the spectral displacement and damage state medians. Furthermore, a convex model of fragility function is established based on the envelope bound convex models of the spectral displacement and damage state medians. A bound loss estimation method is derived by integrating HAZUS-AEBM module with the convex set theory. The loss bounds of a hotel in southern China are obtained and compared to the loss calculated by HAZUS-AEBM method, which locates in the lower half interval of convex analysis results. The uncertainty propagation is analyzed and damage state medians are found to be the most critical factor to the loss. Finally, the PEER's probabilistic loss estimation methodology is also applied to this example to deduce the probability of loss exceeding the bound values of convex analysis results.

Keywords: Convex set theory; convex analysis.

1. Introduction

Past earthquakes have shown that economic and social losses primarily root in damage of buildings. Accurate prediction of building damage and loss is at the heart of reliable estimate of earthquake impacts. Consequently, it is essential to deal with the uncertainties in the process of performance evaluation, from seismic hazard analysis to the assessment of earthquake consequence. The performance parameters are treated as a continuous random variable in the probablity-based method [Miranda *et al.*, 2001, 2006; Aslani and Miranda, 2004, 2005; Krawinkler, 2005, 2006; Reiser *et al.*, 2006; Miranda and Aslani, 2006]. However, when the data are insufficient to support a probabilistic assumption, the limitations and disadvantages of stochastic methods emerge in this situation, and thus decisions made from a pure stochastic approach may be questionable [Elishakoff, 1995; Chandler

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and Lam, 2001; Chen and Collins, 2001; Cornell, 2001; Hamburger *et al.*, 2004]. Alternatively, the set-theoretic, convex description of uncertainty is more appropriate, which does not need any information about how the data are distributed in its domain.

A convex analysis is a method of quantifying uncertainty without resorting to probabilistic concepts. Instead, the uncertainty is characterized by a set of functions with common global characteristics. The convex method of modeling uncertainty is well suited when only scarce information is available, as in the case of earthquake loss estimation of individual buildings. In convex analysis, the discussion of the meanings and existence of solution is avoided, and the local minimum is the global minimum within a convex set [Ben-Haim and Elishakoff, 1990].

In this study, the non probabilistic convex set theory is employed to deal with the uncertainties of spectral displacement, damage state medians, and loss ratio, and a new bound earthquake loss estimation method for individual building is proposed in the framework of HAZUS-AEBM procedure. Finally, the bound loss estimation method and the PEER's probabilistic loss estimation methodology are applied to a hotel in southern China, and the propagation of uncertainty is discussed.

2. Direct Economic Loss Estimation Based on Convex Set Theory

2.1. *Convex analysis method*

The main idea of the set model is that the input and system variables with uncertainty are described by a non probabilistic bound set, in which the information about the likelihood for each realization is the same, as the available information is not sufficient to substantiate another distribution. If the set is convex, it is termed as a convex model. In the convex analysis method, some typical convex set models have been proposed for modeling the uncertainties of variables or functions [Ben-Haim and Elishakoff, 1990]. Among the models, the envelope bound convex model is frequently employed for modeling uncertainties of uncorrelated variables, i.e.

$$\Omega_{\mathrm{EB}} = \{x(t) \in R : \tilde{x}_j(t) \leq x_j(t) \leq \hat{x}_j(t), j = 1, \ldots, r\}, \tag{1}$$

where Ω_{EB} is the convex set; $x(t)$ is the vector of uncertain variable; $\tilde{x}_j(t)$ and $\hat{x}_j(t)$ are the lower and upper bounds of the jth uncertain variable, and R is the set of real numbers.

The determination of the lower and upper bounds of an uncertain variable or function by convex analysis method is an extreme optimum problem with constraints and can be described as follows

$$\begin{gathered} \operatorname{find}(x(t)) \\ \min \text{ or } \max S(x(t)), \\ \text{s.t. } x(t) \in \mathrm{CM} \end{gathered} \tag{2}$$

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where $x(t)$ is the vector of an uncertain variable; $S(x(t))$ is the objective function, and CM is a certain convex model.

Because Eq. (2) is an extreme optimum problem, either Lagrange multiplier method, Kuhn-Tucker method, or sequential programming method can be used to solve this problem.

2.2. *Convex model of fragility function*

Building fragility functions are lognormal functions in HAZUS-AEBM that describe the probability of reaching, or exceeding, structural and nonstructural damage state, given median estimates of spectral response, e.g. spectral displacement [NIBS, 2002]. The conditional probability of being in, or exceeding, a particular damage state ds, given spectral displacement S_d, is defined as

$$P[Ds \geq ds \,|\, S_d] = \Phi\left[\frac{1}{\beta_{ds}}\ln\left(\frac{S_d}{\overline{S_{d,ds}}}\right)\right], \tag{3}$$

where $P[Ds \geq ds \,|\, S_d]$ is the probability of exceeding ds conditioned on spectral displacement S_d; $\overline{S_{d,ds}}$ is the median value of spectral displacement at which the building reaches the threshold of damage state ds; β_{ds} is the standard deviation of the natural logarithm of spectral displacement for damage state ds; and Φ is the standard normal cumulative distribution function.

With the fragility functions, the probability that a structure will be in a certain damage state ds_i can be computed as the arithmetic difference between fragility functions corresponding to two consecutive damage states:

$$PSTR_{ds} = \begin{cases} 1 - P[Ds \geq ds_i \,|\, S_d] & i = 0 \\ P[Ds \geq ds_i \,|\, S_d] - P[Ds \geq ds_{i+1} \,|\, S_d] & 1 \leq i \leq m, \\ P[Ds \geq ds_i \,|\, S_d] & i = m \end{cases} \tag{4}$$

where $PSTR_{ds}$ is the discrete probability that a structure will be in a certain damage state; $i = 0$ corresponds to the state of no damage in the structure, while m denotes the complete damage state in the structure.

2.2.1. *Uncertainty analysis of spectral displacement and damage state medians*

The capacity–demand diagram method is used to derive spectral displacement as an input of fragility analysis module [NIBS, 2002]. Both the demand and capacity spectra involve many uncertain factors that arise from the earthquake phenomenon, structural geometries, material properties, as well as the approximations and assumptions used in establishing structural models for seismic analysis. The uncertainty of these factors leads to great uncertainty on the spectral displacement S_d in Eq. (3).

The medians $\overline{S_{d,ds}}$ of spectral displacement in Eq. (3) is another key parameter that contributes most to the fragility analysis. The guidance of determining damage state medians by pushover curves in HAZUS-AEBM manual is sufficiently vague and flexible. Thus, different users may deduce different damage state medians using this manual [NIBS, 2002]. In addition, the pushover curves also exhibit divergent properties because of the uncertainty of structural model, different lateral load patterns, and some important parameters in the process of pushover analysis. The subjectivity of the guidance of determining $\overline{S_{d,ds}}$ and the uncertainty of pushover curves results in the damage state medians being also as a highly uncertain variable.

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2.2.2. *Fragility analysis based on convex set theory*

In this study, the spectral displacement is determined using nonlinear static procedure. The demand spectrum is obtained based on the response spectrum in Chinese Code for Seismic Design of Buildings (GB50011-2001 2001) herein. The uncertainty of the demand spectrum, which arises from the uncertainties of the maximum of seismic influence coefficient $\alpha_{\max}$ and the characteristic period T_g, and the uncertainty of capacity spectrum, which is caused by different lateral load patterns of pushover analysis and the uncertainties of $\alpha_{\max}$ and T_g in SRSS load pattern, are considered in this section. A bound capacity–demand diagram method proposed by Jia and Duan [2008] as shown in Figs. 1 and 2 is used to deduce the bounds of spectral response. The spectral acceleration $A(\mathrm{m/s^2})$ is plotted against spectral displacement S_d in Figs. 1 and 2. S_d^0 is the spectral displacement S_d calculated by the capacity–demand diagram method, and S_d^u and S_d^l denote the upper and lower bounds of the spectral displacement S_d deduced by the bound capacity–demand diagram method. The bound spectral displacement is an input of fragility analysis instead of the median value in HAZUS-AEBM.

In order to remove the subjective views in determining the medians $\overline{S_{d,ds}}$ of spectral displacement, guidance is provided according to the broad descriptions

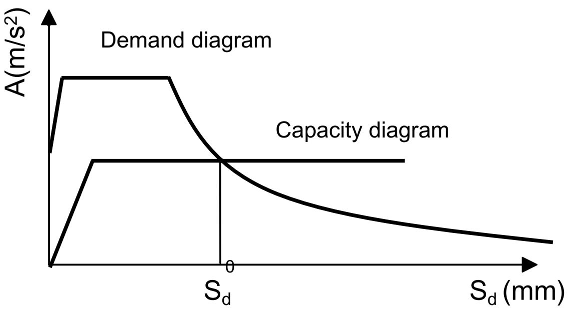

Fig. 1. Capacity–demand diagram method.

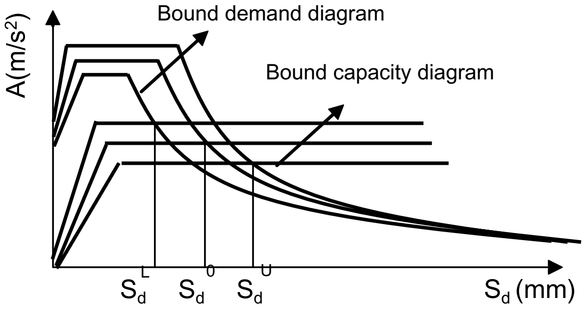

Fig. 2. Bound capacity–demand diagram method.

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Table 1. General guidance for selection of damage state medians.

Damage state	Criteria for selection of damage state medians
Slight	The first structural component yield
Moderate	50% of structural component cracks or 10% of structural component yield
Extensive	75% of structural component cracks or 50% of structural component yield
Complete	90% of structural component cracks or 75% of structural component yield

of structural damage given in HAZUS-MH technical manual to quantify the damage state medians by pushover curves in Table 1. Furthermore, the uncertainties of pushover curves, which result from different lateral load patterns and the uncertainties of $\alpha_{\max}$ and T_g in SRSS load pattern, are considered when $\overline{S_{d,ds}}$ is calculated. The envelope bound convex model as described in Eqs. (5) and (6) is employed to model the uncertainty of the spectral displacement S_d and the medians $\overline{S_{d,ds}}$ of spectral displacement:

$$S_d^* = \left\{S_d^l \le S_d \le S_d^u \mid S_d^l, S_d^u \in R, S_d^l, S_d^u \ge 0\right\} \tag{5}$$

$$\overline{S^*_{d,ds}} = \left\{\overline{S^l_{d,ds}} \le \overline{S_{d,ds}} \le \overline{S^u_{d,ds}} \mid \overline{S^l_{d,ds}}, \overline{S^u_{d,ds}} \in R, \overline{S^l_{d,ds}}, \overline{S^u_{d,ds}} \ge 0\right\}, \tag{6}$$

where S_d^* and $\overline{S^*_{d,ds}}$ is the envelope bound convex model of spectral displacement and its medians, respectively; S_d^u and S_d^l are the upper and lower bounds of the spectral displacement; $\overline{S^u_{d,ds}}$ and $\overline{S^l_{d,ds}}$ are those of the medians of spectral displacement; and R is the set of real numbers.

The conditional probability of being in, or exceeding a particular damage state in Eq. (3) is a nonlinear function of S_d and $\overline{S_{d,ds}}$. As the Hessian matrix of $\Phi(S_d, \overline{S_{d,ds}})$

$$\nabla^2\Phi = \begin{bmatrix} \dfrac{\partial^2\Phi(S_d, \overline{S_{d,ds}})}{\partial S_d^2} & \dfrac{\partial^2(S_d, \overline{S_{d,ds}})}{\partial S_d \partial \overline{S_{d,ds}}} \\ \dfrac{\partial^2\Phi(S_d, \overline{S_{d,ds}})}{\partial \overline{S_{d,ds}} \partial S_d} & \dfrac{\partial^2\Phi(S_d, \overline{S_{d,ds}})}{\partial \overline{S^2_{d,ds}}} \end{bmatrix} \tag{7}$$

is negative definite, $\Phi(S_d, \overline{S_{d,ds}})$ is not a convex function [Wang and Dong, 1987]. Solving the extreme of $\Phi(S_d, \overline{S_{d,ds}})$ turns to be a nonlinear programming, i.e. search for the minimum or the maximum of $\Phi(S_d, \overline{S_{d,ds}})$ when S_d and $\overline{S_{d,ds}}$ varies in the envelope bound convex model by Eqs. (5) and (6). The programming problem is described as follows

$$\begin{gathered} \text{find}(S_d, \overline{S_{d,ds}}) \\ \min \text{or} \max(\Phi(S_d, \overline{S_{d,ds}})) \\ \text{s.t.} \begin{cases} S_d^l - S_d \le 0 \\ S_d - S_d^u \le 0 \\ \overline{S_{d,ds}^l} - \overline{S_{d,ds}} \le 0 \\ \overline{S_{d,ds}} - \overline{S_{d}^u}_{,ds} \le 0. \end{cases} \end{gathered} \tag{8}$$

This problem can be solved by the Kuhn-Tucker method [Wang and Dong, 1987], which changes the inequality constraint to an equality constraint using a slack variable, and then the programming problem with equality constraints can be solved with the readily available Lagrange multiplier method. The Lagrange function and constraints are defined as follows

$$\begin{cases} L = \Phi(S_d, \overline{S_{d,ds}}) + \lambda_1(S_d^l - S_d) + \lambda_2(S_d - S_d^u) + \lambda_3(\overline{S_{d,ds}^l} - \overline{S_{d,ds}}) \\ \qquad + \lambda_4(\overline{S_{d,ds}} - \overline{S_{d,ds}^u}) \\ \partial L/\partial S_d = 0, \partial L/\partial \overline{S_{d,ds}} = 0 \\ \lambda_1 \ge 0, \lambda_2 \ge 0, \lambda_3 \ge 0, \lambda_4 \ge 0 \\ S_d^l - S_d \le 0, S_d - S_d^u \le 0, \overline{S_{d,ds}^l} - \overline{S_{d,ds}} \le 0, \overline{S_{d,ds}} - \overline{S_{d,ds}^u} \le 0 \\ \lambda_1(S_d^l - S_d) = 0, \lambda_2(S_d - S_d^u) = 0, \lambda_3(\overline{S_{d,ds}^l} - \overline{S_{d,ds}}) = 0, \lambda_4(\overline{S_{d,ds}} - \overline{S_{d,ds}^u}) = 0. \end{cases} \tag{9}$$

The only solution is found to be when $S_d = S_d^u$ and $\overline{S_{d,ds}} = \overline{S_{d,ds}^l}$, $\Phi(S_d, \overline{S_{d,ds}})$ is the maximum, and when $S_d = S_d^l$ and $\overline{S_{d,ds}} = \overline{S_{d,ds}^u}$, $\Phi(S_d, \overline{S_{d,ds}})$ reaches its minimum. Then, the boundary points set of $P[Ds \ge ds|S_d]$ is in the range of the set $P^*[Ds \ge ds|S_d]$ defined as

$$P^*[Ds \ge ds|S_d] = \Phi\left[\frac{1}{\beta_{ds}} \ln\left(\frac{S_d^l}{\overline{S_{d}^u}_{,ds}}\right)\right], \quad \Phi\left[\frac{1}{\beta_{ds}} \ln\left(\frac{S_d^u}{\overline{S_{d,ds}^l}}\right)\right], \tag{10}$$

where $P^*[Ds \ge ds|S_d]$ is the boundary points set of $P[Ds \ge ds|S_d]$. Let $PSTR_{ds}^*$ be the set containing the lower and upper boundary points $PSTR_{ds}$, then $PSTR_{ds}^*$ with its two elements is

$$PSTR_{ds}^* = [PSTR_{ds}^l, PSTR_{ds}^u], \tag{11}$$

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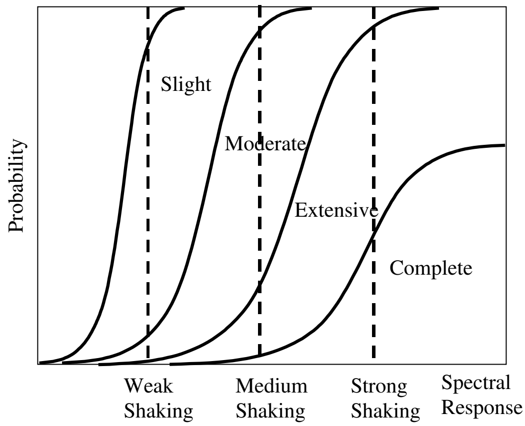

Fig. 3. Common fragility curves.

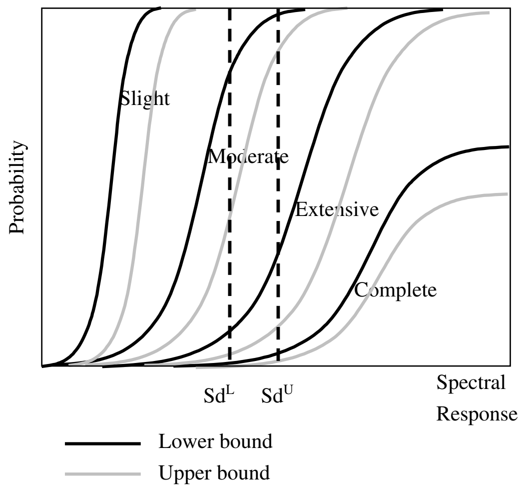

Fig. 4. Bound fragility curves.

where $PSTR^*_{ds}$ is the boundary points set of $PSTR_{ds}$. Figure 3 provides an example of fragility curves for the four damage states used in the HAZUS-AEBM and illustrates differences in damage state probability for three levels of spectral response corresponding to weak, medium, and strong earthquake ground shaking, respectively. The bound fragility curves proposed in this paper are presented in Fig. 4. The bounds of spectral displacement S_d^u and S_d^l are an input for the bound fragility analysis instead of the median value in HAZUS-AEBM, and the fragility curves corresponding to each damage state is also shown with an upper black solid line and lower bound gray solid line because of considering the uncertainty of the medians $\overline{S_{d,ds}}$ of spectral displacement.

2.3. *Earthquake loss estimation based on convex analysis method*

The direct economic loss of structural system in HAZUS-AEBM manual [NIBS, 2002] can be calculated by Eq. (12) after deriving the discrete probability $PSTR_{ds}$ in a certain damage state:

$$EL = RV \times \sum_{ds=2}^{5} (PSTR_{ds} \times STRD_{ds}), \tag{12}$$

where EL is the direct economic loss of structural system; RV is the replacement value of structural system; $PSTR_{ds}$ is the discrete probability in a certain damage state shown in Eq. (4); and $STRD_{ds}$ is the loss ratio for different damage states and can be derived as

$$STRD_{ds} = \frac{RC}{RV}, \tag{13}$$

where RC is the repair cost of different damage states.

2.3.1. *Uncertainty analysis of loss ratio*

The earthquake loss estimation is also closely associated with the loss ratio of different damage states, besides the spectral displacement and damage state medians. The uncertainty of loss ratio mostly depends on the uncertainties of repair cost of different damage states defined in Eq. (13). Next, we discuss the uncertainties of repair cost.

For a certain damage state, different companies may provide different repair process. The uncertainties of repair procedures will lead to uncertainties in repair cost. Furthermore, as the repair procedure is presented, the repair cost is then calculated as follows

$$TC = DC + IC + PP + NC, \tag{14}$$

where TC is the total repair cost; DC is direct repair cost; IC is indirect repair cost; PP is planned profit; and NC is tax cost. DC, IC, and NC are derived as follows

$$DC = LC + MC + FC + ODC \tag{15}$$

$$IC = AC + OIC \tag{16}$$

$$NC = YT + CT + ET, \tag{17}$$

where LC is labor cost; MC is material cost; FC is facility cost; ODC are other costs; AC is construction management cost; OIC are other indirect costs; YT is business tax; CT is city construction management tax; and ET is additional education tax. The regional differences and price fluctuations will cause the uncertainties of total repair cost TC. In addition, the correlation of different components, demand-driven cost inflation after earthquake, the modification of seismic design code, and stress effects of owners also have a strong influence on the uncertainty of total repair cost.

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2.3.2. *Bound loss estimation*

There are very few studies on the variability of the repair cost that can be used to obtain loss function; however, there is no information available on the probability distribution of repair costs of either structural or nonstructural components [Krawinkler, 2005]. Furthermore, the range of loss ratio is often proposed by the individual researchers [Yin, 1996; NIBS, 2002; Porter *et al.*, 2002; Crowley *et al.*, 2005]. In this study, the envelope bound convex model defined in Eq. (18), which does not need any probabilistic information of loss ratio, is applied to model the uncertainty of loss ratio:

$$\begin{aligned} STRD_{ds}^{*} = \{STRD_{ds}^{l} \le STRD_{ds} \le STRD_{ds}^{u} \mid STRD_{ds}^{l}, STRD_{ds}^{u} \in \mathrm{R}, \\ STRD_{ds}^{l}, STRD_{ds}^{u} \ge 0\} \end{aligned} \tag{18}$$

where $STRD_{ds}^{*}$ is the envelope bound convex model of $STRD_{ds}$; and $STRD_{ds}^{l}$ and $STRD_{ds}^{u}$ are the bounds of loss ratio.

Similar procedure discussed in Sec. 2.2 is repeated, and here the bound set of earthquake loss can be deduced by Eqs. (11), (12), and (18) as

$$EL^{*} = \left[RV \times \sum_{ds=2}^{5} (PSTR_{ds}^{l} \times STRD_{ds}^{l}), RV \times \sum_{ds=2}^{5} (PSTR_{ds}^{u} \times STRD_{ds}^{u})\right], \tag{19}$$

where EL^{*} is the boundary points set of EL.

3. Numerical Study

A five-story hotel in southern China is employed as a numerical example to demonstrate the benefits of the proposed methods in earthquake loss estimation. The plan and elevation views of the building are shown in Figs. 5 and 6.

This hotel is a relatively old, reinforced concrete frame building, and was constructed on natural foundation with single footing. The column section is

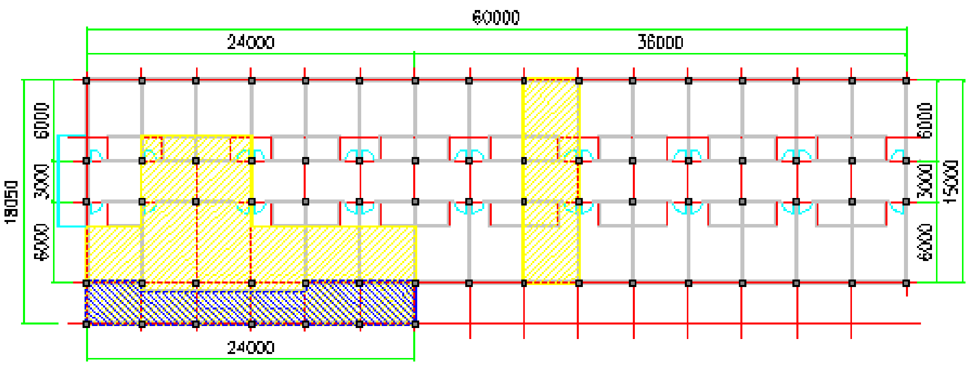

Fig. 5. Plan view of the hotel (mm).

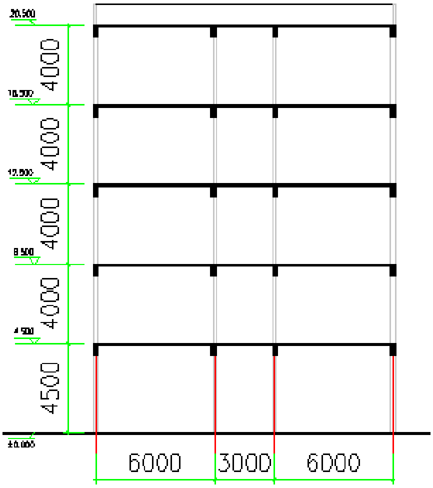

Fig. 6. Elevation view of the hotel (mm).

400×400 mm, and the beam section is 250×500 mm. Concrete in column and beam has nominal strength of $f_{cu} = 30\,\text{N/mm}^2$, and concrete in slab is nominally $25\,\text{N/mm}^2$. The reinforcement steel in column and beam is hot rolled ribbed steel (HRB) bars, and the nominal yielding stress is 335 MPa. The hoop reinforcement steel is hot rolled plain steel bars (HPB), and the nominal yielding stress is 235 MPa. The fundamental period of this building is 0.41 s. The site class is?, and the seismic fortification intensity is 8 degrees according to Chinese Code for Seismic Design of Buildings [GB50011-2001, 2001]. The replacement cost of this building is assumed to be USD 218.85 per square meter.

3.1. *Bound of direct economic loss*

The range of the two parameters are suggested as $\alpha_{\max} = 0.9$–1.10 and $T_g = 0.35$–0.45 s [Chen, 1997; Wu and Gao, 2004]. Five load patterns including the inverse triangular distribution, the uniform distribution, the generalized power distribution, the modal adaptive distribution, and the SRSS loading mode are employed to derive the pushover curves for predicting spectral displacement and its medians. The pushover curves of the hotel subjected to the five load patterns are shown in Fig. 7. The interval of spectral displacement is 115–823 mm after considering the uncertainties of demand and capacity spectrum, and more detailed results of spectral displacement solution are listed in a previous paper [Jia and Duan, 2008].

The pushover curves in Fig. 7 and guidance in Table 1 are combined to obtain the bounds of the medians $\overline{S_{d,ds}}$ of spectral displacement. The boundary points set of $P[Ds \geq ds|S_d]$ is calculated by Eq. (10) after deriving the bounds of the spectral displacement S_d and the medians $\overline{S_{d,ds}}$ of spectral displacement, and then the boundary points set of the discrete probability $PSTR_{ds}$ can be deduced by Eqs. (4)

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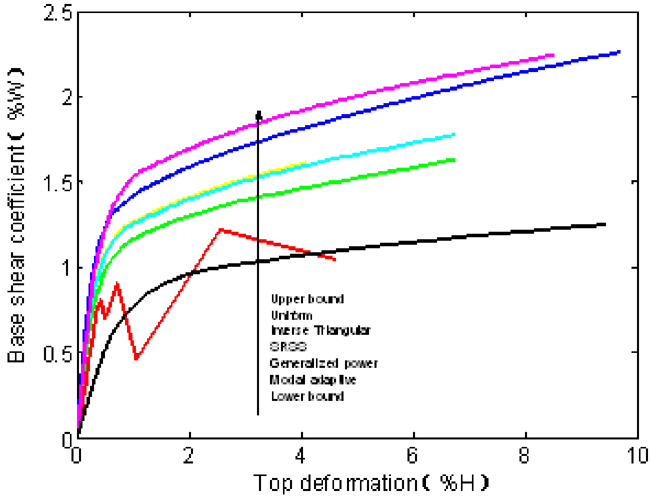

Fig. 7. Pushover curves for the example structure under five load patterns.

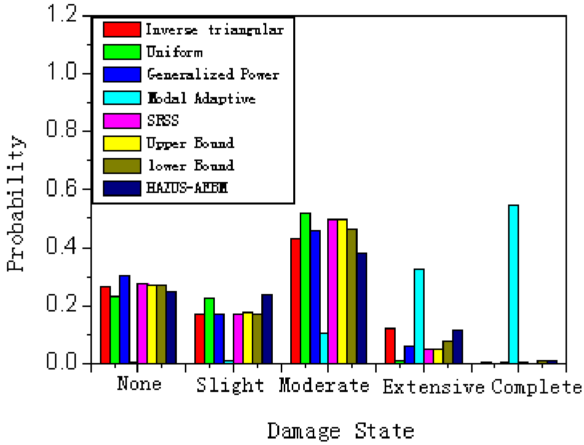

Fig. 8. Damage probability of lower bound of spectral displacement.

and (11). Figures 8–10 show the discrete probability $PSTR_{ds}$ of the structure in a certain damage state corresponding to the upper and lower bounds of spectral displacement, and its median.

As shown in Figs. 8–10, the discrete damage probability, which are in the extensive and complete damage states, increases when the spectral displacement varies from lower bound to the upper bound. The discrete damage probability of load pattern of the modal adaptive distribution in the extensive and complete damage states is much larger than other lateral load patterns, which may be attributed to the pushover curve of the modal adaptive distribution for deriving spectral

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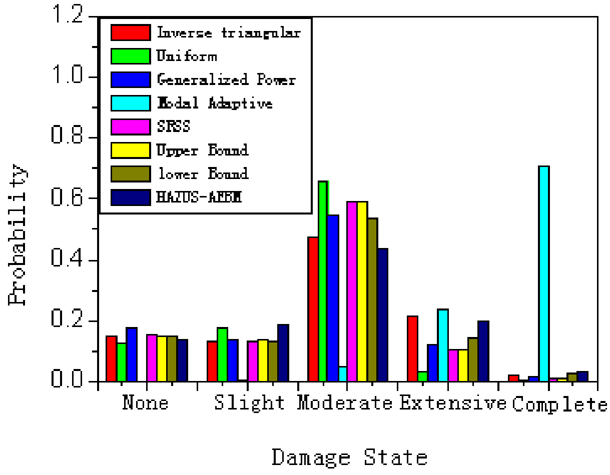

Fig. 9. Damage probability of medians of spectral displacement.

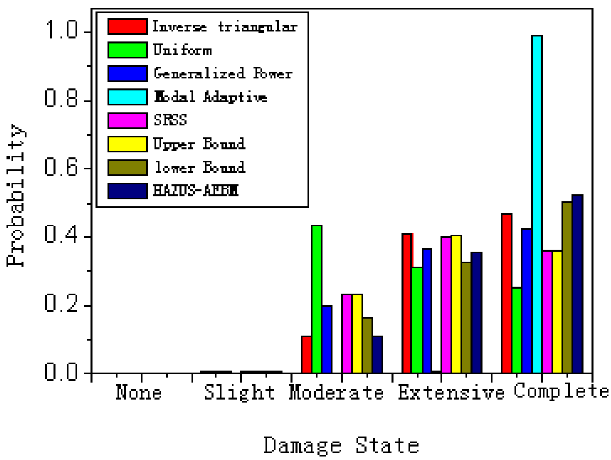

Fig. 10. Damage probability of upper bound of spectral displacement.

displacement and damage state medians is quite different from others lateral load patterns, as shown in Fig. 7.

The lower and upper bounds of loss ratio, $STRD_{ds}^{l}$ and $STRD_{ds}^{u}$, are adopted as, 1–10% for slight damage, 10–40% for moderate damage, 40–80% for extensive damage, and 100% for complete damage [Yin, 1996]. According to the interval of the discrete probability $PSTR_{ds}$ calculated by Eq. (11), the boundary points set EL^{*} of earthquake loss can be derived by Eqs. (12) and (19), and the earthquake loss for the upper and lower bounds, and median of damage probability is shown in Figs. 11–13, respectively.

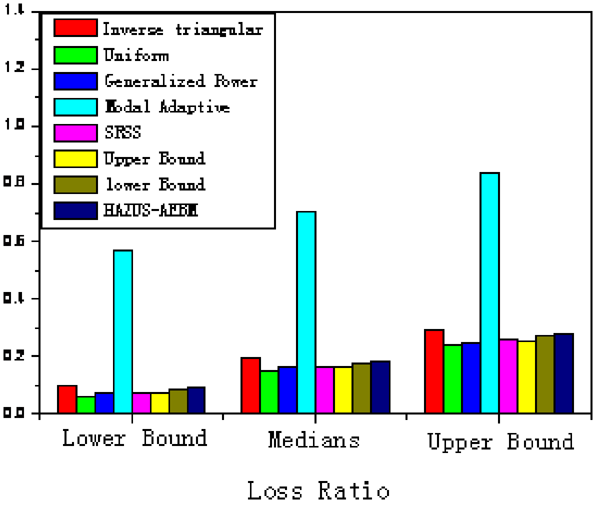

Fig. 11. Loss of lower bound of damage probability.

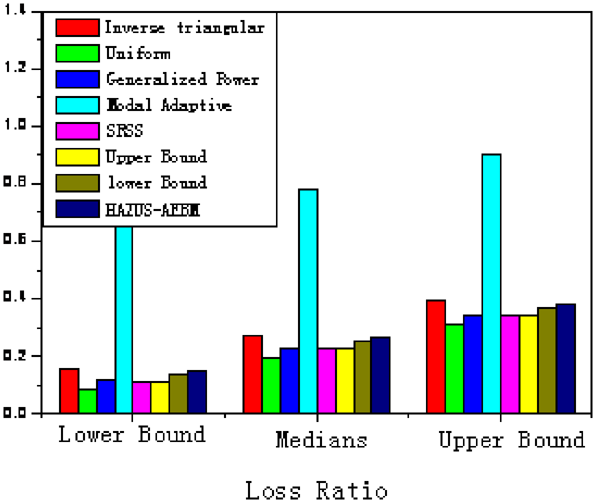

Fig. 12. Loss of medians of damage probability.

Earthquake loss estimation results are very sensitive to the uncertainties of spectral displacement, the medians of spectral displacement, and loss ratio as shown in Figs. 11–13. The direct economic loss of this hotel by convex analysis theory is 0.0571–0.9833 million USD and 0.2732 million USD by HAZUS-AEBM manual methodology, i.e. it locates in the lower half interval of convex analysis results rather than the median. In addition, the economic loss of the modal adaptive distribution is larger than other lateral load patterns, and this phenomenon effected by the damage of the modal adaptive distribution is worse than the others.

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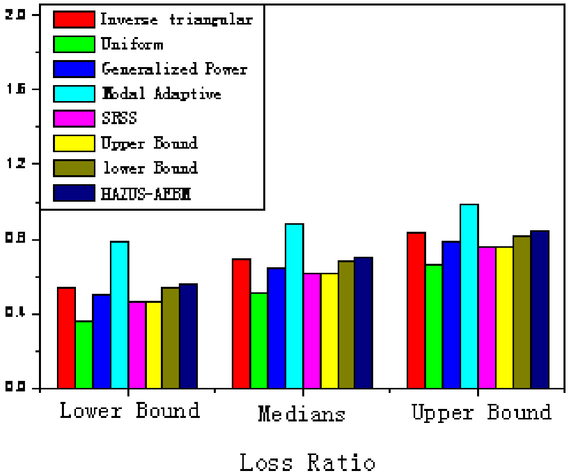

Fig. 13. Loss of upper bound of damage probability.

3.2. *The propagation of uncertainty*

The HAZUS-AEBM system includes structural capacity module, earthquake demand module, fragility analysis module, and economic loss estimation module. Each module requires to input a large amount of data based on an engineering perspective and assumptions that are inherently uncertain, and thus affect the reliability of the results. The main sources of uncertainty considered in this research include:

(1) The uncertainty of demand spectrum, which arises from the uncertainties of the maximum of seismic influence coefficient $\alpha_{\max}$ and the characteristic period T_g.
(2) The uncertainty of capacity spectrum, which is caused by different lateral load patterns of pushover analysis and the uncertainties of $\alpha_{\max}$ and T_g in SRSS load pattern.
(3) The uncertainty of damage state medians, which is caused by different pushover curves.
(4) The uncertainty of loss ratio.

The uncertainty propagation is analyzed by setting all uncertain sources to their deterministic value except for one, which is set first to be an uncertain variable. The results of uncertainty propagation are described in Table 2, and depicted graphically in the tornado diagram of Fig. 14. In Fig. 14, the horizontal and the vertical axes represent the earthquake loss and the uncertain variable, respectively. The effect of each basic variable is shown on bar, and the vertical line at 0.2732 shows the earthquake loss corresponding to HAZUS-AEBM manual methodology. The parameter with the largest impact on earthquake loss is shown on the top of the chart, and the smallest one is at the bottom.

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Table 2. Earthquake loss sensitivity analysis (million USD).

Uncertain sources	Lower bound	Upper bound
Capacity curves		
Load pattern	0.2599	0.3909
Two parameters of SRSS	0.2804	0.4411
Demand spectrum		
$\alpha_{\max}$	0.2324	0.3309
T_g	0.2464	0.4646
Damage state medians	0.1959	0.7777
Loss ratio	0.1527	0.3951

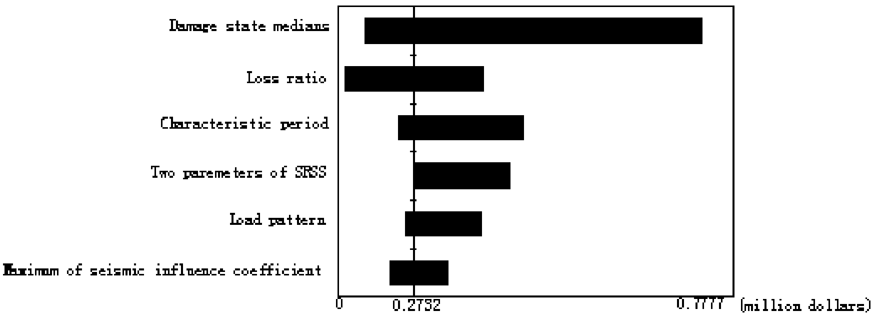

Fig. 14. Tornado diagram analysis of earthquake loss.

The greatest part of loss uncertainty is due to the uncertainty of damage state medians. Secondary contributors include the loss ratio, the characteristic period, and two parameters of the SRSS pattern. The uncertainty in loss associated with different load patterns and the maximum of seismic influence coefficient are small.

3.3. *Probability of the direct economic loss estimation bounds*

The basic equation for performance assessment of PEER can be expressed as

$$\lambda(DV) = \iiint G(DV \mid DM)dG(DM \mid EDP)dG(EDP \mid IM)d\lambda(IM), \quad (20)$$

where $\lambda(DV)$ is the mean annual frequency of exceedance of decision variable DV; $G(DV \mid DM)$ is the probability of exceeding DV conditioned on damage measure DM; $G(DM \mid EDP)$ is the probability of exceeding DM conditioned on engineering demand parameters EDP; $G(EDP \mid IM)$ is the probability of exceeding EDP conditioned on intensity measure IM; and $\lambda(IM)$ is the mean annual frequency of exceedance of intensity measure IM.

In this section, the probability of losing a certain amount in a given scenario $P(L_t \geq l_t \mid IM = im)$ is analyzed, and details are given as per Krawinkler [2005]. The PGA of the seismic fortification intensity 8 degree is 0.4082 g according to Chinese Code for Seismic Design of Buildings [GB50011-2001, 2001], and 20 ground

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motions scaled to 0.4082 g are utilized to derive the mean and the standard deviation of the engineering demand parameters (interstory drift ratio) by nonlinear dynamics history analysis. The medians and the standard deviation of different damage states are extracted from HAZUS-AEBM manual [NIBS, 2002]. The expected value of loss ratio is adopted as, 5.5% for slight damage, 25% for moderate damage, 60% for extensive damage, and 90% for complete damage.

In this study, the probability that the structure collapses under a ground motion with a level of intensity IM is deduced as

$$P(C \mid IM = im) = \int_0^\infty P[C \mid EDP_i = edp_i] \cdot |\, dP(EDP_i > edp_i \mid IM = im)|, \tag{21}$$

where $P(C \mid IM = im)$ is the probability that the structure collapses under intensity IM; $P(EDP_i > edp_i \mid IM = im)$ is the probability of exceeding edp_i when subjected to an intensity measure im; $P[C \mid EDP_i = edp_i]$ is the probability that the structure collapses when engineering demand parameter equals edp_i, and it can be obtained as follows

$$P(C \mid IM = im) = \int_0^\infty P[C \mid EDP_i = edp_i] \cdot |\, dP(EDP_i > edp_i \mid IM = im)|, \tag{22}$$

where $P(Complete \mid EDP_i = edp_i)$ is the probability that the structure reaches the complete damage state when subjected to an engineering demand parameter edp_i and can be calculated with fragility function referenced in Sec. 2.2. CR is the collapse probability when the structure reaches the complete damage state, and is proposed as 10% for this building by HAZUS-MH manual [NIBS, 2002]. Finally, the probability of loss exceeding the bound value of convex analysis results is $P(L_t \geq 0.0571\,\mathrm{M} \mid IM = 0.4082\,\mathrm{g}) = 0.4488$, and $P(L_t \geq 0.9833\,\mathrm{M} \mid IM = 0.4082\,\mathrm{g}) = 0.1121$.

4. Conclusions

This paper proposes an approach to earthquake loss estimation by integrating convex set theory with HAZUS-AEBM module, which is well suited for the case of only scarce seismic information being available. The following conclusions can be drawn from this study:

The uncertainties of spectral displacement, damage state medians, and loss ratio can be described by the envelope bound convex set model. The lower bounds of fragility curves can be obtained by simply setting spectral displacement at minimum and damage state medians at maximum, respectively. Contrarily, the upper bounds of fragility curves can be derived when spectral displacement reach maximum and damage state medians is minimum. Finally, the bounds of loss estimation can be derived by integrating HAZUS-AEBM module with the convex set theory.

The top three contributors to uncertainty in loss are damage state medians, loss ratio, and the characteristic period of response spectrum. The loss is not sensitive to the load patterns of pushover curves and the maximum of seismic influence coefficient.

The loss estimation results of a hotel in southern China shows that the loss by HAZUS-AEBM method locates in the lower half interval of convex analysis results. The probability of loss exceeding the lower and upper bound values of convex analysis results is 0.4488 and 0.1121, respectively.

The convex set theory-based approach requires less information; yet, it yields robust results compared to the probabilistic approach. The simple presentation of results in the form of interval gives the convenience of easy understanding, but it is still a top issue to be addressed about how to employ the bound results of convex analysis to make decisions. It should be also noted that the convex set theory with less information is less refined than probabilistic models, and the probabilistic model can give more accurate information compared to the convex set theory.

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