Short-Time Dynamical Behaviour of Depinning Transition in the Uniformly Frustrated Two-Dimensional XY Model *

NIE Qing-Miao(聂青苗)[1,2**], ZHOU Wei(周石岂)[2], LI Hai-Bin(李海彬)[1], XU Zhi-Jun(徐志君)[1], CHEN Qing-Hu(陈庆虎)[2]

[1]*Department of Applied Physics, Zhejiang University of Technology, Hangzhou 310023*

[2] *Department of Physics, Zhejiang University, Hangzhou 310027*

(Received 25 January 2008)

A short-time dynamic scaling approach is extended to study the depinning transition of the two-dimensional frustrated XY model driven by external currents. We investigate the scaling behaviour of depinning transition in the XY model with three different flux densities $f = 1/2, 1/25, 1/30$. The short-time scaling behaviour in the depinning transition of the two-dimensional XY model is clearly shown up. Besides the critical current, the exponent θ is obtained.

PACS: 64.60.Ht, 74.25.Sv, 74.25.Qt, 75.40.Gb

Recently, the short-time Monte Carlo (MC) method has witnessed great success in the equilibrium phase transition, since Janssen *et al.*[1] discovered that a universal dynamic scaling form exists in the macroscopic short-time regime. A number of MC simulations[2−6] confirmed the existence of such a short-time dynamic behaviour after a microscopic time scale $t_{\rm mic}$, if the system initially at a very high temperature with small or zero magnetization or in a ground state (GS) is relaxed toward the equilibrium state. This short-time dynamic scaling theory provides a new and practical technique for the measurement of the critical temperature as well as dynamic and static critical exponents in the second-order phase transition. Recently, this dynamic approach has been successfully applied or generalized to the weak first-order phase transition [7] as well as KT phase transition.[8] More recently, this approach has been successfully extended to study the non-equilibrium phase transition with small current or field.[9]

Depinning transitions are found in a large variety of physical problems, such as fluid invasion in porous media,[10,11] depinning of charge density waves, [12,13] depinning of flux line in type-II superconductors,[14−19] field-driven motion of domain walls in ferromagnets,[20,21] and elastic media in disordered environment.[22,23] In the mixed state of type-II superconductors, the magnetic field penetrates in the form of flux lines each carrying one flux quantum $\Phi_0 = h/2e$. At zero temperature the system exhibits a depinning transition from a pinned state below the critical driving force F_c or current I_c to a sliding state above F_c or I_c. The property of depinning transition is rather unconventional. Discontinuous curves of the average velocity v versus driving force F were found in 2D and 3D vortex systems at zero temperature[16,17,24] as well as in fully frustrated Josephson junction arrays (JJAs) at low temperatures.[25] However, mean-field theory on charge-density wave model predicted a continuous depinning transition for strong pinning case with a scaling law $v \sim (F - F_c)^\alpha$, and found the exponent $\alpha \approx 1.5$.[13] The renormalization-group (RG) theory on elastic system with quenched disorder gave $\alpha < 1$,[26] by assuming a continuous depinning transition. Simulations on two-dimensional (2D) vortex systems in the presence of random pins found concave upwards force-velocity ($v-F$) curves for weak pinning systems[15,27] and convex upwards ones for strong pinning systems.[27] They are also found in 2D colloid system with pins.[28] Up till now, no final conclusion has been achieved on this topic. As we known, the value of exponent α is important to grasp the property of the depinning transition. However, the determination of the exponent α is based on the accurate evaluation of the critical current I_c or force F_c. The short-time dynamic scaling approach is just one of good ways to obtain the critical current. It is our motivation of this work.

In this paper we apply the short-time dynamic scaling method to study the depinning transition of XY model and analyse its critical behaviour. In particular, we study the time dependence of the average voltage as a function of the applied current. We observe a continuous depinning transition and determine the corresponding critical exponents by short-time dynamic scaling analysis method.

We begin with the Hamiltonian of a 2D XY model

$$H = -J \sum_{\langle ij\rangle} \cos(\theta_i - \theta_j - A_{ij}), \tag{1}$$

*Supported by the National Natural Science Foundation of China under Grant No 10574107, the National Basic Research Programme of China under Grant No 2006CB601003, and the Natural Science Foundation of Zhejiang Province under Grant No Y607472.

**Email: qingmiao_nie78@hotmail.com or nieqingmiao@zjut.edu.cn

where θ_i denotes the phase of the superconducting order parameter on grain i, and J sets the strength of the Josephson coupling between neighbouring superconducting grains, and $A_{ij} \equiv (2e/\hbar)\int_i^j \boldsymbol{A} \cdot d\boldsymbol{l}$ is the integral of the vector potential $\boldsymbol{A}$ from site i to j. We assume a constant, uniform external field $\boldsymbol{B}$ along the z direction, for which the summation of A_{ij} around any unit cell is $\sum_{\text{cell}} A_{ij} = 2\pi f$, where the constant $f = BS/\Phi_0$ is the density of magnetic flux quanta per unit cell, and S is the area. In this work, we consider uniformly frustrated XY model with $f = 1/2, 1/25, 1/30$.

Simulations are performed on $L \times L$ square lattices within resistively-shunted junction (RSJ) dynamics under the fluctuating twist bounding condition (FTBC). External currents are applied in x direction with the current density $\boldsymbol{I} = (i_x, 0)$. The net current from site i to site j is written as the sum of the supercurrent and the normal resistive current,

$$I_{ij} = I_0 \sin(\phi_{ij} = \theta_i - \theta_j - A_{ij} - \mathbf{r}_{ij} \cdot \Delta) + V_{ij}/R, \quad (2)$$

where $I_0 \equiv 2eJ/\hbar$ is the critical current of the single junction, V_{ij} is the voltage drop across the junction, R is the shunt resistance, θ_i is periodic in both directions ($\theta_i = \theta_{i+L\hat{x}} = \theta_{i+L\hat{y}}$). $\boldsymbol{r}_{ij} \equiv r_j - r_i$ is a unit vector from site i to j, and $\boldsymbol{\Delta} = (\Delta_x, \Delta_y)$ is the fluctuating twist variable. Supposed that I_0 and R are the same for all junctions.

The phase variable θ_i on the site i with position vector $\boldsymbol{r}_i$ satisfies[29]

$$\dot{\theta}_i = -\sum_j G_{ij} {\sum_k}' [\sin(\phi_{jk})], \quad (3)$$

where G_{ij} is lattice Green's function, the primed summation is over nearest-neighbour sites (k) of j, and the unit of time is $\hbar/2eRI_0$. The dynamics of $\boldsymbol{\Delta}$ is given by[29]

$$\frac{d\Delta_x}{dt} = \frac{1}{L^2} \sum_{\langle ij \rangle x} \sin(\theta_i - \theta_j - A_{ij} - \Delta_x) - i_x, \quad (4)$$

$$\frac{d\Delta_y}{dt} = \frac{1}{L^2} \sum_{\langle ij \rangle y} \sin(\theta_i - \theta_j - A_{ij} - \Delta_y), \quad (5)$$

where $\sum_{\langle ij \rangle x}$ and $\sum_{\langle ij \rangle y}$ denote the summation over all links in the x direction and y direction, respectively. The current density $i_x \equiv I_x/I_0$ is in units of I_0. The voltage drop is $V = -L\dot{\Delta}_x$. For convenience, units are taken of $I_0 = R = \hbar/2e = J = 1$ in the following.

The applied external current I and resulted voltage drop V in 2D XY model are analogous to the driving force F and vortex velocity v in a superconducting thin film. Thus the following correspondences between the two systems apply: $F \leftrightarrow I$ and $v \leftrightarrow V$.

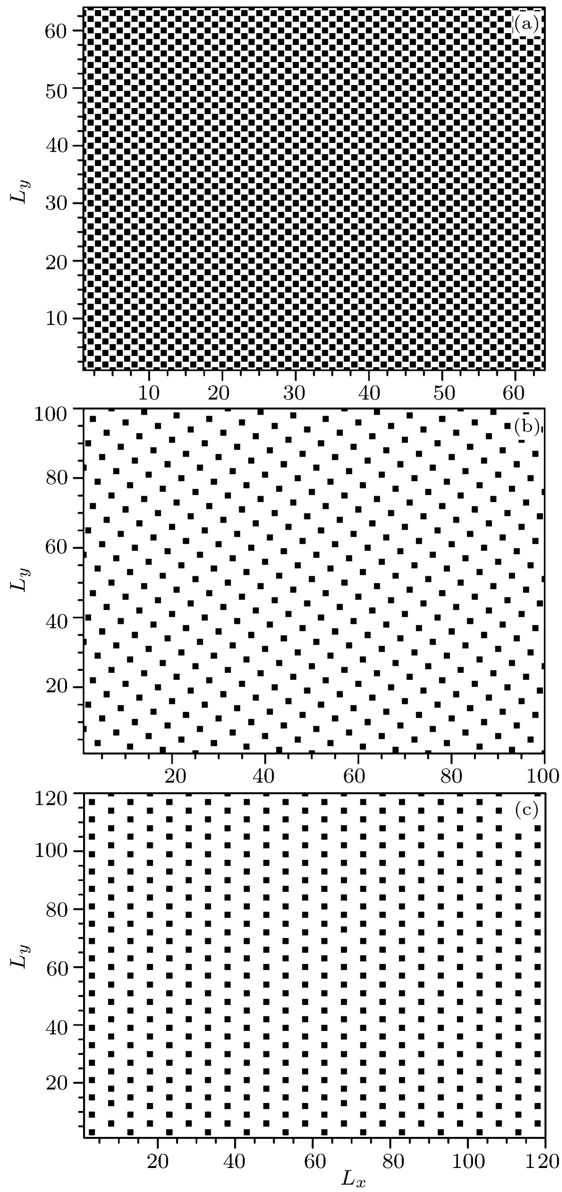

Fig. 1. Vortex distribution in the ground state for (a) $f = 1/2$ and annealing ground state for (b) $f = 1/25$, (c) $f = 1/30$. The corresponding system size is $L = 64$, 100 and 120, respectively.

We perform the short-time dynamic simulations from the ground state for $f = 1/2$ and annealing ground state for 1/25, 1/30 with the fluctuating twist variable $\Delta = (0, 0)$. Figure 1 shows the vortex distribution in these three cases. Analogy to the second-order phase transition, we choose voltage V along x direction as the order parameter. Simulations are performed on a square lattice of area $L \times L$ with current I near the critical current I_c. The evolution of the system driven with current I is investigated by solving the dynamic equations (3)–(5) with a second-order Runge–Kutta algorithm. According to the short-time dynamics scaling theory, after the microscopic time scale t_{mic} the scaling behaviour emerges and the order parameter obeys[5]

$$\langle V(t,i)\rangle = t^{-\theta}G(t^{1/\nu z}i), \tag{6}$$

where $i=(I-I_c)/I_c$ is the reduced current, t is the time, $\theta=\beta/\nu z$, β and ν are the static critical exponents, and z is the dynamical exponent. That means at the critical current I_c, *i.e.* $i=0$, the behaviour of voltage shows a power law relation $\langle V(t,I_c)\rangle \sim t^{-\theta}$, thus the critical exponent θ can be determined.

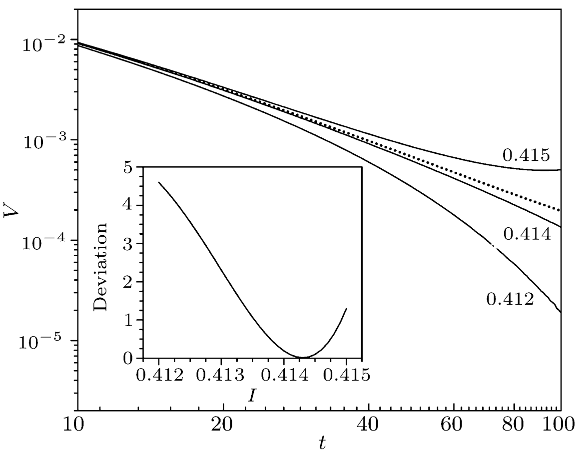

Fig. 2. Log-log plot of the time evolution of voltage $\langle V\rangle$ after a microscopic time scale $t_{\rm mic}=10$ for the FFXY model. The solid lines represent $\langle V\rangle$ for current $I=0.415$, 0.414, and 0.412. The dotted line is at $I_c=0.4142$. The inset displays the deviation of the voltage from the power law with current range from 0.415 to 0.412.

First, we consider XY model with $f=1/2$ (FFXY) and select three values of current which enclose the critical current. Figure 2 shows the decay of the average voltage $\langle V\rangle$ as a function of time at zero temperature in a log–log scale for three different currents. At the onset of the evolution, all curves decrease with time in a similar way. Different behaviour only emerges after a microscopic time scale $t_{\rm mic}=10$. Since for large currents, $\langle V\rangle$ reaches a finite steady value $V>0$, while for small currents, $V=0$. Thus the log–log $\langle V\rangle - t$ curves are concave for small currents and convex for large currents as shown in Fig. 2. Therefore, there is a drastic change of behaviour between $I=0.415$ and $I=0.412$. We can judge that I_c is between 0.415 and 0.412.

In order to precisely determine the critical current I_c, we present the time evolution of voltage $\langle V\rangle$ for current range from $I=0.415$ to $I=0.412$. With the voltage obtained at these three currents, we calculate the values of voltage for current within (0.415, 0.412) with a current step 0.001 by quadratic interpolation. For each value of current, the deviation of the voltage from the power law is calculated as the square deviations $SD=\sum[\langle V(t)\rangle - y(t)]^2$ in the time interval $[10,100]$, where the function $y(t)=C_1t^{-C_2}$ is obtained by linear fitting of log–log $\langle V\rangle - t$ curves in Fig. 2. In the inset of Fig. 2, the deviation of $\langle V\rangle$ is plotted as a function of current. The current at which the square deviation takes the minimum is defined as the critical current I_c. The resulting value of $I_c=0.4142(3)$ is in agreement well with that obtained from theoretical calculation $\sqrt{2}-1$. Furthermore, the voltage at I_c is also plotted in Fig. 2 with a dotted line. The slope of this curve yields the critical exponent $\theta=1.6873(5)$. We therefore conclude that the short-time dynamic scaling analysis method can be used to determine the critical current of depinning transition of 2D XY model. The advantage of short-time scaling approach is spare much more time which is free of critical slowing down. The accurate evaluation of I_c is very important since even a small error in its value can strongly affect the determination of the scaling exponents.

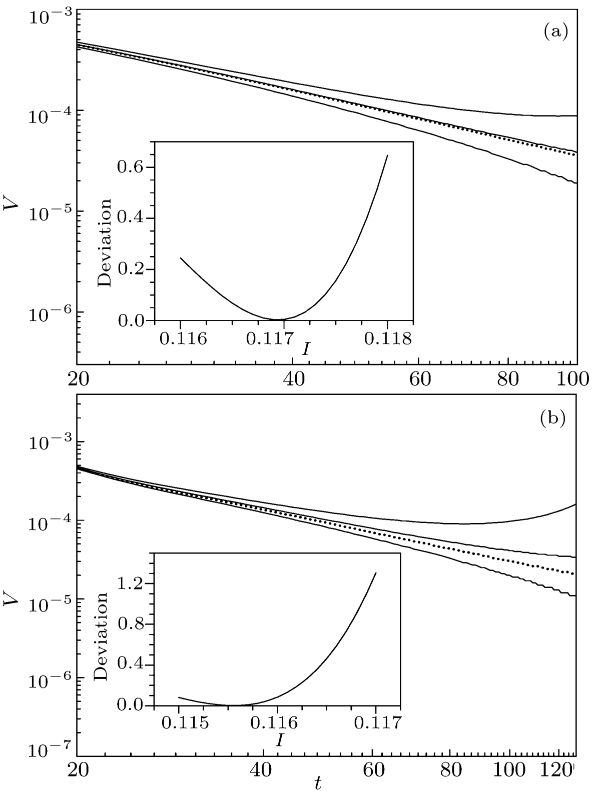

Fig. 3. Log–log plots of the voltage V versus time t with three currents denoted in the figure for (a) $f=1/25$ and (b) $f=1/30$. The dotted lines are at $I_c=0.1168$ for $f=1/25$ and $I_c=0.1156$ for $f=1/30$. The inset displays the deviation of the voltage from the power law with current range from 0.116 to 0.118 (a) and from 0.115 to 0.117 (b).

Following the short-time scaling approach described above, we accurately find the critical current and scaling exponents for $f=1/25$ and 1/30, respectively. All the results are listed in Table 1. The corresponding log–log plots of $V-t$ are presented in Fig. 3. On the other hand, with the voltage at each current obtained from long-time steady-state simulations we observe the continuous depinning transition which obeys $V \sim (I - I_c)^\alpha$ for these flux densities. Figure 4 plots the value of V as a function of $I - I_c$ for $f = 1/25$ in a log–log scale. A power-law behaviour $V \sim (I - I_c)^\alpha$ is shown in Fig. 4. The best fit to the data yields $I_c = 0.1169$, which agrees well with the result obtained from short-time dynamic scaling approach.

Table 1. Critical currents and the critical exponents for the 2D XY model with $f = 1/2, 1/25, 1/30$.

	$f = 1/2$	$f = 1/25$	$f = 1/30$
I_c	0.4142(2)	0.1168(3)	0.1156(4)
θ	1.6873(3)	1.6013(2)	1.6435(2)

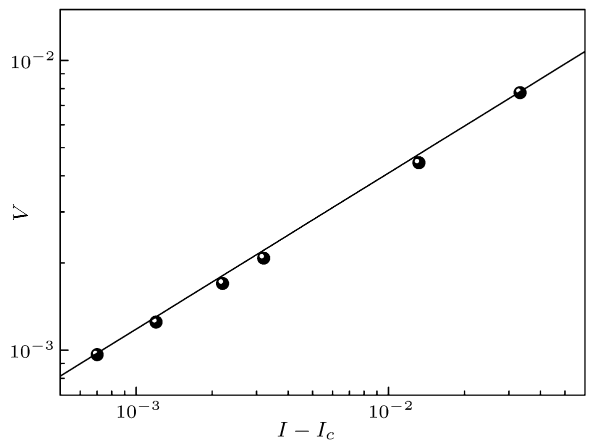

Fig. 4. Log-log plot of the steady-state average voltage as a function of the reduce current $I - I_c$ with $I_c = 0.1169$ for $f = 1/25$.

From Table 1, we observe that the critical current I_c increases with the density of flux f. The more the vortices, the larger the values of f, and the stronger the interaction between vortices, then the higher the depinning potential.[30] Therefore large critical current to make the vortices flow is needed. It is interesting that the exponent θ is roughly independent of the value of flux density. However, we cannot give the value of exponents β, ν and z in the framework of short-time dynamical scheme. With the help of long-time simulation results, we can obtain more information of depinning transition.

In conclusion, we have studied the depinning transition in the square lattice of the frustrated XY model by the short-time dynamic approach. It is observed that the resulting I_cs are consistent with those obtained from the steady-state long time simulations. We therefore conclude that the short-time scaling analysis method can be applicable to the depinning transition of frustrated XY model driven by external current. We also find that the critical current I_c increases with the flux density f.

References

[1] Janssen H K, Chaub B and Schmittmann B 1989 *Z. Phys.* B **73** 539
[2] Li Z B, Schülke L and Zheng B 1995 *Phys. Rev. Lett.* **74** 3396
[3] Zheng B 1998 *Int. J. Mod. Phys.* B **12** 1419
[4] Jaster A 1999 *Phys. Lett.* A **258** 177
[5] Luo H J, Schülke L and Zheng B 1998 *Phys. Rev. Lett.* **81** 180
[6] Chen Q H, Luo M B and Jiao Z K 2001 *Phys. Rev.* B **64** 212403
[7] Schülke L and Zheng B 2000 *Phys. Rev.* E **62** 7482
[8] Zheng B, Schulz M and Trimper S 1999 *Phys. Rev.* E **59** R1351
[9] Nie Q M, Luo M B and Chen Q H 2006 *Phys. Rev.* B **74** 024523
Nie Q M, Luo M B and Chen Q H 2008 *Eur. Phys. J.* B (submitted)
Nie Q M, Luo M B and Chen Q H 2006 *Eur. Phys. J.* B **74** 024523
Nie Q M, Luo M B and Chen Q H 2008 *Phys. Rev.* B (submitted)
[10] Cieplak M and Robbins M 1988 *Phys. Rev. Lett.* **60** 2042
[11] Avramenko A A, Kuznetsov A V, Basok B I and Blinov D G 2005 *Phys. Fluids* **17** 94102
[12] Fisher D S 1983 *Phys. Rev. Lett.* **50** 1486
[13] Fisher D S 1985 *Phys. Rev.* B **31** 1396
[14] Bhattacharya S and Higgins M J 1993 *Phys. Rev. Lett.* **70** 2617
[15] Jensen H J, Brass A, Brechet Y and Berlinsky A J 1988 *Phys. Rev.* B **38** 9235
[16] Jensen N G, Bishop A R and Domínguez D 1996 *Phys. Rev. Lett.* **76** 2985
[17] Reichhardt C, Olson C J and Nori F 1998 *Phys. Rev.* B **58** 6534
[18] Olson C J, Reichhardt C and Vinokur V M 2001 *Phys. Rev.* B **64** 140502
[19] Chandran M, Scalettar R T and Zimányi G T 2002 *Phys. Rev. Lett.* **89** 187001
[20] Bruinsma R and Aeppli G 1984 *Phys. Rev. Lett.* **52** 1547.
[21] Roters L, Hucht A, Lubeck S, Nowak U and Usadel K D 1999 *Phys. Rev.* E **60** 5202
[22] Marchetti M C, Middleton A A, Saunders K and Schwarz J M 2003 *Phys. Rev. Lett.* **91** 107002
[23] Reichhardt C, Olson C J, Martin I and Bishop A R 2003 *Europhys. Lett.* **61** 221
[24] Luo M B and Hu X 2005 *Int. J. Mod. Phys.* B **19** 455
[25] Maeconi V I and Domínguez D 2001 *Phys. Rev. Lett.* **87** 17004
[26] Chauve P, Giamarchi T and Le Doussal P 2000 *Phys. Rev.* B **62** 6241
[27] Faleski M C, Marchetti M C and Middleton A A 1996 *Phys. Rev.* B **54** 12427
[28] Chen J X, Cao Y G and Jiao Z K 2003 *Phys. Lett.* A **318** 146
[29] Kim B J, Minnhagen P and Olsson P 1999 *Phys. Rev.* B **59** 11506
[30] Falo F, Bishop A R and Lomdahl P S 1990 *Phys. Rev.* B **41** 10983