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A new extension of seismic instantaneous frequency using a fractional time derivative

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A B S T R A C T

The seismic instantaneous frequency attribute has been applied to interpret a depositional layer. Instead of the standard instantaneous frequency, the Caputo fractional differential operator has been demonstrated as a more suitable mathematical tool of nonlinear analysis for the depositional layer. Based on the Caputo operator, we propose a fractional extension of instantaneous frequency attribute to detect thin layers of sandstone formations. From numerical analysis of a 25-Hz Ricker wavelet, we found that the fractional instantaneous frequency of 0.99 order keeps an approximation of standard instantaneous frequency and enhances the negative frequency (anomalous frequency spike), which help in more precise geological interpretation. A three-layer wedge model and a case history from the Bohai Bay Basin showed that the 0.99 order instantaneous frequency provides more depositional sandstone information than the standard instantaneous frequency. The anomalous frequency spike, at the merging point of the top and the base reflections or at the tip of the stratum, of 0.99 order instantaneous frequency is more prominent than that of the standard one. The results help to interpret the thickness changes of thin sandstone formations. In practical application, the 0.99 order instantaneous frequency has revealed several deposition phases and lateral heterogeneity of a fluvial reservoir system. Therefore, the 0.99 order instantaneous frequency is a valuable seismic attribute for reservoir characterization.

1. Introduction

Complex seismic trace attributes are the most popular post-stack attributes, which derived from complex-trace analysis (Taner and Sheriff, 1977; Taner et al., 1979). Today, the family of complex seismic trace attributes encompasses a large and varied set of trace and volume attributes, such as instantaneous amplitude, instantaneous phase, instantaneous frequency, relative amplitude, wavelength, dip, and azimuth (Barnes, 2007).

Instantaneous frequency is one of the basic instantaneous attributes which have been used successfully in reservoir characterization (Chopra and Marfurt, 2005; Iturrarán-Viveros, 2012; Rezvandehy et al., 2011; Taner, 2001) and the estimation of quality factor Q (Gao et al., 2011; Matheney and Nowack, 1995; Tonn, 1991). The negative value for instantaneous frequency (anomalous frequency spike) is an interesting enigma in complex seismic trace analysis. In short, frequency anomalies largely correspond to waveform distortion caused by wavelet interference at the merging point of the top and the base reflections or at the tip of the stratum (Hardage et al., 1998; Robertson and Fisher, 1988; Taner et al., 1979; White, 1991). White (1991) offers a simplest explanation that complex signal is a generalization of the phasor representation of simple harmonic motion to signals with time variant amplitudes and periods. During the past thirty years, anomalous frequency spikes have been applied to reservoir characterization (Hardage et al., 1998; Robertson and Nogami, 1984; Rodovich and Oliveros, 1998; Zeng, 2010). Zeng (2010) demonstrated the geologic significance by interpreting anomalous instantaneous frequency.

If $x(t)$ is a seismic trace and $y(t)$ is the Hilbert transform of $x(t)$, then the instantaneous frequency is defined as (Taner et al., 1979)

$$f(t) = \frac{1}{2\pi}\frac{x(t)\frac{dy(t)}{dt} - y(t)\frac{dx(t)}{dt}}{x^2(t) + y^2(t)} \quad (1)$$

which has units of hertz. $f(t)$ is the standard instantaneous frequency (SIF) in order to distinguish from the fractional extension. The key operation of this computation is the first order differential operator. Mainardi (1996) has pointed out that the appearance of time fractional derivatives is expected in wave phenomena. Consequently, if a non-integer order replaces the first order in the differential operator, it is possible that fractional instantaneous frequency shows more depositional information, like sandstone change, in complex layer.

During the decade, the fractional calculus is one of the most intensively developing areas of mathematical analysis, which is the theory of integrals and derivatives of arbitrary order (Podlubny, 1999). One of the main advantages of fractional calculus is that the fractional

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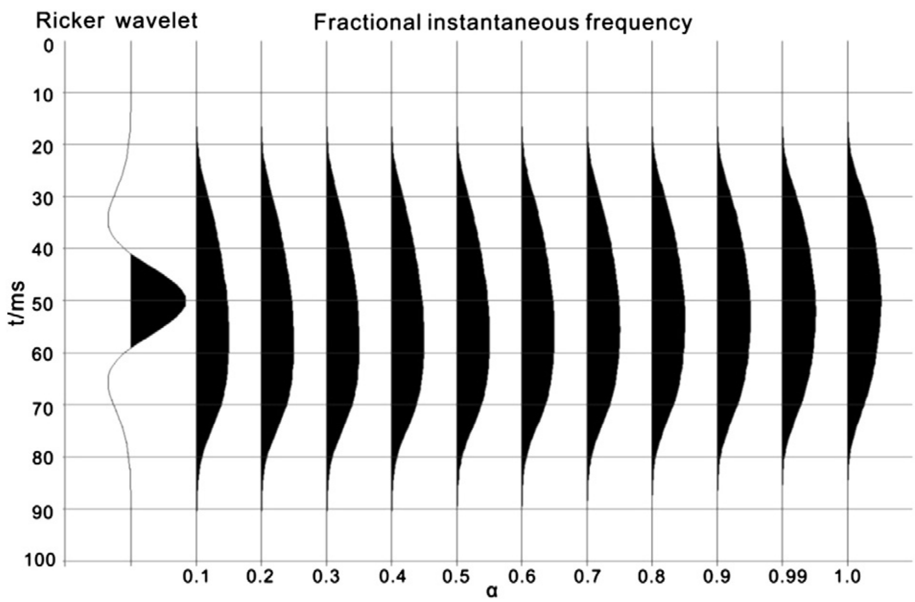

Fig. 1. Different FIF results of 25-Hz Ricker wavelet. Simply speaking, the shapes between orders of 0.99 and 1 are most similar.

derivatives provide an excellent approach for the description of memory and hereditary properties of various materials and processes (Hilfer, 2000), which have been used in various nonlinear fields (Xu and Tan, 2006). The subsurface depositional layer can be treated as a complex nonlinear system (Harms and Tackenberg, 1972; Herrman et al., 2001). To be specific, fractional derivatives appear in constant-Q model and Biot theory, related to the memory effects in viscoelastic fluid and porous rocks at seismic frequencies (Gurevich and Lopatnikov, 1995; Lu and Hanyga, 2005; Mainardi, 1996; Mainardi and Tomirotti, 1997). Considering the memory effects of wave propagation in complex nonlinear layer, we propose to replace the differential operator with the Caputo fractional differential operator as an extension of instantaneous frequency to help characterize depositional model. Unlike the standard differential operator, the fractional operator increases in length as time increases, because it must keep the memory effects (Carcione, 2009). The Caputo fractional differential operator is one of the fractional derivative concepts developed by Caputo (1967), which has been widely applied in both engineering and computation fields.

In this paper, we first involve in the derivation of fractional instantaneous frequency based on the Caputo derivative, and then discuss the geological significance of its anomalous frequency spikes for their interpretive value as indicators of sandstone thickness change.

2. Method

2.1. Fractional differential operator

Suppose that $\forall\ \alpha > 0, t > a, \alpha, s, t \in R$, the Caputo differential operator of fractional differentiation of order α is defined as (Caputo, 1967)

$$ {}_a^C D_t^{\alpha a} z(t) := \begin{cases} \dfrac{1}{\Gamma(n-\alpha)} \displaystyle\int_a^t (t-\tau)^{n-\alpha-1} z^{(n)}(\tau)\,d\tau & n-1<\alpha<n\in N \\ \dfrac{d^n}{dt^n} z(t) & \alpha = n \in N, \end{cases} \tag{2} $$

where $z^{(n)}(\tau)$ denotes the derivative of order n, and $\Gamma(x)$ is a Gamma function. ${}_a^C D_t^{\alpha}$ denotes the Caputo differential operator, which can be abbreviated as D^{α}. Superscript C means that the operator is introduced by the Italian mathematician Caputo (1967). Superscript α denotes the fractional differentiation of order α. Subscripts a and t denote the lower and the upper bounds of the integration. In a causal system, a is an initial time. For simplicity we consider the model case $a = 0$ below.

2.2. Fractional instantaneous frequency

Joining Eq. (1) and Eq. (2), we define the fractional extension of instantaneous frequency (FIF) as

$$ f_{\alpha}(t) := \begin{cases} \dfrac{1}{2\pi} \dfrac{x(t) D^{\alpha} y(t) - y(t) D^{\alpha} x(t)}{e^2(t) + \varepsilon^2 e_{\max}^2} & 0<\alpha<1, \quad 0<\varepsilon<1 \\ \dfrac{1}{2\pi} \dfrac{x(t)\dfrac{dy(t)}{dt} - y(t)\dfrac{dx(t)}{dt}}{e^2(t) + \varepsilon^2 e_{\max}^2} & \alpha = n = 1, \\ e^2(t) = x^2(t) + y^2(t) & \\ e_{\max}^2 = \max\left[e^2(t)\right] & \end{cases} \tag{3} $$

which also has units of hertz. Suffix α denotes the fractional instantaneous frequency of order α, D^{α} denotes the Caputo differential operator. $\varepsilon^2 e_{\max}^2$ denotes a small damping factor to reduce large spikes of noise (Gao et al., 1999; Matheney and Nowack, 1995). The damping factor ε is set to be 0.05 in this study.

2.3. Numerical scheme

Let $0 < \alpha < 1$, τ denotes a time step, f_n denotes the n-layer function $f(t_n)$, $t_n = n\tau$, then

$$ \begin{aligned} D^{\alpha} z(t) &= \frac{1}{\Gamma(1-\alpha)} \int_0^{t_n} \frac{z'(\tau)}{(t_n-\tau)^{\alpha}} d\tau = \frac{1}{\Gamma(1-\alpha)} \sum_{k=1}^{n} \int_{t_{k-1}}^{t_k} \frac{z'(\tau)}{(t_n-\tau)^{\alpha}} d\tau \\ &= \frac{1}{\Gamma(1-\alpha)} \sum_{k=1}^{n} \frac{z_k - z_{k-1}}{\tau} \int_{t_{k-1}}^{t_k} \frac{1}{(t_n-\tau)^{\alpha}} d\tau \\ &= \frac{1}{\Gamma(1-\alpha)} \sum_{k=1}^{n} \frac{z_k - z_{k-1}}{\tau} \frac{1}{1-\alpha} \tau^{1-\alpha} \left[(n-k+1)^{1-\alpha} - (n-k)^{1-\alpha} \right], \end{aligned} \tag{4} $$

where $z'(\tau)$ is the first derivative, and the approximation for the first derivative is the backward difference scheme.

Table 1
The peak frequency value in different FIF of 25-Hz Ricker wavelet.

Order	Time(ms)	The peak frequency (Hz)
0.1	60	0.0668
0.2	61	0.1832
0.3	61	0.4154
0.4	61	0.8536
0.5	60	1.6495
0.6	59	3.0502
0.7	58	5.4548
0.8	57	9.4945
0.9	55	16.1575
0.99	54	25.6278
0.999	54	26.8122
0.9999	54	26.9333
1	50	26.6580

3. Application to models

3.1. Ricker wavelet

In practical applications, we should select a fractional order matching seismic data. Therefore, we first analyzed the fractional instantaneous frequency of 25-Hz Ricker wavelet to understand the variations of the non-integer order operators. Here the 25-Hz Ricker wavelet is set to 101 time samples (101 ms). To compare instantaneous frequencies of different orders, Fig. 1 shows eleven fractional orders of 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.99, and 1. Simply speaking, the shapes between orders of 0.99 and 1 are most similar. After numerical analysis, Table 1 shows that the value of the peak frequency increases as non-integer order increases, and the position of the peak frequency shifts from 50 ms to 60 ms. In functional analysis, the first order is a demarcation

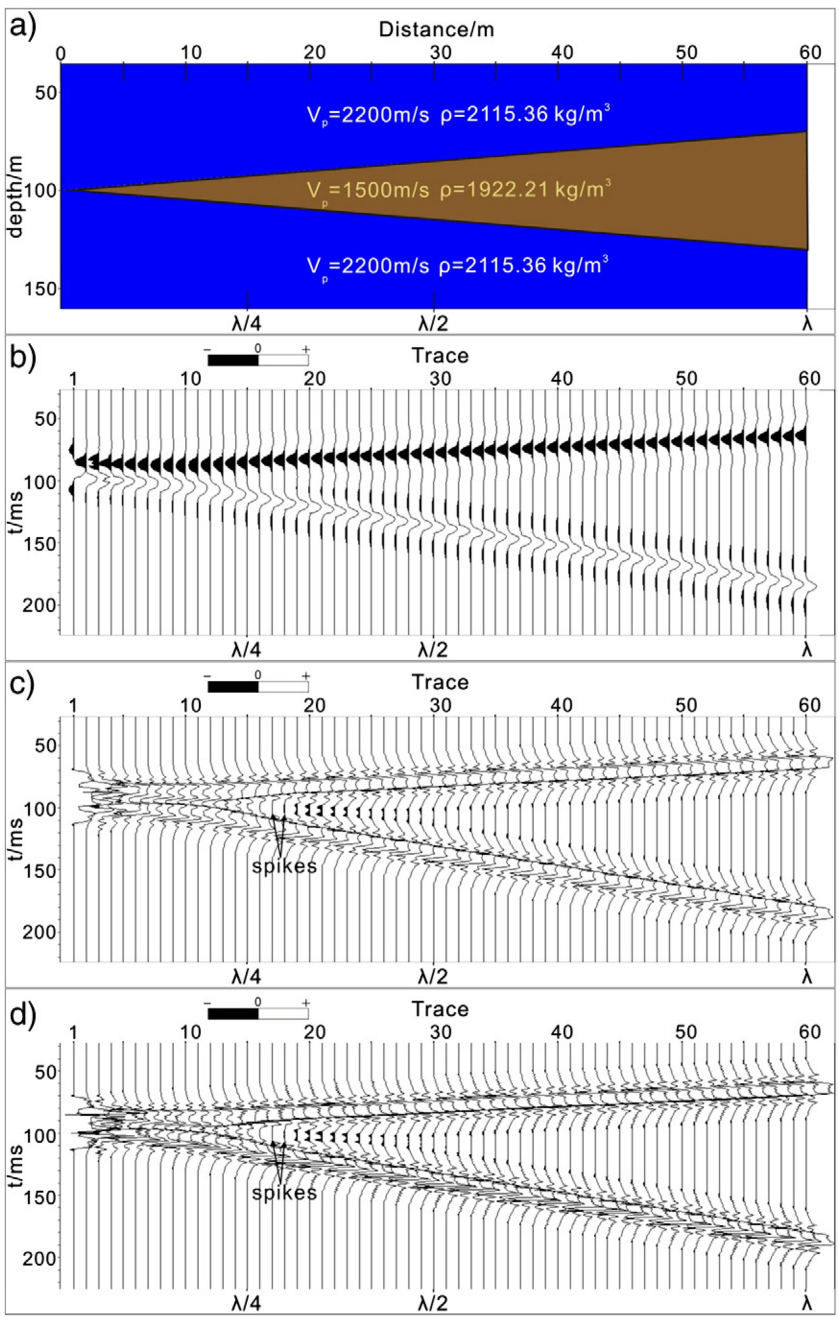

Fig. 2. The spikes of the FIF in three wedge model. (a) A geological model; (b)synthetic seismic section using a 25 Hz Ricker wavelet; (c) the SIF section; (d) the 0.99 order FIF section.

point of a functional (Podlubny, 1999). For example, the p-norm is not differentiable while $p < 1$, but is continuously differentiable while $p > 1$. In other words, the peak frequency of SIF (order of 1) is independent, and the peak frequencies of FIF (order of 0.1–0.9999) reveal a continuous increase trend. To keep physical meanings of instantaneous frequency, both the value and position of FIF should be almost the same as the SIF. In Table 1, the order of 0.99, 0.999 and 0.9999 are three possible options. Note that the peak frequency of 0.99 order FIF is smaller than SIF, and both the peak frequencies of 0.999 and 0.9999 order FIF are larger than SIF. Because the peak frequency of SIF is a breaking point, and highlighting of the negative frequency is of interest in this study, the order of 0.99 is better than the order of 0.999 or 0.999. Based on the above analysis, we recommended that fractional order is 0.99 in this study.

3.2. Synthetic seismogram

Robertson and Nogami (1984) used a wedge model to highlight the interpretive significance of the instantaneous frequency attribute. A similar wedge model is presented in Fig. 2a. The thinning layer has low acoustic impedance (a sandstone formation in shallow Bohai Bay) with equal but opposite sign of reflection coefficients at the top and the base of the layer. A 25-Hz peak frequency Ricker wavelet is convolved with the model to generate noise-free synthetic seismograms (Fig. 2b).

Zeng (2010) showed that anomalous instantaneous frequency spikes, at the merging point of the top and the base reflections of a wedge or at the tip of the wedge, were resulted from certain bed-thickness ranges around the tuning thickness. Both for the standard and the 0.99 order instantaneous frequency section (Fig. 2c and d), some spikes appear at the center of a seismically thick bed where the top and base reflections start to merge. To show the detail of the spikes, traces 17 and 18 were zoomed in. Fig. 3 shows that the spikes are pronounced in the 0.99 order section than in the standard section. The potential reason is keeping the memory effects of wave propagation in complex nonlinear layer that directly requires the fractional operator increases in length as time increases (Carcione, 2009). Based on this comparison, the 0.99 order instantaneous frequency reveals more potential to detect the variation of sandstone thickness.

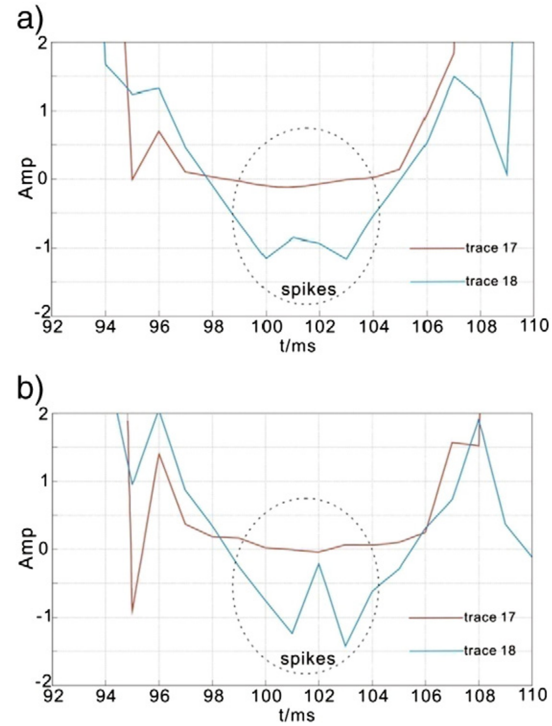

Fig. 3. The detail of (a) the standard and (b) the 0.99 order anomalous instantaneous frequency spikes in traces 17 and 18.

4. Case history

The reservoirs in the KL10-1 oilfield are dominated by fluvial channels and shallow-water deltaic systems, where multi-phase channels are stacked together with variable thicknesses of sandstones (Yu et al., 2009). Fig. 4a depicts a high-quality seismic section over the KL10-1 oilfield, which was acquired in 2008. The signal-to-noise ratio is high, with a vertical resolution of about 12.5 m in the shallow interval of interest. Horizons T1 and T3 represent the bottom and the top of the Minghuazhen Formation. Horizon T2 is the top of the Lower Minghuazhen Formation. Four shifted horizons were created by adding isochore of 16 ms, 38 ms, 48 ms, and 72 ms to the T2 horizon, which represent four fluvial depositional phases (Wang et al., 2012). In Fig. 4a, a red ellipse encloses an interval of 1000 m lentoid channel sandstone (referred to as "sandstone A"), which belongs to the T2 + 38 ms phase. Fig. 4b and c shows the standard and the 0.99 order instantaneous frequency, respectively. The initial and final times of integration in the fractional derivatives are top and bottom times of the Minghuazhen Formation, in order to ensure the causality of the same layer.

In Fig. 4, we marked 9 spikes (S1–S9) to compare the detection effect between the standard and the 0.99 order instantaneous frequency sections. Over all, spikes of thickness variation show more clearly in the 0.99 order instantaneous frequency section than in the standard section detecting the sandstone channel. Spikes S1–S4 are inside of sandstone A. These four spikes of positive and negative frequency reveal the process of channel deposition. A comparison of the amplitude value of those spikes reveals that S3 > S1 > S4 > S2. The spacing between spikes is S1S2 = S2S3 = 350 m, and S3S4 = 250 m (Fig. 5a). In our geological views, these phenomena indicate four changes of thickness in sandstone A (Fig. 5a). Spikes S5–S9 show a smaller negative value in the standard instantaneous frequency section, but a larger negative value in the 0.99 order section. For example, S9 shows that a weak negative value connects with two strong negative values in about 400 m channel thin bed, which indicates lateral heterogeneity (Fig. 5b).

5. Conclusion

By attempting to extend to instantaneous frequency using a fractional time derivative form, it is demonstrated that the fractional instantaneous technique is a potential tool to detect a sandstone channel because of keeping the memory effects of wave propagation in sedimentary layer.

In this study, we propose a new scheme of the fractional instantaneous frequency using the Caputo operator, and show the interpretive value of an anomalous instantaneous frequency to characterize a sandstone channel. From the different non-integer results of the 25-Hz Ricker wavelet, we select a suitable fractional order of instantaneous frequency of 0.99 in this study. The 0.99 order instantaneous frequency keeps an approximation of standard instantaneous frequency and enhances the negative frequencies.

The anomalous instantaneous frequency spikes technique is applied to a wedge model and the case history in Bohai Bay Basin. The 0.99 order instantaneous frequency spikes reveal more details of sandstone formation than the standard instantaneous frequency spikes.

The fractional extension of instantaneous frequency has a great potential of being a significant seismic attribute in future.

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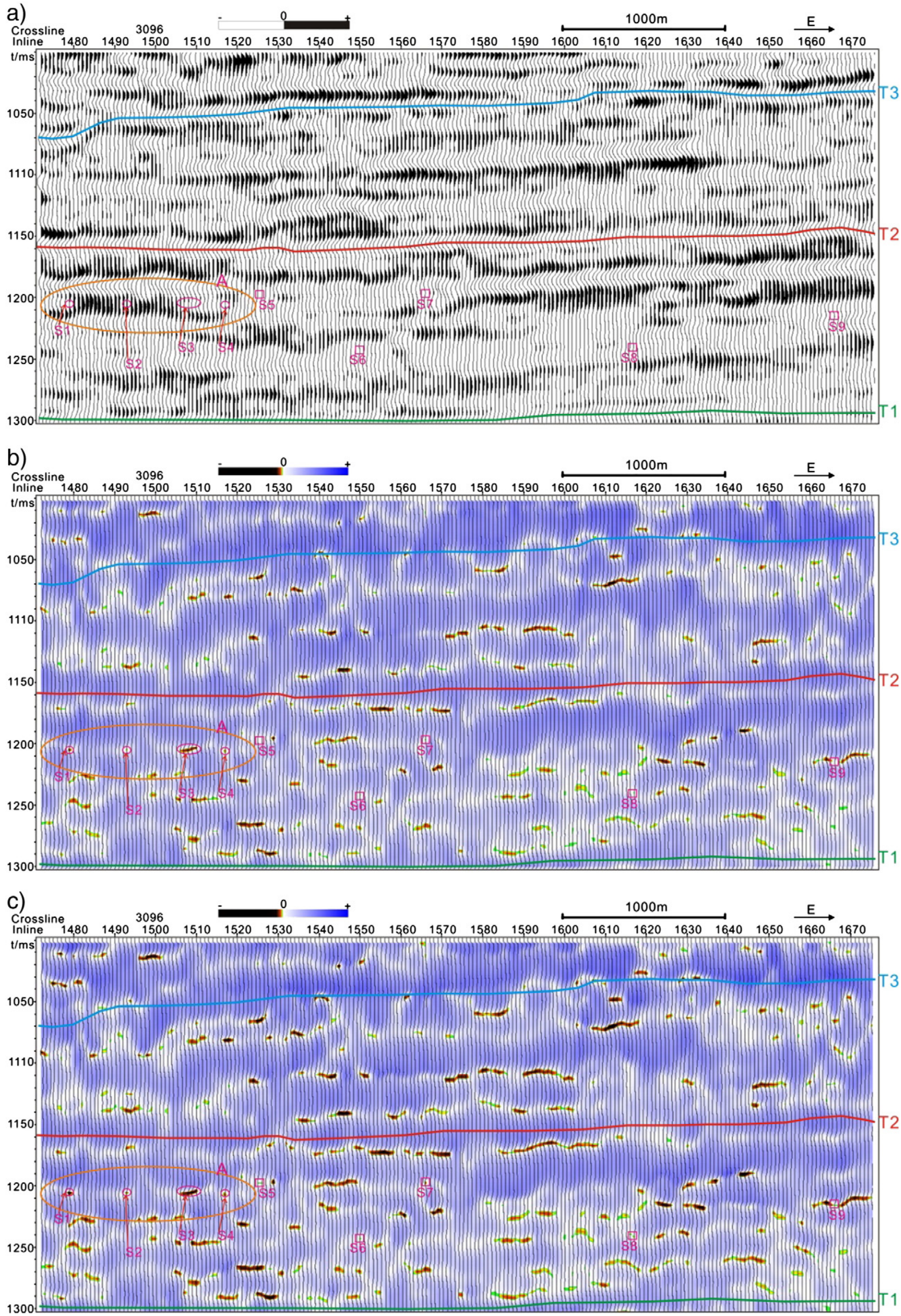
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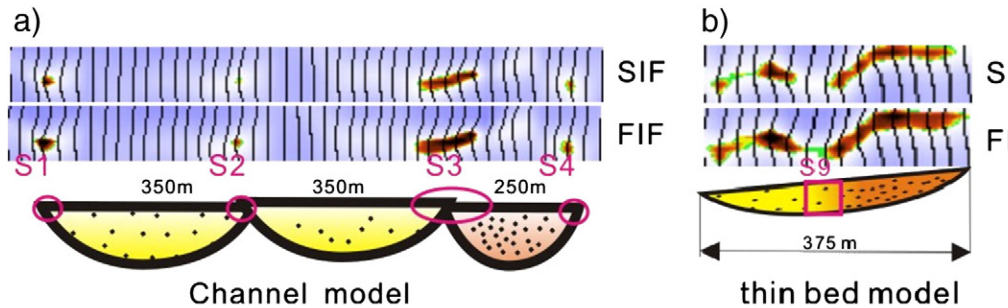

Fig. 5. Geological interpretations of anomalous instantaneous frequency spikes in the SIF and 0.99 order FIF sections. (a) An ideal sedimentary model of sandstone A; (b) an ideal lateral heterogeneity model of the thin bed. Note that the amplitude of spikes in the FIF section is larger than the SIF section.

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Fig. 4. The anomalous instantaneous frequency spikes to detect the thickness changes of channel sandstone. (a) An amplitude section of Minghuazhen Formation in the KL10-1 oilfield, Bohai Bay; (b) anomalous frequency spikes of the SIF section; (c) anomalous frequency spikes of the 0.99 order FIF section.