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A framework of an anisotropic elastoplastic model for clays

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ABSTRACT

A framework of an anisotropic elastoplastic model is proposed that has potential applications to different types of clays. The model adopted the form of work dissipation based yield surface of Dafalias. A surface configuration parameter is introduced to the yield surface in addition to that of the plastic potential surface. This idea was motivated by the fact that different types of clays may have a reasonably close slope of critical state line in stress space but different shear strengths. The resulting yield surface is compared with yield surface data for 17 natural clays. The overall level of agreement is satisfactory.

1. Introduction

In an elastoplastic model, an initial yield surface has to be defined that changes according to the hardening rules. In order to accurately capture the yield points along different stress paths, the assumed yield surface should have a suitable shape and size. One of the convenient ways to develop an elastoplastic model for a particular type of clay, is to obtain the yield surface by curve fitting the yield points obtained from the stress probing tests. Though the level of agreement between the test data for this particular clay and the simulations by such a model could be satisfactory, it may not perform well for a different type of clays and would have a limited range of applications. On the other hand, if there had been a form of yield surface flexible enough to fit the yield points of different types of clays, then the resulting model would have great potential for simulating the mechanical behavior of different types of clays. In this paper, such an effort in developing a model will be described.

2. Yield surface and plastic potential surface

Various forms of yield surfaces have been proposed; however, not all of them are physically reasonable. To help evaluate different forms of yield surfaces, the following two assumptions are made: (1) the yield surface has to be smooth enough so that the derivatives of the yield surface with respect to stresses at any point on the surface are definite; (2) the stress state inside the yield surface has to be physically admissible. In particular, the minimum mean stress, if tension, has to be sustainable by the clay.

The bullet-shaped yield surface of the Original Cam-clay model (Roscoe et al., 1963) is not smooth at the tip. Later, the Modified Cam-clay model (Roscoe and Burland, 1968) was developed with an elliptical yield surface, and is widely used for simulating the mechanical behavior of clays. Several well known clay models (Kavvadas, 1982; Anandarajah and Dafalias, 1986; Banerjee and Yousif, 1986; Dafalias, 1986, 1987; Kaliakin and Dafalias, 1990; Crouch and Wolf, 1992; Whittle and Kavvadas, 1994) are extensions of the Modified Cam-clay model. Some models (for example, Crouch and Wolf, 1992) adopted a shape parameter R ($\geq$2.0) in extending the Modified Cam-clay yield surface to the tension side. However, a very large R has to be adopted in order to capture the low shear strength of some clays, even for some reconstituted clays. In such cases, the maximum tensile mean stress predicted by the model could be very large and is probably not admissible for reconstituted clays which usually have little tensile strength. On the other hand, seldom has there been any model developed to simulate the tensile behavior of clays because it is of little engineering significance. These models with the parameter R were developed more for the purpose of capturing the low shear strength than that of describing the tensile strength.

There are models utilizing composite yield surfaces. In the state boundary surface model (Schofield, 1980), the Modified Cam-clay yield surface, Hvorslev rupture surface, and tensile fracture surface are combined together. The model does not allow any tension to develop and all the stress states within the yield surface are physically admissible. It suffers from the drawback that the whole surface is not smooth. The composite yield surface proposed by Dafalias and Herrmann (1986) that has two ellipses and one hyperbola is

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smooth. It uses a surface configuration parameter as adjustment at the tension side. This surface configuration parameter is difficult to be calibrated through its physical meaning, the tensile strength. The expression of the composite surface is very complicated and was later simplified as a single ellipse (Kaliakin and Dafalias, 1989).

Dafalias (1986, 1987) proposed an anisotropic elastoplastic model based on a work dissipation assumption with a contribution coupling the volumetric and deviatoric plastic strain rates. It gives a simple way to account for the effect of anisotropy through a state variable α. The previously mentioned two physical reasonings are also satisfied. Therefore, the form of Dafalias' yield surface is adopted here.

If an associative flow rule is assumed, then the parameter M (the slope of the critical state line in the mean stress-deviator stress space) is also a surface configuration parameter for the yield surface. If M is large, the predicted shear strength would be high, which is in contradiction to the fact that different types of clays may have a reasonably close M but different shear strengths. On the other hand, if the assumption of the associative flow rule is dropped and M is treated as a surface configuration parameter for the plastic potential surface only, while another parameter N is introduced as a surface configuration parameter for the yield surface, the situation is quite different. While the slope of the critical state line, M, is kept, the parameter N can be calibrated to better capture the shear strength. Physically, M represents a frictional constant at the critical state where the magnitude of the deviator stress q needed to keep the clay flowing continuously is the product of M with the effective pressure p. The parameter M only states a region (in fact a line) for the critical states in the p–q space and the final critical state point is not determined in general cases. The ultimate q, which could be higher or lower than the one predicted by the associative model, can be predicted by the nonassociative model with a suitable value of N. The parameter N, in fact, represents a measure of the shear strength over the anisotropic line. This point will be further explained.

In the current model, the same form of expression is adopted for both the plastic potential surface and the yield surface, and the two surfaces share the same anisotropic line, but the plastic potential surface has the surface configuration parameter M while the yield surface has the surface configuration parameter N. In general, M and N are not equal, and the flow rule is nonassociative. If $M = N$, it degenerates to an associative model. Usually clays behave differently on the compression side and extension side, thus $M_c(M_e)$ and $N_c(N_e)$ are adopted as model parameters.

It may be pointed out that Dafalias et al. (2002, 2006) also used the same function for the yield and plastic potential surfaces, but their main purpose was to simulate the strain softening behavior during undrained compression shearing for normally anisotropically consolidated clays by using N smaller than M. In their formulation, a constant N was used to keep the model simple. While N acts as a geometric restriction for the peak shear strength on the compression side, its meaning on the extension side is not clear. In this proposed model, N_c and N_e are used in order to simulate the shear strengths on the compression and extension sides, respectively, for different types of clays. The nonassociative flow rule can still be adopted even for clays without showing strain softening behavior.

3. Description of formulations

The purpose here is to outline the framework of developing an elastoplastic model with the previously mentioned yield surface. In what follows, all stresses are effective and compressive stresses are positive. The three stress invariants I, J_α and S_α (or I, J_α and θ) are defined in terms of the stress tensor σ_{ij} and the anisotropic tensor α_{ij} as

$$I = \sigma_{kk}, \qquad J_\alpha^2 = \frac{1}{2}s_{ij}^\alpha s_{ij}^\alpha, \qquad S_\alpha^3 = \frac{1}{3}s_{ij}^\alpha s_{jk}^\alpha s_{ki}^\alpha,$$

$$\sin(3\theta) = \frac{3\sqrt{3}}{2}\left(\frac{S_\alpha}{J_\alpha}\right)^3, \quad -\frac{\pi}{6} \le \theta \le \frac{\pi}{6} \tag{1}$$

where $s_{ij}^\alpha = s_{ij} - (1/3)I\alpha_{ij}$ is the deviatoric stress tensor with respect to the anisotropic direction, different from the deviatoric stress tensor $s_{ij} = \sigma_{ij} - (1/3)I\delta_{ij}$ which is with respect to the mean stress direction; δ_{ij} is the Kronecker delta. In the triaxial space, the mean stress p and deviator stress q are defined as

$$p = \frac{1}{3}I, \qquad q = \sqrt{3}J, \qquad J^2 = \frac{1}{2}s_{ij}s_{ij} \tag{2}$$

The initial anisotropic tensor α_{ij}^0 in the p–q space is assumed to be determined by the formation stress σ_{ij}^0:

$$\alpha_{ij}^0 = \frac{s_{ij}^0}{p^0}, \quad p^0 = \frac{1}{3}\sigma_{kk}^0, \quad s_{ij}^0 = \sigma_{ij}^0 - p^0\delta_{ij} \tag{3}$$

Clays consolidated with a constant stress ratio, such as isotropically consolidated clays, are special cases. Or α_{ij}^0 can be identified by some suitable procedure. The current anisotropic tensor α_{ij} is determined by the initial anisotropic tensor α_{ij}^0 and hardening rule for α_{ij}. The anisotropic quantity in the p–q space is $\alpha = \sqrt{(3/2)\alpha_{ij}\alpha_{ij}}$. Correspondingly, the anisotropic tensor in the I–J space is $\beta_{ij} = \alpha_{ij}/\left(3\sqrt{3}\right)$ and the anisotropic quantity is $\beta = \alpha/\left(3\sqrt{3}\right)$. The yield surface F adopted is

$$F(\sigma_{ij}, \beta_{ij}, I_0) = J_\alpha^2 - (T^2 - \beta^2)I(I_0 - I) = 0 \tag{4}$$

where I_0 represents the isotropic hardening variable, and is the value of I at the intersection of the anisotropic line and the yield surface; $T = N/\left(3\sqrt{3}\right)$ is the surface configuration parameter for the yield surface in the I–J space; and N is defined as

$$N(\theta) = \frac{2k}{(1+k) - (1-k)\sin(3\theta)}N_c, \quad k = \frac{N_e}{N_c} \tag{5}$$

The corresponding plastic potential surface is

$$G(\sigma_{ij}, \beta_{ij}, I_\alpha) = J_\alpha^2 - (S^2 - \beta^2)I(I_\alpha - I) = 0 \tag{6}$$

where $S = M/\left(3\sqrt{3}\right)$ is the surface configuration parameter for the plastic potential surface in the I–J space, and M is defined as

$$M(\theta) = \frac{2l}{(1+l) - (1-l)\sin(3\theta)}M_c, \quad l = \frac{M_e}{M_c} \tag{7}$$

The stress variable I_α has the value of I at the intersection of the anisotropic line and the plastic potential surface. As the yield surface and the plastic potential surface are related by the current stress σ_{ij} on the yield surface, I_α can be calculated as

$$I_\alpha = \frac{J_\alpha^2}{(S^2 - \beta^2)I} + I \tag{8}$$

It can be verified that both F and G have the same form of the yield surface proposed by Dafalias (1986, 1987).

In the current formulation, the internal plastic variables are I_0 and α_{ij}, evolutions of which are specified by relevant isotropic hardening rule and anisotropic hardening rule, respectively, which are open to be proposed. Geometrically, the isotropic hardening rule is used to control the size of the yield surface while the anisotropic hardening rule is to control the rotation and distortion of the yield surface.

In the next step, standard procedures of developing an elastoplastic model are followed. First, the loading index has to be defined with the plastic modulus obtained from the consistency condition for the yield surface. Then the plastic strain rate can be calculated

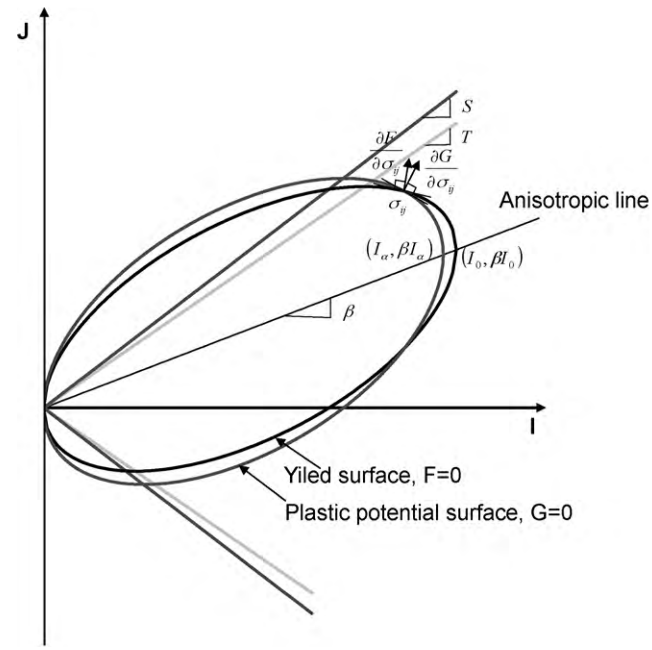

Fig. 1. Anisotropic elastoplastic model.

by the loading index and the normal of the plastic potential surface. By decomposing the total strain rate into elastic and plastic parts, and combing the expressions for the elastic and plastic strain rates, one obtains the classical form of the elastoplastic stress strain relationship.

The yield surface and the plastic potential surface share the same anisotropic line and have similar sizes, shown in Fig. 1. As a result, the direction of the plastic strain rate is not too much different from the normal direction of the yield surface. In the SANICLAY model (Dafalias et al., 2006), different anisotropic state variables are used for the yield surface and plastic potential surface, and the angle between the direction of the plastic strain rate and the normal direction of the yield surface is not small in their illustration of the model. It is also interesting to note that, for the isotropic case, the model with N smaller than M is similar to the nonassociative model studied by Banerjee and Stipho (1978) where the Original Cam-clay yield surface was adopted as the yield surface and the Modified Cam-clay yield surface as the plastic potential surface. In their study, it was found that the nonassociative model gave better simulations than the Modified Cam-clay model for a reconstituted Kaolin clay.

4. Role of *N*

The role of N may be illustrated by a simple model based on the above framework. It uses the same isotropic hardening rule as the Modified Cam-clay model, but no evolution of anisotropy is assumed for simplicity. Assume that a clay has the material parameters of critical state soil mechanics as follows: $\lambda = 0.20$, $\kappa = 0.05$, $M_c = 1.00$, $M_e = 0.80$, and $\nu = 0.30$. Undrained triaxial compression and extension tests were simulated on the clay with different overconsolidation ratios, 1.0, 1.5 and 4.0. In all cases, it is assumed that the initial anisotropy α^0 is 0.23 and the anisotropy does not evolve during the shearing tests. The ratio of the initial radial stress to the initial axial stress is 0.80. The numerical results are shown in Fig. 2, with values of N being slightly smaller than M, the same as M, and slightly larger than M. For the compression tests, $N_c = 0.95$, 1.00 and 1.05, and for the extension tests, $N_e = 0.75$, 0.80 and 0.85. The yield surface with parameters $(\alpha, N_c, N_e) = (0.23, 1.00, 0.80)$ is also plotted as reference.

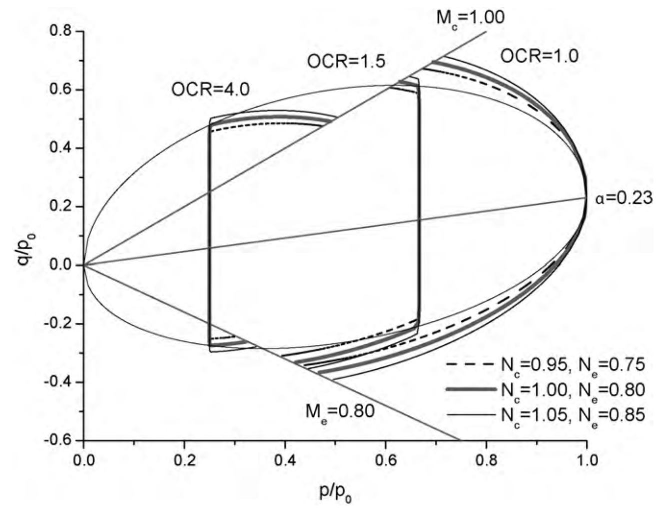

Fig. 2. Role of *N*.

It can be seen that the shear strength, higher or lower than the one predicted by the corresponding associative model, are captured by a suitable value of N, larger or smaller than M, under normally consolidated and overconsolidated conditions. In the present case, a small variation of N was selected to make the overall plotting clear. However, it can be expected that a larger difference of undrained shear strength would be obtained if N is much different from M. Compared to the models adopting the shape parameter R, the current model is more flexible and can theoretically predict the shear strength larger than that predicted by the associative model. In addition, it avoids the possible unrealistically large value of the predicted tensile strength for reconstituted clays with low shear strength, as has already been mentioned. Compared to the SANICLAY model (Dafalias et al., 2006) that has a constant N, the proposed model may be more flexible in simulating the behavior of different types of clays, especially for those with shear strength on the extension side quite different from that on the compression side. From the illustration of the current model, it may appear as a slight modification of the model proposed by Dafalias (1986, 1987), but it is capable of capturing the shear strengths of clays more accurately.

5. Comparison of yield surfaces for different types of clays

Diaz-Rodriguez et al. (1992) presented data on normalized yield surfaces for 17 different natural clays with angles of internal friction ϕ varying from 17.5° to 43°. It could be considered as one of the most comprehensive databases on the shapes of yield surface of natural clays. In determining the yield surface data, the shape and size of yield surfaces could have changed during the stress probing tests, which is not discussed here. In addition, it is assumed for simplicity that the test data are accurate in the following discussions. The properties of these natural clays are shown in Table 1, where the parameters M_c and M_e were calculated by assuming that the critical state region is a Mohr-Coulomb type failure surface with zero apparent cohesion. The parameters for fitting the yield surface data are also shown in Table 1.

The yield surface with parameters (α, N_c, N_e) and the one with (α, M_c, M_e) are compared with the yield surface data in each case, as shown in Fig. 3. The yield surface with parameters (α, N_c, N_e) is plotted with thick dark line, while the one with (α, M_c, M_e) is with thin line. In some cases, the two coincide. In a few cases, the yield surface with (α, N_c, N_c) is also plotted with dashed line for reference. Note that the yield surfaces are normalized by the vertical

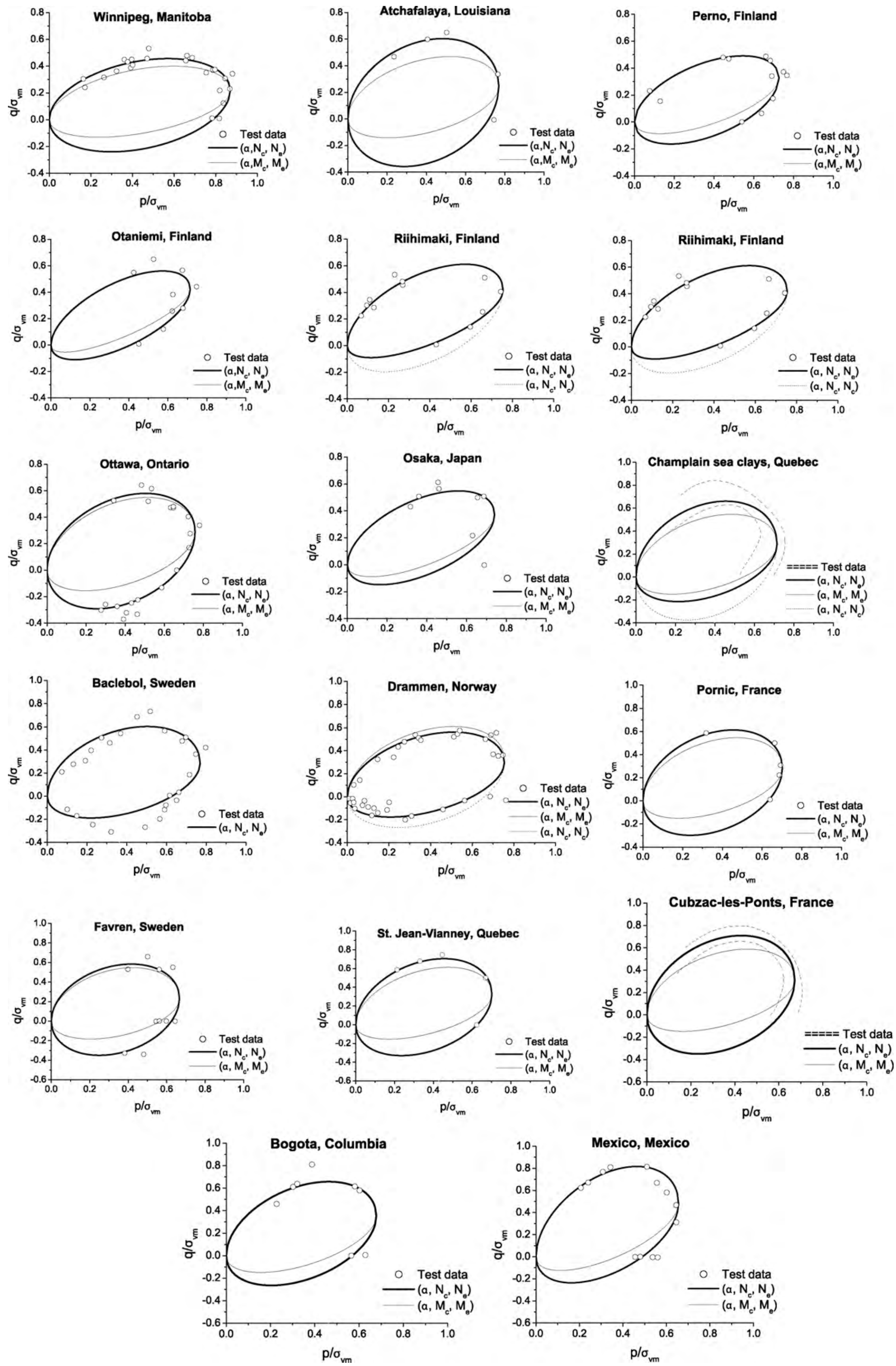

Fig. 3. Comparisons of yield surfaces for natural clays.

Table 1
Properties of natural clays and parameters for the yield surfaces (Diaz-Rodriguez et al., 1992).

Clay site	ϕ (°)	M_c	M_e	α	N_c	N_e
Winnipeg, Manitoba	17.5	0.67	0.55	0.25	0.80	0.80
Atchafalaya, Louisiana	23	0.90	0.69	0.32	1.25	1.25
Perno, Finland	23	0.90	0.69	0.45	0.90	0.90
Otaniemi, Finland	25	0.98	0.74	0.59	0.98	0.90
Riihimaki, Finland	27	1.07	0.79	0.55	1.07	0.79
St. Louis, Quebec	25	0.98	0.74	0.35	1.25	1.10
Ottawa, Ontario	27	1.07	0.79	0.38	1.15	1.15
Osaka, Japan	25	0.98	0.74	0.50	0.98	0.90
Champlain sea clays, Quebec	27–30, say, 28.5	1.13	0.82	0.40	1.45	1.00
Backebol, Sweden	30	1.20	0.86	0.37	1.20	0.86
Drammen, Norway	30	1.20	0.86	0.39	1.10	0.86
Pornic, France	29	1.16	0.83	0.40	1.35	1.25
Favren, Sweden	32	1.29	0.90	0.35	1.40	1.40
St. Jean-Vianney, Quebec	32	1.29	0.90	0.46	1.55	1.40
Cubzac-les-Ponts, France	32	1.29	0.90	0.46	1.65	1.50
Bogota, Columbia	35	1.42	0.96	0.52	1.42	1.30
Mexico, Mexico	43	1.76	1.11	0.73	1.76	1.45

preconsolidation stress σ_{vm}. The level of agreement for the overall comparison is satisfactory between the yield surface with (α, N_c, N_e) and test data. However, the yield surface with (α, M_c, M_e) is not able to fit the test data as well in most cases. In addition, the yield surface with (α, N_c, N_c) may not be as good as that with (α, N_c, N_e) when compared to the test data, such as for Riihimaki clay.

6. Additional discussions

It may be pointed out that other similar forms of yield surface could be used under this proposed model framework. However, the yield surface of Dafalias (1986, 1987) is one of the best choices. Even with the current form of yield surface, the model formulations can have some options. For example, the plastic potential surface can be chosen with $I_\alpha = I_0$. In this case, the direction of the plastic strain rate is determined by the derivatives of the plastic potential surface at the intersection of the surface with a mapping line. This mapping line connects the stress origin and the current stress point. Such a set of formulations could serve approximately the same role in simulating the clay behavior as the current one. Other variations in the formulations are also possible, such as different anisotropic variables for the yield and plastic potential surfaces, as in the SANICLAY model where possible 'hook' like undrained stress path prediction could be avoided for undrained extension shearing test on a normally anisotropically consolidated clay (Dafalias et al., 2006). In the current presentation, the model has been kept as simple, flexible, and traditional as possible, and may be viewed as a modification of the anisotropic critical state model by Dafalias (1986, 1987).

7. Conclusions

The parameter M, which specifies the critical states in stress space, does not necessarily determine the shear strength of clays. Thus, a parameter N was introduced to the yield surface to better capture the shear strength. The resulting yield surface has shown great flexibility in fitting the yield surface data for a large number of clays. Therefore, the framework of the anisotropic elastoplastic model with this yield surface has potential applications to different types of clays.

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