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Analysis of Convertible Bond Value Based on Integration of Support Vector Machine and Copula Function

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The pricing of convertible bonds (CB) is still a problem that needs to be addressed because it is a kind of hybrid financial instrument. This article proposed a novel method with support vector machine (SVM) integrated to copula function. Unlike existing single-factor or bi-factor pricing models based on corporate value and the underlying stock price, respectively, this model can cope with many limitations on the pricing of CB, such as nonlinearity, the departure from normality, multivariate joint normality distribution, market incompleteness, and so on. And above all, the new model exhibited great flexibility in that copula function can portray dependence structure between the underlying stock price and interest rate and that SVM can further tackle nonlinear relationship among variables. Moreover, the integration of SVM and copula function rendered the sensitivity analysis more convenient and accurate. Empirical analysis showed that the proposed model enhanced generation ability of out-of-sample, with satisfactory robustness and mark increase in pricing accuracy and hedging effectiveness compared with the traditional models.

Keywords Bivariate dependence structure; Convertible bond (CB); Copula function; Statistical learning theory; Support vector machine (SVM).

Mathematics Subject Classification 68Q32; 68T05; 62H20.

1. Introduction

Most convertible bonds (CB) have many embedded options compared to the straight bonds, such as conversion option, call option, put option, option to lower the conversion price, and even to accumulate interest rate, and so on. These options are mainly American and path-dependent, and make the pricing considerably complicated, thus resulting in the increasing difficulties in CB pricing (Kihn, 1996).

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So far, there are, in general, two categories of pricing models of CBs, namely single-factor and bi-factor models which are based on corporate value and corporate stock price, respectively. Ingersoll (1977) and Brennan and Schwartz (1977) applied Black-Scholes option pricing theory to CB pricing and constructed a single-factor corporate value model. By adding interest rate volatility into the model above, Brennan and Schwartz (1980) deeply improved the model and proposed the bi-factor corporate value model. On the other hand, McConnell and Schwartz (1986) first presented another kind of CB pricing model with corporate stock being the underlying asset, and Ho and Pfeffer (1996) developed a corresponding bi-factor stock pricing model with interest rate being another underlying variable. Tsiveriotis and Fernandes (1998), Duffie and Singleton (1999), and Takahashi et al. (2001) expanded simplified structured model by adding default probability and recovery rate. Ammann and Seiz (2006) employed the modified Black-Scholes European option pricing model, and Yigitbasioglu and Alexander (2006) proposed that a nonlinear, multi-factor, reduced-form, equity-linked default model leads to a set of nonlinear partial differential complementarity equations governed by the volatility path. Chambers and Lu (2007) presented a binomial tree model for pricing convertible bonds, different from Das and Sundaram (2006) through differences in the specification of the correlation between interest rates and stock prices, and found that the correlation between the stock price and interest rate levels is especially important in the pricing of CBs of financial institutions. And above all, by taking into account the characteristics of CBs in the Chinese market and using the basic theories and methods of financial engineering, Zheng and Lin (2004) and Lai et al. (2005) presented different pricing models and made corresponding empirical studies, which all showed that, compared with the theoretical prices, CBs in China were to some extent underpriced.

However, the poor performance of most of the models for CB pricing above is attributed to a lot of restrictions or assumptions, such as linearity, normality distribution, constant correlation between the underlying stock price and interest rate, fixed default probability and recovery rate, market completeness, and so on, most of which were far from the market reality and neglected complicated interaction among options within CBs.

As flexible analysis tools for tackling the problems mentioned above, support vector machines (SVM) and copula function have seldom been applied to CB pricing (except for Shen et al., 2010). Therefore, this article is aimed at proposing a novel method of CB pricing based on integration of SVM and copula function, which will absorb some previous study viewpoints mentioned above and make an attempt to the innovation of CB pricing. By illustrating dependence structure among the important features using a copula function and training SVM through the data of selected features along with the dependence structures that significantly affect CB value in Chinese financial markets, this approach can predict CB value with more accuracy and determine hedging ration effectively. The remainder of this article is organized as follow. The methodologies are described in Sec. 2 with the Clayton copula and LS-SVM mainly introduced. Section 3 presents empirical analysis of application of the model proposed to CB pricing and hedging. Section 4 concludes.

2. A Model Based on Integration of SVM and Copula Function

The framework of the model construction contains the copula choosing and the SVM optimization consists of two steps, shown in Fig. 1. Step 1 explores the

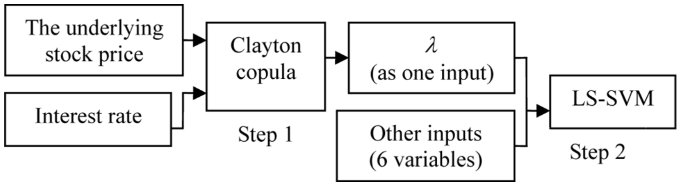

Figure 1. The framework of the integration of LS-SVM and the Clayton copula used for analysis of convertible bond value.

dependence structure of the underlying stock price and interest rate through a copula function. LS-SVM is then established in Step 2, with the outcome of Step 1 as one of its inputs, to investigate CB's price.

2.1. *Dependence Structure Analysis Using Copula Function*

Here, only the bivariate copula is considered. According to Nelsen (1999), its definition is described as follows.

A two-dimensional copula C is a real function defined on $I^2 = [0,1] \times [0,1]$, with range $I = [0,1]$, such that

$$\text{for every } (u, v) \text{ of } I^2, \quad C(u,0) = 0 = C(0,v), \quad C(u,1) = u, \quad C(1,v) = v;$$

$$\text{for every rectangle } [u_1, u_2] \times [v_1, v_2] \text{ in } I^2, \text{ with } u_1 \leq u_2 \quad \text{and} \quad v_1 \leq v_2,$$
$$C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0.$$

As such, it can represent the joint distribution function of two standard uniform random variables U_1 an U_2:

$$C(u, v) = P(U \leq u, V \leq v).$$

This feature can be used to re-write copulas for the joint distribution function of two (even nonuniform) random variables. The most interesting fact about copulas in this sense is Sklar's theorem (Sklar, 1959): Let $F(s,t)$ be a joint distribution function with continuous marginals $F_1(s)$ and $F_2(t)$. Then there exists a unique copula such that

$$F(s, t) = C(F_1(s), F_2(t)). \tag{1}$$

Conversely, if C is a copula and $F_1(s)$ and $F_2(t)$ are continuous univariate distributions, $F(s,t) = C(F_1(s), F_2(t))$ is a joint distribution function with marginals $F_1(s)$ and $F_2(t)$.

The theorem suggests then to represent the multiplicity of joint distributions consistent with given marginals through copulas. There are mainly three kinds of copulas, namely normal copula, extreme value copula, and Archimedean copula, and three copulas are commonly used in financial risk management with the specification as follows:

$$\text{Normal copula:} \quad C(u,v) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{(1-\theta^2)}} \exp\left\{\frac{(s^2 - 2\theta st + t^2)}{2(1-\theta^2)}\right\} ds\, dt$$
$$= \Phi_\theta(\Phi^{-1}(u), \Phi^{-1}(v));$$

Gumbel copula: $C(u, v) = \exp\{-[(-\ln(u))^\theta + (-\ln(v))^\theta]^{\frac{1}{\theta}}\}$, $\theta \geq 1$;
Clayton copula: $C(u, v) = (u^{-\theta} + v^{-\theta} - 1)^{-\frac{1}{\theta}}$.

Copulas are linked to nonparametric association measures, namely dependence measure by useful relationships. As an example, Kendall's τ may be proved to be

$$\tau = 4\iint_{[0,1]^2} C(u, v)dC(u, v) - 1. \tag{2}$$

Here, assume that the bivariate vectors of the underlying stock price and interest rate exhibit continuous marginals so that each copula is defined in a unique way. Now, it seems hard to characterize exactly each distribution law of such vectors' variables, and moreover, Durrleman et al. (2000) showed that a misspecification of marginals leads to a biased estimation of the copula function. Such a problem leads us to directly consider the copula function without specifying any marginal law. Therefore, rather than specifying given marginals ex ante and inducing then the related copula function, a given copula function is specified consistent with the studied empirical dependence structure.

To be able to fit conveniently estimated copulas, one-parameter Archimedean copulas are chosen here. Specifically, the choice restricts to the Clayton copula because, according to Servigny and Renault (2002), the correlation between assets in the falling market becomes much higher than that of the rising market, and the Clayton copula depicts the down tail well. Moreover, the Clayton copula functions exhibit the nice property with an analytical expression such that its related Kendall's tau is a function of its parameter θ. Namely, there exists the expression:

$$\tau = \theta/(\theta + 2), \tag{3}$$

where τ is observed here.

This simplified framework leads us to an estimation method of copulas based on dependence measures: the dependence measure used here is Kendall's tau τ statistic. Therefore, the parameter's value can be computed directly given a value of Kendall's tau such as $\theta = 2\tau/(1 - \tau)$. When it is not the case, the equation above may be solved numerically to get the parameter's value.

After the copula function has been determined, an approximated correlation, denoted by λ, can be here expressed referring to the definition of the low-tail dependence (Zhang, 2002):

$$\lambda \approx \frac{C(u, v)}{(u + v)/2} = 2C(u, v)/(u + v). \tag{4}$$

The framework in Eq. (4) constitutes a reduced measure to be used for consideration of the dynamics of dependence structure between the stochastic variables s and t.

As for estimation of λ, there exists two approaches, namely empirical distribution function and quantile-based equation. This section will discuss the former, with the later explored in Sec. 3.4.

With regard to empirical distribution function, it can be expressed:

$$F_1(s) = \frac{1}{l}\sum_{i=1}^{l} \vartheta(s - s_i), \quad \text{where } \vartheta(x) = \begin{cases} 1, & \text{if } s > 0 \\ 0, & \text{otherwise} \end{cases},$$

where s_i is the sample of stochastic variable S, and $F_1(s)$ is its empirical distribution function. Under the copula framework, u is equal to $F_1(s)$. Likely, v is equal to $F_2(t)$, calculated through the same methodology as $F_1(s)$. Then, the λ in Eq. (4) can be computed by:

$$\lambda \approx 2C(F_1(s), F_2(t))/(F_1(s) + F_2(t)). \tag{5}$$

Obviously, this approach can only apply to estimate the past value of λ.

2.2. *LS-SVM for Nonlinear CB Price Estimation*

Vapnik (1995) proposed the notion of the support vector machine (SVM) as a new generation learning system for small samples based on recent advances in statistical learning theory. It overcomes the shortcomings of the traditional pattern recognition and neural network recognition algorithm, such as large samples, "the curse of dimensionality", local optimization, over-fitness, etc., and has a higher recognition rate and better generalization performance. It is now being established as one of the standard tools for machine learning and data mining, such as pattern recognition and regression analysis (Cristianini and Shawe-Taylor, 2000). Then, the least square support machine (LS-SVM) is introduced below (Suykens and Vandewalle, 1999).

Given a training set, $\{x_i, y_i\}$, $i = 1, 2, \ldots, l$, where the input data x is assumed to be a compact domain in a Euclidean space R^n and the output data y is assumed to be a closed subset of R. Learning from data can be viewed as approximation of a multivariate function $f(x)$ that represents the relation between the input x and output y. By some nonlinear mapping $\phi(x)$, the input x is mapped onto a hypothesis space (or feature space) in which the learning machine (algorithm) selects a certain function $f(x)$.

According to the learning theory, for constructing a nonlinear LS-SVM, the decision function takes the following form:

$$f(x) = w \cdot \phi(x) + b.$$

Then, the decision function can be estimated by taking the following quadratic programming:

$$\begin{aligned} &\text{minimize} \quad \frac{1}{2}\|w\|^2 + \gamma \sum_{i=1}^{l} e_i^2 \\ &\text{subject to} \quad y_i = w^T \phi(x_i) + b + e_i, \end{aligned} \tag{6}$$

where the first term $\|w\|^2$ is called the regulation term, γ is the regularization constant which plays a trading-off between the regularization performance and the empirical error, and e_i is the error variables.

According to the generalized representer theorem (Evgeniou et al., 2000), the solution of $f(x)$ can usually be expressed with regard to the basic elements of Hilbert space with appositively defined kernel function $K(\cdot, \cdot)$:

$$K(x_i, x_j) = \phi(x_i)^T \phi(x_j), \quad i, j = 1, 2, \ldots, l$$

Then, there exists a linear equation group:

$$\begin{bmatrix} 0 & \mathbf{1}^T \\ \mathbf{1} & \mathbf{K} + \gamma^{-1}\mathbf{I} \end{bmatrix} \begin{bmatrix} b \\ \boldsymbol{\alpha} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{Y} \end{bmatrix}, \tag{7}$$

where $\mathbf{1} = [1, \ldots, 1]^T$, $\mathbf{Y} = [y_1, \ldots, y_l]^T$, $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_l]^T$, and $\mathbf{I}$ is unit matrix.

By taking Eq. (7), $\boldsymbol{\alpha}$ and b can be obtained, and the resulting LS-SVM model for nonlinear function estimation can be then denoted as follows:

$$f(x) = \sum_{i=1}^{l} \boldsymbol{\alpha}_i K(x_i, x) + b. \tag{8}$$

Any symmetric function which satisfies Mercer conditions can be used as the kernel function here (Wahba, 1999), and different kernel functions can also be constructed into variant learning machines.

The kernel functions most used presently are:

Polynomial kernel: $K(x_i, x_j) = [(x_i, x_j) + 1]^d$;

Radial basis function (RBF): $K(x_i, x_j) = \exp(-\|x_i - x_j\|^2/\sigma^2)$;

Sigmoid kernel: $K(x_i, x_j) = \tanh[v(x_i, x_j) + c]$, where $v > 0$, $c < 0$.

And, any non negative linear combination of Mercer kernels is still a Mercer one. Once the kernel is chosen, the parameters γ and σ^2 (e.g., sig2) need to be assigned next.

2.3. *Integration of SVM and Copula Function*

With the help of copula function, dependence structure of the underlying stock and interest rate can be explored. As one of inputs into the SVM, the dependence structure will exhibit significant impact on the CB's price. The process of modeling, detailed in Fig. 1, consists of two steps. Step 1 is responsible for the exploration of dependence structure of the underlying stock price and interest rate, denoted by the λ in Eq. (5), and LS-SVM modeling is completed in Step 2, with CB's price outputted.

3. Empirical Analysis

"Xin-gang CB" (Code: 110003), discussed below, is issued by a steel company "XINYU IRON STEEL GROUP CO., LTD." in China, with the underlying asset "Xin-gang stock" (Code: 600782). The data patterns are collected within the time period 09/08/2008-12/31/2010, and 549 data patterns are obtained with some inaccessible data eliminated. At the same time, these data patterns are divided into two parts: one part is used for the train set with 425 data patterns between 09/08/2008-06/30/2010, and the other is used for the test set with 124 data patterns within 07/01/2010-12/31/2010.

Most of data patterns are collected from the special data base: ESSET/DB and the experiments below are implemented through MATLAB software.

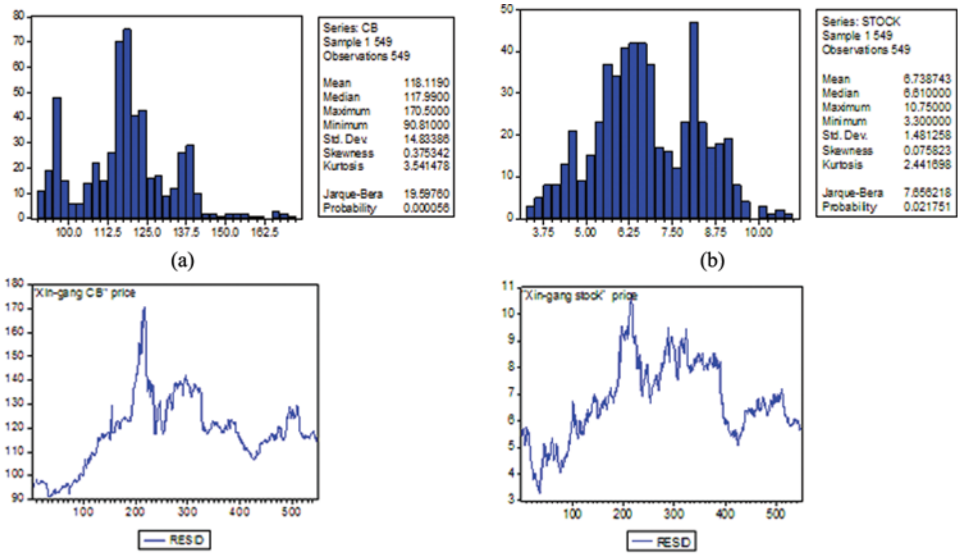

Figure 2. The descriptive statistical analysis of the data mainly involved in the histogram and stats and the line graph of the prices of "Xin-gang CB" and its underlying stock, "Xin-gang stock," respectively. (color figure available online)

3.1. *The Descriptive Statistics Analysis of the Data*

The descriptive statistical analysis of the data is done underneath to obtain an instruction for the feature selection and the parameter determination of LS-SVM and copula function. The analysis is mainly involved in investigating the prices of "Xin-gang CB" and its underlying stock, "Xin-gang stock" because the value of CB varies with its underlying asset price tempestuously and principally (Fig. 2).

The kurtosis values of the prices of "Xin-gang CB" and "Xin-gang stock" are 3.54 and 2.44, respectiveiy, and a lot of prominent accumulations appear in their line graphs correspondingly. In unit root test, the values of augmented Dickey-Fuller test statistic are -2.052236 (prob. 0.2645) and -1.728280 (prob. 0.4164), respectively, and display a unit root in all olthe two kinds of prices.

All of these characteristics above mean that the two kinds of prices both tolerate the departure from the nomai distribution and the linearity and stationarity, which are all dominantly used as assumptions by the traditional theories, and that some improvements must be conducted to adapt the value analysis of CB to the financial market, with the integration of SVM and copula function just discussed here.

3.2. *Feature Selection and Kernel Parameter Determination of LS-SVM*

First, estimation of the dependence structure is done. As a known feature, the dependence structure λ is first calculated. Under the notion of Kendall's tau τ, it can be estimated empirically and the parameter θ of Clayton copula can be then deduced through Eq. (3): $\theta = 2\tau/(1-\tau)$, with the λ being naturally calculated using Eq. (5). After these values of the λ has been calculated, they naturally fall into two parts according to the corresponding set, the train set or the test set, of the underlying stock price and interest rate.

Except for the λ, there are other features to be selected again. According to the previous study and the references described in Sec. 1, factors (e.g., features) that affect CB value can be summarized as follows: initial stock price, life time, volatility of the underlying stock, risk-free interest rate, credit risk premium, compound rate, call price, call conditions, put price, put conditions, conversation price, conversation ratio, conversation price adjustment condition, etc.

Actually, under most scholars' consideration, six other variables are selected here, namely, the underlying stock price, beta (i.e., the systematic risk measure for the stock), convertible premium rate, the yield of straight bond corresponding to the CB, the time to maturity, and the stock market index. Therefore, the seven variables consist of elements of the input vectors, which implies x is seven-dimensional. Of course, y is necessarily the CB's historical price.

As the dynamics of financial time series are strongly nonlinear, it is intuitively believed that nonlinear kernel functions could achieve better performance than the linear kernels. In this article, the RBF function is used as the kernel function of LS-SVMs, because the RBF kernel tends to give good performance under general smoothness assumptions. Consequently, it is especially useful if no additional knowledge of the data is available.

In order to make an LS-SVM model, two extra parameters: the regularization parameter, γ and the bandwidth of the RBF kernel, σ^2 (sig2) need to be determined first. By using optimization command "bay_optimize" to the training set, the two parameters can be optimized, respectively. Accounting for the disadvantage of LS-SVMs in sparsity, the least relevant support vectors must be removed in order to obtain sparsity by "sparselssvm" command. After these necessary processes, an LS-SVM combined with copula, shown in Fig. 1, is established.

3.3. *Estimation of CB Price*

Using the test set, the CB price can be estimated. The prediction performance is evaluated using the following statistical metrics, namely, the normalized mean squared error (NMSE) and the mean absolute error (MAE), described, respectively, as follows:

$$\mathrm{NMSE} = \frac{1}{\sigma^2 l}\sum_{i=1}^{l}(y_i - f(x_i))^2, \quad \mathrm{MAE} = \frac{1}{l}\sum_{i=1}^{l}|y_i - f(x_i)|,$$

where σ^2 is the normalized squared error of the data, not the same notion as the σ^2 in the radial basis function (RBF).

At the same time, for the sake of performance comparison of various models, a single LS-SVM model is also set up. The testing result is showed in Table 1, with a mark advantage in the model proposed over the single LS-SVM.

3.4. *Stress Testing and Hedging*

Stress testing and hedging ratio discussion are explored below for the deep analysis of CB value based on the integration of LS-SVM and copula function.

Table 1
The performance comparison between LS-SVM and the model based on the integration of LS-SVM with copula function

Models	NMSE	MAE
LS-SVM	0.1513	0.6734
LS-SVM integrated with copula function	0.0259	0.3711

First, scenario needs to be first described. Here, some illustrations for using copulas are given as risk estimation tools. Knowing now the chosen Clayton copulas characterizing the studied dependence structures, such a characterization to realize a scenario analysis could be used.

Only scenarios which could be unfavorable to the CB are considered. To address this problem, tremendous fluctuation in price of the underlying stock or interest rate must be taken into account, and the impact of change in dependence structure on the CB's price is also be quantified. In practice, the following probability is given:

$$P(V > v \mid U \leq u) = P(T > t \mid S \leq s) = \frac{P(V > v, U \leq u)}{P(U \leq u)} = \frac{u - C(u, v; \theta)}{u},$$

where $(u, v) \in [0, 1]^2$, (S, T) is here the bivariate vector of the underlying stock price and interest rate respectively. Moreover, (U, V) corresponds to the uniform transformation of (S, T) on the subset $[0, 1]^2$ given the observed marginals (i.e., empirically estimated on data with Deheuvels, 1981 method). The formula above implies a scenario where the underlying stock price is in declining condition accompanied with an increase in the interest rate, all of which are disadvantageous to the CB.

In particular, the following quantile-quantile dependence measure is preferred:

$$P(V > q_\alpha \mid U \leq q_\alpha) = \frac{q_\alpha - C(q_\alpha, q_\alpha; \theta)}{q_\alpha} = \alpha, \tag{9}$$

where α represents the required probability level or critical level and q_α is the related quantile.

Therefore, a scenario analysis could lead us to consider, for example, a disturbing probability level of 10% or, differently, a stress scenario or a crisis situation with a probability level of 1%. Then, with q_α calculated using Eq. (9), the λ in Eq. (4) is correspondingly changed into:

$$\lambda \approx C(q_\alpha, q_\alpha)/q_\alpha. \tag{10}$$

Correspondingly, this approach in Eq. (10) is suitable for estimating the future value of λ, in contrast to Eq. (5), and the estimated expectation value of the λ using Eq. (10) is helpful to determine the hedging ration below.

Secondly, the hedging ratio is then discussed.

According to Eq. (8), take the partial derivative of the output $f(x_i)$ with respect to the input x_{ik} (Cao et al., 2003):

$$\frac{\partial y_i}{\partial x_{ik}} = \frac{\partial(\sum_{j=1}^{l} \alpha_j k(x_j, x_i) + b)}{\partial x_{ik}} = \frac{\partial(\sum_{j=1}^{l} \alpha_j k(x_j, x_i))}{\partial x_{ik}} + \frac{\partial b}{\partial x_{ik}}$$
$$= \sum_{j=1}^{l} \alpha_j \frac{\partial k(x_j, x_i)}{\partial x_{ik}}, \quad i = 1, 2, \ldots, l; \quad k = 1, 2, \ldots, n.$$

To the Gaussian kernel $K(x_i, x) = \exp\left(-\frac{1}{\sigma^2} \sum_{l=1}^{n} (x_{il} - x_{jl})^2\right)$, then

$$\frac{\partial k(x_j, x_i)}{\partial x_{ik}} = -\frac{2}{\sigma^2}(x_{ik} - x_{jk}) \cdot \exp\left(-\frac{1}{\sigma^2} \sum_{l=1}^{n} (x_{il} - x_{jl})^2\right).$$

In this case,

$$\frac{\partial y_i}{\partial x_{ik}} = -\frac{2}{\sigma^2} \sum_{j=1}^{l} \alpha_j (x_{ik} - x_{jk}) \exp\left(-\frac{1}{\sigma^2} \sum_{l=1}^{n} (x_{il} - x_{jl})^2\right). \tag{11}$$

As can be shown in Eq. (11), the derivative of the output to the inputs can be calculated for any x_{ik}, and the value depends on the input x_{ik}, the support vector x_j as well as the converged Lagrange multipliers α_j.

It is imperative that the CB should be hedged because it suffers from the exposure to the changes in not only the underlying stock price but also the λ. This means that all the changes in the underlying stock price and the λ must be considered (Das and Sundaram, 2006).

Here, the *Delta*, δ_S, and δ_λ, are taken into account which represent the partial derivative of the convertible bond price with respect to a change in the price of the underlying stock and the λ, respectively. Suppose P to be the price of CB and S the price of the underlying stock. Now, δ_S can be expressed as below:

$$\delta_S = \frac{\partial P}{\partial S} \approx \frac{1}{l} \sum_{i=1}^{l} \left| \frac{\partial y_i}{\partial S_i} \right|,$$

where x_{ik} stands for S_i $(i = 1, 2, \ldots, l)$, with δ_S calculated using Eq. (11).

Likely, there exists:

$$\delta_\lambda = \frac{\partial P}{\partial \lambda} \approx \frac{1}{l} \sum_{i=1}^{l} \left| \frac{\partial y_i}{\partial \lambda_i} \right|.$$

Thus, according to Taylor one-order expansions of P, the ΔP can be expressed:

$$\Delta P \approx \delta_S \cdot \Delta S + \delta_\lambda \cdot \Delta \lambda.$$

Under assumptions that no transaction costs exist and position in short is permitted, the *delta*-neutral hedging strategy demands that h units of the underlying stocks should be sold short to hedge the exposure of the CB. Here, the *delta*-neutral hedging strategy must be modified to account for the change in the λ.

Table 2
The hedging performance comparison of the model proposed, based on the integration of LS-SVM with copula function, with other models including Simple regression (OLS) and Black-Scholes equation. The model proposed is listed under different scenarios, namely $\alpha = 1\%$ and $\alpha = 10\%$, with the same δ_S value

	δ_S value	δ_λ value		H value
Simple regression (OLS)	0.7321	–		76.9%
Black-Scholes equation	0.6147	–		70.3%
The model based on the integration	1.2126	$\alpha = 1\%$	0.7982	85.1%
of LS-SVM with Clayton copula		$\alpha = 10\%$	1.1727	84.7%

Thus, once the change in the CB price happens, the number of the underlying stock sold short is:

$$h = \frac{\Delta P}{\Delta S} \approx \frac{\delta_S \cdot \Delta S + \delta_\lambda \cdot \Delta\lambda}{\Delta S} = \delta_S + \frac{\Delta\lambda}{\Delta S}\delta_\lambda \approx \frac{1}{l}\left(\sum_{i=1}^{l}\left|\frac{\partial y_i}{\partial S_i}\right| + \frac{\Delta\lambda}{\Delta S} \cdot \sum_{i=1}^{l}\left|\frac{\partial y_i}{\partial \lambda_i}\right|\right),$$

where h stands for the hedging ration, which consists of two parts and considers all the changes in the price of the underlying stock and the dependence structure between it and interest rate.

The hedging effectiveness measurement is defined (Satyanarayan, 1998):

$$H = 1 - \frac{EXP_2}{EXP_1}, \tag{12}$$

where EXP_1 and EXP_2 represent the exposure of CB before and after hedging respectively.

According to the criterion given in Eq. (12), the hedging performance of the model proposed under modified *delta*-neutral hedging is listed in Table 2, which is the average of H values within the hedge period. Intuitively, the ability of the model proposed is compared with conventional methods including simple regression (e.g., OLS) (Johnson, 1960) and Black–Scholes equation (Black and Scholes, 1973). The corresponding results are also shown in Table 2.

4. Conclusions

It is a beneficial attempt to integrate SVM and copula function to analyze the value of CB and this can effectively strengthen their advantages in tackling nonlinear, nonnormal, and nonstationary financial issues. This article used the integration to deal with embedded options and their interaction which greatly influence the value of CB, and the integration conveniently made the dependence structure between the underlying stock price and interest rate into one component of the input vectors of LS-SVM, which carried out satisfactory results as illustrated in Table 1. Moreover, the model proposed helped to analyze exposure of CBs caused by changes in the underlying stock price and dependence structure between it and interest rate and accurately determinate the hedging ration just illustrated in Table 2.

As a preliminary attempt, the methodology proposed has to be improved next in two respects. First, the optimization of parameters for all the θ in copula functions and the γ and the bandwidth of the RBF kernel, σ^2, in LS-SVM should be implemented in the consistent principles and solution frameworks; otherwise, the optimization of parameters in these two tools can't achieve consistent objects. Second, other kinds of SVM should be explored to overcome the drawbacks of the LS-SVM in that the sparse solution of support vectors in LS-SVM is not satisfactory.

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