A Special Case of Multi-way Relay Channel: When beamforming is not applicable

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***Abstract*—In this paper, we study a special case of multi-way relaying channel, to which traditional beamforming cannot achieve the best performance due to insufficient antennas. A new transmission protocol is proposed by aligning the messages from the same pair with the help of relay precoding. As a result, inter-pair interference can be avoided and intra-pair interference can be coped with by using network coding. Then analytic results, such as the ergodic sum rate and the outage probability, are developed for the proposed protocol. The numerical results are also provided to demonstrate the performance of our proposed scheme. To improve the diversity gain of the proposed scheme, an optimal scheme is also presented.**

***Index Terms*—Network coding, multi-way relay, beamforming, ergodic sum rate, outage probability.**

I. Introduction

As an efficient technique to extend the coverage and improve the system performance, relaying transmission has drawn much attention in the wireless communication research field. In particular, there exists extensive work about two-way relaying channel, which has many important applications in communications [1], [2]. Due to the broadcasting nature of radio propagating, network coding has been proposed as a promising method to significantly increase the throughput for two-way relaying channel [3], [4], [5]. The key idea of network coding is to allow the relay to broadcast the mixture of messages to destinations, and then each destination obtains the desired message by subtracting its self-interference from the mixture. As a result, information exchange between two source nodes can be completed in two time slots, whereas the conventional protocol needs four time slots.

Multi-way relay channel is a natural extension of two-way relaying channel, where multiple pairs of source nodes exchange information with their partners. For such a scenario, beamforming is an efficient approach to eliminate inter-pair interference, and to achieve high spectral efficiency [6], [7]. However, there is a strict constraint that the number of relay antennas is larger than or at least the same as the number of sources with single antenna. Otherwise, there are not enough degrees of freedom to remove inter-pair interference [8].

In the next generation communication systems, such as LTE-Advanced systems, Type I and Type II relays are defined as two categories of relaying technologies. As defined in the present standardizations, Type I relay has the independent cell ID, and the right for radio resource management (RRM). Thus Type I relay essentially can be considered as a low-power base station. To improve the transmission efficiency, it is possible for the users to communicate with each other via Type I relays instead of the base stations in the future cellular systems. Due to the low cost for installation, enough relays could be located to satisfy the high-speed requirement of the users, which is a difficult problem for base stations. Therefore, we focus on the multi-way relay channel to introduce a realistic spacial multiplexing technique for the future wireless communication system, which shows the advantages of our work.

In this paper, we consider a multi-way relaying scenario with M pairs of single-antenna sources and one multi-antenna relay. Particularly we focus on a special case where the number of relay antennas is less than the number of sources, to which traditional beamforming cannot be applied. The main contribution of this paper is to propose a new transmission protocol for such a challenging scenario. Specifically, the use of proposed protocol can ensure that multi-pair information exchange can be accomplished within two time slots, which yields high multiplexing gains. On the other hand, for such a special case, conventional beamforming has to rely on time sharing due to insufficient antennas, which results in loss of system performance. By carefully designing the precoding matrices at the relay, two messages from the same pair could be aligned to each other, and each node can obtain the message from its partner by eliminating self-interference from the mixture. Note that such a protocol can be extended to general multi-way relaying channels in a straightforward way. In addition to simulations, the proposed protocol is also evaluated by analytic results, including ergodic sum rate, outage probability, as well as their simplified approximations at the high signal-to-noise (SNR) region. Both the analytical and numerical results demonstrate that the proposed protocol can achieve better performance than the comparable ones. Moreover, an optimal proposed scheme is also provided, which can further improve the diversity gain of proposed scheme.

This work was supported in part by National Natural Science Foundation of China (Grant No. 61072058), the State Major Science and Technology Special Projects (Grant No. 2010ZX03003-003-01), the Fok Ying Tong Education Foundation Application Research Projects (Grant No. 122005) and the Program for New Century Excellent Talents in University.

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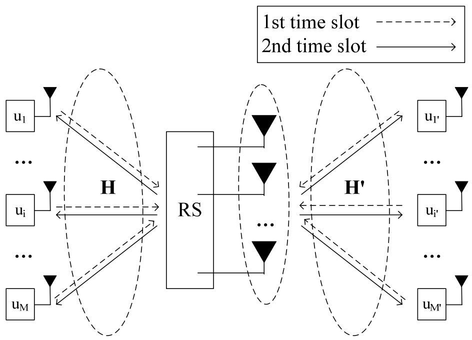

Fig. 1. Multi-pair two-way relay channels scenario for the proposed protocol.

II. Protocol Description

Consider a communication scenario with M pairs of single-antenna users and one relay equipped with N antennas, where each pair exchanges information via the relay. For notation simplicity, we denote u_i and u_i' as the two users from the same pair as shown in Fig. 1. For such a scenario, it is well known that traditional zero forcing (ZF) beamforming can be applied when $N \geq 2M$; however, it become challenging for the case $N < 2M$ since the relay does not have enough degrees of freedom to serve all $2M$ users simultaneously. In this section, we only focus on the case with $N = 2M-1$, and the proposed scheme can be extended to a general case with $N > 2M-1$ as shown in Section IV.

All nodes are subjected to the half-duplex constraint, and time division duplexing (TDD) is used because of its simplicity. Only time synchronization is assumed in this paper, similar to [4]. Due to the symmetry of TDD systems, the incoming channel and forwarding channel are assumed to be symmetric. All channels are considered as quasi-static Rayleigh fading, and the channel fading for each link is independent and identically distributed (i.i.d) with each other. It is assumed that the relay has the access to all source-relay channel information, while each user dose not have to know the global channel information.

The proposed network coding transmission consists of two time slots. During the first time slot, all the users transmit messages to the relay simultaneously, and thus the relay receives

$$\mathbf{y_r} = \sum_{i=1}^{M} \mathbf{h}_i s_i + \sum_{i=1}^{M} \mathbf{h'}_i s_i' + \mathbf{n_r}, \tag{1}$$

where s_i is the message sent from the user u_i, $\mathbf{y_r}$ is the $N \times 1$ observation vector at the relay, $\mathbf{n_r}$ is the relay additive Gaussian noise vector, $\mathbf{h}_i$ is the $N \times 1$ channel vector between the user u_i and the relay, s_i' and $\mathbf{h'}_i$ are defined similarly for the user u_i'.

The amplify-forward (AF) strategy will be considered in this paper. The key idea of the proposed protocol is to design the precoding matrices at the relay, where two messages from the same pair will be grouped together and inter-pair interference will be eliminated. Specifically for the i-th pair, the precoding matrix $\mathbf{Q}_i$ can be designed as

$$\mathbf{Q}_i = (\mathbf{I}_N - \mathbf{P}_i(\mathbf{P}_i^H \mathbf{P}_i)^{-1}\mathbf{P}_i^H)_{N\times N},\ i \in 1, \cdots, M, \tag{2}$$

where $\mathbf{P}_i$ is a $N \times (2M-2)$ submatrix of the channel matrix $\mathbf{H} = [\mathbf{h}_1 \cdots \mathbf{h'}_M]_{N\times 2M}$ by removing its i-th and $(i+M)$-th columns, i.e $\mathbf{P}_i = [\mathbf{h}_1 \cdots \mathbf{h}_{i-1}\ \mathbf{h}_{i+1} \cdots \mathbf{h}_M\ \mathbf{h}_1' \cdots \mathbf{h}_{(i-1)}'\ \mathbf{h}_{(i+1)}' \cdots \mathbf{h}_M']$. Apparently $\mathbf{Q}_i$ is a orthogonal projection matrix generated from $\mathbf{P}_i$, which is a submatrix of the channel matrix. Due to the definition of orthogonal projection matrix, the null space dimension for $\mathbf{Q}_i$ is $(2M-2)$. Therefore, the number of relay antennas must satisfy the constraint that $N \geqslant 2M-1$, which ensures that the dimension of the signal space of $\mathbf{Q}_i$ is larger or equal to one. Prior to transmission, then the relay uses $\mathbf{Q}_i$ to group two messages from the i-th pair as following

$$\mathbf{Q}_i\mathbf{y_r} = \mathbf{Q}_i\mathbf{h}_i s_i + \mathbf{Q}_i\mathbf{h}_i' s_i' + \mathbf{Q}_i\mathbf{n_r}, \tag{3}$$

where the facts $\mathbf{Q}_i\mathbf{h}_k = \mathbf{0}$ and $\mathbf{Q}_i\mathbf{h}_k' = \mathbf{0}(i \neq k)$ have been used.

During the second time slot, the mixed messages are broadcasted to all the users,

$$\mathbf{t_r} = \sum_{i=1}^{M} \alpha_i \mathbf{Q}_i \mathbf{y_r}, \tag{4}$$

where α_i is the factor for power normalization, $\alpha_i = 1/\sqrt{\|\mathbf{Q}_i\mathbf{h}_i\|_2^2 + \|\mathbf{Q}_i\mathbf{h}_i'\|_2^2 + (1/\rho)\|\mathbf{Q}_i\|_2^2}$, and $\|\mathbf{A}\|_2$ is the norm of $\mathbf{A}$, defined as $\|\mathbf{A}\|_2 = \left(\sum\limits_i \sum\limits_j {a_{ij}}^2\right)^{\frac{1}{2}}$, and ρ is denoted as SNR. Note that equal power allocation is adopted at the relay for simplicity. As a result, the user u_i receives

$$r_i = \mathbf{h}_i^H \mathbf{t_r} + n_i = \sum_{j=1}^{M} \alpha_j \mathbf{h}_i^H \mathbf{Q}_j \mathbf{y_r} + n_i, \tag{5}$$

where n_i is the additive Gaussian noise at the user u_i. Due to the conjugate symmetry of Hermite matrix $\mathbf{Q}_i$, it can be easily shown that $\mathbf{h}_i^H\mathbf{Q}_j = \mathbf{0}$, for $i \neq j$. Thus the observation at the user u_i is shown as

$$r_i = \alpha_i \mathbf{h}_i^H \mathbf{Q}_i \mathbf{h}_i s_i + \alpha_i \mathbf{h}_i^H \mathbf{Q}_i \mathbf{h}_i' s_i' + \alpha_i \mathbf{h}_i^H \mathbf{Q}_i \mathbf{n_r} + n_i. \tag{6}$$

As shown in (6), the desired message could be detected by the user u_i if two coefficients $\alpha_i \mathbf{h}_i^H \mathbf{Q}_i \mathbf{h}_i$ and $\alpha_i \mathbf{h}_i^H \mathbf{Q}_i \mathbf{h}_i'$ are informed. Since the relay has the access to global channel information, such coefficients will be calculated at the relay and transmitted to the users. As a result, the proposed scheme only needs to transmit $4M$ coefficients in total. Provided that global information is assumed at each user, $2M^2N$ coefficients are required to be transmitted, which is much larger than our proposed scheme. Thus the proposed scheme can further reduce the system resource spending on channel information transmission.

Similar to physical layer network coding, each destination can first subtract its own information from the observation and then detect the message from its partner. In the following section, we will develop some analytic results for the performance achieved by our proposed protocol for a special case where the relay equipped with $(2M-1)$ antennas.

III. Performance Analysis for the Proposed Protocol

In this section, we use the ergodic sum rate and outage probability as the criteria to evaluate the performance of the proposed protocol. The ergodic sum rate demonstrates the averaged system throughput a transmission scheme can achieve and the outage probability shows the reception reliability given a targeted data rate. Note that in this section, we only focus on the case with $N = 2M - 1$.

A. Ergodic sum rate analysis

From the definition, the ergodic sum rate can be obtained by finding the expectation of the mutual information over all possible channel state. The ergodic sum rate for the user u_i can be shown as

$$R_i = \frac{1}{2}\mathcal{E}\left\{\log_2\left(1+\frac{\rho\alpha_i^2\|\mathbf{h}_i^H\mathbf{Q}_i\mathbf{h}_i'\|_2^2}{\alpha_i^2\|\mathbf{h}_i^H\mathbf{Q}_i\|_2^2+1}\right)\right\}. \tag{7}$$

When $N = 2M - 1$, $\mathbf{Q}_i\mathbf{h}_i$ and $\mathbf{Q}_i\mathbf{h}_i'$ are aligned in the same direction. Combined with the conjugate symmetry and idempotence of $\mathbf{Q}_i$, the numerator in (7) can be derived as

$$\|\mathbf{h}_i^H\mathbf{Q}_i\mathbf{h}'_i\|_2^2 = \|(\mathbf{Q}_i\mathbf{h}_i)^H\mathbf{Q}_i\mathbf{h}'_i\|_2^2 = \|\mathbf{Q}_i\mathbf{h}_i\|_2^2\|\mathbf{Q}_i\mathbf{h}'_i\|_2^2. \tag{8}$$

Substituting (8) and the expression of α_i into (7), R_i can be simplified as

$$R_i = \frac{1}{2}\mathcal{E}\left\{\log_2\left(1+\frac{\rho^2\|\mathbf{Q}_i\mathbf{h}_i\|_2^2\|\mathbf{Q}_i\mathbf{h}'_i\|_2^2}{2\rho\|\mathbf{Q}_i\mathbf{h}_i\|_2^2+\rho\|\mathbf{Q}_i\mathbf{h}'_i\|_2^2+\|\mathbf{Q}_i\|_2^2}\right)\right\}. \tag{9}$$

In order to obtain the closed-form expression for the ergodic sum rate, it is important to obtain the probability density function (PDF) of R_i. Define $z_1 = \|\mathbf{Q}_i\mathbf{h}_i\|_2^2$ and $z_2 = \|\mathbf{Q}_i\mathbf{h}'_i\|_2^2$, then the joint distribution of the two parameters is provided in the following lemma.

Lemma *1:* The two variables, z_1 and z_2, are independent identically exponentially distributed and their joint PDF can be shown as following

$$f(z_1, z_2) = e^{-(z_1+z_2)},\ z_1, z_2 \geqslant 0. \tag{10}$$

Proof: See Appendix A. ■

Based on the conclusion in Lemma 1, we can present the following theorem about the upper and lower bounds of the system ergodic sum rate.

Theorem *2:* When $N = 2M-1$, the system ergodic sum rate R for the proposed protocol can be bounded as

$$\frac{M}{\ln 2}\left[e^{\frac{1}{2\rho}}Ei\left(-\frac{1}{2\rho}\right)-e^{\frac{1}{\rho}}Ei\left(-\frac{1}{\rho}\right)\right] < R < \frac{2M}{\ln 2}\left[e^{\frac{1}{2\rho}}Ei\left(-\frac{1}{2\rho}\right)-e^{\frac{1}{\rho}}Ei\left(-\frac{1}{\rho}\right)\right], \tag{11}$$

where $Ei(\cdot)$ denotes the exponential-integral function.

Proof: See Appendix B. ■

Furthermore, the ergodic sum rate for point-to-point direct transmission can be expressed as

$$R_{DR} = \int_0^\infty \log_2(1+\rho z)\,e^{-z}dz = \frac{e^{\frac{1}{\rho}}}{\ln 2}\left[-Ei\left(-\frac{1}{\rho}\right)\right]. \tag{12}$$

Thus the difference between the ergodic sum rates of two protocols can be bounded as

$$R - MR_{DR} > \frac{Me^{\frac{1}{2\rho}}}{\ln 2}Ei\left(-\frac{1}{2\rho}\right) \approx \frac{Me^{\frac{1}{2\rho}}}{\ln 2}[\ln(2\rho)-\mathcal{C}], \tag{13}$$

where $\mathcal{C}$ denotes the Euler's constant. When $\rho > \frac{1}{2}e^{\mathcal{C}}$, we can obtain that

$$R > MR_{DR}, \tag{14}$$

which shows that the proposed protocol can achieve M times larger ergodic sum rate than the Peer-to-Peer direct transmission scheme, if the SNR is high enough. Taking dual-hop transmission scenario and the existence of inter-pair interference into account, the capacity gain for the proposed protocol could be significant.

B. Outage Probability Analysis

In this part, we focus on the outage probability for a single user since the inter-pair interference is eliminated. The amplify forward strategy is employed at the relay, thus the outage probability P_i for user u_i can be defined as the probability that the data rate of dual-hop channel is less than the targeted data rate, which can be shown as

$$P_i = P\{R_i < R_t\}, \tag{15}$$

where R_t is the data rate. From the definition of data rate in (9), P_i can be further derived as

$$P_i = P\left\{\frac{\rho^2 z_1 z_2}{2\rho z_1+\rho z_2+1} < 2^\gamma\right\}, \tag{16}$$

where $\gamma = 2^{2R_t} - 1$. By using the PDF of z_i proposed in Lemma 1, we can have the following theorem about outage probability of the proposed protocol.

Theorem *3:* When $N = 2M - 1$, the outage probability for the user u_i achieved by the proposed protocol can be shown as

$$P_i = (1-e^{-\frac{\gamma}{\rho}})\Gamma(2,\frac{2\gamma}{\rho}) + (\frac{2\rho\gamma^2+1}{4\rho\gamma})^2\sqrt{\frac{2\gamma}{\rho}}e^{\frac{2\gamma}{\rho}} W_{-\frac{3}{2},-1}(\frac{2\gamma}{\rho}) + \frac{2\rho\gamma^3+1}{4\rho^3\gamma}e^{-\frac{2\gamma}{\rho}}W_{-1,-\frac{1}{2}}(\frac{2\gamma}{\rho}), \tag{17}$$

where $\Gamma(a, x)$ is the incomplete Gamma function, and $W_{\lambda,\mu}(z)$ is the Whittaker function. Furthermore, the approximation of outage probability for high SNR can be given as

$$P_i = 1 - e^{-\frac{\gamma}{\rho}} \approx \frac{\gamma}{\rho}. \tag{18}$$

Proof: See Appendix C. ■

As shown in Theorem 3, the diversity gain for a single-user in the proposed protocol is 1, which is the same as the Peer-to-Peer direct transmission scheme. In other words, each user can communicate with its partner as if there is no other pairs transmitting at the same time. Therefore such property demonstrates that the proposed protocol is reliable for multi-pair information exchanging.

IV. Extension of the Proposed Scheme When $N > 2M - 1$

Previously we focused on the case with $N = 2M-1$, where $\mathbf{Q}_i$ can be viewed as an orthogonal projection matrix of the $N \times 2M$ matrix $\mathbf{P}_i$. As shown in the proof of Lemma 1, $\mathbf{Q}_i$ is equal to $\mathbf{q}_i\mathbf{q}_i^H$, and it is easy to show that the dimension of the null space of $\mathbf{P}_i$ is 1 and $\mathbf{q}_i$ is from the null space. When the number of relay antennas $N > 2M - 1$, the dimension of the null space of $\mathbf{P}_i$ becomes $(N - 2M + 2)$. By using the basis vectors from such a subspace, denoted as $\mathbf{q}_{i,k}$, $1 \leq k \leq (N - 2M + 2)$, we can construct $(N - 2M + 2)$ projection matrix, denoted as $\mathbf{Q}_{i,k}$

$$\mathbf{Q}_{i,k} = \mathbf{q}_{i,k}\mathbf{q}_{i,k}^H \quad (k = 1, \cdots, N - 2M + 2). \tag{19}$$

When $\mathbf{Q}_{i,k}$ is selected as the precoding matrix, the received SNRs for the i-th pair are

$$SNR_{i,k} = \frac{\rho^2 \left\|\mathbf{h}_i^H \mathbf{Q}_{i,k}\mathbf{h}'_i\right\|_2^2}{2\rho \left\|\mathbf{Q}_{i,k}\mathbf{h}_i\right\|_2^2 + \rho \left\|\mathbf{Q}_{i,k}\mathbf{h}'_i\right\|_2^2 + 1},$$
$$SNR_{i',k} = \frac{\rho^2 \left\|\mathbf{h}_i^H \mathbf{Q}_{i,k}\mathbf{h}'_i\right\|_2^2}{\rho \left\|\mathbf{Q}_{i,k}\mathbf{h}_i\right\|_2^2 + 2\rho \left\|\mathbf{Q}_{i,k}\mathbf{h}'_i\right\|_2^2 + 1}. \tag{20}$$

To improve the system transmission performance, it is necessary to chose an appropriate precoding matrix $\tilde{\mathbf{Q}}_i$ among $\mathbf{Q}_{i,k}$ for the i-th pair. Since the system performance is largely impacted by the worst user's performance of each pair, the precoding matrix can be selected by using the following rule

$$\tilde{\mathbf{Q}}_i = \arg \max_{\tilde{\mathbf{Q}}_i \in \Sigma} \min \{SNR_{i,k}, SNR_{i',k}\},$$
$$k \in \{1, \cdots, N - 2M + 2\}, \tag{21}$$

where $\Sigma = \{\mathbf{Q}_{i,1}, \cdots, \mathbf{Q}_{i,N-2M+2}\}$ is the set of precoding matrices. We can have the following corollary for the outage performance of such a scheme.

Corollary *4:* The use of the selection criterion in (21) can ensure that the diversity gain $(N - 2M + 2)$ is achievable for all users.

Proof: See Appendix D. ■

Such a result shows that the extra diversity gain can be achieved by increasing the number of user antennas, and the proposed selection approach is optimal in term of diversity gain.

V. Numerical Results

In this section, the performance of the proposed transmission protocol is evaluated by using Monte-Carlo simulations. The comparable scheme is the physical layer network coding (PNC) based on time sharing. The reason to use PNC as the comparable scheme is that the scenario with $N = 2M - 1$ is addressed, to which beamforming is not applicable. Regarding to the application of PNC to the addressed scenario, the time-sharing approach is used, where we first randomly choose a pair and PNC is applied to realize two-way relaying within the chosen pair. Since the proposed protocol is based on the AF strategy, AF has also been applied to PNC.

In Fig. 2 and Fig. 3, the ergodic sum rate and the outage probability of the proposed scheme are shown as functions of

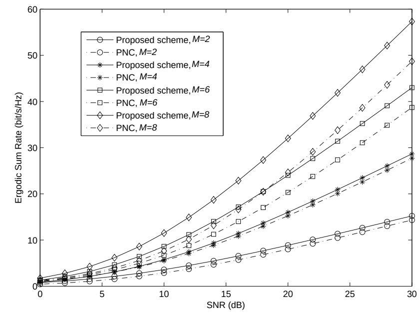

Fig. 2. Ergodic sum rate vs SNR for the proposed scheme and PNC. $M = 2, 4, 6, 8$ and $N = 2M - 1$.

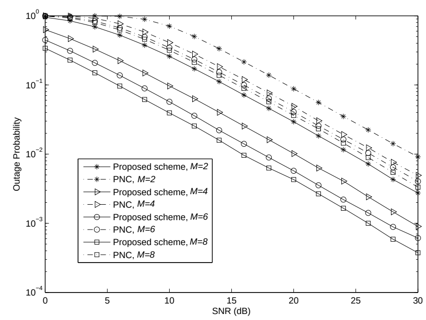

Fig. 3. Outage probability vs SNR for the proposed scheme and PNC. $M = 2, 4, 6, 8$, $N = 2M - 1$ and $R_t = 2$ bit/s/Hz.

SNR, where the number of user pairs is set as $M = 2, 4, 6, 8$ and the targeted date rate is set as $R_t = 2$ bit/s/Hz. As shown in the figures, the proposed scheme can achieve higher ergodic sum rate than the PNC scheme. In addition, the slopes of curves for the ergodic sum rates increase faster than the comparable scheme as M increased, which means that the gap between the two schemes can be further enlarged. Moreover, the difference of the outage probability performance between the two schemes widens as M increases. Such a significant gain of transmission performance can be explained as following. When the number of user pairs is increased, the time sharing based approach requires more time slots to accomplish information exchange among the multiple pairs, but the proposed transmission protocol still requires only two time slots. As a result, the proposed scheme can achieve higher throughput gain and more robust performance than PNC scheme when M increases.

The proposed scheme is also compared with the traditional beamforming. Note that beamforming is used as a comparable scheme for the scenario with $N > 2M - 1$, and not applicable for the case of $N \leqslant 2M - 1$. Specifically the traditional zero-forcing (ZF) beamforming scheme is modified for such a scenario with $N = 2M$, and requires two time slots for M user pairs as well. Assuming all source-relay channel information is available for the relay, the precoding matrix for the ZF beamforming scheme can be given as $\mathbf{W} = (\mathbf{H}_2^H)^{-1}\mathbf{H}_1^{-1}$, where $\mathbf{H}_1 = [\mathbf{h}_1 \cdots \mathbf{h}_M \ \mathbf{h}'_1 \cdots \mathbf{h}'_M]$ and $\mathbf{H}_2 = [\mathbf{h}'_1 \cdots \mathbf{h}'_M \ \mathbf{h}_1 \cdots \mathbf{h}_M]$, and $\mathbf{h}_i$ is the $N \times 1$ channel vector between the user u_i and the relay, while $\mathbf{h}'_i$ is defined

$$M\iint_{z_1,z_2}\left[\log_2(\rho z_1+1)+\log_2(\rho z_2+1)-\log_2(2\rho z_1+\rho z_2+1)\right]e^{-(z_1+z_2)}dz_1dz_2<R$$
$$<M\iint_{z_1,z_2}\left[\log_2(2\rho z_1+1)+\log_2(\rho z_2+1)-\log_2(2\rho z_1+\rho z_2+1)\right]e^{-(z_1+z_2)}dz_1dz_2. \tag{26}$$

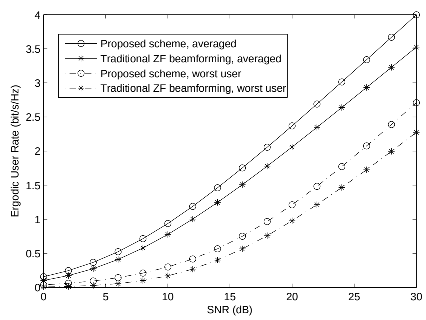

Fig. 4. Averaged ergodic user rate and ergodic user rate for the worst user, $M=4$, $N=2M$.

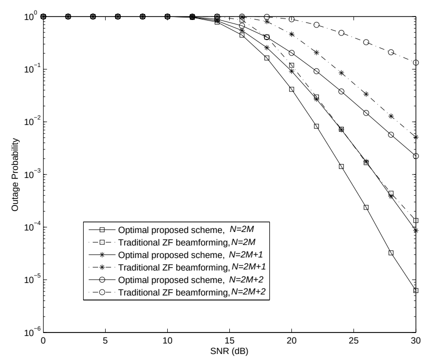

Fig. 5. Outage probability vs SNR for the proposed scheme and traditional ZF beamforming with different settings of relay antennas, $M=4$ and $R_t=5$ bit/s/Hz.

similarly for the user u_i'.

The averaged and the worst ergodic user rates are shown as the functions of SNR in Fig. 4. The number of user pairs is $M=4$, and the number of relay antennas is $N=2M$. As can be observed from the figure, the proposed protocol can achieve larger averaged ergodic user rate than traditional ZF beamforming, and a similar result is also provided for the user with the worst performance. In Fig. 5, the optimal proposed scheme is compared with traditional ZF beamforming, where the number of user pairs is $M=4$, the number of relay antennas is $N=2M,\ 2M+1,\ 2M+2$, and the targeted data rate is $R_t=5$ bit/s/Hz. As shown in the figure, the diversity gain of optimal proposed scheme grows larger as the number of relay antennas increase. Moreover, the optimal scheme always achieves better robust performance than the comparable scheme.

VI. Conclusion

In this paper, we proposed a network coding protocol for multi-pair two-way relay channels, and the precoding matrices have been carefully designed to remove inter-pair interference in the system. the proposed scheme is still efficient for a special case where beamforming is not applicable. Analytical results, such as ergodic sum rate and outage probability, have been developed for the proposed transmission protocol in a specific case. The performance comparisons with PNC and ZF beamforming are also provided respectively. Both the analytical and numerical results show that the proposed protocol is spectral efficient, and can achieve higher performance gains. To enlarge the diversity gain of the proposed scheme, the optimization is also studied, and evaluated by simulation.

Appendix A
Proof of Lemma 1

When $N=2M-1$, the dimension of non-null space for the precoding matrix $\mathbf{Q}_i$ is 1. Since the precoding matrix is an idempotent matrix, i.e. $\mathbf{Q}_i\mathbf{Q}_i=\mathbf{Q}_i$, its only non-zero eigenvalue is one. Therefore, the eigenvalue decomposition of $\mathbf{Q}_i$ can be shown as $\mathbf{Q}_i=\mathbf{q}_i\mathbf{q}_i^H$, where $\mathbf{q}_i$ is the eigenvector of the matrix corresponding to the eigenvalue 1. Then we rewrite the expression of $\|\mathbf{Q}_i\mathbf{h}_i\|_2^2$ as

$$\|\mathbf{Q}_i\mathbf{h}_i\|_2^2=(\mathbf{Q}_i\mathbf{h}_i)^H(\mathbf{Q}_i\mathbf{h}_i)=\mathbf{h}_i^H\mathbf{Q}_i\mathbf{h}_i=|\mathbf{h}_i^H\mathbf{q}_i|^2, \tag{22}$$

and $\|\mathbf{Q}_i\mathbf{h}_i'\|_2^2$ can be derived as a similar expression. By using the fact that unitary transform does not change the statistical property of Gaussian matrices, it can be shown that $\mathbf{h}_i^H\mathbf{q}_i$ and $\mathbf{h}_{i'}^H\mathbf{q}_{i'}$ are still identically and independent complex Gaussian distributed, from which the lemma can be easily proved.

Appendix B
Proof of Theorem 2

Since the messages from each pair are aligned to different directions which are orthogonal to each other, there is no inter-pair interference during the transmission process. Hence the system ergodic sum rate R can be treated as the sum of $2M$ channel ergodic sum rates from the relay to users, which can be shown as

$$R=\sum_{i=1}^{M}R_i+\sum_{i'=1}^{M}R_{i'}. \tag{23}$$

By using the PDF provided in Lemma 1, the ergodic sum rate R_i for user $u_{i'}$ can be given as

$$R_i=\frac{1}{2}\iint_{z_1,z_2}\log_2\Big(1+\frac{\rho^2z_1z_2}{2\rho z_1+\rho z_2+1}\Big)e^{-(z_1+z_2)}dz_1dz_2. \tag{24}$$

$$
\begin{aligned}
P_i &= P\left(z_2 > \frac{2\gamma_i}{\rho}\right) P\left(0 < z_1 < \frac{\rho\gamma_i z_2 + 1}{\rho^2 z_2 - 2\rho\gamma_i}\right) + P\left(0 < z_2 < \frac{2\gamma_i}{\rho}\right) P\left(z_1 > \frac{\rho\gamma_i z_2 + 1}{\rho^2 z_2 - 2\rho\gamma_i}\right) \\
&= \int_{\frac{2\gamma_i}{\rho}}^{\infty} z_2 e^{-z_2}\left(1 - e^{-\frac{\rho\gamma_i z_2 + 1}{(\rho^2 z_2 - 2\rho\gamma_i)}}\right) dz_2 + \int_0^{\frac{2\gamma_i}{\rho}} z_2 e^{-z_2} e^{-\frac{\rho\gamma_i z_2 + 1}{v(\rho^2 z_2 - 2\rho\gamma_i)}} dz_2 \\
&= (1 - e^{-\frac{\gamma}{\rho}})\Gamma(2, \frac{2\gamma}{\rho}) + (\frac{2\rho\gamma^2 + 1}{4\rho\gamma})^2 \sqrt{\frac{2\gamma}{\rho}} e^{\frac{2\gamma}{\rho}} W_{-\frac{3}{2},-1}(\frac{2\gamma}{\rho}) + \frac{2\rho\gamma^3 + 1}{4\rho^3\gamma} e^{-\frac{2\gamma}{\rho}} W_{-1,-\frac{1}{2}}(\frac{2\gamma}{\rho}). \qquad (28)
\end{aligned}
$$

Considering the receive SNR at each user is independent and identically distributed, the system ergodic sum rate can be expressed exactly as

$$
\begin{aligned}
R = M \iint_{z_1, z_2} \big[&\log_2(2\rho z_1 + \rho z_2 + \rho^2 z_1 z_2 + 1) \\
&- \log_2(2\rho z_1 + \rho z_2 + 1)\big] e^{-(z_1+z_2)} dz_1 dz_2. \qquad (25)
\end{aligned}
$$

Then we can derive (26), and thus the system ergodic sum rate can be bounded as

$$
\begin{aligned}
\frac{M}{\ln 2}&\left[e^{\frac{1}{2\rho}} Ei\left(-\frac{1}{2\rho}\right) - e^{\frac{1}{\rho}} Ei\left(-\frac{1}{\rho}\right)\right] < R < \\
&\frac{2M}{\ln 2}\left[e^{\frac{1}{2\rho}} Ei\left(-\frac{1}{2\rho}\right) - e^{\frac{1}{\rho}} Ei\left(-\frac{1}{\rho}\right)\right], \qquad (27)
\end{aligned}
$$

and the theorem is proved.

Appendix C
Proof of Theorem 3

Recall the expression of receive SNR in Lemma 1, the outage probability P_i equals to the probability of the occurrence of the events in (28), which is based on the total probability theorem.

As shown in (28), for high SNR, we have $\frac{\gamma}{\rho} \to 0$, and $\Gamma(2, \frac{2\gamma}{\rho}) \to 1$. Recalling the expression of $W_{\lambda,\mu}(\frac{2\gamma}{\rho})$,

$$
W_{\lambda,\mu}(\frac{2\gamma}{\rho}) = e^{-\frac{\gamma}{\rho}} \left(\frac{2\gamma}{\rho}\right)^{\frac{\mu+1}{2}} U\left(\frac{1}{2} + \mu - \lambda, 1 + 2\mu; \frac{2\gamma}{\rho}\right). \qquad (29)
$$

When $\frac{\gamma}{\rho} \to 0$, it can be easily obtained that $e^{-\frac{\gamma}{\rho}} \to 1$, and $\left(\frac{2\gamma}{\rho}\right)^{\frac{\mu+1}{2}} \to 0$. Based on the Maclaurin series for the Confluent Hypergeometric Function of the Second Kind, $U\left(\frac{1}{2} + \mu - \lambda, 1 + 2\mu; \frac{2\gamma}{\rho}\right)$ goes to a finite constant when $\frac{2\gamma}{\rho}$ goes to 0. Then we can derive that $W_{\lambda,\mu}(\frac{2\gamma}{\rho}) \to 0$ when $\frac{2\gamma}{\rho}$ goes to 0. Thus the following approximation can be derived as

$$
P_i = 1 - e^{-\frac{\gamma}{\rho}} \approx \frac{\gamma}{\rho}. \qquad (30)
$$

And the proof for the expression of outage probability P_i is completed.

Appendix D
Proof of Corollary 4

The corollary can be easily proved by showing that the worst user for each pair can have the diversity order $(N - 2M + 2)$. Define $P(i)_{k,worst}$ as the worst outage probability when $\mathbf{Q}_{i,k}$ is the precoding matrix for the i-th pair. The outage probability of the worst user for such a selection approach can be derived as

$$
\begin{aligned}
P(i)_{worst} &= \min\{P(i)_{1,worst}, \cdots, P(i)_{k,worst}\} \\
&= \left[P(i)_{1,worst}\right]^{N-2M+2}, \qquad (31)
\end{aligned}
$$

where the last equation follows from the fact that unitary transform does not change the statistical property of a Gaussian matrix and therefore $SNR_{i,k}$ is independent to $SNR_{i,m}$ for $k \neq m$. Furthermore $P(i)_{k,worst}$ can be bounded as

$$
P(i)_{k,worst} \leqslant P(i)_{k,u} + P(i)_{k,u'} \approx \left(\frac{2\gamma}{\rho}\right), \qquad (32)
$$

where $P(i)_{k,u}$ and $P(i)_{k,u'}$ are the outage probabilities for the i-th pair when $\mathbf{Q}_{i,k}$ is used, and the approximation is from Theorem 3. Combining the above two equations, the corollary is proved.

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