Engineering Computations

A novel image denoising algorithm in wavelet domain using total variation and grey theory

Hong-jun Li Zhi-min Zhao Xiao-lei Yu

A novel image denoising algorithm in wavelet domain using total variation and grey theory

Hong-jun Li
College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, People's Republic of China
Zhi-min Zhao
College of Science, Nanjing University of Aeronautics and Astronautics, Nanjing, People's Republic of China, and CSIRO Materials Science and Engineering, Highett, Australia, and
Xiao-lei Yu
College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, People's Republic of China

Received 30 August 2009
Revised 12 January 2010
Accepted 14 January 2010

Abstract

Purpose – The traditional total variation (TV) models in wavelet domain use thresholding directly in coefficients selection and show that Gibbs' phenomenon exists. However, the nonzero coefficient index set selected by hard thresholding techniques may not be the best choice to obtain the least oscillatory reconstructions near edges. This paper aims to propose an image denoising method based on TV and grey theory in the wavelet domain to solve the defect of traditional methods.

Design/methodology/approach – In this paper, the authors divide wavelet into two parts: low frequency area and high frequency area; in different areas different methods are used. They apply grey theory in wavelet coefficient selection. The new algorithm gives a new method of wavelet coefficient selection, solves the nonzero coefficients sort, and achieves a good image denoising result while reducing the phenomenon of "Gibbs."

Findings – The results show that the method proposed in this paper can distinguish between the information of image and noise accurately and also reduce the Gibbs artifacts. From the comparisons, the model proposed preserves the important information of the image very well and shows very good performance.

Originality/value – The proposed image denoising model introducing grey relation analysis in the wavelet coefficients selecting and modifying is original. The proposed model provides a viable tool to engineers for processing the image.

Keywords Image processing, Mathematical modelling, Programming and algorithm theory

Paper type Research paper

1. Introduction

Image denoising is an important task for further process of images like segmentation, feature, extraction, texture, analysis, etc. Image denoising refers to suppressing the noise

This work was supported by the National Natural Science Foundation of China (NO.10172043), the Aeronautics Science Foundation of China (NO.05G52047), the Specialized Research Fund for the Doctoral Program of Higher Education of China (NO. 20093218110024), and International Science and Technology Cooperation Grant (NO.BZ2008060).

Engineering Computations: International Journal for Computer-Aided Engineering and Software
Vol. 27 No. 7, 2010
pp. 863-877

0264-4401
DOI 10.1108/02644401011073692

while retaining the edges and other important structures as much as possible. In recent years, the partial different equation (PDE) technique and wavelet transforms have been intensively used in image process. The variation model based on PDE has been applied in image process successfully; it maintains the image edge very well while smoothening the noise. Another method wavelet transform is a novel multi-analysis method; it is an effective tool of image denoising based on the character of focusing the detail of the image.

Total variation (TV) model and wavelet threshold are two kinds of effective methods. In recent years, increasing number of scholars work on the combined area and proposed some relational researches (Xu and Osher, 2005; Chan and Zhou, 2007; Bhoi and Meher, 2008; Feng and Yang, 2006; Starck *et al.*, 2005; Vese and Osher, 2003). Xu and Osher (2005) used the wavelet thresholding procedure on TV model; it brought a good image denoising result with little signals loss while only considered the relationship on the thresholding techniques and modified the thresholding method. Later, Chan and Zhou (2007) proposed a TV wavelet thresholding procedure; this method reconstructed images through selecting and modifying the retained wavelet coefficients. It had fewer oscillations near edges while the noise was smoothed. The paper considered to select another index to have the same given number of nonzero coefficients which returns less oscillatory, not to analyze the character of coefficients in location directly. Bhoi and Meher (2008) proposed a method which was combined by wavelet coefficients and the TV model. It selected the nonzero wavelet coefficient index set by standard hard thresholding techniques, and then changes their values so that the reconstructed images have a minimized TV norm. However, the nonzero coefficients index set selected by hard thresholding techniques may not the best choice to obtain the least oscillatory reconstructions near the edges. All this motivates us to select and modify the nonzero wavelet coefficients in the thresholding procedure subject to minimizing the TV norm of restored images.

In this work, we analyze the relationship between the TV and wavelet transform and motivate by the defect of traditional methods. We aim to design a thresholding procedure to select and modify the wavelet coefficients to achieve fewer edge artifacts while retaining sharp edges. So, this paper contains some researches on the wavelet coefficients inter-scale and intra-scale; and selects wavelet coefficients by using the grey relational analysis. It proposes a better thresholding procedure when considering the coefficients relationship both in scale and location. Several numerical experiments demonstrate that the algorithm we proposed is able to access to the superior denoising result and reduce the edge oscillations than traditional methods.

The rest of the paper is organized as follows: Section 2 introduces the basic knowledge of the TV model. In section 3, we introduce the basic knowledge of wavelet transform. In section 4 we present our new algorithm and show its steps. In section 5, we discuss the parameters set in the experiment. In section 6, we discuss the results of the experiment. The conclusions of this paper are presented in section 7.

2. TV model

In recent years, there have been a number of important models and methods about the variational image regularization research issues. The most typical one is the TV regularization model. It has been proposed by Rudin *et al.* (1992) also known as Rudin, Osher, Fatime (ROF) model or TV model, which consists of minimizing the following functional:

$$\min_u E(u) = \int_\Omega |\nabla u| \, dxdy + \frac{\lambda}{2}\int_\Omega (u - u_o)^2 dxdy \tag{1}$$

where $u_0 \in L^2(\Omega)$ is noisy image; $u \in L^2(\Omega), \Omega \subset R^n$ has continuum boundary; ∇u and $|\nabla u|$ are image gradient and the gradient mode, respectively. The first item at right of Equation (1) is the TV norm, which depends on the image range of variation; the second is approximation function, which controls the differences of image u and noised image u_0. There is no specific definition of how to obtain the optimal point of balance. Only through the changes in the Lagrange multiplier λ to adjust the regularizing and approximate parts; when λ tends to zero, the regularizing function is objective function, the image tends to a constant, the image is over-smooth so that the edge is seriously blurred; when λ tends to infinity, the image will be sharp oscillation. Therefore, the choice of λ cannot be too small or too large.

The solution is obtained by finding a steady state solution of a time dependent partial differential equation, which is the evolution of the following Euler-Lagrange equations:

$$\begin{cases} u(x,y,t) = div\left(\dfrac{\nabla u}{|\nabla u|}\right) + \lambda(u_0 - u) \\ u(x,y,0) = u_0 \end{cases} \tag{2}$$

3. Wavelet transforms

3.1 Wavelet transform in image

The 2-D discrete wavelet transform is generated by 1-D discrete wavelet transform (Chui, 1992a, b; Mallet, 1999). In the process, the image is decomposed into four sub-images: LL, LH, HL, and HH. We further decompose the LL sub-image and can obtain another four sub-images in next scale, we stop decomposing when it reaches the wavelet scale we need. Figure 1 gives the image decomposed by wavelet in scale 3. LL is the smooth part while HH, LH, HL are detail parts. The wavelet gives a good expression to image denoising. By the analysis of coefficients after transform and the proper reconstruction, it realizes the denoising process at the end.

3.2 The characters of wavelet coefficients in different areas

There are different statistical characters between image and noise. The image energy is mainly in the low frequency area and the detailed area in the high frequency,

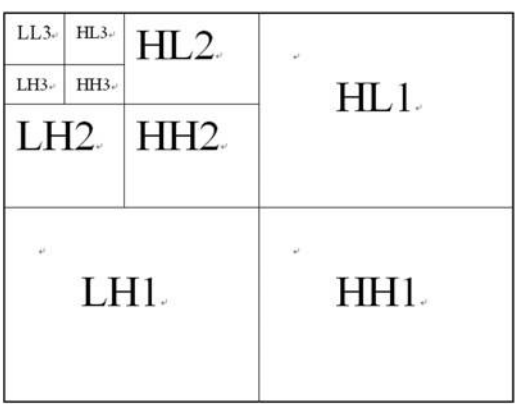


Figure 1.
The image decomposed in different sub-band with three scales

corresponding to big value wavelet coefficients. The noise energy spreads around almost everywhere and corresponds to the small value wavelet coefficients.

In traditional methods, a threshold value was set to distinguish which was useful. If the wavelet coefficient was bigger than the threshold value, it would be an image signal, while in case of the opposite it would be the noise and would be deleted. This was how the traditional methods achieved the image denoising process. However, the noise is dense on finest scale in wavelet. It is difficult to distinguish which wavelet coefficient corresponds to the edge or the noise. Image can be divided into "smoothness area" and "edge area"; "smoothness area" has small change on grey level and "edge area" has dramatic change on grey level. But the noise in image fluctuates at random with small range. So, the statistical of wavelet coefficients in these areas are obviously different.

Image can be divided into four parts by multi-scale wavelet decompose: low frequency, horizontal direction, vertical direction, and diagonal direction. These wavelet coefficients reflect the approximate contour image in low frequency and the details of image in high frequency separately. So, all the wavelet coefficients are divided into low frequency area and high frequency area. We analyze the character of wavelet coefficients in different areas, choose different algorithm to process, overcome the traditional thresholding algorithm disadvantage, and improve the algorithm more accurately.

4. New algorithm

4.1 The character of relation on wavelet coefficients

There are two kinds of relations of wavelet coefficients: intra-scale relation and inter-scale relation. Intra-scale relation is in each direction while inter-scale relation is in each scale. The image has strong relation while noise has little or no relation. In the traditional method, wavelet shrinkage has been proposed. The shrinkage coefficients are nonlinear, and the shrinkage process reduces the low value and maintains the high value. The aim of algorithm is to reduce the value of wavelet coefficient which is generated by noise and maintains the coefficient relation generated by the image.

In the wavelet transform (Mehdi and Hossein, 2009; Mencattini *et al.*, 2008), there is no obvious relation between the coefficients in the neighborhood, but the absolute and square value of the coefficients present a Markov distribution. We model wavelet coefficients of the image within sub-bands as a realization of a doubly stochastic process. Specifically, the wavelet coefficients are assumed to be conditionally independent zero-mean Gaussian random variables, given their variances. These variances are modeled as identically distributed, highly correlated random variables. For estimation purposes, we approximate wavelet coefficients as locally independent and identically distributed. The variance of coefficients which comes from each direction in subbands has strong relations. When the wavelet is decomposed, the range of wavelet coefficient changes as the image Lipschitz exponent changes. When the noise Lipschitz exponent is smaller than zero, the wavelet coefficients decrease while the scale increase; when the image edge Lipschitz exponent is bigger than zero, the wavelet coefficients increase and the scale increase. This character is useful in calculating the diffuse filter coefficients in wavelet domain. Therefore, the wavelet coefficients can be analyzed in different scales by the relationship of their absolute values.

In this paper, we use the character of inter-scale and intra-scale dependencies between wavelet coefficients, and apply the grey theory (Deng, 1982, 1996; Hong *et al.*, 2006; Ma *et al.*, 2004, 2005) into wavelet coefficient selection. The different characters of coefficients in different areas distinguish the wavelet coefficients very well, and obtain a perfect image denoising result.

4.2 Noise detection based on grey relational analysis

Noise detection is the key step in image denoising, the grey of noise is the addition of image grey and noise grey. When an image pixel is disturbed by noise, its grey value is different from the coefficients in the neighborhood. However, the biggest grey value is not always the noise pixel; in the noiseless image, smooth area or edge area also has the extreme value. So, it is easy to make a mistake while considering this pixel as noise. To our experience, if the pixel has little relation to coefficients in the neighborhood, the pixel may be a noise; otherwise useful signals are expert to a single pixel.

There are many methods (Feng and Yan, 2006; Zhang *et al.*, 2009; Ni *et al.*, 2008) to detect noise pixel, mainly by the statistic of coefficients in neighborhood, calculate the mean value and deviation value of neighborhood grey and set a threshold value to detect noise. Also, one should take the statistic maximum and minimum values in neighborhood as a candidate of noise pixel and set a threshold value to improve the precision.

On the analysis above, in order to reduce the detection time and reduce the possibility of leak and miss detection, we proposed a new method. The method is based on the different decay speeds of image and noise in each scale, and analyzes the grey relation of coefficients between different scales and locations. The method we proposed can obtain the noise position accurately.

4.3 New algorithm model

Chan and Wong (1998) and Chan and Zhou (1998) used wavelet hard thresholding in image denoising after the TV model process. Although the image denoising result is improved, the Gibbs' phenomenon is not reduced. The traditional TV model in the wavelet domain uses thresholding directly in the selection of coefficients and lead the Gibbs' phenomenon exist. To overcome this defection, Chan and Zhou (2007) proposed another model which improves the method of wavelet coefficients selection. The new algorithm reduces the Gibbs' phenomenon and improves the image denoising result. It mentions that "selecting nonzero wavelet coefficients in the thresholding procedure, location is also important" in Chan and Zhou (2007). Being enlightened by this, we propose a new image denoising algorithm which succeeds in selecting of nonzero wavelet coefficients with local relation.

In this paper, we divide the wavelet into two parts: low frequency area and high frequency area, in different areas use the different methods. We apply grey theory in wavelet coefficient selection. The new algorithm aims to give a new method of wavelet coefficients selection, to solve the nonzero coefficients sort, and to achieve a good image denoising result while reducing the phenomenon of Gibbs.

4.3.1 Different algorithm in different area. The observed noisy image f is polluted by white Gaussian noise. The image model as:

$$\tilde{f} = f + \varepsilon \tag{3}$$

where $\tilde{f}$ is noisy image. ε is a sample of a zero-mean white Gaussian field of variance σ^2.

In this paper, we first decomposed the noisy image $\tilde{f}$ and obtained wavelet coefficients $c_{j,k}$, where j and k are decomposition scale and direction, respectively.

In the traditional method, the threshold is used to select wavelet coefficients by the value of λ:

$$M = \{\hat{c}_{j,k} : |c_{j,k}(\tilde{f})| > \lambda\} \tag{4}$$

However, the discontinuous points in an image may present the phenomenon of Gibbs. To alleviate this phenomenon, Rudin *et al.* (1992) proposed a TV method. It is defined as:

$$TV(f) = \int_{\Omega} |\nabla f| dx \tag{5}$$

In this paper, we use high frequency wavelet coefficients in expert finest scale coefficients, and reconstruct other coefficients to gain a low frequency image f':

$$f' = W^{-1}\theta W(\tilde{f}) \tag{6}$$

where θ is defined as:

$$\theta := \{x : |x| \in c_{j-1,k}\} \tag{7}$$

Low frequency image f' contains a little noise, so we use TV minimization to process and obtain the image g. The reason we chose TV minimization is that it can suppress noise effectively while retaining edges. This has been demonstrated in the literature, such as Chan and Wong (1998), Chan and Zhou (1998) and Rudin (1992). In the high frequency, we apply the grey relation theory to deal with the wavelet coefficients.

4.3.2 Grey relation theory in the scale of wavelet coefficients. We used the sub-scale high frequency wavelet coefficients of image f' and image g to calculation the grey relation of coefficients. $c_{j-1,k}$ and $c'_{j-1,k}$ are a sequence of sub-scale high frequency wavelet coefficients in each image. So one can define the sequence of wavelet coefficients as:

$$c_{j-1,k} := \{c_{j-1,k}(1), c_{j-1,k}(2), c_{j-1,k}(3), \ldots, c_{j-1,k}(n)\} \tag{8}$$

$$c'_{j-1,k} := \{c'_{j-1,k}(1), c'_{j-1,k}(2), c'_{j-1,k}(3), \ldots, c'_{j-1,k}(n)\} \tag{9}$$

Then one obtains the initial value and difference value sequence. The initial value is defined as:

$$\bar{c}_{j-1,k} = c_{j-1,k}/c_{j-1,k}(1) \tag{10}$$

$$\bar{c}'_{j-1,k} = c'_{j-1,k}/c'_{j-1,k}(1) \tag{11}$$

So the difference value sequence is:

$$\Delta_i = \{|\bar{c}'_{j-1,n}(1) - \bar{c}_{j-1,n}(1)|, \ldots |\bar{c}'_{j-1,n}(\underline{n}) - \bar{c}_{j-1,n}(\underline{n})|\} \tag{12}$$

To use the difference value sequence of wavelet coefficients, we can define a grey relation as ξ:

$$\xi(n) = (\min(\Delta_i) + \lambda \max(\Delta_i))/(\Delta_i(n) + \lambda \max(\Delta_i)) \tag{13}$$

where λ is set as 0.5.

So the total relation of two sequences of wavelet coefficients can be defined as follows:

$$\bar{\xi} = \frac{1}{n}\sum_{j=1}^{n} \xi_n(j) \tag{14}$$

4.3.3 Grey relation theory in the location of the wavelet coefficient. We use the sub-scale grey relation of wavelet coefficient to modify the finest wavelet coefficient. In the finest area we apply the window of size 2×2 to analyze each coefficient $c(m, n)$. Grey relation of coefficients in each direction can be defined as:

$$\xi_h = \frac{c(m, n-1)}{c(m, n-1) + |c(m, n) - c(m, n-1)|} \tag{15}$$

$$\xi_v = \frac{c(m-1, n)}{c(m-1, n) + |c(m-1, n) - c(m, n)|} \tag{16}$$

$$\xi_d = \frac{c(m-1, n-1)}{c(m-1, n-1) + |c(m, n) - c(m-1, n-1)|} \tag{17}$$

where ξ_h, ξ_v, ξ_d are the key values of the horizontal direction, vertical direction, diagonal direction, respectively, in wavelet high frequency location.

Through the analysis above we can further recognize the relation between the coefficients in the wavelet domain. We combine the inter-scale and intra-scale dependencies to modify the wavelet coefficients between different scales and different locations. This algorithm can capture dependencies between wavelet coefficients deeply. To this full analysis, we can get a good result in divided wavelet coefficients.

4.3.4 Noise detection. Comparing with the grey relation of these wavelet coefficients, we can distinguish the noise position from signal. So, we define a new select coefficient method, defined as:

$$P_\sigma(\tilde{f}_h) = W^{-1}\theta_c W(\tilde{f}_h) \tag{18}$$

where,

$$\theta_c := \left\{ \begin{array}{ll} k_1 c_{j,k}, & \xi_{\min} \geq \bar{\xi} \\ 0, & \xi_{\max} \leq \bar{\xi} \\ k_2 c_{j,k}, & others \end{array} \right\} \tag{19}$$

$$\xi_{\min} = \min(\xi_h, \xi_v, \xi_d) \tag{20}$$

$$\xi_{\max} = \max(\xi_h, \xi_v, \xi_d) \tag{21}$$

$\tilde{f}_h$ are the wavelet coefficients in finest scale. $\xi_{\min}$ and $\xi_{\max}$ are the minimum and maximum of grey relational values in a neighborhood, respectively.

In Equation (19), the first row is the texture of the image while the grey relation is kept at a high level in each direction. The second row is the noise of the image while the grey relation is at a low level in all directions. The last row is the edge or small texture of the image while they have character of direction, and so the grey relation may fluctuate in some direction. The parameter of k_1 is set to 1 normally, and k_2 is reduced to zero while the noise of image is increased. From the experiment we find that the value of k_2 is in the region [0 1]. To reduce algorithm complexity, we it set to 0.5.

4.3.5 Obtain denoised image. In the analysis above we obtain the wavelet coefficients in low area and high area, and also reconstruct these coefficients and obtained a new denoised image.

4.4 New algorithm flow

The flow of the algorithm proposed in this paper is shown in Figure 2.

5. Parameter analyzing

Diffusion filter in the wavelet domain can avoid the phenomenon of subsection in image. The image denoising result is related with wavelet basis and decomposing scale. In the following part, we will discuss the influence on image denoising.

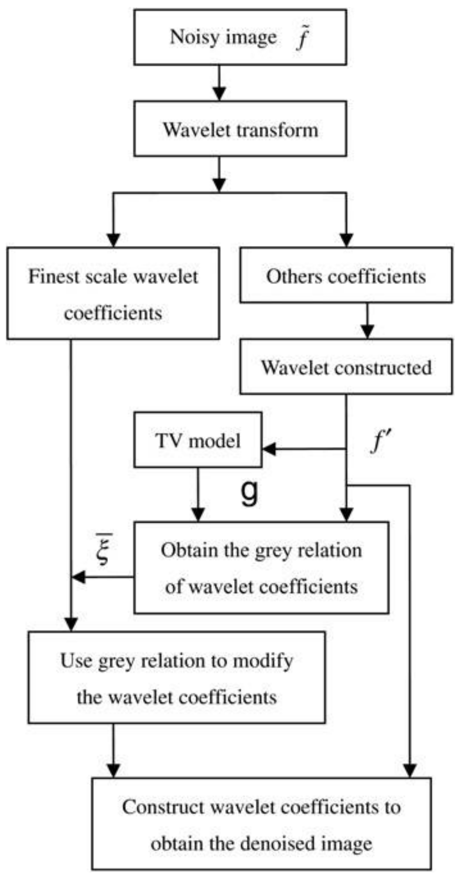


Figure 2.
Chart of algorithm flow

5.1 The influence of decomposition of scale

In this paper, a large number of experiments prove whether the decomposition of scale influences the image denoising result. In the experiment, we choose three kinds of decomposing scales: [0], [0, 1], [0, 1, 3]. The image denoising result is shown in Table I.

Table I gives the influence of image denoising result on a different scale decomposition. As the decomposition of scale increases, the image denoising result is improved, and also the complexity of algorithm is increased. By considering all, we choose [0, 1, 3] scale as our decomposition scale.

5.2 The influence of wavelet basis

The influence of wavelet basis on image denoising can be recognized as its filter banks influence. So, we choose several normal kinds of filters to analyze whether it influences the image denoising result. In the experiment, we use "Barbara" image with $\sigma = 30$ white Gaussian noise. Figure 3 gives the result of comparing result. The "black point" (in Figure 3) represents the five filter banks we mentioned in this paper. The five filter banks left to right are "9-7," "v9-7," "5-3," "Burt," and "pkva." The experiment results show that the filter banks have little influence on algorithm in this paper. Hence, we have chosen "9-7" and "pkva" filter banks as they are better than others.

6. Experimental results and performance evaluation

In this paper, we first decompose image into two parts: low frequency area and high frequency area. In the low frequency area, we use the TV minimization to deal with the

σ		20	30	40	50
Scale [1]	WHT	23.8	20.7	18.1	15.8
	TVHD	27.3	25.5	24.5	23.7
Scale [0,1]	WHT	24.7	22.2	20.3	19.4
	TVHD	27.4	25.6	24.6	23.8
Scale [0,1,3]	WHT	24.7	22.2	20.5	19.4
	TVHD	27.5	25.7	24.6	23.8

Note: WHT: wavelet hard thresholding

Table I. The influence of different scale decompose

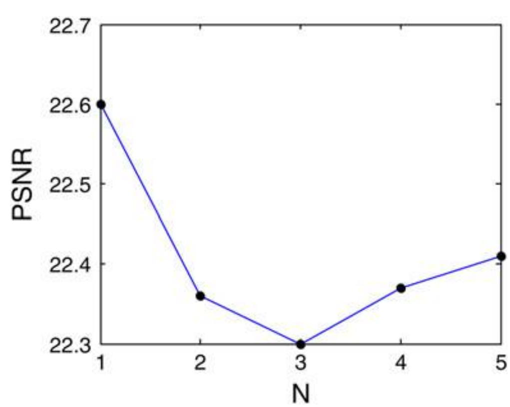


Figure 3. Influence of algorithm on different filters

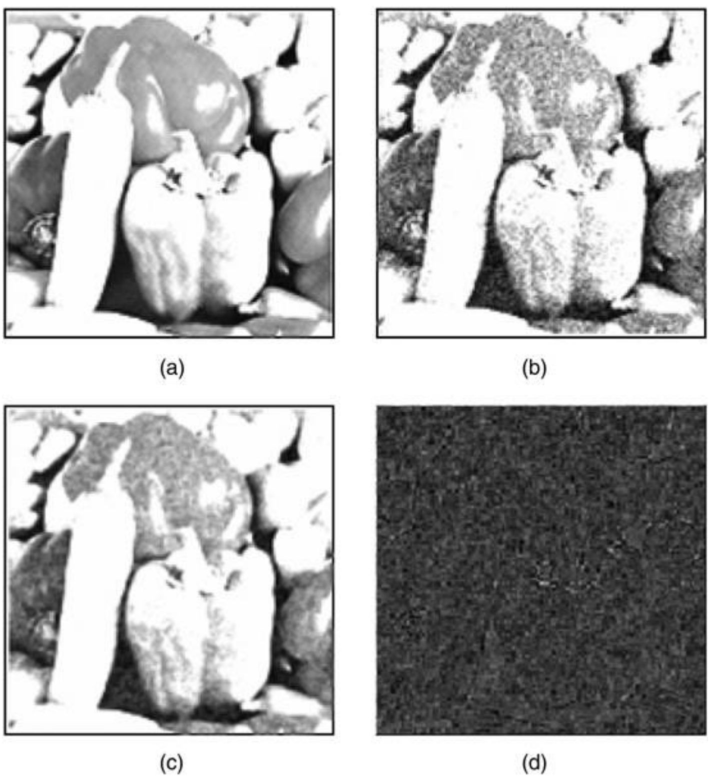
low frequency coefficients and obtain a new low frequency image. The new low frequency image keeps the information of the original, and also restrains the noise in image.

6.1 Experiment analysis in low frequency area

There is a little noise in the low frequency area. The noise can be retained by TV algorithm. Figure 4 gives the result of the low-frequency area processed by the TV model. The residual image contains little edge information. This is caused by the leak directions in wavelet transform and the mean value in area can't be maintained when the wavelet coefficients are small. In the analysis of low frequency area, we conclude the drawback of algorithm. In this paper, we introduce the grey relation theory to our algorithm, and solve this phenomenon.

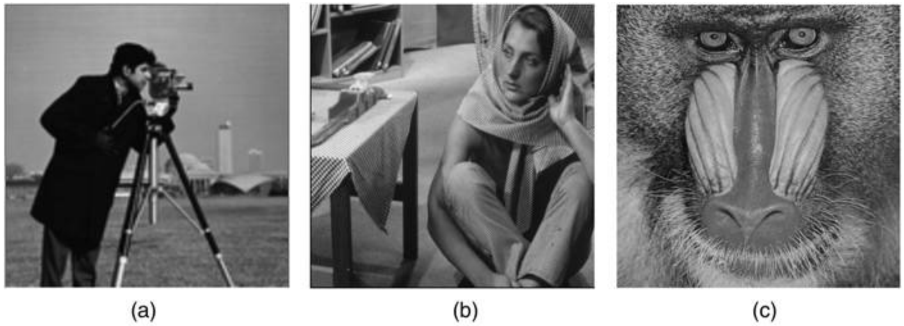
6.2 Experimental analysis on our proposed method

In this paper, we use a large number of tests in several kinds of image denoising algorithm. Figure 5 gives some of the images for test. In order to prove our algorithm is



Notes: (a) Low frequency area in original image; (b) low frequency area in noisy image; (c) low frequency area in our algorithm; (d) the residual part in our algorithm

Figure 4.
Low frequency area processing result



Notes: (a) Cameraman; (b) Barbara; (c) baboon

Figure 5.
Test images

superior to that of others, we use the peak signal to noise ratio (PSNR) as a comparison criterion. The value of PSNR can express the result of image, the big value of PSNR means that the image is in good condition:

$$PSNR = -10 \times \log 10((f - f_d)^2/(M \times N)) \tag{22}$$

where $M \times N$ is the size of image, f and f_d are the original and filtered images, respectively.

The experiment compares two kinds of image denoising results. Figure 6 gives the different noise of standard deviation denoising result under different types of images. We show three images: "cameraman," "Barbara," and "baboon" (in Figure 6). The "circle" mark line is the expression of the algorithm TV hard thresholding (TVHD)

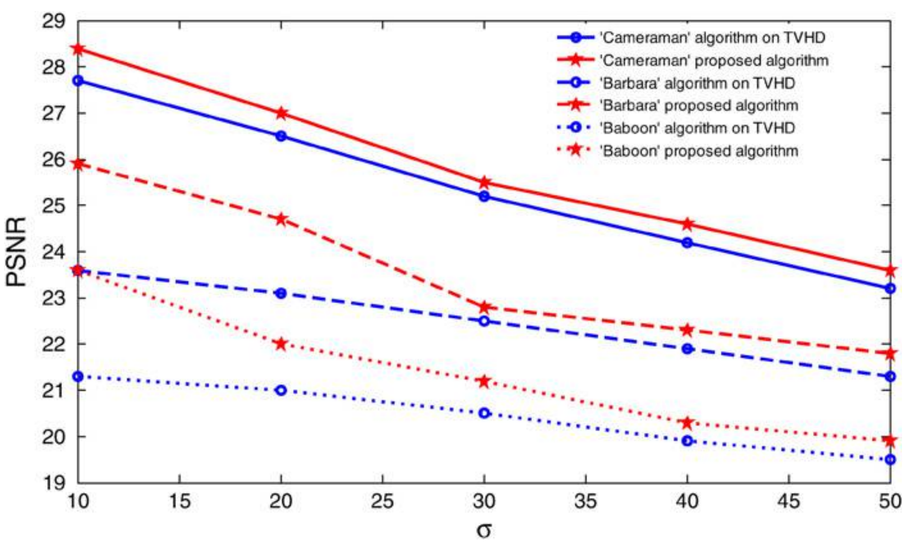


Figure 6.
Comparison with two algorithms

proposed in Chan and Zhou (2000, 2007) and the "star" mark line is the algorithm we proposed. Our proposed method is superior in denoising result.

In Figure 7, we show the "peppers" image denoising result:

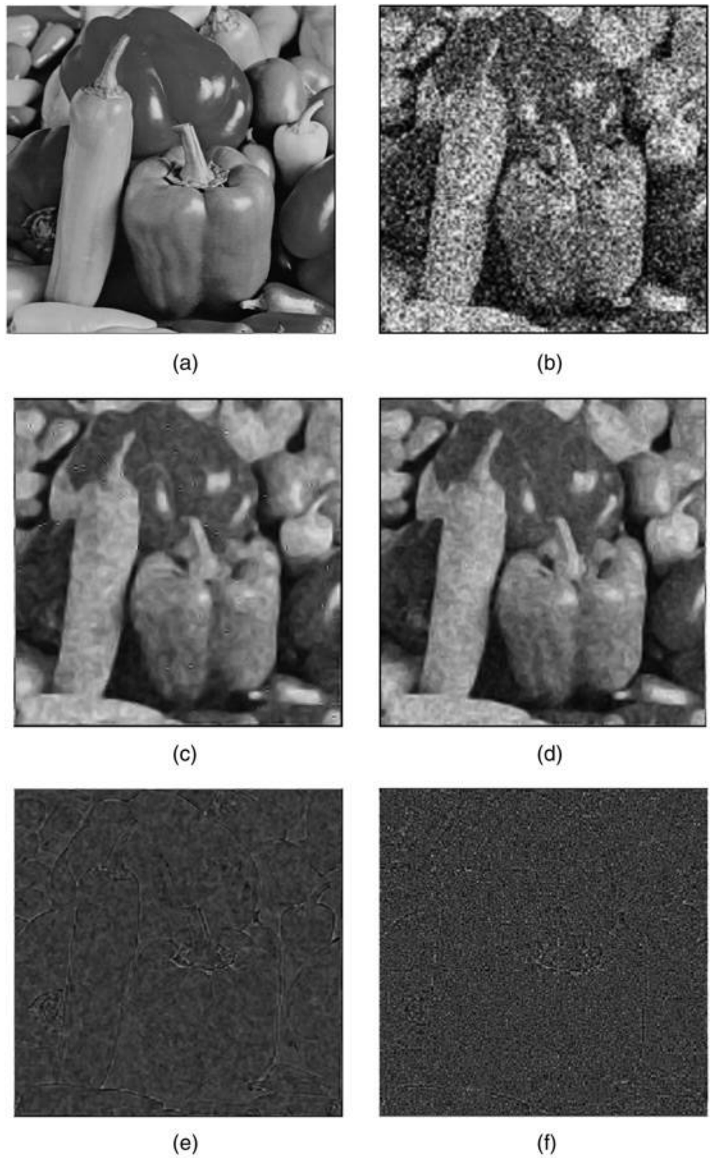


Figure 7.
Image denoising based on different algorithms

Notes: (a) Original image; (b) noisy image; (c) algorithm on TVHD; (d) proposed in this paper; (e) the residual part in algorithm on TVHD; (f) the residual part in proposed Algorithm

- original image;
- noisy image;
- denoised image on TVHD;
- denoised image by our algorithm proposed in this paper;
- the residual part in algorithm on TVHD; and
- the residual part in proposed algorithm.

Moreover, it is seen that some cartoon and texture parts are still present in the residual part (in Figure 7(e)). By the comparing experiment, we can see that our the algorithm based on grey relational analysis can obtain successful results. More precisely, we give the quantitative description of the performance of the two methods. We utilize the structural similarity proposed by Wang and Bovik (2002) to evaluate the structural information quality based on the two methods. Then, the structural similarities between the several methods denoised images and the standard "Peppers" test image are calculated. The results are shown in Figure 8. From the curves (in Figure 8), we can see that the denoised image on our method is more similar to the original image than the TVHD model and wavelet hard thresholding. Hence, it can be concluded that the algorithm we proposed can maintain the image edge and obtain perfect results in vision.

7. Conclusions

In this paper, in the analysis of the traditional TV model image denoising in the wavelet domain we propose an improved algorithm based on grey relation theory. The new algorithm can select and locate noise coefficients in good condition. In this paper, we decomposed image into two parts: low frequency area and high frequency area, and analyze the wavelet coefficients by different methods. The new algorithm can improve image denoising result and reduce the phenomenon of Gibbs in image.

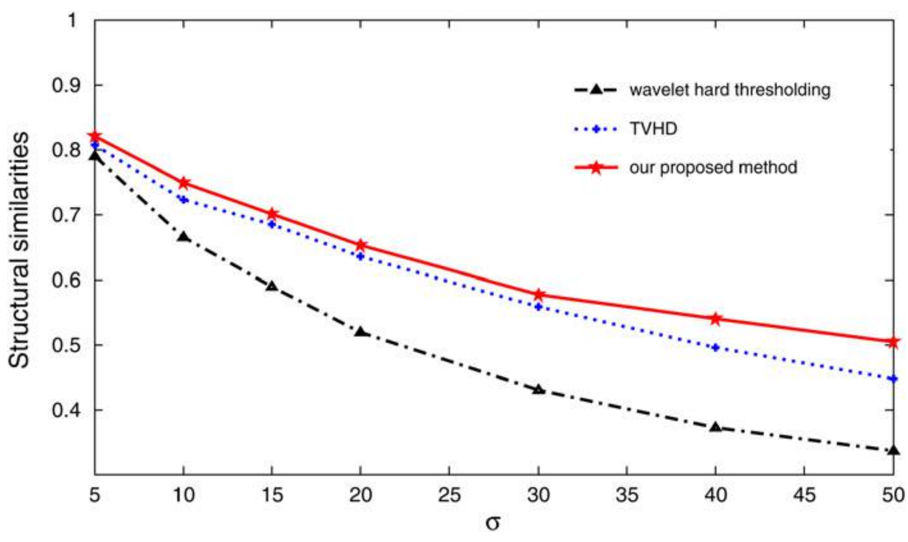


Figure 8.
The structural similarities between the denoised image and the standard image

References

Bhoi, N. and Meher, S. (2008), "Total variation based wavelet domain filter for image denoising", *Proceedings of the 2008 First International Conference on Emerging Trends in Engineering and Technology, Nagpur, Maharashtra*, IEEE Computer Society, Washington, DC, pp. 20-5.

Chan, T.F. and Wong, C.K. (1998), "Total variation blind deconvolution", *IEEE Transactions of Image Progressing*, Vol. 7 No. 3, pp. 370-5.

Chan, T.F. and Zhou, H.M. (1998), "Feature preserving lossy image compression using nonlinear PDE'S", *SPIE Proceedings on Advanced Signal Processing Algorithms, San Diego, CA*, IEEE Computer Society, Washington, DC, pp. 316-27.

Chan, T.F. and Zhou, H.M. (2000), "Total variation improved wavelet thresholding in image processing", *Processing to the 2000 International Conference on Image Processing, Vancouver*, pp. 391-4.

Chan, T.F. and Zhou, H.M. (2007), "Total variation wavelet thresholding", *Journal of Scientific Computing*, Vol. 32 No. 2, pp. 315-41.

Chui, C.K. (1992a), *An Introduction to Wavelet*, Academic Press, New York, NY.

Chui, C.K. (1992b), *Wavelet: A Tutorial in Theory and Applications*, Academic Press, New York, NY.

Deng, J. (1982), "Control problems of grey system", *System and Control Letters*, Vol. 1 No. 5, pp. 288-94.

Deng, J. (1996), *The Basic Method of Grey System*, Huazhong University of Science and Technology Press, Wuhan.

Feng, D. and Yan, J. (2006), "An algorithm of infrared imaging edge detection based on grey relational analysis", *Infrared Technology*, Vol. 28 No. 3, pp. 161-4.

Feng, L. and Yang, X.-C. (2006), "Based on the total variation and wavelet hard threshold de-noising images", *Signal Processing*, Vol. 22 No. 6, pp. 917-9.

Hong, J., Tao, J. and Li, G. (2006), "Noise reduction of digital image based on grey correlation degree", *Journal of Wuhan University of Technology*, Vol. 30 No. 4, pp. 639-41.

Ma, M., Hao, C. and Han, P. (2004), "Image fidelity criterion based on relational analysis", *Journal of Computer Aided Design Computer Graphics*, Vol. 16 No. 7, pp. 978-83.

Ma, M., Tian, H.P. and Hao, C.Y. (2005), "Image denoising using grey relational analysis in spatial domain", *Visual Communications and Image processing*, Vol. 5960, pp. 335-42.

Mallet, S. (1999), *A Wavelet Tour of Signal Processing*, 2nd ed., Academic Press, New York, NY.

Mehdi, N. and Hossein, N.-p. (2009), "Image denoising in the wavelet domain using a new adaptive thresholding function", *Neurocomputing*, Vol. 72 Nos 4-6, pp. 1012-25.

Mencattini, A., Salmeri, M., Lojacono, R., *et al.* (2008), "Mammographic images enhancement and denoising for breast cancer detection using dyadic wavelet processing", *IEEE Transactions on Instrumentation and Measurement*, Vol. 57 No. 7, pp. 1422-30.

Ni, C., Li, Q. and Xia, L.Z. (2008), "A novel method of infrared image denoising and edge enhancement", *Signal Processing*, Vol. 88 No. 6, pp. 1606-14.

Rudin, L., Osher, S. and Fatemi, E. (1992), "Nonliner total variation based noise removal algorithm", *Proceedings of the Eleventh Annual International Conference of the Center for Nonlinear Studies on Experimental Mathematics, Los Alamos, NM*, Elsevier North-Holland, Amsterdam, pp. 259-68.

Starck, J.L., Elad, M. and Donoho, D.L. (2005), "Image decomposition via the combination of sparse representations and a variational approach", *IEEE Transactions of Image Progressing*, Vol. 14 No. 10, pp. 1570-82.

Vese, L.A. and Osher, S.J. (2003), "Modeling textures with total variation minimization and oscillating patterns in image processing", *Journal of Scientific Computing*, Vol. 19 No. 1, pp. 553-72.

Wang, Z. and Bovik, A.C. (2002), "A universal image quality index", *IEEE Signal Processing Letters*, Vol. 9 No. 3, pp. 81-4.

Xu, J. and Osher, S. (2005), "Iterative regularization and nonlinear inverse scale space applied to wavelet based denoising", *IEEE Transactions of Image Progressing*, Vol. 16 No. 2, pp. 534-44.

Zhang, Z., Ma, S.L., Liu, H., *et al.* (2009), "An edge detection approach based on directional wavelet transform", *Computer & Mathematics with Applications*, Vol. 57 No. 8, pp. 1265-71.

Corresponding author
Zhi-min Zhao can be contacted at: zhaozhimin@nuaa.edu.cn

This article has been cited by:

1. Hongjun Li, Ching Y. Suen. 2016. A novel Non-local means image denoising method based on grey theory. *Pattern Recognition* **49**, 237-248. [CrossRef]
2. Hong-jun Li, Zheng-guang Xie, Wei HuAn image compression method using sparse representation and grey relation 53-56. [CrossRef]
3. Li Hong-jun, Hu Wei, Xie Zheng-guang, Wang Wei. 2013. Image denoising method based on grey relational threshold. *Grey Systems: Theory and Application* **3**:2, 191-200. [Abstract] [Full Text] [PDF]
4. H.-J. Li, Z.-M. Zhao, X.-L. Yu. 2012. Grey theory applied in non-subsampled Contourlet transform. *IET Image Processing* **6**, 264. [CrossRef]