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Scheme for opportunistic spectrum access in cognitive radio

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Abstract: In this study, an opportunistic spectrum access model with learning strategy is presented for cognitive radio system. Consider a hidden Markov model in learning process, where the ternary hypothesis testing scheme is proposed to perform sensing with the goodness of fit testing. By using a gradient method, the secondary user can estimate the channel patterns and keep up with the variations of the primary user activities. An opportunistic access channel capacity is introduced to evaluate the quality of service of the objective licensed channel. Also, a partially observable Markov decision process framework is presented to exploit spectrum holes. Further analysis shows that, unlike the binary hypothesis testing where the idle state is always protected, the idea of the proposed ternary hypothesis testing puts both the idle and busy state in the same position, which reflects the real state of the licensed channel more precisely. Simulation results indicate that the proposed ternary hypothesis testing scheme outperforms the conventional binary hypothesis testing for both the goodness of fit testing and the energy detection.

1 Introduction

Cognitive radio (CR) is a promising wireless communication technology to improve the efficient usage of the licensed spectrum [1]. As one of the basic operation models for CR system, opportunistic spectrum access (OSA) enables secondary users (SUs) to access licensed spectrum without causing harmful interference to primary users (PUs) [2–4]. For the OSA model, a SU usually deploys spectrum sensing technique to detect the PU transmission state, and then adapts channel parameters to the changing environment. Based on the estimate of channel pattern of the PU, the SU decides whether to choose current channel or switch to another one. Finally, the SU exploits spectrum holes to transmit data. The above process consists of an intelligent cognition cycle.

Most of the related work has shown the perfect performance of spectrum sensing [5, 6]. When the SU estimates the channel pattern, a hidden Markov model (HMM) has been used in learning process [7, 8], in which the maximum-likelihood (ML) estimation can be achieved by using the gradient method. Moreover, In [9–11] gave some simple ways to compute the probability of the observation sequence under the given model. Lee and Akyildiz [12] provided an optimal spectrum sensing framework by choosing proper observation time and transmission time. An efficient channel selection algorithm called myopic sensing policy was proposed in [13]. For the rapidly variations of PU activities, by considering the transition state, the proposed ternary hypothesis testing scheme can reduce too much access and exit delay [14]. Monahan [15] developed partially observable Markov decision process (POMDP) framework and presented algorithms for computing optimal solutions to POMDP. As pointed out in [16], a learning-based OSA model over data-centric PU network divided a frame into a channel learning subframe and a channel access subframe, where the SU decided to perform spectrum sensing or data transmission during each slot adaptively. However, most of existed work assumed that the SU detected channel during each slot only with two results, idle or busy, but never considered the switch between them, which leads to lower performances especially in the rapid variations of the PU activity.

In this paper, we consider a cognition cycle which contains four processes: spectrum sensing, learning process, channel selection and channel access. First, the SU detects the current channel during each slot with the following three results: idle, busy and the transition between idle and busy, where the goodness of fit testing is performed in spectrum sensing. Second, assuming the lengths of the PU idle and busy times follow exponential distribution, the SU estimates channel pattern by using HMM. Then, the SU decides to choose the current channel or switch to another one based on the opportunistic access channel capacity. Finally, according to the POMDP model, the SU accesses the channel to transmit data. Simulation results indicate that compared with the conventional binary hypothesis testing scheme, our proposed scheme achieves higher channel utilisation without increasing collision rate by using ternary hypothesis testing.

The rest of paper is organised as follows. Section 2 describes the adaptive sensing framework for the OSA model. In Section 3, the opportunistic access is presented with the proposed ternary hypothesis testing scheme. In Section 4, we present some representative simulation results. Finally, this paper is concluded in Section 5.

2 System model

2.1 PU model

The PU network has a license to use M frequency channels, and each of them has a bandwidth of W. Fig. 1 illustrates a two-state continuous-time Markov chain to describe the PU activity in a licensed channel. The channel state alternates between idle (i.e. 0) and busy (i.e. 1), whereas the lengths of idle and busy time are independent of each other with exponentially distributed with the mean $1/\alpha$ and $1/\beta$, respectively. Consider an additive white Gaussian noise (AWGN) channel, the received sample follows $N(\mu, \sigma^2)$, where μ denotes the amplitude of the PU signal and σ^2 denotes the noise variance. Hence, the PU channel pattern is completely determined by a vector with four parameters α, β, μ and σ, that is, $\boldsymbol{\lambda} = (\alpha, \beta, \mu, \sigma) \in \boldsymbol{\Lambda}$. It is assumed that the channel pattern varies so slowly that the PU activity is ergodic during a frame.

2.2 SU model

As was shown in Fig. 1, each frame is divided as a channel learning subframe and a channel access subframe, which consists of N_L slots and N_A slots, respectively [16]. It is assumed that the clocks of the SUs will be maintained synchronisation by themselves and are also independent with those of the PUs. During the channel learning subframe, the SU should estimate the channel pattern and adapt channel parameters to the changing environment in order to access licensed channel in an optimal way. Although during the channel access subframe, the SU either performs sensing or transmits data based on the most recent sensing results as well as the prior knowledge of channel pattern in the preceding channel learning subframe.

In the channel learning subframe, a SU detects licensed channel by using the goodness of fit testing for a slot duration of T. Based on the ternary hypothesis testing, the observation during each slot is one of the following three results: idle, busy and the transition state. By modelling an HMM, the SU estimates channel pattern $\boldsymbol{\lambda}$ from the sensing results. Then, the SU calculates opportunistic access channel capacity C_{op} and decides to choose current channel or switch to another one by comparing C_{op} with the given threshold C_t, which could provide sufficient access opportunities to support quality of service (QoS) requirements.

In the channel access subframe, the SU decides to perform spectrum sensing or data transmission during each slot adaptively according to the rate of the PU activity variations. By using a POMDP framework, the SU could prevent unnecessary sensing to maximise channel utilisation as much as possible, while keeping the rate of collision with the PU at a low level. In Section 3, we will explain both the channel learning algorithm and the channel access algorithm in detail.

3 Opportunistic access with learning strategy

3.1 Spectrum sensing

The SU performs spectrum sensing on a frequency channel for each slot during the channel learning subframe. Let $\boldsymbol{Y} = \{Y_1, Y_2, \ldots, Y_i, \ldots, Y_n\}$ denote n samples at local receiver in a slot. Then, $Y_i = X_i + N_i$, where X_i is a PU signal and N_i is an AWGN. When there is no PU signal transmission, $Y_i \sim N(0, \sigma^2)$. Denote $f_0(y)$ as the probability distribution function (pdf) of Y_i and denote $F_0(y)$ as the cumulative distribution function (cdf) of Y_i. If the PU is active, $Y_i \sim N(\mu, \sigma^2)$. Denote $f_1(y)$ as the pdf of Y_i and denote $F_1(y)$ as the cdf of Y_i. Let $F_Y(y)$ denote the empirical distribution function (edf) of $\boldsymbol{Y}$, which is defined as

$$F_Y(y) = \left|\left\{i | Y_i \leq y, 1 \leq i \leq n\right\}\right|/n \tag{1}$$

where $|\bullet|$ denotes the cardinality of the finite set.

In this paper, the ternary hypothesis testing scheme is proposed to perform sensing with the goodness of fit testing, which is described as

H_0: $\boldsymbol{Y}$ is an independent and identically distributed (i.i.d.) sequence following a cdf with $F_0(y)$

H_1: $\boldsymbol{Y}$ is an i.i.d. sequence following a cdf with $F_1(y)$

H_2: $\boldsymbol{Y}$ is not an i.i.d. sequence following a cdf with either $F_0(y)$ or $F_1(y)$ (i.e. there exists a switch in channel state

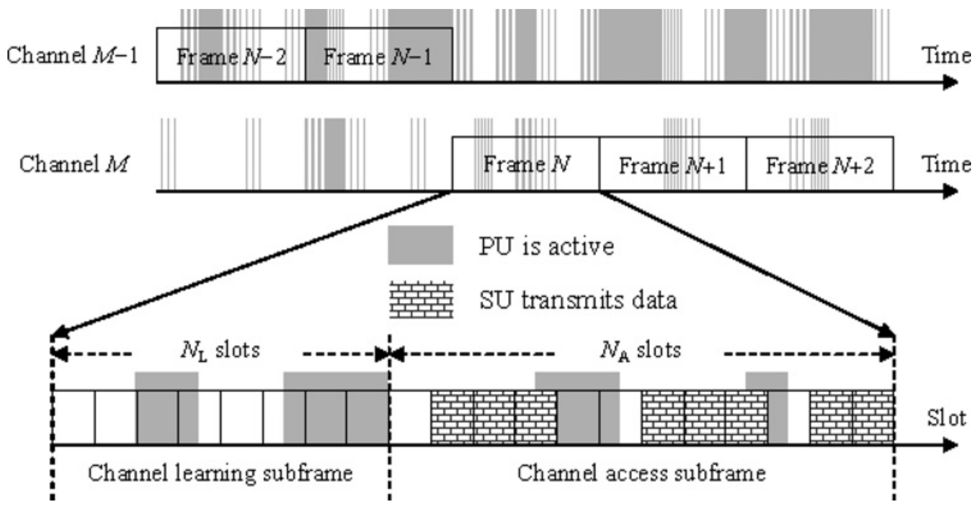

Fig. 1 *Frame structure of the proposed adaptive sensing scheme*

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that $\boldsymbol{Y}$ follows a cdf with $tF_0(y)/T + (T-t)F_1(y)/T$, $(0 < t < T)$, where t depends on the time point of switch).

The Anderson–Darling (AD) test, one of the widely used goodness of fit testing in statistics, is always used to perform sensing [17]. Under H_0, the AD test statistic W_0^2 is defined as

$$W_0^2 = n\int_{-\infty}^{+\infty} \left(F_Y(y) - F_0(y)\right)^2 \psi\left(F_0(y)\right)\, \mathrm{d}F_0(y) \tag{2}$$

where $\Psi(t) = 1/[t\,(1-t)]$. In the AD test, the H_0 is rejected if $W_0^2 > \xi_0$, where ξ_0 is a threshold that the false alarm probability under H_0 is at a desired level γ_0

$$\Pr\left\{W_0^2 > \xi_0 | H_0\right\} = \gamma_0 \tag{3}$$

It was shown in [6] that the distribution of W_0^2 under H_0 does not depend on the noise distribution $F_0(y)$ at all and is more or less independent of n. Furthermore, as $n \to +\infty$, the distribution of the test statistic W_0^2 under H_0 converges to the following limited distribution (see (4))

As pointed out in [18], the speed of convergence to the above limited distribution given in (4) is so rapidly that for any realistic situation $(n > 5)$, one can use the limited distribution instead of the distribution of W_0^2 to determine $\Pr\{W_0^2 \le \xi_0\}$.

Similarly, we can define the test statistic W_1^2 as

$$W_1^2 = n\int_{-\infty}^{+\infty} \left(F_Y(y) - F_1(y)\right)^2 \psi\left(F_1(y)\right)\, \mathrm{d}F_1(y) \tag{5}$$

In the AD test, H_1 is rejected if $W_1^2 > \xi_1$, where ξ_1 is a threshold that the probability of miss under H_1 is at a desired level γ_1

$$\Pr\left\{W_1^2 > \xi_1 | H_1\right\} = \gamma_1 \tag{6}$$

Theorem 1: Suppose that $\boldsymbol{S}$ is the sample space of $\boldsymbol{Y}$, when n is sufficient large, the sets $\left\{\boldsymbol{Y}|W_0^2 \le \xi_0\right\}$, $\left\{\boldsymbol{Y}|W_1^2 \le \xi_1\right\}$ and $\left\{\boldsymbol{Y}|W_0^2 > \xi_0,\ W_1^2 > \xi_1\right\}$ are a partition of the sample space $\boldsymbol{S}$.

Proof 1: See Appendix for the proof.

In the ternary hypothesis testing, it is necessary to exactly separate the set $\left\{\boldsymbol{Y}|W_0^2 \le \xi_0\right\}$ and $\left\{\boldsymbol{Y}|W_1^2 \le \xi_1\right\}$. Fortunately, this can be ensured by selecting an appropriate n. As a result, according to the above theorem, the solution for the ternary hypothesis testing scheme can be calculated as Fig. 2.

In contrast, we can naturally derive the solution for the binary hypothesis testing scheme as Fig. 3.

Based on the ternary hypothesis testing, the SU performs sensing during each slot with three observations: idle, busy and the transition state. Compared with the binary hypothesis testing scheme, the proposed ternary hypothesis testing scheme is much more sensitive to the switch between idle and busy, which will have a great significance to the upcoming channel learning and access processes.

Algorithm 1
For each slot: if $W_0^2 \le \xi_0$
then accept H_0 (i.e.,the idle state).
else if $W_1^2 \le \xi_1$
then accept H_1 (i.e., the busy state).
else
accept H_2 (i.e., the transition state).

Fig. 2 *Ternary hypothesis testing scheme*

Algorithm 2
For each slot: if $W_0^2 \le \xi_0$
then accept H_0 (i.e., the idle state).
else
then accept H_1 (i.e., the busy state).

Fig. 3 *Binary hypothesis testing scheme*

3.2 Learning process

Unlike Markov models where each state is exactly corresponded to an observable event, we discuss the concept of HMM where the observation is a probabilistic function of the state. A HMM is a doubly embedded stochastic process with an underlying stochastic process that is hidden, but can only be observed through another set of stochastic processes that produce the sequence of observations.

Consider a channel learning subframe as an HMM. The PU activities are the hidden state to the SU, which can only be inferred from noisy observations. Then, the state transition probabilities depend on the changing rate of PU activity (i.e. α and β) and the observation probability are related to the detection probabilities, which in turn are determined mainly by the mean and variance of the PU signal (i.e. μ and σ). Suppose that the initial state distribution is equal to the stationary state distribution. Hence, the state transition probability distribution $\boldsymbol{P}$, the observation symbol probability distribution under given state $\boldsymbol{Q}$ and the initial state probability distribution $\boldsymbol{\pi}$ are the function of the channel pattern $\boldsymbol{\lambda}$. By using HMM, the log-likelihood function of the received observation sequence $\boldsymbol{O} = (O_1, O_2, \ldots, O_{NL})$ can be calculated under the given channel pattern, that is, $\ln(\Pr\{\boldsymbol{O}|\boldsymbol{\lambda}\})$. In order to adjust the model parameter $\boldsymbol{\lambda} = (\alpha, \beta, \mu, \sigma)$ to maximise $\ln(\Pr\{\boldsymbol{O}|\boldsymbol{\lambda}\})$, the SU updates the estimate of the channel pattern towards the gradient direction in each iteration.

To model the hidden Markov process, the state reflects the PU activities during the slot. It is assumed that a slot is short enough so that the PU activity does not change more than once within each slot. Accordingly, the state of slot n, denoted by S_n, is simply defined as an ordered pair of the PU activities at the beginning and the end of slot n. Then,

$$\Pr\left\{W_0^2 \le \xi_0\right\} = \frac{\sqrt{2\pi}}{\xi_0}\sum_{j=0}^{+\infty}\binom{-1/2}{j}(4j+1)\exp\left(-(4j+1)^2\pi^2/(8\xi_0)\right)\int_0^{+\infty}\exp\left(\xi_0/\left(8\left(w^2+1\right)\right) - (4j+1)^2\pi^2w^2/(8\xi_0)\right)\mathrm{d}w \tag{4}$$

S_n is one of four possible states from the state space {(0,0), (0,1), (1,0), (1,1)}. If the state is (0,0) or (1,1), the PU remains inactive or active all along a slot, whereas the state is (0,1) or (1,0), the PU activity changes once during a slot. During the channel learning subframe, the sequence of channel states is formulated by $\boldsymbol{S} = (S_1, S_2, \ldots, S_{NL})$. The observation result, denoted by O_n, reflects the SU sensing results in slot n. In order to detect the switch of channel state between idle and busy more efficiently, in this paper, the observation space is $\{0, \xi, 1\}$ during the channel learning subframe. If the observation is 0 or 1, the SU decides that the channel is idle or busy during the slot. On the other hand, if the observation is ξ, the SU decides that there exists a switch in channel state within the slot.

Now, we determine $\boldsymbol{P}$, $\boldsymbol{Q}$, $\boldsymbol{\pi}$, respectively. $P_{k,\,l}(i, j)$ denotes the state transition probability from state (k, l) to state (i, j), (i.e. $P_{k,\,l}(i, j) = \Pr\{S_{n+1} = (i, j) \mid S_n = (k, l)\}$). Note that the PU activity at the end of current slot is the same as the beginning of the next slot. Based on the prior knowledge of channel pattern $\boldsymbol{\lambda} = (\alpha, \beta, \mu, \sigma)$, the state transition probability matrix is given as

$$\boldsymbol{P} = \left\{P_{k,\,l}(i,\ j)\right\} = \begin{bmatrix} \mathrm{e}^{-\alpha T} & 1-\mathrm{e}^{-\alpha T} & 0 & 0 \\ 0 & 0 & 1-\mathrm{e}^{-\beta T} & \mathrm{e}^{-\beta T} \\ \mathrm{e}^{-\alpha T} & 1-\mathrm{e}^{-\alpha T} & 0 & 0 \\ 0 & 0 & 1-\mathrm{e}^{-\beta T} & \mathrm{e}^{-\beta T} \end{bmatrix} \tag{7}$$

We define $Q_{i,\,j}(o)$ as the probability that the observation O_n is o under the given state S_n is (i, j), (i.e. $Q_{i,\,j}(o) = \Pr\{O_n = o \mid S_n = (i,\ j)\}$). Recall that $Q_{0,\,0}(0) = \Pr\left\{W_0^2 \le \xi_0 | H_0\right\}$ can be calculated as (4). Krishnamurthy [8] gives the upper bound of $Q_{1,\,1}(0) = \Pr\left\{W_0^2 \le \xi_0 | H_1\right\}$ $\forall n$ satisfied with (23), $\exists \lambda > 0$, such that

$$\Pr\left\{W_0^2 \le \xi_0 | H_1\right\} \simeq \frac{\mathrm{e}^{-\lambda C\sqrt{n}} E\left(\mathrm{e}^{\lambda B_n}\right)}{\mathrm{e}^{-\lambda\sqrt{\xi_0}}} \tag{8}$$

where

$$C = \sqrt{\int_{-\infty}^{+\infty} \left(F_1(y) - F_0(y)\right)^2 \psi\left(F_0(y)\right)\, \mathrm{d}F_0(y)}$$

is a constant and

$$B_n = \sqrt{n\int_{-\infty}^{+\infty} \left(F_Y(y) - F_1(y)\right)^2 \psi\left(F_0(y)\right)\, \mathrm{d}F_0(y)}$$

In Appendix, it is derived that $Q_{1,\,0}(0) = \Pr\left\{W_0^2 \le \xi_0 | H_2\right\}$ is approximately equal to $\Pr\left\{W_0^2 \le 3\xi_0 | H_1\right\}$ for large n, which can be calculated by (8). Similarly, we determine the rest of observation probability in matrix. Consequently, the observation probability matrix $\boldsymbol{Q}$ is derived as (see (9))

$$\boldsymbol{Q} = \left\{Q_{i,\,j}(o)\right\} = \begin{bmatrix} \Pr\left\{W_0^2 \le \xi_0 | H_0\right\} & \Pr\left\{W_0^2 \le 3\xi_0 | H_1\right\} & \Pr\left\{W_0^2 \le 3\xi_0 | H_1\right\} & \Pr\left\{W_0^2 \le \xi_0 | H_1\right\} \\ \Pr\left\{W_0^2 > \xi_0,\ W_1^2 > \xi_1 | H_0\right\} & \Pr\left\{W_0^2 > 3\xi_0 | H_1\right\} + \Pr\left\{W_1^2 > 3\xi_1 | H_0\right\} & \Pr\left\{W_0^2 > 3\xi_0 | H_1\right\} + \Pr\left\{W_1^2 > 3\xi_1 | H_0\right\} & \Pr\left\{W_0^2 > \xi_0,\ W_1^2 > \xi_1 | H_1\right\} \\ \Pr\left\{W_1^2 \le \xi_1 | H_0\right\} & \Pr\left\{W_1^2 \le 3\xi_1 | H_0\right\} & \Pr\left\{W_1^2 \le 3\xi_1 | H_0\right\} & \Pr\left\{W_1^2 \le \xi_1 | H_1\right\} \end{bmatrix} \tag{9}$$

The initial state distribution is denoted by $\pi_{i,\,j}$, (i.e. $\pi_{i,\,j} = \Pr\{S_1 = (i,\ j)\}$). It is assumed that the initial state distribution is equal to the stationary state distribution. Hence, we have

$$\boldsymbol{\pi} = \left\{\pi_{i,\,j}\right\} = \begin{bmatrix} \mathrm{e}^{-\alpha T}\left(1-\mathrm{e}^{-\beta T}\right)/\left(2-\mathrm{e}^{-\alpha T}-\mathrm{e}^{-\beta T}\right) \\ \left(1-\mathrm{e}^{-\alpha T}\right)\left(1-\mathrm{e}^{-\beta T}\right)/\left(2-\mathrm{e}^{-\alpha T}-\mathrm{e}^{-\beta T}\right) \\ \left(1-\mathrm{e}^{-\alpha T}\right)\left(1-\mathrm{e}^{-\beta T}\right)/\left(2-\mathrm{e}^{-\alpha T}-\mathrm{e}^{-\beta T}\right) \\ \left(1-\mathrm{e}^{-\alpha T}\right)\mathrm{e}^{-\beta T}/\left(2-\mathrm{e}^{-\alpha T}-\mathrm{e}^{-\beta T}\right) \end{bmatrix} \tag{10}$$

Since a general HMM is completely described by $\boldsymbol{P}_{4\times 4}$, $\boldsymbol{Q}_{3\times 4}$, $\boldsymbol{\pi}_{4\times 1}$, which is only dependent on the parameter $\boldsymbol{\lambda} = (\alpha, \beta, \mu, \sigma)$, the ML estimation is able to find the real channel pattern. In this paper, denote $\ln(\Pr\{\boldsymbol{O}|\boldsymbol{\lambda}\})$ as the log-likelihood function of the observation sequence $\boldsymbol{O}$ under the given channel pattern $\boldsymbol{\lambda}$, which can be calculated as

$$\Pr\{\boldsymbol{O}|\boldsymbol{\lambda}\} = \sum_{\boldsymbol{S}=\left(S_1, S_2, \ldots, S_{N_\mathrm{L}}\right)} \pi_{S_1} Q_{S_1}\left(O_1\right) \prod_{i=2}^{N_\mathrm{L}} P_{S_{i-1}}\left(S_i\right) Q_{S_i}\left(O_i\right) \tag{11}$$

As a result, the real channel pattern can be achieved by

$$\hat{\boldsymbol{\lambda}} = \arg\max_{\boldsymbol{\lambda}\in\boldsymbol{\Lambda}} \ln\left(\Pr\{\boldsymbol{O}|\boldsymbol{\lambda}\}\right) \tag{12}$$

To reduce the computational complexity, the gradient method is adopted with recursive algorithm that updates the estimate of the channel pattern once in each frame. In frame n, the gradient method updates the estimate as

$$\hat{\boldsymbol{\lambda}}(n) = \hat{\boldsymbol{\lambda}}(n-1) + \varepsilon(n)\nabla\ln\left(\Pr\{\boldsymbol{O}(n)|\hat{\boldsymbol{\lambda}}(n-1)\}\right) \tag{13}$$

where $\varepsilon\ (n)$ is the step size for frame n and $\nabla\ln(\Pr\{\boldsymbol{O}|\boldsymbol{\lambda}\})$ is the gradient of $\ln(\Pr\{\boldsymbol{O}|\boldsymbol{\lambda}\})$, that is

$$\nabla\ln\left(\Pr\{\boldsymbol{O}|\boldsymbol{\lambda}\}\right) = \left(\frac{\partial\ln\left(\Pr\{\boldsymbol{O}|\boldsymbol{\lambda}\}\right)}{\partial\alpha}, \frac{\partial\ln\left(\Pr\{\boldsymbol{O}|\boldsymbol{\lambda}\}\right)}{\partial\beta}, \frac{\partial\ln\left(\Pr\{\boldsymbol{O}|\boldsymbol{\lambda}\}\right)}{\partial\mu}, \frac{\partial\ln\left(\Pr\{\boldsymbol{O}|\boldsymbol{\lambda}\}\right)}{\partial\sigma}\right) \tag{14}$$

The gradient in (14) can be derived from the partial derivatives of $\ln(\Pr\{\boldsymbol{O}|\boldsymbol{\lambda}\})$ with respect to α, β, μ, σ, respectively, by using the forward–backward method [7].

Note that, in each iteration, the gradient method updates the estimate of the channel pattern towards the gradient direction

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of $\ln(\Pr\{\boldsymbol{O}|\boldsymbol{\lambda}\})$. Over multiple frames, the estimate gradually converges to the real channel pattern. Unfortunately, the gradient method can only find a local optimal point. Hence, we should choose the initial estimate that is close to the real channel pattern.

3.3 Channel selection

Based on the estimate of the channel pattern in frame n, the SU decides whether to choose the current channel or switch to another one in the next frame. In this paper, we introduce the opportunistic access channel capacity as an indicator of QoS, which is denoted as

$$C_{\text{op}} = \frac{\beta}{\alpha+\beta} W \log_2\left(1 + \frac{\mu^2}{\sigma^2}\right) \tag{15}$$

The SU compares C_{op} with the given threshold C_{t} to decide which action the SU takes. If $C_{\text{op}} \geq C_{\text{t}}$, then the SU stays on the current channel. Although $C_{\text{op}} < C_{\text{t}}$, the SU continues to access the current channel during the following channel access subframe and then switches to the next available frequency channel in the next frame.

3.4 Channel access

During the channel access subframe, consider a POMDP framework – a generalisation of a Markov decision process which permits uncertainty regarding the state of a Markov process and allows state information acquisition [15, 19]. In the POMDP model, the SU combines the most recent sensing result with the prior knowledge of channel pattern estimated in the preceding of channel learning subframe to decide whether to transmit data or perform sensing in each slot. The proposed channel access scheme should maximise channel utilisation as much as possible, while keeping the rate of collision with the PU at a low level.

Similar to the HMM, the states, the observations and the actions are described in a POMDP model. The definition of the state is the same as that in the HMM and the state transition probabilities can be calculated from the channel pattern estimated in the channel learning subframe. The action of slot n, is denoted by A_n from the action space $\{0, 1\}$. If $A_n = 0$, the SU chooses to perform sensing during the slot n. Whereas $A_n = 1$, the SU decides to transmit data in slot n. During the channel access subframe, a sequence of the actions is given by $\boldsymbol{A} = (A_1, A_2, \ldots, A_{NA})$. Considering that the SU could not perform sensing as well as transmit data at the same time, the observation probabilities in the POMDP depend on both the channel patterns and the actions, that is, $Q_{i,j,m}(o) = \Pr\{O_n = o|S_n = (i, j), A_n = m\}$. If the SU performs sensing during the slot n (i.e. $A_n = 0$), the observation probabilities is equal to the sensing result in the channel learning subframe. On the other hand, if the SU transmits data during the slot n (i.e. $A_n = 1$), the observation O_n is a null observation (i.e. $Q_{*,*,1}(\varnothing) = 1$). Hence, the observation space for a channel access subframe is $\{0, \xi, 1, \varnothing\}$.

The belief vector for the slot n is denoted by $\boldsymbol{\pi}(n) = (\pi_{0,0}(n), \pi_{0,1}(n), \pi_{1,0}(n), \pi_{1,1}(n))$, where $\pi_{i,j}(n)$ represents the probability of the state (i, j) in slot n, which is influenced by both the previous observations and the actions (i.e. $\pi_{i,j}(n) = \Pr\{S_n = (i, j) \mid \boldsymbol{\pi}(1), A_1, A_2, \ldots, A_{n-1}, O_1, O_2, \ldots, O_{n-1}\}$), where the initial belief vector $\boldsymbol{\pi}(1)$ is the stationary distribution as shown in (10). If $A_n = 1$, then the decision-making algorithm obtains no information about the real state and the belief vector evolves according to the state transition probability. On the other hand, if $A_n = 0$, in addition to the state transition, the sensing result is also taken into account by the decision-making algorithm. Accordingly, by using Bayes' theorem, the belief vector in slot n is updated from the belief vector in slot $n-1$ as

$$\pi_{i,j}(n) = \frac{\sum_{(k,l)} P_{k,l}(i, j) Q_{k,l,m}(o) \pi_{k,l}(n-1)}{\sum_{(i,j)} \sum_{(k,l)} P_{k,l}(i, j) Q_{k,l,m}(o) \pi_{k,l}(n-1)} \tag{16}$$

The SU should try to maximise the channel utilisation while keeping the collision rate at a low level during the channel access subframe. The policy is a function that maps the belief vector to the next action. Among the policies, we aim to find the optimal policy that can maximise the following optimal value function

$$\begin{aligned} V_{N_{\text{A}}}(\boldsymbol{\pi}) &= \max_{A_{N_{\text{A}}} \in \{0,1\}} \left\{ \sum_{(i,j)} \pi_{i,j}(N_{\text{A}}) R\big((i,j),\ A_{N_{\text{A}}}\big) \right\} \\ V_n(\boldsymbol{\pi}) &= \max_{A_n \in \{0,1\}} \left\{ \sum_{(i,j)} \pi_{i,j}(n) R\big((i, j),\ A_n\big) \right. \\ &\quad \left. + \sum_{\{0,\xi,1\}} \sum_{(i,j)} \sum_{(k,l)} P_{k,l}(i, j) Q_{k,l}(o) \pi_{k,l}(n) V_{n+1}(\boldsymbol{\pi}) \right\} \end{aligned} \tag{17}$$

where $R((i, j), A_n)$ denotes a reward function. The reward $R((0,0), 1)$ should be a positive value, since the SU could successfully transmit data without causing harmful interference to PU. The reward $R((0,1), 1)$, $R((1,0), 1)$, $R((1,1), 1)$ should be a negative value, since the collision occurs when the SU transmit data. The rest of reward (i.e. $R((*,*), 0)$) should be chosen to zero or a slight negative value, since the time is consumed without data transmission.

Although the optimal policy can be calculated by dynamic programming recursion, it could not satisfy the real time operation because of the high computational complexity [16]. To choose an appropriate action with lower complexity, for each slot n, we can empirically reduce the optimal policy to

$$A_n = \begin{cases} 1, & 1 - \pi_{0,0}(n) \leq \delta \\ 0, & 1 - \pi_{0,0}(n) > \delta \end{cases} \tag{18}$$

where δ denotes the decision threshold [20, 21].

Now, two evaluate parameters (i.e. the channel utilisation and the collision rate) is set to reflect the performances of the proposed scheme. The channel utilisation is defined as the proportion of slots that the SU could successfully transmit data without causing harmful interference to PU, that is

$$U = \frac{\left|\{n|S_n = (0, 0), \quad A_n = 1, \quad 1 \leq n \leq N_{\text{A}}\}\right|}{N_{\text{L}} + N_{\text{A}}} \tag{19}$$

whereas the collision rate is defined as the proportion of slots that the SU transmits data when the licensed channel is occupied by the PU, that is

$$C = \frac{\left|\{n|S_n \neq (0, 0), \quad A_n = 1, \quad 1 \leq n \leq N_{\text{A}}\}\right|}{\left|\{n|S_n \neq (0, 0)\}\right|} \tag{20}$$

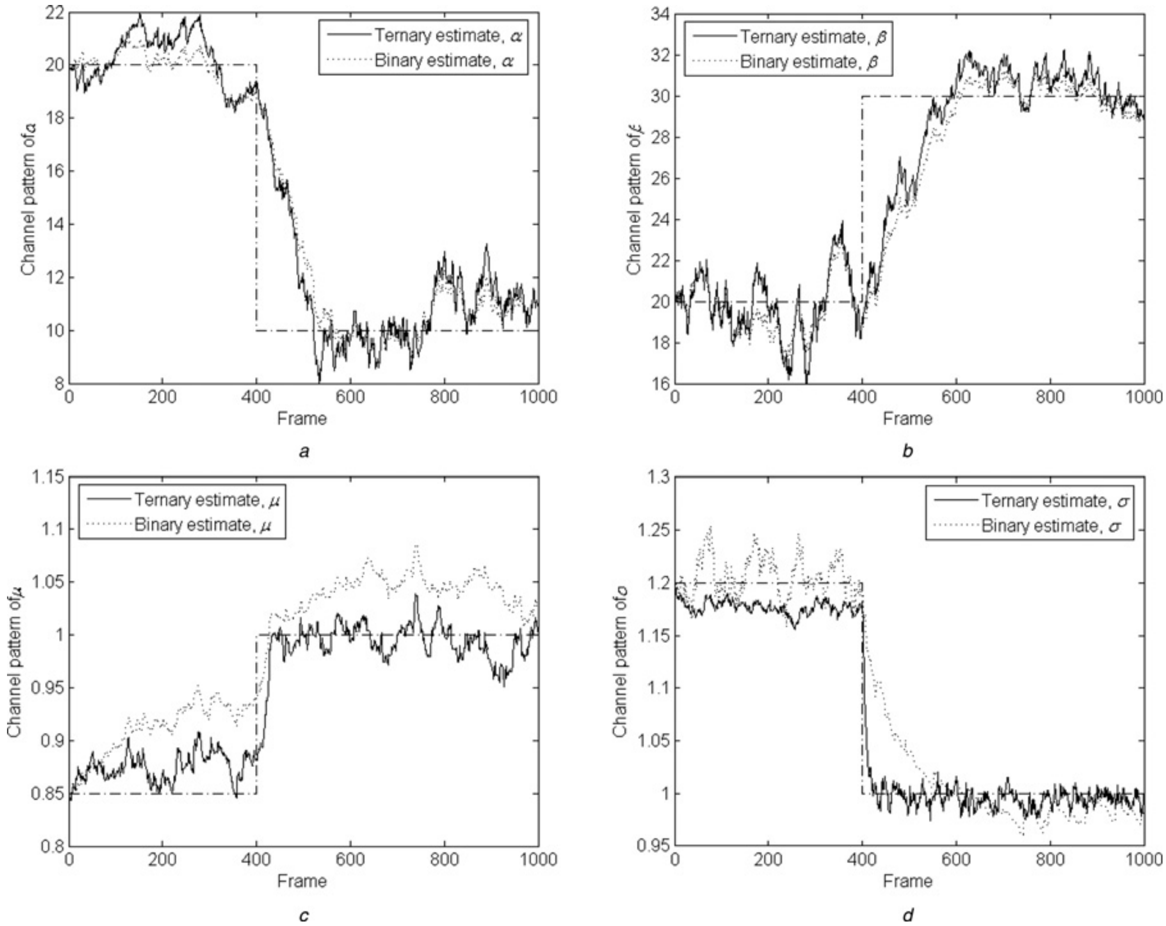

Fig. 4 *Estimate of channel pattern λ = (α, β, μ, σ) during the channel learning subframe*

a Estimate of channel parameter α
b Estimate of channel parameter β
c Estimate of channel parameter μ
d Estimate of channel parameter σ

where |•| denotes the cardinality of the finite set, as defined in (1). In the next section, we will compare the ternary hypothesis testing scheme with the conventional binary one based on the two parameters above.

Finally, it is necessary to explain how the proposed ternary hypothesis testing scheme can reduce the access and exit delay. Consider that the current state S_n is (1, 0), for the ternary hypothesis testing scheme, the SU performs sensing and the observation result is most likely equal to ξ, then updates the belief vector towards the idle state, which implies the SU to transmit data in the next slot. However, for the binary hypothesis testing scheme, in this situation, the observation result is always given by 1, then the SU updates the belief vector towards the busy state, which consequently leads to unnecessary sensing in slot $n+1$. Similarly, such improvement achieved by the ternary hypothesis testing scheme will also be occurred when the current state S_n is (0, 1). Therefore unlike the binary hypothesis testing scheme where the idle state is protected, the idea of the proposed ternary hypothesis testing scheme also considers the transition state, which reflects the real state of the licensed channel more precisely.

4 Simulation results

In this section, we first consider the performance of the channel learning scheme and the channel access scheme, respectively, then combine both schemes to evaluate the utilisation of the time-varying channel pattern, at last consider the performances with different traffic models. Suppose that the length of a frame is set about 1 s, the length of a slot T is set about 1 ms and each slot is generated by 16 samples when the SU performs sensing. There are 100 slots in a channel learning subframe and 900 slots in a channel access subframe, respectively. In this paper, we compares the ternary hypothesis testing with the binary hypothesis testing by using the goodness of fit testing and the energy detection to perform sensing, respectively. The thresholds for the goodness of fit testing are set by $\xi_0 = 1.121$ and $\xi_1 = 1.410$. In addition, the energy detection is performed compared with the goodness of fit testing, as shown in [5]. If the PU is inactive during the slot, the received signal follows the central χ^2 distribution with the mean $n\sigma^2$. On the other hand, if the PU is active, the received signal follows the non-central χ^2distribution with the mean $n\sigma^2$ and the non-centrality parameter $n\mu^2/\sigma^2$ [22]. When sample size n is sufficient large, according to the central limit theorem, the χ^2 distribution can well be approximated by norm distribution. Also, the thresholds for the energy detection are set by $\zeta_0 = 18.94$ and $\zeta_1 = 23.77$. We use a constant step size, $\varepsilon(n) = 10^{-3}$ for the recursive algorithm. To simplify the model, assume that the SU does not switch to another channel when making channel selection.

In Fig. 4, we illustrate how well the SU estimates the time-varying channel pattern by using channel learning

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scheme. Consider the situation that the channel pattern changes in frame 400th. Before the frame 400th, the real channel pattern $\boldsymbol{\lambda} = (20\,\text{Hz},\ 20\,\text{Hz},\ 0.85,\ 1.2)$. After the frame 400th, the real channel pattern $\boldsymbol{\lambda} = (10\,\text{Hz},\ 30\,\text{Hz},\ 1,\ 1)$. As illustrated in these figures, the SU could well keep up with the variations of the channel pattern except for the parameter μ with binary hypothesis testing scheme. In fact, for the binary hypothesis testing scheme, the channel parameter μ has less influence on the probability distribution of $\boldsymbol{Q}$, which consequently leads to more sensitive to the randomness of the exponential distribution. In addition, the convergence rate of the proposed ternary hypothesis testing scheme is faster than that of the binary hypothesis testing scheme. In general, when the channel pattern changes, the estimate of the channel pattern will approach to the real channel pattern within about 200 frames and then fluctuates around it because of the constant step size. We can further use the adaptive step size to improve the speed and the accuracy of convergence of the channel pattern.

In Fig. 5, we compare the ternary hypothesis testing scheme with the conventional binary hypothesis testing scheme by evaluating the channel utilisation and collision rate, respectively. In order to focus on the performance of the channel access scheme regardless of the influence on the estimation error of the channel learning scheme, we assume that the channel pattern $\boldsymbol{\lambda} = (10\,\text{Hz},\ 30\,\text{Hz},\ 1,\ 1)$ remains the same over the time and is known to the SU. It can be seen that with the increasing of decision threshold δ, the performance of the channel utilisation improves as well as the collision rate decreases. Furthermore, the proposed ternary hypothesis testing scheme achieves better channel utilisation than that of the conventional binary hypothesis testing without at the expense of the collision rate. Simulation result indicates that compared with the binary hypothesis testing, the proposed ternary hypothesis testing scheme improves the average of channel utilisation by 5% for the goodness of fit testing and 6% for the energy detection, respectively. However, the collision rate of the proposed ternary hypothesis testing as well as the traditional binary hypothesis testing alternate up and down randomly, and the average difference of collision rate remains within only 1%.

Then, we combine the channel learning scheme and the channel access scheme to evaluate the performance of channel utilisation. In Fig. 6, consider the real channel pattern is $\boldsymbol{\lambda} = (10\,\text{Hz},\ 30\,\text{Hz},\ 1,\ 1)$ during 1000 frames and assume the initial channel pattern is $\boldsymbol{\lambda_0} = (20\,\text{Hz},\ 20\,\text{Hz},$

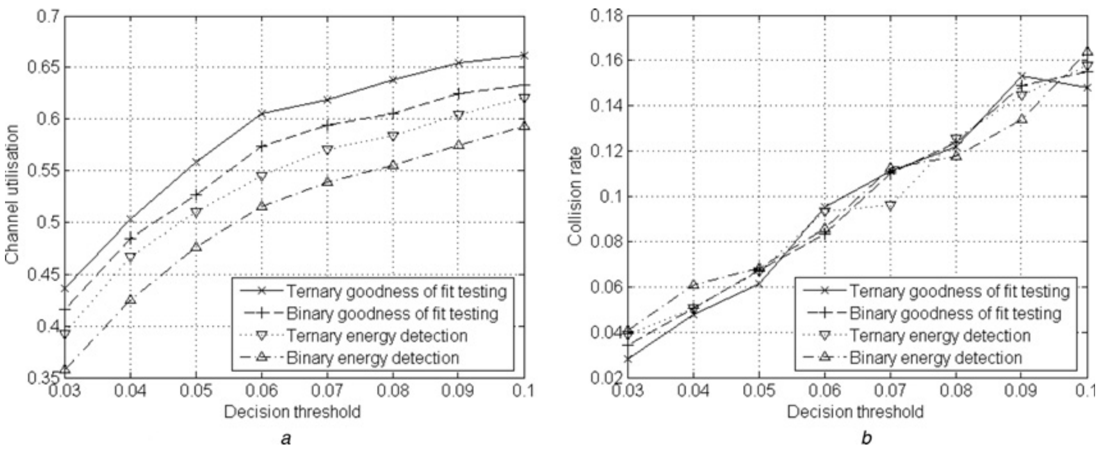

Fig. 5 *Comparison of performance during the channel access subframe*

a Comparison of the channel utilisation
b Comparison of the collision rate

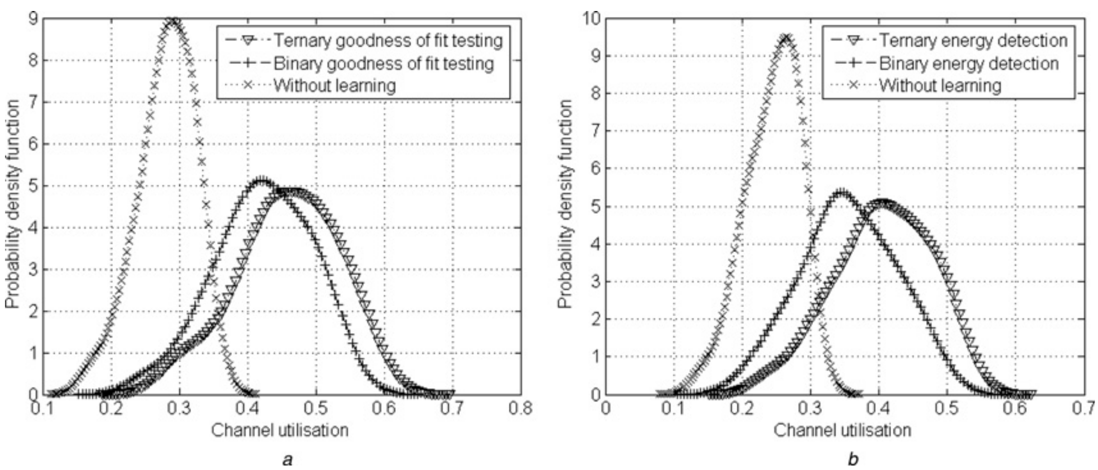

Fig. 6 *Comparison of pdf of the channel utilisation*

a Goodness of fit testing
b Energy detection

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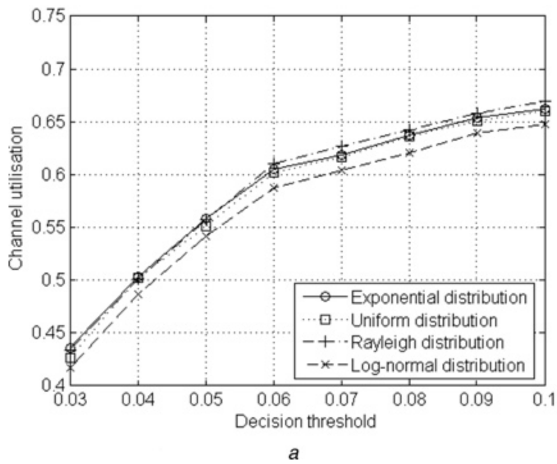

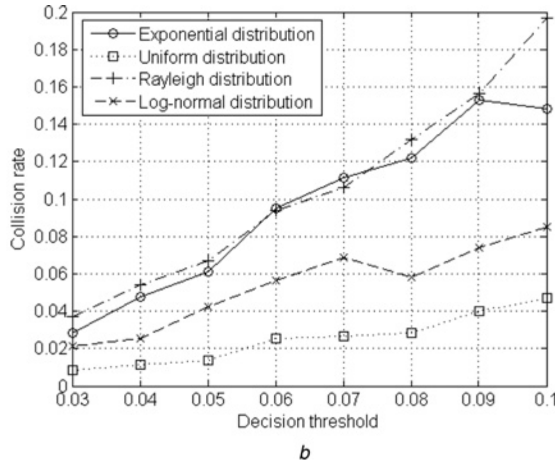

Fig. 7 *Comparison of performance with different traffic patterns*
a Comparison of the channel utilisation
b Comparison of the collision rate

0.85, 1.2). We estimate the channel utilisation in each frame and describe the pdf over 1000 frames. The decision threshold δ is set about 0.05. By comparing these two subfigures, it can be observed that the goodness of fit testing outperforms the energy detection for the channel utilisation. Since the proposed learning scheme requires additional N_L slots for the channel learning subframe, for fairness in comparison, the proposed scheme without learning contains $N_L + N_A$ slots for the channel access subframe within a frame. It can be seen from Fig. 6*a* that when the SU performs sensing with goodness of fit testing, the proposed ternary hypothesis testing scheme achieves the average of channel utilisation (i.e. 0.46) considerably higher than the proposed scheme without learning (i.e. 0.28), and binary hypothesis testing (i.e. 0.42). Whereas the SU performs sensing with the energy detection, as shown in Fig. 6*b* that the proposed ternary hypothesis testing scheme achieves the average of channel utilisation (i.e. 0.40) considerably higher than the proposed scheme without learning (i.e. 0.24), and the binary hypothesis testing (i.e. 0.36).

Finally, we consider a more realistic situation that the PU traffic patterns are not exponentially distributed, and the real patterns are unknown to the SU. Compared with exponential distribution, in our simulations, the PU traffic pattern are simulated by uniform distribution, Rayleigh distribution and logarithmic normal distribution, respectively. However, because of the non-after-effect advantage of exponential distribution, the SU estimates the PU activities and access licensed channel by the same way. The average lengths of idle and busy time are set to be equal that different traffic models can be compared under fair conditions. In Fig. 7, it can be shown that even though the real traffic models are different, the channel utilisation remains almost the same. On the other hand, the collision rate is influenced by different traffic patterns. To the best of our knowledge, unlike the ML estimation that the real traffic pattern should be known to the SU, in the realistic situation, it is convenient to use the linear minimum mean-square estimation that the SU only needs to know the first- and second-order moment of channel patterns. As a result, we can naturally come to a conclusion that the proposed algorithm can fit the realistic situations well even when the real traffic patterns are unknown to the SU.

5 Conclusion

In this paper, an OSA model is proposed for CR system, which divided a frame into the channel learning subframe and the channel access subframe. We consider a channel learning subframe as a HMM, where the ternary hypothesis testing scheme is proposed to perform sensing with the goodness of fit testing. Then the SU estimates the channel pattern and decides whether to choose current channel or switch to another one. By using a POMDP framework in the channel access subframe, the SU could either perform sensing or transmit data based on the belief vector. Simulation results indicate that the proposed ternary hypothesis testing achieves high channel utilisation compared with the conventional binary hypothesis testing, whereas keeping the collision rate at a low level. Future work will consider more practical system and we hope that the proposed scheme provide insights for the design of multiuser OSA model with energy efficiency.

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8 Appendix

8.1 Proof of sufficient condition for sample size n

To prove the sufficient condition for n, we first calculate low bound of $W_0^2 + W_1^2$ as

$$\begin{aligned} W_0^2 + W_1^2 &= n\int_{-\infty}^{+\infty}\left(F_Y(y) - F_0(y)\right)^2\psi\left(F_0(y)\right)\,\mathrm{d}F_0(y) \\ &\quad + n\int_{-\infty}^{+\infty}\left(F_Y(y) - F_1(y)\right)^2\psi\left(F_1(y)\right)\,\mathrm{d}F_1(y) \\ &\geq \frac{n}{2}\int_{-\infty}^{+\infty}\left(F_1(y) - F_0(y)\right)^2\psi_{\min}\left(F_*(y)\right)f_{\min}(y)\,\mathrm{d}y \end{aligned} \tag{21}$$

where $\Psi_{\min}(F_*(y)) = \min\{\Psi(F_0(y)), \Psi(F_1(y))\}$ and $f_{\min}(y) = \min\{f_0(y), f_1(y)\}$. Note that

$$\int_{-\infty}^{+\infty}\left(F_1(y) - F_0(y)\right)^2\psi_{\min}\left(F_*(y)\right)f_{\min}(y)\,\mathrm{d}y \tag{22}$$

is a constant which is independent of $\boldsymbol{Y}$. Furthermore, if

$$n > 2\left(\xi_0 + \xi_1\right)/\int_{-\infty}^{+\infty}\left(F_1(y) - F_0(y)\right)^2\psi_{\min}\left(F*(y)\right)f_{\min}(y)\,\mathrm{d}y \tag{23}$$

Equation (21) can be simplified as

$$W_0^2 + W_1^2 > \xi_0 + \xi_1 \tag{24}$$

Derived from (24), it can be naturally concluded that with respect to $\boldsymbol{Y} \in \boldsymbol{S}$

$$\left\{\boldsymbol{Y}|W_0^2 \leq \xi_0\right\} \cap \left\{\boldsymbol{Y}|W_1^2 \leq \xi_1\right\} = \varnothing \tag{25}$$

As a result, the sets $\left\{\boldsymbol{Y}|W_0^2 \leq \xi_0\right\}$, $\left\{\boldsymbol{Y}|W_1^2 \leq \xi_1\right\}$ and $\left\{\boldsymbol{Y}|W_0^2 > \xi_0,\ W_1^2 \leq \xi_1\right\}$ are a partition of the sample space $\boldsymbol{S}$.

8.2 Derived of $\Pr\left\{W_0^2 \leq \xi_0|H_2\right\}$

To derive the closed form of $\Pr\left\{W_0^2 \leq \xi_0|H_2\right\}$, we rewrite the test statistic W_0^2 under H_2 as

$$\begin{aligned} W_0^2 &= n\int_{-\infty}^{+\infty}\left(F_Y(y) - F_0(y)\right)^2\psi\left(F_0(y)\right)\,\mathrm{d}F_0(y) \\ &= n\int_{-\infty}^{+\infty}\left(\frac{T-t}{T}\left(F_{Y_0}(y) - F_0(y)\right) + \frac{t}{T}\left(F_{Y_1}(y) - F_0(y)\right)\right)^2 \\ &\quad \times \psi\left(F_0(y)\right)\,\mathrm{d}F_0(y) \end{aligned} \tag{26}$$

where $F_{Y0}(y)$ denotes the edf with the normal population $N(0, \sigma^2)$ and $F_{Y1}(y)$ denotes the edf with the normal population $N(\mu, \sigma^2)$. Note that $(F_{Y1}(y) - F_0(y))$ is much larger with probability than $(F_{Y0}(y) - F_0(y))$ under the condition that n is large enough. Since the time point t is random during the slot, to simplify the computation, we calculate the mean of W_0^2 in place of the marginal distribution that

$$\begin{aligned} E\left(W_0^2\right) &= \int_0^T\int_{-\infty}^{+\infty}\frac{\beta \mathrm{e}^{-\beta t}}{1 - \mathrm{e}^{-\beta T}}n\left(\frac{t}{T}\left(F_{Y_1}(y) - F_0(y)\right)\right)^2 \\ &\quad \times \psi\left(F_0(y)\right)\,\mathrm{d}F_0(y)\,\mathrm{d}t \end{aligned} \tag{27}$$

Consider that α and β are sufficient small compared with the time duration of slot, (27) can well be approximated by

$$E\left(W_0^2\right) = \frac{n}{3}\int_{-\infty}^{+\infty}\left(F_{Y_1}(y) - F_0(y)\right)^2\psi\left(F_0(y)\right)\,\mathrm{d}F_0(y) \tag{28}$$

Therefore it can be found that $\Pr\left\{W_0^2 \leq \xi_0|H_2\right\}$ is approximately equal to $\Pr\left\{W_0^2 \leq 3\xi_0|H_1\right\}$.