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Another two dark energy models motivated from Károlyházy uncertainty relation

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Abstract The Károlyházy uncertainty relation indicates that there exists a minimal detectable cell δt^3 over the region t^3 in Minkowski space-time. Due to the energy-time uncertainty relation, the energy of the cell δt^3 cannot be less δt^{-1}. Then we get a new energy density of metric fluctuations of Minkowski spacetime as δt^{-4}. Motivated by the energy density, we propose two new dark-energy models. One model is characterized by the age of the universe and the other is characterized by the conformal age of the universe. We find that in the two models, the dark energy mimics a cosmological constant in the late time.

1 Introduction

The study of dark energy has become one of the most active fields in modern cosmology. Considerable efforts have been made to explore the nature of dark energy [1–7]. Recently, the agegraphic dark energy (ADE) [8] and new agegraphic dark-energy (NADE) [9] models are motivated from the Károlyházy uncertainty relation [10–14] which tells us that the time t in Minkowski space-time cannot be known to a better accuracy than [10–14] (see Ref. [15] for a recent review)

$$\delta t = \beta t_p^{2/3} t^{1/3}. \tag{1}$$

Here β is a dimensionless constant, t_p is the reduced Planck time. In this paper, we adopt the units $c = \hbar = 1$. Following [16, 17], Eq. (1) indicates that for a length scale t, there exists a minimal detectable cell $\delta t^3 \sim t_p^2 t$ over a region t^3. The time-energy uncertainty relation indicates that the energy of the minimal cell cannot be smaller than [16–18]

$$E_{\delta t^3} \sim t^{-1}. \tag{2}$$

Then the energy density of the metric fluctuations of the Minkowski space-time is [16–18]

$$\rho_q \sim \frac{E_{\delta t^3}}{\delta t^3} \sim \frac{1}{t_p^2 t^2}. \tag{3}$$

Motivated by the equation, the energy density of ADE was proposed to be [8]

$$\rho_q = \frac{3n^2 M_p^2}{T^2}.$$

Here n is a dimensionless constant parameter, $M_p = (8\pi G)^{-1/2}$ and T is the age of the universe

$$T = \int_0^t dt' = \int_0^a \frac{da}{Ha}, \tag{4}$$

where a is the scale factor, $H \equiv \dot{a}/a$ is the Hubble parameter and a dot denotes the derivative with respect to the cosmic time t. However, it is found that there exist some implicit inconsistences in the model that ADE tracks the matter during the matter-dominated epoch [8, 9] and the ability of ADE in deriving the accelerated expansion contradicts the existence of the radiation/matter-dominated epoch [19, 20].

In order to address the drawbacks, the NADE model was proposed. In NADE model, the energy density of dark energy was proposed to be [9]

$$\rho_q = \frac{3n^2 M_p^2}{\eta^2},$$

where η is the conformal age of the universe

$$\eta = \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da}{Ha^2}. \tag{5}$$

The NADE model is very successful in fitting the observation data [22, 23]. However, in the NADE model, there is

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one thing, which makes us uneasy: that the conformal age η is not a physical scale and can be rescaled arbitrarily.

We find that there may exist other ways out of the difficulties in the ADE model. The relation (3), which plays the key role in the above motivations, is based on the Károlyházy uncertainty relation (1) and Eq. (2). Equation (2) is motivated from the energy-time uncertainty relation. However, the energy-time uncertainty relation may indicate the other natural result. The Károlyházy uncertainty relation (1) tells us that the time t fluctuates with the amplitude $\delta t \sim t_p^{2/3} t^{1/3}$. Due to the energy-time uncertainty relation, this indicates the existence of energy

$$E \sim \delta t^{-1} \sim \frac{1}{t_p^{2/3} t^{1/3}}. \tag{6}$$

Actually, in [16], the above energy has been proposed and supposed to distribute uniformly over the volume t^3. Then the density energy $\rho_q \sim t_p^{-2/3} t^{-10/3}$ was derived [16]. Yet, this density energy cannot be used to motivate a dark-energy model since the corresponding fractional energy density $\Omega_q \sim \rho_q t_p^2/H^2 \sim t_p^{4/3}/t^{4/3}$ is a decreasing function of t. However, if we suppose that the energy given in Eq. (6) is distributed homogeneously over the minimal detectable cell $\delta t^3 \sim t_p^2 t$, then we get the energy density ρ_q associated with the fluctuation of the Minkowski space-time as

$$\rho_q \sim \frac{E}{\delta t^3} \sim \frac{1}{\delta t^4} \sim \frac{1}{t_p^{8/3} t^{4/3}}. \tag{7}$$

Fortunately, we find that from the above energy density, new dark-energy models can be motivated.

Based on Eq. (7), in Sect. 2, we propose a new dark-energy model characterized by the age of the universe to address the drawbacks in the ADE model. In Sect. 3, following the motivation of NADE, we motivate a new dark-energy model characterized by the conformal age of the universe from Eq. (7). Finally, the summary is given.

2 New model characterized by age of the universe

Based on Eq. (7), following the motivation of the ADE model we may propose a new model of dark energy characterized by the age of the universe as

$$\rho_q = \frac{3n^{2/3} M_p^{8/3}}{T^{4/3}}, \tag{8}$$

where n is a constant parameter and T is the age of the universe defined in Eq. (4). The corresponding fractional energy density is

$$\Omega_q \equiv \frac{n^{2/3} M_p^{2/3}}{T^{4/3} H^2}. \tag{9}$$

Let us show whether the dark energy (8) can drive the present accelerated expansion of the universe or not. In the matter-dominated (MD) epoch, since $H = \frac{2}{3t} \propto a^{-3/2}$, we have $\Omega_q \propto T^{2/3} \propto a$. Then the fractional energy density (9) increases during the MD epoch and the dark energy (8) becomes dominated eventually. So the tracking behavior of ADE during the MD epoch does not exist in this model. From Eq. (8), we have

$$\dot\rho_q + \frac{4}{3}\frac{\rho_q}{T} = 0. \tag{10}$$

By comparing the equation with the conservation law of the dark energy $\dot\rho_q + 3H(1+w_q)\rho_q = 0$ and using Eq. (9), we can get the equation of state (EoS) parameter w_q as

$$w_q = -1 + \frac{4}{9}\Omega_q^{3/4}\sqrt{\frac{H}{nM_p}}. \tag{11}$$

We can derive from Eq. (11) that in the dark-energy-dominated (DED) epoch the parameter w_q is given by

$$w_q \simeq -1 + \frac{4}{9}\frac{1}{(nM_pT)^{1/3}}, \tag{12}$$

since approximately $\Omega_q \simeq 1$ and $H^2 \simeq \frac{n^{2/3} M_p^{2/3}}{T^{4/3}}$. Then, with the increasing of T, w_q approaches -1 and the dark energy (8) mimics a cosmological constant to drive the accelerated expansion. So, from the above analysis, we know that the dark energy (8) has at least a reasonable qualitative behavior.

Now considering the flat Friedmann–Robertson–Walker (FRW) universe with the dark energy (8) and pressureless matter, the corresponding Friedmann equation reads

$$H^2 = \frac{1}{3M_p^2}(\rho_m + \rho_q). \tag{13}$$

Here ρ_m is the energy density of the matter and the corresponding conservation law is

$$\dot\rho_m + 3H\rho_m = 0. \tag{14}$$

Then using Eqs. (9), (13), (8), and (14), we find that the evolution of the fractional energy density (9) is governed by the two equations

$$\frac{d\Omega_q}{da} = \frac{3}{a}\Omega_q(1-\Omega_q)\left(1 - \frac{4}{9}\Omega_q^{3/4}\sqrt{\frac{\tilde H}{\tilde n}}\right), \tag{15}$$

$$\frac{d\tilde H}{da} = -\frac{3\tilde H}{2a}\left(1 - \Omega_q + \frac{4}{9}\Omega_q^{7/4}\sqrt{\frac{\tilde H}{\tilde n}}\right), \tag{16}$$

where

$$\tilde H \equiv \frac{H}{H_0}, \qquad \tilde n \equiv \frac{nM_p}{H_0}, \tag{17}$$

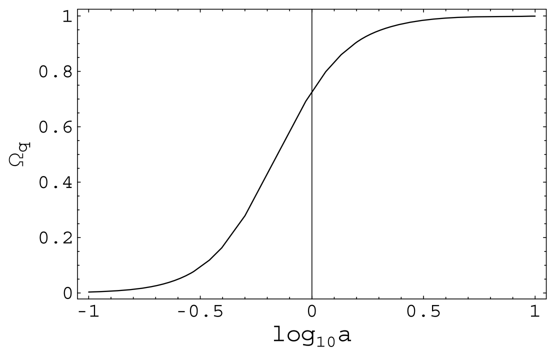

Fig. 1 The evolution of Ω_q for the dark energy (8) versus $\log_{10} a$ with the initial condition $\Omega_{q0} = 0.728$ and $\tilde{n} = 20$

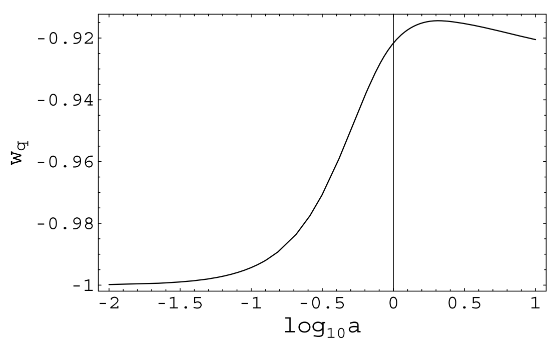

Fig. 2 The evolution of w_q for the dark energy (8) versus $\log_{10} a$ with the initial condition $\Omega_{q0} = 0.728$ and $\tilde{n} = 20$

and the subscript 0 denotes the present value of the corresponding parameter. By choosing $a_0 = 1$, $\tilde{n} = 20$ and the initial condition $\Omega_{q0} = 0.728$, we solve the equations and display the evolution of Ω_q with respect to $\log_{10} a$ in Fig. 1. The result displayed in Fig. 1 tells us that the dark energy (8) is negligible in the early universe and becomes dominant in late times, which is consistent with the analysis in the last paragraph.

Using Eq. (17), we may rewrite Eq. (11) as

$$w_q = -1 + \frac{4}{9}\Omega_q^{3/4}\sqrt{\frac{\tilde{H}}{\tilde{n}}}\ . \tag{18}$$

Then using the above equation and the numerical solution obtained by solving Eqs. (15) and (16) with $a_0 = 1$, $\tilde{n} = 20$ and $\Omega_{q0} = 0.728$, we display the evolution of w_q with respect to $\log_{10} a$ in Fig. 2. We also plot the current value of w_q versus $\tilde{n}$ with fixed $\Omega_{q0} = 0.728$ in Fig. 3. We see that $w_{q0} \lesssim -0.89$ as $\tilde{n} \gtrsim 10$. Therefore the EoS parameter is consistent with the Wilkinson Microwave Anisotropy Probe (WMAP) observation [21], as $\tilde{n}$ is taken to be a number of order ten.

Furthermore, let us calculate the shift parameter R, which characterizes the position of the first peak of the cosmic microwave background spectrum and is defined as [24]

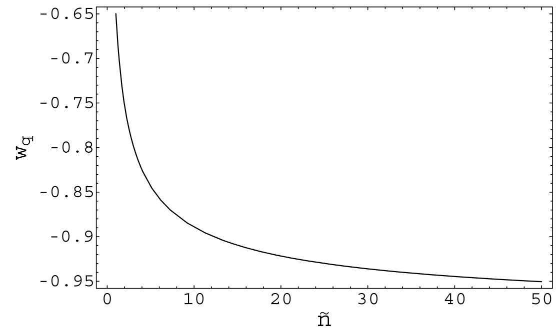

Fig. 3 The current equation of state parameter w_q versus the parameter $\tilde{n}$ with fixed $\Omega_{q0} = 0.728$

$$R = \sqrt{\Omega_{m0}} \int_0^{z_*} \frac{dz}{\tilde{H}(z)}. \tag{19}$$

Here z_* is the redshift of decoupling. The 7-year WMAP observations tell us that $z_* = 1091.3 \pm 0.91$ at 1σ confidence level [21]. In this paper, we choose $z_* = 1091$. Then, using the numerical solution of Eqs. (15) and (16) with $a_0 = 1$, $\tilde{n} = 20$ and $\Omega_{q0} = 0.728$, we obtain the value of R to be

$$R = 1.722. \tag{20}$$

The 7-year WMAP observations tell us $R = 1.725 \pm 0.018$ at 1σ confidence level [21]. So the dark-energy model (8) with $\tilde{n} = 20$ is consistent with the 7-year WMAP observations.

3 New model characterized by conformal age of universe

In order to eliminate the inconsistences in the ADE model, the authors in Ref. [9] proposed the NADE model by replacing the age T with the conformal age η. The NADE model is very successful in fitting the observation data [22, 23]. Motivated by this, from Eq. (7) we propose a dark-energy model characterized by the conformal age of the universe as

$$\rho_q = \frac{3n^{2/3}M_p^{8/3}}{\eta^{4/3}}, \tag{21}$$

where η is the conformal age of universe defined in Eq. (5). The corresponding fractional energy density is

$$\Omega_q = \frac{n^{2/3}M_p^{2/3}}{\eta^{4/3}H^2}. \tag{22}$$

Let us consider the flat FRW universe filled with dark energy (21) and pressureless matter. In the MD epoch, the energy conservation equation of the matter, Eq. (14), tells us $H^2 \propto a^{-3}$. Substituting this into Eq. (5), we have $\eta \propto \sqrt{a}$. Then Eq. (22) tells us that in the MD epoch, $\Omega_q \propto a^{7/3}$. Then we know that the fractional energy density (22) increases in the matter-dominated epoch and eventually, the dark energy (21) becomes dominant. From Eqs. (21), (5), and (22) and the conservation equation $\dot{\rho}_q + 3H(1 + w_q)\rho_q = 0$, we can easily get the EoS parameter as

$$w_q = -1 + \frac{4}{9}\frac{\Omega_q^{3/4}}{a}\sqrt{\frac{H}{nM_p}}. \tag{23}$$

In the DED epoch, since approximately $\Omega_q \simeq 1$ and $H^2 \simeq \frac{n^{2/3}M_p^{2/3}}{\eta^{4/3}}$, from Eq. (23) we have

$$w_q \simeq -1 + \frac{4}{9}\frac{1}{a(nM_p\eta)^{1/3}}. \tag{24}$$

So, as the expansion of the universe, w_q approaches -1 and the dark energy (21) mimics a cosmological constant. The above analysis makes us believe that qualitatively the dark energy (21) is a reasonable model.

In order to confirm the qualitative analysis in the last paragraph, now let us survey the evolution of the fractional energy density (22) quantitatively. Using Eqs. (5), (13), (21), (22), and (14), we find that the evolution of Ω_q is governed by the two equations

$$\frac{d\Omega_q}{da} = \frac{3}{a}\Omega_q(1-\Omega_q)\left[1 - \frac{4}{9}\frac{\Omega_q^{3/4}}{a}\sqrt{\frac{\tilde{H}}{\tilde{n}}}\right], \tag{25}$$

$$\frac{d\tilde{H}}{da} = -\frac{3\tilde{H}}{2a}\left[1 - \Omega_q + \frac{4}{9}\frac{\Omega_q^{7/4}}{a}\sqrt{\frac{\tilde{H}}{\tilde{n}}}\right], \tag{26}$$

where $\tilde{H} \equiv H/H_0$ and $\tilde{n} \equiv nM_p/H_0$. Still choosing $a_0 = 1$, $\tilde{n} = 20$ and $\Omega_{q0} = 0.728$, we solve the equations numerically and display the evolution of Ω_q with respect to $\log_{10} a$ in Fig. 4. The result displayed in Fig. 4 tells us that the dark energy (21) is negligible in the early universe and becomes dominant in the late universe, which confirms the qualitative analysis in the last paragraph.

By rewriting Eq. (23) as

$$w_q = -1 + \frac{4}{9}\frac{\Omega_q^{3/4}}{a}\sqrt{\frac{\tilde{H}}{\tilde{n}}}, \tag{27}$$

we display the evolution of w_q versus $\log_{10} a$ in Fig. 5 with $a_0 = 1$, $\tilde{n} = 20$ and $\Omega_{q0} = 0.728$. From Fig. 5, we know that w_q approaches -1 in the future, which confirms the qualitative analysis. With $a_0 = 1$, we find that the curve representing the current value of w_q defined in Eq. (27) versus $\tilde{n}$ with $\Omega_{q0} = 0.728$ is just that displayed in Fig. 3. So, as in the last section, from Fig. 3 we may conclude that the EoS parameter of the dark energy (21) is consistent with the WMAP

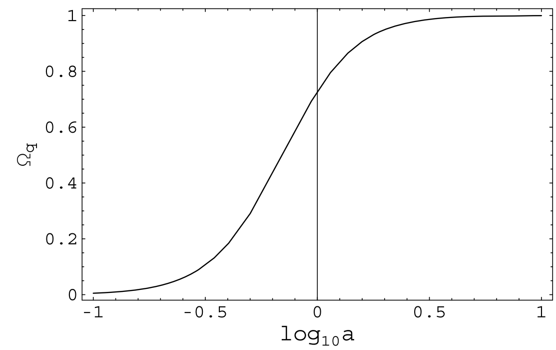

Fig. 4 The evolution of Ω_q for the dark energy (21) versus $\log_{10} a$ with the initial condition $\Omega_{q0} = 0.728$ and $\tilde{n} = 20$

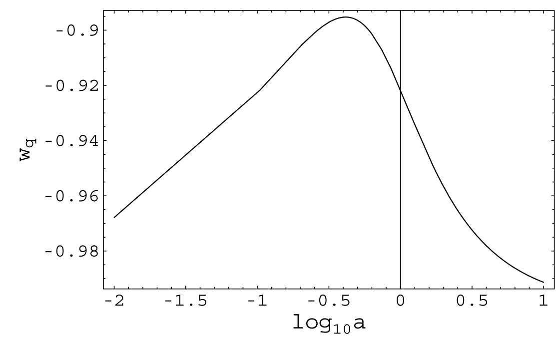

Fig. 5 The evolution of w_q for the dark energy (21) versus $\log_{10} a$ with the initial condition $\Omega_{q0} = 0.728$ and $\tilde{n} = 20$

observation [21], as $\tilde{n}$ is taken to be a number of order ten. Further, with $a_0 = 1$, $\tilde{n} = 20$ and $\Omega_{q0} = 0.728$, we obtain the value of the shift parameter R as

$$R = 1.716. \tag{28}$$

So the dark-energy model (21) with $\tilde{n} = 20$ is in agreement with the 7-year WMAP observations ($R = 1.725 \pm 0.018$) [21].

4 Summary

We propose two new models of dark energy based on the Károlyházy uncertainty relation (1). The two models are different from the ADE [8] and NADE [9] models. The ADE and NADE models are motivated from Eq. (3) which is deduced from Eq. (1) by taking the energy of the minimal detectable cell δt^3 to be t^{-1}, while the two models in this note are motivated from Eq. (7), which is obtained by arguing the energy of the cell δt^3 to be δt^{-1}. Both Eq. (3) and Eq. (7) are the natural results of the relation (1) and the energy-time uncertainty relation. So, following the motivation of the ADE and NADE models, we motivate the two models from Eq. (7).

It is well known that there exist implicit inconsistences in the model of ADE [8, 9, 19, 20]. In fact, it is in order to eliminate the inconsistences of ADE that the model of NADE is proposed [9]. We find that in the two models in the note, the dark energy has the reasonable behavior that the dark energy is negligible in the MD epoch and eventually becomes dominant to drive the accelerated expansion, and no inconsistences exist in the two models. Particularly, the model proposed in Sect. 2 is characterized by the age of the universe T, not the unphysical scale η.

By calculating the shift parameter, we compare the two models with the observational data. We find that the two models with $\tilde{n} = 20$ fit the 7-year WMAP data well. We hope that the two models can shed new light on solving the dark-energy problem.

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