

Phase-modulated Waveform Design Using Maximum Mutual Information Criterion*

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Abstract — The optimal radar waveform for target information extraction had been derived to be the water-filling solution using maximum Mutual information (MI) criterion, however, the optimal waveform is not a constant-envelope signal. In order to make full use of the system radiated power, a phase-iterative method to design the constant-envelope phase-modulated waveform is proposed. By minimizing the Euclidean distance between the designed waveform and the optimal one, the proposed method can achieve a small MI loss. The results of extensive simulations demonstrate that the MI loss of our approach can be less than 0.5nat when the signal duration is greater than 1 μ s.

Key words — Waveform design, Mutual information (MI), Radar, Target information extraction.

I. Introduction

The radar system can achieve the target detection, tracking and recognition by analyzing the received echo, so it is very important to select or design the appropriate transmitted waveform to extract the information of targets, which can improve the target detection and recognition efficiency. Therefore, the waveform design based on the given properties of the target and clutter plays the very important role in the smart radar signal processing in recent years. Additionally, as the resolution of radar system improves increasingly, the assumption of point-target will not hold because the spatial extent occupied by the observed target exceeds one resolution cell, and then, the extended target model is proposed to accurately represent the behavior of observed targets. In this paper, a phase-modulated waveform design based on the water-filling solution is concerned in order to fully exploit the radiated power.

Much earlier work has been presented on the techniques of radar waveform optimization and design in the past decades. In Refs.[1, 2], the optimal transmitter-receiver design methods under the constraints of signal energy and bandwidth were proposed according to the Signal-to-noise ratio (SNR) or Signal-to-interference ratio (SINR) criteria for the target detection. In Ref.[3], waveform designs created maximizing the average distance between the different targets echoes were used to im-

prove the performance of a closed-loop radar system applied to target recognition. In Refs.[1, 4], the MI between the received measurement and the target response was applied to be the criteria of the optimal waveform design for the target information extraction problem. However, the assumption of an arbitrary waveform used in Refs.[1–4] is not appropriate for a practical radar system because it is extremely difficult to implement. The maximum waveform modulus constraint is more suitable for the practical radar systems, and a constant modulus waveform can fully exploit the power of the transmitter^[5]. The constant modulus constraint was previously discussed and adopted in the optimal waveform design for improving target detection^[6–8]. In Ref.[9], a method based on phase-modulated signal was proposed to exploit the transmit capability, whose effect would be deteriorated in the heavy clutter. In Ref.[10], the design of unimodular sequences with good autocorrelation properties was solved by minimizing the Integrated sidelobe level (ISL) of sequences.

In this paper, we propose a phase-iterative method to design the phase-modulated waveform in order to make full use of the system radiated power. The original contribution of this paper is that the method can ensure a small MI loss and make the designed waveform approximate the water-filling solution as quickly as possible.

The paper is organized as follows. In Section II, we present the simplified signal model and the optimal waveform design. In Section III, we introduce a phase-iterative method for designing the phase-modulated waveform. In Section IV, we present the performance results and discuss the proposed waveform design algorithm. Our conclusions are given in Section V.

II. System Model and Optimal Waveform Design

The block diagram in Fig.1 illustrates the simplified signal model with random target in signal-dependent clutter. In the model, $x(t)$ is a complex-valued transmitted waveform with finite duration T and energy, $h(t)$ is a Gaussian extended target ensemble with Energy spectral variance (ESV) $\sigma_H^2(f)$, $w(t)$ is a zero-mean complex Wide sense stationary (WSS) Gaus-

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sian random process with known Power spectral density (PSD) $P_{ww}(f)$, $n(t)$ is the zero-mean additive WSS Gaussian noise with known PSD $P_{nn}(f)$. From the previous model assumptions, the received signal is given by

$$y(t) = x(t) * h(t) + x(t) * w(t) + n(t) \quad (1)$$

The MI $I(y(t); h(t)|x(t))$ between the received radar echo $y(t)$ and the target ensemble $h(t)$ given a transmitted signal $x(t)$ is adopted to measure the amount of extended target information^[1], which can be further given by

$$I(y(t); h(t)|x(t)) = T \int_{-B/2}^{B/2} \ln \left[1 + \frac{2|X(f)|^2 \sigma_H^2(f)}{TP_{nn}(f) + 2|X(f)|^2 P_{ww}(f)} \right] df \quad (2)$$

where $X(f)$ is the Fourier transform of transmitted signal $x(t)$, B is the operation bandwidth. Considering the transmitted energy constraint, the water-filling waveform which can maximize $I(y(t); h(t)|x(t))$ was given in Ref.[4], whose Energy spectral density (ESD) $\varepsilon_{opt}(f)$ is described by

$$\begin{aligned} \varepsilon_{opt}(f) &= |X_{opt}(f)|^2 \\ &= \max[0, -R(f) + \sqrt{R^2(f) + S(f)(A - D(f))}] \end{aligned} \quad (3)$$

where

$$\begin{aligned} D(f) &= \frac{TP_{nn}(f)}{2\sigma_H^2(f)} \\ R(f) &= \frac{TP_{nn}(f)(2P_{ww}(f) + \sigma_H^2(f))}{4P_{ww}(f)(P_{ww}(f) + \sigma_H^2(f))} \\ S(f) &= \frac{TP_{nn}(f)\sigma_H^2(f)}{2P_{ww}(f)(P_{ww}(f) + \sigma_H^2(f))} \end{aligned} \quad (4)$$

A is a constant and depends on the energy constraint $\int_{-B/2}^{B/2} |X(f)|^2 df = E$. Therefore, the maximum MI $I(y(t); h(t)|x(t))$ is given by

$$I_{\max}(y(t); h(t)|x(t)) = T \int_{-B/2}^{B/2} \ln \left[1 + \frac{2\varepsilon_{opt}(f)\sigma_H^2(f)}{TP_{nn}(f) + 2\varepsilon_{opt}(f)P_{ww}(f)} \right] df \quad (5)$$

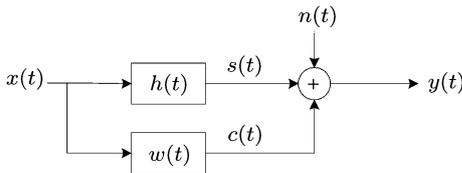


Fig. 1. Signal model with random target in signal-dependent clutter

III. Phase-modulated Waveform Design

From the result of optimal ESD given in Eq.(3), the transmitted signal can be synthesized by using Durbin's method as described in Ref.[11] in accordance with the optimal ESD.

However, the transmitter of actual radar system is power-limited, the designed signal can not ensure the peak power transmission in the duration time because it is amplitude-modulated, so this method can not fully exploit the power of the transmitter. In order for the transmitter to work under maximum power within the pulse duration, the modulus of the transmitted waveform should be a constant that equals the maximum waveform modulus, so that the phase-modulated signal with constant envelop is employed. The stationary phase method can be used to design the phase-modulated signal with a large time-bandwidth product^[12]. However, it is difficult to obtain the designed signal in accordance with the arbitrary auto-correlation function. Based on these considerations, a phase-iterative algorithm is proposed for use in designing a phase-modulated waveform that can approach the optimal transmitted waveform.

We assume that $u_{pm}(t)$ is a complex phase-modulated waveform given by

$$u_{pm}(t) = ce^{j\varphi(t)}, \quad t \in [-T/2, T/2] \quad (6)$$

where c is the constant-valued modulus related to the transmitter power. The ESD of signal $u_{pm}(t)$ will then be

$$\varepsilon_{pm}(f) = |U_{pm}(f)|^2 = \left| \int_{-T/2}^{T/2} u_{pm}(t) e^{-j2\pi ft} dt \right|^2 \quad (7)$$

where $U_{pm}(f)$ is the Fourier transform of $u_{pm}(t)$. The sampled signal of $u_{pm}(t)$ in vector form is given by

$$\mathbf{u} = c[e^{j\varphi_0}, e^{j\varphi_1}, \dots, e^{j\varphi_{N-1}}]^T \quad (8)$$

and the phase vector is denoted as

$$\boldsymbol{\varphi} = [\varphi_0, \varphi_1, \dots, \varphi_{N-1}]^T \quad (9)$$

where $[\cdot]^T$ is the transposed operator, $N = T/T_s$, T is the signal duration time and T_s is the sampling interval.

Our objective is to design the signal $u_{pm}(t)$ that can make the corresponding MI I_{pm} achieve or approximate the maximum value of MI I_{\max} given in Eq.(5). The amplitude of $u_{pm}(t)$ is constant, therefore, the problem can be stated as "finding the appropriate phase vector $\boldsymbol{\varphi}$ that can make the MI loss between the optimal waveform and phase-modulated waveform as small as possible", which is defined by

$$\Delta I = I_{\max} - I_{pm} \quad (10)$$

From the definition of MI in Eq.(2), it is difficult to obtain straightforward minimization of ΔI by designing $\boldsymbol{\varphi}$. In Appendix, we have derived that the inequation

$$\Delta I \leq \tilde{K}' \sqrt{\int_{-B/2}^{B/2} (\varepsilon_{opt}(f) - \varepsilon_{pm}(f))^2 df} \quad (11)$$

is held, where \tilde{K}' is a constant related to the signal duration time, the ESV of target and the PSD of additive noise. Therefore, we define the penalty function

$$G(\boldsymbol{\varphi}) = \int_{-B/2}^{B/2} (\varepsilon_{pm}(f) - \varepsilon_{opt}(f))^2 df \quad (12)$$

to describe the ESD difference between the phase-modulated waveform and the optimal transmitted waveform. Our next objective is to minimize the ESD difference $G(\boldsymbol{\varphi})$ within the passband $[-B/2, B/2]$ by optimizing the phase vector $\boldsymbol{\varphi}$. From

the result in Eq.(11), the approximation between the optimal signal and phase-modulated signal can also promise a small MI loss. Because the ESD difference $G(\varphi)$ can converge to zero when the signal duration T is infinite^[13], the I_{pm} will equal the I_{max} in Eq.(11), therefore, the minimization of the ESD difference $G(\varphi)$ in Eq.(12) can make I_{pm} approach the upper bound I_{max} as closely as possible.

The function defined in Eq.(12) can be obtained using the Discrete Fourier Transform of $u(t)$, which is

$$G(\varphi) = F_s \sum_{m=0}^{M-1} \left(\left| eT_s \sum_{n=0}^{N-1} e^{j(\varphi_n - 2\pi f_m n T_s)} \right|^2 - \varepsilon_{opt}(f_m) \right)^2 \quad (13)$$

where $M = B/F_s$, $f_m = -B/2 + mF_s$, $e = \sqrt{E/T}$, E is the transmitted signal energy and F_s is the sampling interval in the frequency domain. Due to the nonlinearity, it is difficult to find an analytical solution to minimize $G(\varphi)$. To solve this problem, a phase-iterative method in which the calculation in each step is analytical is proposed. This method of updating φ can also keep the ESD difference $G(\varphi)$ decreasing monotonically with each iterative step. Besides, the optimal phase element φ_k that can minimize $G(\varphi)$ must satisfy $\partial G(\varphi)/\partial \varphi_k = 0$ and $\partial^2 G(\varphi)/\partial \varphi_k^2 > 0$, where $\partial G(\varphi)/\partial \varphi_k$ is the first-order partial derivative of $G(\varphi)$ with respect to phase φ_k which indicates the phase of $u(t)$ at the k_{th} sampling time, and $\partial^2 G(\varphi)/\partial \varphi_k^2$ is the second-order partial derivative. Therefore, the phase-iterative algorithm for the phase-modulated baseband waveform design can be summarized as follows.

- (1) Initialise the phase vector φ , e.g. $\varphi^{(0)} = [0, 0, \dots, 0]^T$.
- (2) Let the initial number be $p = 0$, and calculate the ESD difference $G^{(0)}$.
- (3) $p = p + 1$.
- (4) Set k from 0 to $N - 1$, numerically solve the equation $\partial G(\varphi)/\partial \varphi_k^{(p)} = 0$ and find the solution $\varphi_k^{(p)}$, which satisfies $\partial^2 G(\varphi)/\partial (\varphi_k^{(p)})^2 > 0$.
- (5) Update $\mathbf{u}_{pm}^{(p)} = ce^{j\varphi^{(p)}}$, $G^{(p)} = G(\mathbf{u}_{pm}^{(p)})$ and $\delta = G^{(p)} - G^{(p-1)}$.
- (6) If $\delta > D$, where D is a predefined threshold, go to the step (3); otherwise, $\mathbf{u}_{pm}^{(p)}$ is the solution.

Since $G(\varphi)$ decreases monotonically with each iterative step, the phase-modulated waveform ESD $\varepsilon_{pm}(f)$ can approximate the optimal transmitted ESD $\varepsilon_{opt}(f)$ as much as possible.

IV. Simulations and Discussions

1. A numerical example and statistical result

We now consider the following scenario. The bandwidth B is 10MHz, the energy of transmitted signal E is 10^4 joules, the signal duration time T is $2\mu s$, the sampling interval T_s is $1/(20B)$, the cut-off threshold D is 10^{-5} , and the additive noise is white with PSD $P_{nn}(f) = 1$ for $f \in [-B/2, B/2]$. The upper subfigure of Fig.2 shows the ESV of the random target impulse response; the lower subfigure shows the PSD of the clutter channel. Fig.3 shows the optimal waveform (dashed curve) and the designed phase-modulated waveform (solid curve) for extended target information extraction. As expected, the iterative solution designed by our method and the water-filling solution are very similar. Besides, the

transmitted waveform places as less energy as possible into the frequency band in which the clutter is significant to de-emphasize the clutter effect. The MI under the designed phase-modulated signal is $I_{pm} = 25.0$ nats, while the maximal MI under the optimal waveform is $I_{max} = 25.2$ nats, so the MI loss is $\Delta I = 0.2$ nat. For the same case corresponding to Fig.2, an improvement over the LFM signal is 7.7nats because of $I_{LFM} = 17.5$ nats. Fig.4 shows the variations of the MI I_{pm} (the left longitudinal axis of the coordinates) and the ESD difference $G(\varphi)$ (the right longitudinal axis of the coordinates) over 10 iterative steps. We can see that with decreasing $G(\varphi)$ the corresponding MI under the designed phase-modulated signal, shown by the dashed curve, converges to the maximum MI and that it achieves 0.25nat loss at the 10th iteration. From the above results, it can be concluded that the phase-modulated waveform designed by our proposed phase-iterative algorithm can achieve satisfactory MI approximation to the maximum MI. To further evaluate the performance of our proposed phase-iterative method, we performed Monte Carlo simulations with 1000 times. In each trial, the ESV of target and the power spectrum of clutter are random generated. Fig.5 shows the results of average ESD difference and MI loss of the Monte Carlo simulation. As shown in Fig.5, the average MI loss decreases and converges to zero with increased signal duration because the monotonically-decreasing ESD difference $G(\varphi)$ is much smaller with the longer signal duration.

2. Initialization of phase-iterative algorithm

The initialization of the phase vector φ in the proposed

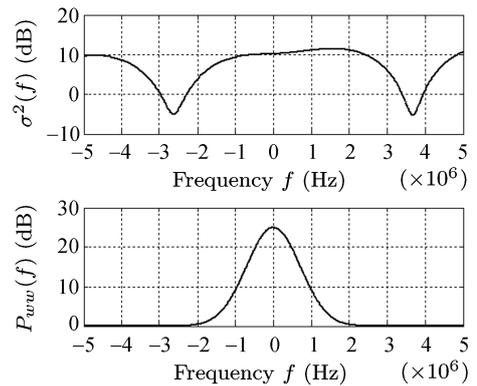


Fig. 2. ESV of extended target and PSD of clutter

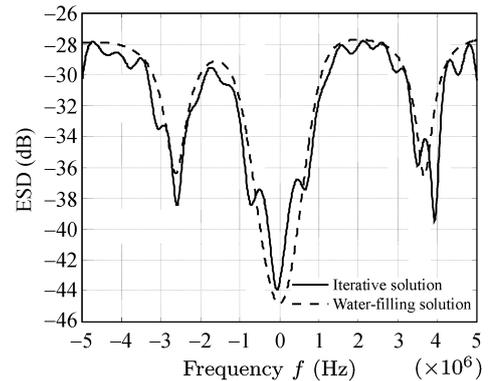


Fig. 3. ESD of the Optimal transmitted signal and the phase-modulated signal

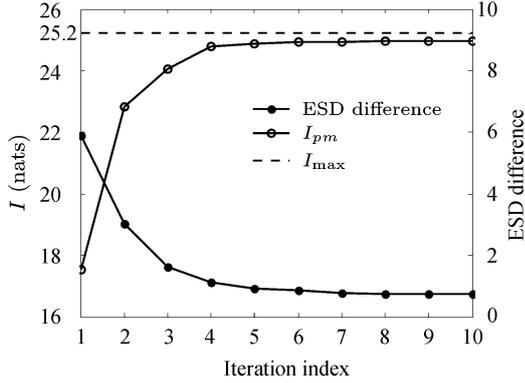


Fig. 4. MI and ESD difference versus number of iterations

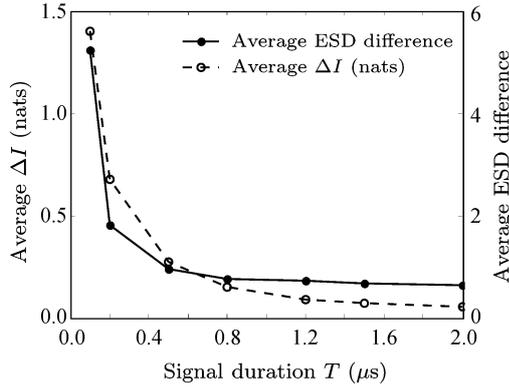


Fig. 5. Average ESD difference and MI difference

phase-iterative algorithm is a considerable problem that can affect the convergence of the iterative solutions. In our method, the phase vector φ of the phase-modulated signal with constant envelop $u_{pm}(t)$ is generally initialized to zeros and will be adjusted within the iterations to reduce the ESD difference $G(\varphi)$. Because the phase term of $u_{pm}(t)$ contains the spectrum characteristics of $U_{pm}(f)$, the phase vector φ can also be initialized to the angle of optimal signal $u_{opt}(t)$, which is the inverse Fourier transform of the optimal transmitted spectrum $U_{opt}(f)$. In this way, more energy of $u_{pm}(t)$ can be concentrated at a frequency band in which the optimal transmitted ESD $\varepsilon_{opt}(f)$ is much larger. However, from the

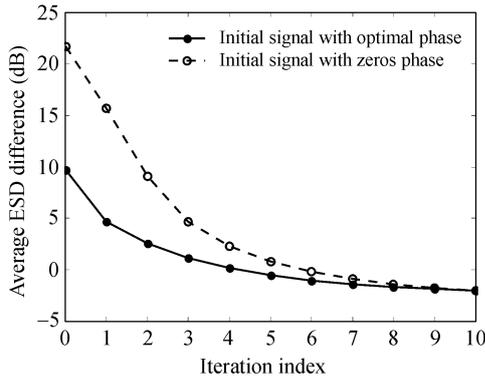


Fig. 6. Average ESD difference with different initializations

definition of ESD, the phase of $u_{opt}(t)$ can not be directly obtained from the optimal ESD $\varepsilon_{opt}(f)$. We therefore as-

sume that the phase of the optimal spectrum $U_{opt}(f)$ is represented by an independent sampling of a uniform distribution on $[-\pi, \pi]$. Fig.6 shows the results of a Monte Carlo simulation with 1000 target samples and clutter power spectrum samples when the signal duration T is assumed to be $2\mu s$. As seen in Fig.6, the phase initialization of signal $u_{pm}(t)$ by the optimal phase, which is the angle of the optimal signal $u_{opt}(t)$, can achieve a better ESD approximation with fewer iterations.

V. Conclusions

The problem of transmitted waveform design for target information extraction in the signal-dependent clutter has been the subject of much research. In this paper, we present a new phase-iterative method for designing the phase-modulated waveform. Based on the optimal transmitted ESD, the Euclidean distance between the designed waveform and the optimal one can be minimized by sequentially adjusting the phase of transmitted signal in the each iterative step, which can ensure the approximation of MI to the maximum MI. The numerical simulations show that the ESD of the phase-modulated waveform is close to the optimal ESD and that the MI loss can be less than $0.5nat$ when the signal duration is longer than $1\mu s$.

Appendix

In Section II, the maximum MI between the received radar echo and the target impulse response is given by

$$I_{max} = T \int_{-B/2}^{B/2} \ln \left[1 + \frac{2\varepsilon_{opt}(f)\sigma_H^2(f)}{TP_{nn}(f) + 2\varepsilon_{opt}(f)P_{ww}(f)} \right] df \quad (14)$$

Therefore, when the transmitted waveform is a phase-modulated signal $u_{pm}(t)$, the corresponding MI can be expressed as

$$I_{pm} = T \int_{-B/2}^{B/2} \ln \left[1 + \frac{2\varepsilon_{pm}(f)\sigma_H^2(f)}{TP_{nn}(f) + 2\varepsilon_{pm}(f)P_{ww}(f)} \right] df \quad (15)$$

For simplicity, we let

$$A(f) = \frac{2\varepsilon_{opt}(f)\sigma_H^2(f)}{TP_{nn}(f) + 2\varepsilon_{opt}(f)P_{ww}(f)} \quad (16)$$

$$B(f) = \frac{2\varepsilon_{pm}(f)\sigma_H^2(f)}{TP_{nn}(f) + 2\varepsilon_{pm}(f)P_{ww}(f)} \quad (17)$$

From Eqs.(14) and (15), the MI difference is defined as

$$\begin{aligned} I_{max} - I_{pm} &= T \int_{-B/2}^{B/2} \ln \frac{1 + A(f)}{1 + B(f)} df \\ &= -T \int_{-B/2}^{B/2} \ln \left(1 + \frac{B(f) - A(f)}{1 + A(f)} \right) df \end{aligned} \quad (18)$$

Applying the Taylor series expansion to Eq.(18), we have

$$\begin{aligned} I_{max} - I_{pm} &\approx -T \int_{-B/2}^{B/2} \frac{(B(f) - A(f))/(1 + A(f))}{1 + (B(f) - A(f))/(1 + A(f))} df \\ &= T \int_{-B/2}^{B/2} \frac{A(f) - B(f)}{1 + B(f)} df \\ &\leq T \int_{-B/2}^{B/2} (A(f) - B(f)) df \end{aligned} \quad (19)$$

However,

$$\int_{-B/2}^{B/2} (A(f) - B(f)) df \leq \int_{-B/2}^{B/2} \frac{2T\sigma_H^2(f)}{P_{nn}(f)} |\varepsilon_{opt}(f) - \varepsilon_{pm}(f)| df \quad (20)$$

Applying the Schwartz Inequality to Eq.(20), we can further obtain

$$\left(\int_{-B/2}^{B/2} (A(f) - B(f)) df \right)^2 \leq \int_{-B/2}^{B/2} \left(\frac{2T\sigma_H^2(f)}{P_{nn}(f)} \right)^2 df$$

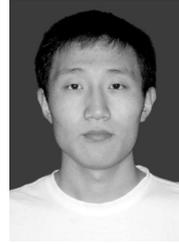
$$\int_{-B/2}^{B/2} (\varepsilon_{opt}(f) - \varepsilon_{pm}(f))^2 df \quad (21)$$

Here, we let $\tilde{K}' = T \sqrt{\int_{-B/2}^{B/2} (2T\sigma_H^2(f)/P_{nn}(f))^2 df}$. In Section II, the signal duration T , the ESV $\sigma_H^2(f)$ and the PSD of the noise $P_{nn}(f)$ are assumed to be known, \tilde{K}' will be a constant. Thus, we have

$$I_{\max} - I_{pm} \leq \tilde{K}' \sqrt{\int_{-B/2}^{B/2} (\varepsilon_{opt}(f) - \varepsilon_{pm}(f))^2 df} \quad (22)$$

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