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Adaptive leader-following consensus control of multi-agent systems using model reference adaptive control approach

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Abstract: This study is devoted to the adaptive consensus problem of multi-agent systems with partly unknown parameters and bounded external disturbances, under the guidance of an active leader with a reference input signal. Firstly, a distributed adaptive protocol is proposed for the system without disturbances by adopting the model reference adaptive control method. Then it is extended to the disturbed system by adding an adaptive disturbance compensator, based on the estimated upper and lower bounds of unknown disturbance. For the above two cases, sufficient conditions are, respectively, given to ensure that all agents can eventually track the prescribed leader. A numerical simulation illustrates the effectiveness of the proposed adaptive consensus protocol.

1 Introduction

In recent years, cooperative control of multi-agent systems has received considerable attention because of its broad application in many areas including cooperative control of unmanned air and underwater vehicles, formation control, flocking of mobile vehicles, distributed optimisation of multiple robotic systems and scheduling of automated highway systems. In cooperative control, consensus problem is well accepted as one of the most important and fundamental issues, to which there are mainly two control strategies: the behaviour-based (or leaderless) method [1–5] and the leader-following approach [1, 6–11]. In particular, leader-following consensus means that all follower agents eventually reach an agreement on the state or output of a preassigned leader, which specifies a desired objective for all other agents to follow and is usually independent of its followers.

In the literature of leader-following consensus for linear multi-agent systems, some researchers have focused on the distributed protocol design and consensus condition analysis of first- or second-order multiple agents with an active leader, whose model is taken as the double- or single-integrator dynamics. For example, in [6], the leader-following consensus problem was studied for the network of first-order follower agents with a second-order leader, whose state could not be completely measured and the acceleration information might be partially unknown. Further, Hong *et al.* [7] developed some neighbour-based rules, consisting of distributed controllers and observers, to address the leader-following consensus problem for second-order follower agents, under the assumptions that the velocity of a given second-order leader would not be measured in real-time and its acceleration input was known to all agents. In [8], the consensus problem was addressed for the second-order multi-agent system to track a first-order leader with a desired constant velocity, and meanwhile the non-uniform time-varying communication delays were taken into account. Recently, the multi-agent system with general linear dynamics has also been considered. Ni and Cheng [10] investigated the leader-following consensus problem for multiple follower agents modelled by an identical linear differential equation, and the leader described by a linear nominal system. To be specific, it was required that the state matrix of the dynamic leader must be the same as that of all follower agents, which implied that the dynamic equations of agents were identical to that of the leader under the zero-input condition. The role and motivation of this requirement are clear, but it may be unsatisfied in some applications, and sometimes accurate parameters of the system dynamics are also hard or even unable to obtain. Meanwhile, an autonomous leader described by a given nominal system without input items cannot present adjustable and flexible desired trajectories. On the other hand, a real network of multiple agents is usually in uncertain environments with various external disturbances and stochastic communication noises, which may cause the network system to diverge or oscillate. Therefore the consensus problem of such a disturbed system is of vital necessity, and has attracted the attention of some researchers [11–16].

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Motivated by the above observations, we study the adaptive leader-following consensus problem of linear multi-agent systems with external disturbances, under the guidance of a more active leader, described by a linear differential equation with a reference input signal. Different from [10], the state matrix of the leader is not required to be equal to that of the agents, and meanwhile the dynamics of the agents are supposed to be different. In addition, some system matrices of the leader and follower agents are allowed to be unknown, considering the difficulties in obtaining plant parameters accurately. Unfortunately, the existing neighbour-based linear protocols in [6–11] are not applicable to the above leader-following system, because of the difference between system matrices of the leader and those of the follower agents, and thus the model reference adaptive control (MRAC) method [17–20] is adopted in the protocol design. In particular, for the multi-agent system with and without bounded external disturbances, we propose two consensus protocols together with some adaptive updating laws, which are developed to estimate the information of unknown system matrices and the bounds of disturbance on-line. Then, the performance of a closed-loop system is analysed to derive consensus conditions on the interaction topology such that all agents can asymptotically track the active leader with a desired reference signal. Finally, a numerical example is included to validate the accuracy of our theoretical results.

The remainder of the paper is organised as follows. In Section 2, the problem to be solved is stated, and some preliminaries from algebraic graph theory are presented. In Section 3, adaptive protocols are proposed and the consensus conditions are derived. Simulation results are given in Section 4, and Section 5 concludes the paper.

2 Problem statement and preliminaries

2.1 Problem statement

Consider a multi-agent system consisting of N follower agents with the ith one modelled by the following linear dynamic system subject to unknown disturbances

$$\dot{x}_i(t) = A_i x_i(t) + B_i(u_i(t) + d_i(t)), \quad i = 1, \ldots, N \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ is the state of the ith agent, $u_i(t) \in \mathbb{R}^m$ is the control input or protocol and the continuous vector function $d_i(t) \in \mathbb{R}^m$ represents the external disturbance with bounded peak value. The dynamics of the leader or model is given by

$$\dot{x}_0(t) = A_0 x_0(t) + B_0 r(t) \tag{2}$$

where $x_0(t) \in \mathbb{R}^n$ is the state, $r(t) \in \mathbb{R}^q$ is the bounded reference signal and A_0 is the stable state matrix.

Remark 1: Among the system matrices of (1) and (2), only the exact information on input matrices B_i $(i = 1, \ldots, N)$ and the reference signal $r(t)$ will be used in the protocol design. This indicates that the dynamics of leader and the system matrices of the follower agents are allowed to be partly unknown.

The leader-following consensus problem is stated as follows: design distributed protocols for N agents using the local information, such that all agents reach consensus on their states with the given leader asymptotically, that is

$$\lim_{t \to \infty} (x_i(t) - x_0(t)) = 0, \quad i = 1, \ldots, N \tag{3}$$

It is obvious that the state matrices of the follower agents and the leader are different, which results in the invalidation of the existing neighbour-based linear consensus protocols in [6–11]. Therefore we employ the MRAC method to the consensus protocol design and performance analysis of the above leader-following system with partly unknown system matrices, by regarding the leader (2) as a reference model. In order to drive all agents to eventually track the leader, some necessary assumptions on system matrices of N follower agents and the leader must be made, motivated by the standard assumption in MRAC [20].

Assumption 1: Assume that there exist H_i^* and K_i^* such that $A_0 = A_i + B_i H_i^{*\mathrm{T}}$ and $B_0 = B_i K_i^{*\mathrm{T}}$ hold.

The ideal matrices H_i^* and K_i^* are generally unavailable, since A_i, A_0 and B_0 may be unknown. On the other hand, for the bounded disturbance $d_i(t)$, there must exist constant upper and lower bounds $\bar{d}_i^*$ and $\underline{d}_i^*$, respectively, but the vectors $\bar{d}_i^*$ and $\underline{d}_i^*$ are also usually unobtainable. Thus, we use $H_i(t)$, $K_i(t)$, $\bar{d}_i(t)$ and $\underline{d}_i(t)$ to denote the estimates of H_i^*, K_i^*, $\bar{d}_i^*$ and $\underline{d}_i^*$ at time instant t, respectively. In fact, the above true values are 'artificial' quantities required only for analytical purposes, that will not appear in the consensus protocol design.

2.2 Algebraic graph theory

Undirected graphs are used to model the interaction topologies among N follower agents. Let $\mathcal{G}_a = (\mathcal{V}_a, \mathcal{E}_a)$ be an undirected graph of order N with the set of nodes $\mathcal{V}_a = \{v_1, v_2, \ldots, v_N\}$, the set of undirected edges $\mathcal{E}_a \subseteq \mathcal{V}_a \times \mathcal{V}_a$. In graph $\mathcal{G}_a$, node v_i represents the ith follower agent, and the undirected edge (v_i, v_j) represents that information is transmitted between agents i and j. Then, if the edge (v_i, v_j) or equivalently (v_j, v_i) exists, we say that agents i and j are neighbours. An undirected path is a sequence of ordered edges of the form (v_{i_1}, v_{i_2}), $(v_{i_2}, v_{i_3}), \ldots$ in an undirected graph, where $v_{i_j} \in \mathcal{V}_a$. If there is a path from every node to every other one, the undirected graph is said to be connected. Note that for an undirected graph that is not connected, there must exist more than one connected component.

To describe the effect of leader to follower agents, let node v_0 represent the leader. Since the leader would not be affected by the follower agents, the connection edges between the leader and the agents are directed, and the corresponding edge set is labelled by $\mathcal{E}_l$. In the graph, a directed edge from v_i to v_0, (v_i, v_0), represents that the information is transmitted from the leader to the ith agent, and it means that the leader is a neighbour of agent i. For convenience of expression, let $\bar{\mathcal{V}} = \{\mathcal{V}_a \cup v_0\}$ be the node set consisting of all agents and the leader. For each follower agent in this leader-following system, its set of neighbours can also include the leader and does so whenever the leader is within the agent's neighbourhood. Summarising the above notations, the set of neighbours of agent i is $\mathcal{N}_i = \{v_j \in \bar{\mathcal{V}} : (v_i, v_j) \in \{\mathcal{E}_a \cup \mathcal{E}_l\}\}$, and notation $\mathcal{N}_0 = \{v_j \in \mathcal{V}_a : (v_j, v_0) \in \mathcal{E}_l\}$ represents the set of follower agents that are directly connected to the leader.

In order to realise the leader-following consensus, we make the following assumption on the interaction topology.

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Assumption 2: At least one agent in each connected component of $\mathcal{G}_a$ is connected to the leader.

3 Main results

In this section, distributed adaptive consensus protocols will be designed and consensus conditions will be given for the leader-following multi-agent system (1) and (2) with partly unknown parameters. To quantitatively analyse the consensus performance of the system, we define $\epsilon_i(t) = x_i(t) - x_0(t)$ and $e_{i,j}(t) = x_i(t) - x_j(t)$ $(i,j = 1, \ldots, N)$ to measure the disagreements of the agents and the leader and the differences between the agents, respectively. The motivation of defining new variables $\epsilon_i(t)$ and $e_{i,j}(t)$ is clearly shown in the following lemma.

Lemma 1: Under Assumption 2, if $x_0(t)$, $\epsilon_i(t)$ $(v_i \in \mathcal{N}_0)$ and $e_{i,j}(t)$ $((v_i, v_j) \in \mathcal{E}_a)$ are bounded, then all $x_i(t)$ $(i = 1, \ldots, N)$ are bounded. Further, if $\epsilon_i(t) = 0$ $(v_i \in \mathcal{N}_0)$ and $e_{i,j}(t) = 0$ $((v_i, v_j) \in \mathcal{E}_a)$ hold, then $x_i(t) - x_0(t) = 0$ is satisfied for all $i = 1, \ldots, N$.

Proof: For any node v_{k_0}, $k_0 \in \{1, \ldots, N\}$, if it is connected to the leader, then there exists a constant M to satisfy $\|x_{k_0}(t) - x_0(t)\| \le M$ according to the condition that $\epsilon_i(t)$ $(v_i \in \mathcal{N}_0)$ is bounded, from which it yields that

$$\|x_{k_0}(t)\| \le M + \|x_0(t)\|, \quad v_{k_0} \in \mathcal{N}_0 \tag{4}$$

Otherwise, for $v_{k_0} \notin \mathcal{N}_0$, it must belong to a connected component of undirected graph $\mathcal{G}_a$, in which there is at least one agent connected to the leader by Assumption 2, with the label l_{k_0} $(\neq k_0)$. Then inequality $\|x_{l_{k_0}}(t) - x_0(t)\| < M$ follows. In the above connected component, the path from nodes k_0 to l_{k_0} is supposed to be (v_{k_0}, v_{k_1}), $(v_{k_1}, v_{k_2}), \ldots, (v_{k_L}, v_{l_{k_0}})$, where L satisfies $L + 1 \le N - 1$. In addition, by the condition that $e_{i,j}(t)$ $((v_i, v_j) \in \mathcal{E}_a)$ are bounded, there exists M_1 such that $\|x_{k_h}(t) - x_{k_{h+1}}(t)\| \le M_1$ holds for $h = 0, \ldots, L$, where $k_{L+1} = l_{k_0}$. Therefore it is derived that

$$\begin{aligned}
&\|x_{k_0}(t) - x_0(t)\| \\
&\quad \le \|x_{k_0}(t) - x_{k_1}(t)\| + \cdots + \|x_{k_L}(t) - x_{k_{L+1}}(t)\| \\
&\qquad + \|x_{l_{k_0}}(t) - x_0(t)\| \\
&\quad \le (L+1)M_1 + M
\end{aligned} \tag{5}$$

from which it is immediate that

$$\|x_{k_0}(t)\| \le (N-1)M_1 + M + \|x_0(t)\|, \quad v_{k_0} \notin \mathcal{N}_0 \tag{6}$$

Summarising the above analysis, we can conclude that $x_{k_0}(t)$ is bounded for any $k_0 \in \{1, \ldots, N\}$, by inequalities (4) and (6). Further, if $\epsilon_i(t) = 0$ $(v_i \in \mathcal{N}_0)$ and $e_{i,j}(t) = 0$ $((v_i, v_j) \in \mathcal{E}_a)$, it can be easily verified that $x_i(t) = x_0(t)$ holds for $i = 1, \ldots, N$ by setting $M = M_1 = 0$ in (5). □

First of all, we consider the multi-agent system without external disturbances, that is, $d_i(t) \equiv 0$ in (1).

3.1 *Multi-agent system without disturbances*

We adopt the MRAC method in the protocol design, by regarding leader (2) as a desired reference model. Here, the MRAC goal is achieved by updating the estimates of unknown relation matrices between agents and the reference model using the local consensus errors, and will be established by the Lyapunov stability theory.

To be specific, the consensus protocol of multi-agent system (1) with $d_i(t) \equiv 0$ is proposed as

$$u_i(t) = H_i^{\mathrm{T}}(t)x_i(t) + K_i^{\mathrm{T}}(t)r(t) \tag{7}$$

with updating laws of estimates $H_i(t)$ and $K_i(t)$

$$\begin{aligned}
\dot{K}_i(t) &= -r(t) \sum_{v_j \in \mathcal{N}_i} (x_i(t) - x_j(t))^{\mathrm{T}} P B_i \\
\dot{H}_i(t) &= -x_i(t) \sum_{v_j \in \mathcal{N}_i} (x_i(t) - x_j(t))^{\mathrm{T}} P B_i
\end{aligned} \tag{8}$$

where $\mathcal{N}_i$ is the neighbour set of agent i, including the follower agents and the leader, and $P \in \mathbb{R}^{n \times n}$ is a positive definite matrix satisfying $A_0^{\mathrm{T}}P + PA_0 = -Q < 0$. The existence of P is guaranteed by the fact that A_0 is a stable matrix. Note that only the local information, determined by the interaction topology, is used in the adaptive consensus protocol (7) and (8). Meanwhile, A_0 is not required to be known exactly, but with some necessary information to obtain the matrix P in (8).

Denote $\tilde{H}_i(t) = H_i(t) - H_i^*$ and $\tilde{K}_i(t) = K_i(t) - K_i^*$. Substituting protocol (7) into the multi-agent system (1) with $d_i(t) \equiv 0$ leads to

$$\begin{aligned}
\dot{x}_i(t) &= A_i x_i(t) + B_i[H_i^{\mathrm{T}}(t)x_i(t) + K_i^{\mathrm{T}}(t)r(t)] \\
&= (A_i + B_i H_i^{*\mathrm{T}})x_i(t) + B_i\tilde{H}_i^{\mathrm{T}}(t)x_i(t) \\
&\quad + B_i K_i^{*\mathrm{T}} r(t) + B_i \tilde{K}_i^{\mathrm{T}}(t) r(t) \\
&= A_0 x_i(t) + B_0 r(t) + B_i \tilde{H}_i^{\mathrm{T}}(t) x_i(t) + B_i \tilde{K}_i^{\mathrm{T}}(t) r(t)
\end{aligned} \tag{9}$$

Then, combining (9) and (2), it is derived that

$$\dot{\epsilon}_i(t) = A_0 \epsilon_i(t) + B_i \tilde{H}_i^{\mathrm{T}}(t) x_i(t) + B_i \tilde{K}_i^{\mathrm{T}}(t) r(t) \tag{10}$$

$$\begin{aligned}
\dot{e}_{i,j}(t) &= A_0 e_{i,j}(t) + B_i \tilde{H}_i^{\mathrm{T}}(t) x_i(t) + B_i \tilde{K}_i^{\mathrm{T}}(t) r(t) \\
&\quad - B_j \tilde{H}_j^{\mathrm{T}}(t) x_j(t) - B_j \tilde{K}_j^{\mathrm{T}}(t) r(t)
\end{aligned} \tag{11}$$

According to the previous development, we now present the leader-following consensus result for the multi-agent system without external disturbances.

Theorem 1: For the multi-agent system (1) with $d_i(t) \equiv 0$, if at least one follower agent in each connected component of $\mathcal{G}_a$ is connected to the leader (2), then N agents asymptotically achieve consensus with the leader under the adaptive protocol (7) and (8), that is, $\lim_{t \to \infty}(x_i(t) - x_0(t)) = 0$, $i = 1, \ldots, N$.

To show Theorem 1, the following lemma is adopted from the literature.

Lemma 2 (Barbalat's Lemma [20])*:* Assume that $g(\cdot) : \mathbb{R} \mapsto \mathbb{R}$ is uniformly continuous and integrable, that is

$$\int_0^\infty g(t)\mathrm{d}t < \infty$$

Then, it holds that $\lim_{t \to \infty} g(t) = 0$.

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Proof of Theorem 1: Take the Lyapunov function candidate

$$V(t) = V_1(t) + V_2(t) \tag{12}$$

with

$$V_1(t) = \sum_{v_i \in \mathcal{N}_0} \epsilon_i^{\mathrm{T}}(t) P \epsilon_i(t) + \frac{1}{2} \sum_{(v_i, v_j) \in \mathcal{E}_a} e_{i,j}^{\mathrm{T}}(t) P e_{i,j}(t) \tag{13}$$

and

$$V_2(t) = \sum_{i=1}^{N} \mathrm{tr}(\tilde{K}_i^{\mathrm{T}}(t)\tilde{K}_i(t) + \tilde{H}_i^{\mathrm{T}}(t)\tilde{H}_i(t)) \tag{14}$$

As stated in the updating law (8), matrix P is positive definite, and satisfies $A_0^{\mathrm{T}}P + PA_0 = -Q < 0$.

By (10) and (11), we first compute the time derivative of $V_1(t)$ as

$$\begin{aligned}
\dot{V}_1(t) = {} & \sum_{v_i \in \mathcal{N}_0} \epsilon_i^{\mathrm{T}}(t)(PA_0 + A_0^{\mathrm{T}}P)\epsilon_i(t) \\
& + 2\sum_{v_i \in \mathcal{N}_0} \epsilon_i^{\mathrm{T}}(t)PB_i[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)] \\
& + \sum_{(v_i,v_j)\in\mathcal{E}_a} \left\{ \frac{1}{2} e_{i,j}^{\mathrm{T}}(t)(PA_0 + A_0^{\mathrm{T}}P)e_{i,j}(t) \right. \\
& + e_{i,j}^{\mathrm{T}}(t)PB_i[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)] \\
& \left. - e_{i,j}^{\mathrm{T}}(t)PB_j[\tilde{H}_j^{\mathrm{T}}(t)x_j(t) + \tilde{K}_j^{\mathrm{T}}(t)r(t)] \right\}
\end{aligned} \tag{15}$$

Since the interaction graph of N follower agents is undirected, $(v_i, v_j) \in \mathcal{E}_a$ implies $(v_j, v_i) \in \mathcal{E}_a$. Then, combining with the fact $e_{i,j}(t) = -e_{j,i}(t)$, we have

$$\begin{aligned}
& - \sum_{(v_i,v_j)\in\mathcal{E}_a} e_{i,j}^{\mathrm{T}}(t)PB_j[\tilde{H}_j^{\mathrm{T}}(t)x_j(t) + \tilde{K}_j^{\mathrm{T}}(t)r(t)] \\
& = \sum_{(v_i,v_j)\in\mathcal{E}_a} e_{i,j}^{\mathrm{T}}(t)PB_i[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)]
\end{aligned}$$

from which it follows that

$$\begin{aligned}
\dot{V}_1(t) = {} & \sum_{v_i \in \mathcal{N}_0} \epsilon_i^{\mathrm{T}}(t)(PA_0 + A_0^{\mathrm{T}}P)\epsilon_i(t) \\
& + 2\sum_{v_i \in \mathcal{N}_0} \epsilon_i^{\mathrm{T}}(t)PB_i[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)] \\
& + \sum_{(v_i,v_j)\in\mathcal{E}_a} \left\{ \frac{1}{2} e_{i,j}^{\mathrm{T}}(t)(PA_0 + A_0^{\mathrm{T}}P)e_{i,j}(t) \right. \\
& \left. + 2e_{i,j}^{\mathrm{T}}(t)PB_i[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)] \right\} \\
= {} & -\sum_{v_i \in \mathcal{N}_0} \epsilon_i^{\mathrm{T}}(t)Q\epsilon_i(t) - \frac{1}{2}\sum_{(v_i,v_j)\in\mathcal{E}_a} e_{i,j}^{\mathrm{T}}(t)Qe_{i,j}(t) \\
& + 2\sum_{v_i \in \mathcal{N}_0} \epsilon_i^{\mathrm{T}}(t)PB_i[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)] \\
& + 2\sum_{(v_i,v_j)\in\mathcal{E}_a} e_{i,j}^{\mathrm{T}}(t)PB_i[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)]
\end{aligned} \tag{16}$$

Now, it is obtained that

$$\begin{aligned}
\dot{V}(t) = {} & \dot{V}_1(t) + \dot{V}_2(t) \\
= {} & -\sum_{v_i \in \mathcal{N}_0} \epsilon_i^{\mathrm{T}}(t)Q\epsilon_i(t) - \frac{1}{2}\sum_{(v_i,v_j)\in\mathcal{E}_a} e_{i,j}^{\mathrm{T}}(t)Qe_{i,j}(t) \\
& + 2\sum_{v_i \in \mathcal{N}_0} \epsilon_i^{\mathrm{T}}(t)PB_i[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)] \\
& + 2\sum_{(v_i,v_j)\in\mathcal{E}_a} e_{i,j}^{\mathrm{T}}(t)PB_i[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)] \\
& + 2\sum_{i=1}^{N} \mathrm{tr}(\tilde{K}_i^{\mathrm{T}}(t)\dot{K}_i(t) + \tilde{H}_i^{\mathrm{T}}(t)\dot{H}_i(t))
\end{aligned} \tag{17}$$

By the facts

$$\begin{aligned}
& \epsilon_i^{\mathrm{T}}(t)PB_i[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)] \\
& \quad = \mathrm{tr}\{[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)]\epsilon_i^{\mathrm{T}}(t)PB_i\} \\
& e_{i,j}^{\mathrm{T}}(t)PB_i[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)] \\
& \quad = \mathrm{tr}\{[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)]e_{i,j}^{\mathrm{T}}(t)PB_i\}
\end{aligned}$$

it yields that

$$\begin{aligned}
& 2\sum_{v_i \in \mathcal{N}_0} \epsilon_i^{\mathrm{T}}(t)PB_i[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)] \\
& \quad + 2\sum_{(v_i,v_j)\in\mathcal{E}_a} e_{i,j}^{\mathrm{T}}(t)PB_i[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)] \\
& = 2\sum_{v_i \in \mathcal{N}_0} \mathrm{tr}\{[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)]\epsilon_i^{\mathrm{T}}(t)PB_i\} \\
& \quad + 2\sum_{(v_i,v_j)\in\mathcal{E}_a} \mathrm{tr}\{[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)]e_{i,j}^{\mathrm{T}}(t)PB_i\} \\
& = 2\sum_{i=1}^{N} \mathrm{tr}\{[\tilde{H}_i^{\mathrm{T}}(t)x_i(t) + \tilde{K}_i^{\mathrm{T}}(t)r(t)] \\
& \quad \times \sum_{v_j \in \mathcal{N}_i} (x_i(t) - x_j(t))^{\mathrm{T}} PB_i\}
\end{aligned} \tag{18}$$

Applying the above equation and updating law (8) into (17), we have

$$\begin{aligned}
\dot{V}(t) = {} & -\sum_{v_i \in \mathcal{N}_0} \epsilon_i^{\mathrm{T}}(t)Q\epsilon_i(t) - \frac{1}{2}\sum_{(v_i,v_j)\in\mathcal{E}_a} e_{i,j}^{\mathrm{T}}(t)Qe_{i,j}(t) \\
\leq {} & 0
\end{aligned} \tag{19}$$

which implies that $\epsilon_i(t)$ $(v_i \in \mathcal{N}_0)$, $e_{i,j}(t)$ $((v_i, v_j) \in \mathcal{E}_a)$, $K_i(t)$ and $H_i(t)$ are bounded. Meanwhile, $x_0(t)$ is also bounded, since the reference signal $r(t)$ is bounded. Then, according to Lemma 1, all $x_i(t)$ $(i = 1, \ldots, N)$ are bounded, and then $\dot{\epsilon}_i(t)$ $(v_i \in \mathcal{N}_0)$ and $\dot{e}_{i,j}(t)$ $((v_i, v_j) \in \mathcal{E}_a)$ are all bounded by (10) and (11). Therefore $\sum_{v_i \in \mathcal{N}_0} \epsilon_i^{\mathrm{T}}(t)Q\epsilon_i(t) + (1/2)\sum_{(v_i,v_j)\in\mathcal{E}_a} e_{i,j}^{\mathrm{T}}(t)Qe_{i,j}(t)$ is uniformly continuous. On the other hand, since $V(t)$ is non-increasing and bounded from below by zero, it has a non-negative limit $V(\infty)$ as $t \to \infty$.

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Summarising the above analysis, we have

$$\int_0^{\infty}\left[\sum_{v_i\in\mathcal{N}_0}\epsilon_i^{\mathrm{T}}(t)Q\epsilon_i(t)+\frac{1}{2}\sum_{(v_i,v_j)\in\mathcal{E}_a}e_{i,j}^{\mathrm{T}}(t)Qe_{i,j}(t)\right]\mathrm{d}t$$
$$=-\int_0^{\infty}\dot{V}(t)\mathrm{d}t=V(0)-V(\infty)\leq V(0)<\infty \qquad (20)$$

Then, it is immediate from Lemma 2 that $\lim_{t\to\infty}\epsilon_i(t)=0$ $(v_i\in\mathcal{N}_0)$ and $\lim_{t\to\infty}e_{i,j}(t)=0$ $((v_i,v_j)\in\mathcal{E}_a)$, which results in $\lim_{t\to\infty}(x_i(t)-x_0(t))=0$ $(i=1,\ldots,N)$ by the proof of Lemma 1. □

3.2 Multi-agent system with bounded disturbances

For the multi-agent system (1) with unknown bounded disturbance $d_i(t)$, the consensus protocol is designed as

$$u_i(t)=H_i^{\mathrm{T}}(t)x_i(t)+K_i^{\mathrm{T}}(t)r(t)-[(I-\sigma_i(t))\bar{d}_i(t)+\sigma_i(t)\underline{d}_i(t)] \qquad (21)$$

and $\sigma_i(t)$ is defined by

$$\sigma_i(t)=\mathrm{diag}\{\sigma_{i1}(t),\ldots,\sigma_{im}(t)\}$$
$$\sigma_{ik}(t)=\begin{cases}0, & \sum_{v_j\in\mathcal{N}_i}(x_i(t)-x_j(t))^{\mathrm{T}}PB_{ik}\geq 0\\ 1, & \sum_{v_j\in\mathcal{N}_i}(x_i(t)-x_j(t))^{\mathrm{T}}PB_{ik}<0\end{cases} \qquad (22)$$

where, B_{ik} is the kth column of matrix B_i, $k=1,\ldots,m$. The updating laws of $K_i(t)$ and $H_i(t)$ are given by (8), and the estimated upper and lower bounds of disturbance are updated by the following adaptive schemes

$$\dot{\bar{d}}_i(t)=\gamma_{i1}(I-\sigma_i(t))B_i^{\mathrm{T}}P\sum_{v_j\in\mathcal{N}_i}(x_i(t)-x_j(t))$$
$$\dot{\underline{d}}_i(t)=\gamma_{i2}\sigma_i(t)B_i^{\mathrm{T}}P\sum_{v_j\in\mathcal{N}_i}(x_i(t)-x_j(t)) \qquad (23)$$

where γ_{i1} and γ_{i2} $(i=1,\ldots,N)$ are positive constants. Substituting protocol (21) into the multi-agent system (1) yields

$$\dot{x}_i(t)=A_0x_i(t)+B_0r(t)+B_i\tilde{H}_i^{\mathrm{T}}(t)x_i(t)+B_i\tilde{K}_i^{\mathrm{T}}(t)r(t)-B_i[(I-\sigma_i(t))\bar{d}_i(t)+\sigma_i(t)\underline{d}_i(t)]+B_id_i(t) \qquad (24)$$

For the convenience of analysis, we denote $d_i(t)=[d_{i1}(t)\ldots d_{im}(t)]^{\mathrm{T}}$, $\bar{d}_i(t)=[\bar{d}_{i1}(t)\ldots\bar{d}_{im}(t)]^{\mathrm{T}}$, $\underline{d}_i(t)=[\underline{d}_{i1}(t)\ldots\underline{d}_{im}(t)]^{\mathrm{T}}$, $\bar{d}_i^*=[\bar{d}_{i1}^*\ldots\bar{d}_{im}^*]^{\mathrm{T}}$ and $\underline{d}_i^*=[\underline{d}_{i1}^*\ldots\underline{d}_{im}^*]^{\mathrm{T}}$.

Theorem 2: For the multi-agent system (1) with partly unknown parameters and bounded disturbance $d_i(t)$, if at least one follower agent in each connected component of $\mathcal{G}_a$ is connected to the leader (2), then N agents asymptotically achieve consensus with the leader under the adaptive protocol (21) together with updating laws (8) and (23), that is, $\lim_{t\to\infty}(x_i(t)-x_0(t))=0$, $i=1,\ldots,N$.

Proof: Take the Lyapunov function candidate as

$$\bar{V}(t)=V(t)+V_3(t)$$

with $V(t)$ given by (12), and $V_3(t)$ defined by

$$V_3(t)=\sum_{i=1}^{N}\frac{\tilde{\bar{d}}_i^{\mathrm{T}}(t)\tilde{\bar{d}}_i(t)}{\gamma_{i1}}+\sum_{i=1}^{N}\frac{\tilde{\underline{d}}_i^{\mathrm{T}}(t)\tilde{\underline{d}}_i(t)}{\gamma_{i2}} \qquad (25)$$

where $\tilde{\bar{d}}_i(t)=\bar{d}_i(t)-\bar{d}_i^*\triangleq[\tilde{\bar{d}}_{i1}(t)\ldots\tilde{\bar{d}}_{im}(t)]^{\mathrm{T}}$ and $\tilde{\underline{d}}_i(t)=\underline{d}_i(t)-\underline{d}_i^*\triangleq[\tilde{\underline{d}}_{i1}(t)\ldots\tilde{\underline{d}}_{im}(t)]^{\mathrm{T}}$.

By the proof of Theorem 1, computing the time derivative of $\bar{V}(t)$ leads to

$$\dot{\bar{V}}(t)=-\sum_{v_i\in\mathcal{N}_0}\epsilon_i^{\mathrm{T}}(t)Q\epsilon_i(t)-\frac{1}{2}\sum_{(v_i,v_j)\in\mathcal{E}_a}e_{i,j}^{\mathrm{T}}(t)Qe_{i,j}(t)$$
$$+2\sum_{v_i\in\mathcal{N}_0}\{-\epsilon_i^{\mathrm{T}}(t)PB_i[(I-\sigma_i(t))\bar{d}_i(t)+\sigma_i(t)\underline{d}_i(t)]$$
$$+\epsilon_i^{\mathrm{T}}(t)PB_id_i(t)\}+2\sum_{(v_i,v_j)\in\mathcal{E}_a}\{-e_{i,j}^{\mathrm{T}}(t)PB_i$$
$$\times[(I-\sigma_i(t))\bar{d}_i(t)+\sigma_i(t)\underline{d}_i(t)]+e_{i,j}^{\mathrm{T}}(t)PB_id_i(t)\}$$
$$=-\sum_{v_i\in\mathcal{N}_0}\epsilon_i^{\mathrm{T}}(t)Q\epsilon_i(t)-\frac{1}{2}\sum_{(v_i,v_j)\in\mathcal{E}_a}e_{i,j}^{\mathrm{T}}(t)Qe_{i,j}(t)$$
$$+2\sum_{v_i\in\mathcal{N}_0}\left\{-\epsilon_i^{\mathrm{T}}(t)P\sum_{k=1}^{m}B_{ik}[(1-\sigma_{ik}(t))\bar{d}_{ik}(t)\right.$$
$$\left.+\sigma_{ik}(t)\underline{d}_{ik}(t)]+\epsilon_i^{\mathrm{T}}(t)P\sum_{k=1}^{m}B_{ik}d_{ik}(t)\right\}$$
$$+2\sum_{(v_i,v_j)\in\mathcal{E}_a}\left\{-e_{i,j}^{\mathrm{T}}(t)P\sum_{k=1}^{m}B_{ik}[(1-\sigma_{ik}(t))\bar{d}_{ik}(t)\right.$$
$$\left.+\sigma_{ik}(t)\underline{d}_{ik}(t)]+e_{i,j}^{\mathrm{T}}(t)P\sum_{k=1}^{m}B_{ik}d_{ik}(t)\right\}$$
$$=-\sum_{v_i\in\mathcal{N}_0}\epsilon_i^{\mathrm{T}}(t)Q\epsilon_i(t)-\frac{1}{2}\sum_{(v_i,v_j)\in\mathcal{E}_a}e_{i,j}^{\mathrm{T}}(t)Qe_{i,j}(t)$$
$$-2\sum_{i=1}^{N}\sum_{v_j\in\mathcal{N}_i}(x_i(t)-x_j(t))^{\mathrm{T}}P\sum_{k=1}^{m}B_{ik}$$
$$\times[(1-\sigma_{ik}(t))\bar{d}_{ik}(t)+\sigma_{ik}(t)\underline{d}_{ik}(t)]$$
$$+2\sum_{i=1}^{N}\sum_{v_j\in\mathcal{N}_i}(x_i(t)-x_j(t))^{\mathrm{T}}P\sum_{k=1}^{m}B_{ik}d_{ik}(t) \qquad (26)$$

where the adaptive law (8) and the fact $e_{i,j}(t)=-e_{j,i}(t)$ have been used. From the condition $\underline{d}_{ik}^*\leq d_{ik}(t)\leq\bar{d}_{ik}^*$ $(k=1,\ldots,m)$ and the definition of $\sigma_{ik}(t)$ by (22), it is

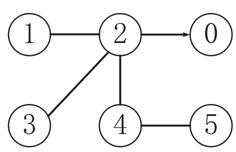

Fig. 1 *Interaction graph of five follower agents and one leader*

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immediate that

$$
\begin{aligned}
&2\sum_{i=1}^{N}\sum_{v_j\in\mathcal{N}_i}(x_i(t)-x_j(t))^{\mathrm{T}}P\sum_{k=1}^{m}B_{ik}d_{ik}(t)\\
&\quad\le 2\sum_{i=1}^{N}\sum_{v_j\in\mathcal{N}_i}(x_i(t)-x_j(t))^{\mathrm{T}}P\\
&\quad\quad\times\sum_{k=1}^{m}B_{ik}[(1-\sigma_{ik}(t))\bar{d}_{ik}^{*}+\sigma_{ik}(t)\underline{d}_{ik}^{*}]
\end{aligned}\tag{27}
$$

Applying the above inequality into (26) yields

$$
\begin{aligned}
\dot{V}(t)\le{}&-\sum_{v_i\in\mathcal{N}_0}\epsilon_i^{\mathrm{T}}(t)Q\epsilon_i(t)-\frac{1}{2}\sum_{(v_i,v_j)\in\mathcal{E}_a}e_{i,j}^{\mathrm{T}}(t)Qe_{i,j}(t)\\
&-2\sum_{i=1}^{N}\sum_{v_j\in\mathcal{N}_i}(x_i(t)-x_j(t))^{\mathrm{T}}P\sum_{k=1}^{m}B_{ik}\\
&\times[(1-\sigma_{ik}(t))\tilde{\bar{d}}_{ik}(t)+\sigma_{ik}(t)\tilde{\underline{d}}_{ik}(t)]
\end{aligned}\tag{28}
$$

from which it follows that

$$
\begin{aligned}
\dot{V}(t)\le{}&-\sum_{v_i\in\mathcal{N}_0}\epsilon_i^{\mathrm{T}}(t)Q\epsilon_i(t)-\frac{1}{2}\sum_{(v_i,v_j)\in\mathcal{E}_a}e_{i,j}^{\mathrm{T}}(t)Qe_{i,j}(t)\\
&-2\sum_{i=1}^{N}\sum_{v_j\in\mathcal{N}_i}(x_i(t)-x_j(t))^{\mathrm{T}}P\sum_{k=1}^{m}B_{ik}[(1-\sigma_{ik}(t))\tilde{\bar{d}}_{ik}(t)\\
&+\sigma_{ik}(t)\tilde{\underline{d}}_{ik}(t)]+2\sum_{i=1}^{N}\sum_{k=1}^{m}\left(\frac{\dot{\bar{d}}_{ik}(t)\tilde{\bar{d}}_{ik}(t)}{\gamma_{i1}}+\frac{\dot{\underline{d}}_{ik}(t)\tilde{\underline{d}}_{ik}(t)}{\gamma_{i2}}\right)\\
={}&-\sum_{v_i\in\mathcal{N}_0}\epsilon_i^{\mathrm{T}}(t)Q\epsilon_i(t)-\frac{1}{2}\sum_{(v_i,v_j)\in\mathcal{E}_a}e_{i,j}^{\mathrm{T}}(t)Qe_{i,j}(t)\\
\le{}&0
\end{aligned}\tag{29}
$$

where the following equivalent version of adaptive updating law (23) is applied in the second step

$$
\dot{\bar{d}}_{ik}(t)=\gamma_{i1}(1-\sigma_{ik}(t))B_{ik}^{\mathrm{T}}P\sum_{v_j\in\mathcal{N}_i}(x_i(t)-x_j(t))
$$

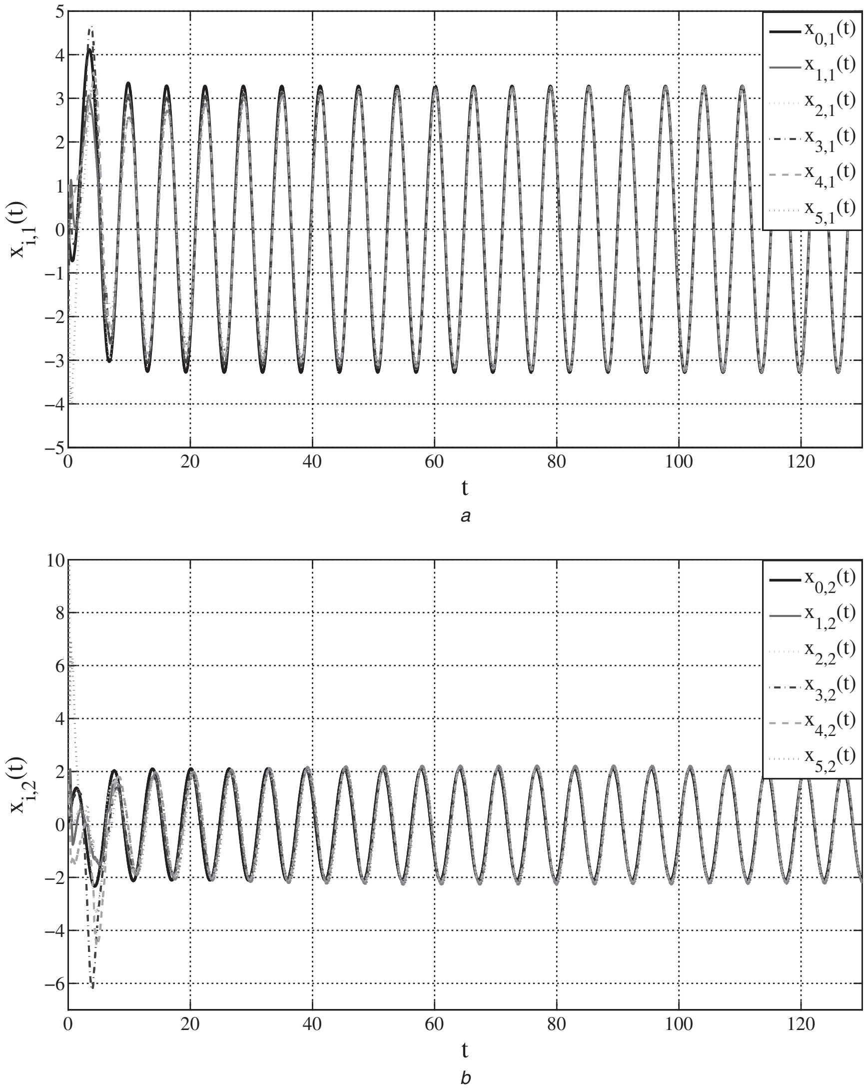

Fig. 2 *State trajectories of five follower agents and the leader*

a First element: $x_{i,1}(t)$

b Second element: $x_{i,2}(t)$

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$$\dot{\underline{d}}_{ik}(t) = \gamma_{i2}\sigma_{ik}(t)B_{ik}^{\mathrm{T}}P\sum_{v_j\in\mathcal{N}_i}(x_i(t)-x_j(t)),\ k=1,\ldots,m$$

By adopting the same proof lines of Theorem 1, the consensus result can be established by summarising the above results and Lemmas 1, 2. □

4 Simulation results

We provide a simulation example to illustrate the leader-following consensus performance of multi-agent systems with bounded disturbances under the proposed adaptive protocol. In particular, a network consisting of five follower agents and one leader is considered. In models (1) and (2), the system matrices are

$$\begin{aligned} A_1 &= A_2 = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}, \quad B_3 = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}, \\ A_4 &= A_5 = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}, \quad B_4 = B_5 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}, \\ A_0 &= \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 1 & -2 \\ 4 & 1 \end{bmatrix} \end{aligned} \tag{30}$$

$x_i(t) = [x_{i,1}(t)\ \ x_{i,2}(t)]^{\mathrm{T}}$ $(i = 0, 1, 2, 3, 4, 5)$, $d_i(t)$ is the bounded random white noise and the reference signal is $r(t) = [\sin(t)\cos(t)]^{\mathrm{T}}$. It can be easily verified that Assumption 1 is satisfied. The interaction graph of five agents and the leader is shown in Fig. 1, which satisfies the topology condition of Theorem 2. The initial state of the leader is given by $x_0(0) = [0\ \ 0]^{\mathrm{T}}$, and the initial states of five agents are $x_1(0) = [-1\ 1]^{\mathrm{T}}$, $x_2(0) = [0\ 2]^{\mathrm{T}}$, $x_3(0) = [-2\ 0]^{\mathrm{T}}$, $x_4(0) = [1\ 1.5]^{\mathrm{T}}$ and $x_5(0) = [-3\ 2.5]^{\mathrm{T}}$.

Take the initial values of the updating matrices and vectors in adaptive laws (8) and (23) as $K_i(0) = H_i(0) = 0$ and $\bar{d}_i(0) = \underline{d}_i(0) = 0$ for $i = 1, \ldots, 5$. Then, under the consensus protocol (21) together with adaptive laws (8) and (23), the robust leader-following consensus performance is presented in Fig. 2. Obviously, the five agents can track the leader with reference signal $r(t)$ within 130 s, even in the presence of unknown disturbances, which validates the effectiveness of the proposed adaptive protocol.

5 Conclusions

This paper has addressed the robust leader-following consensus problem of linear multi-agent systems with partially unknown parameters and bounded disturbances. Different from the previous related work, in this paper, a reference signal is preassigned as the input of an active leader whose state matrix is not required to be equal to that of the follower agents, and meanwhile, the unknown external disturbances are also taken into account. For such a leader-following system, it turns out that the MRAC method together with the adaptive disturbance compensator could be nicely employed to the consensus protocol design. It is shown that under the proposed adaptive protocols, all agents can eventually track the prescribed leader accurately even in the presence of external disturbances, if at least one follower agent in each connected component of interaction graph is connected to the leader.

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