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A biased proportional navigation guidance law with large impact angle constraint and the time-to-go estimation

You-An Zhang[1], Guo-Xin Ma[1,2] and Hua-Li Wu[1]

Abstract

For large impact angle control problem (here, the "large impact angle" means the impact angle in the closed interval from -180° to 180°), estimating the time-to-go accurately is the key of impact time and impact angle control guidance (ITIACG). The objectives of this paper are to construct a new impact angle control guidance (IACG) law suitable for large impact angle control and present a time-to-go estimation procedure for the new IACG law suitable for designing ITIACG law. The constructed IACG law is a biased proportional navigation guidance law with large impact angle constraint, the rule of the cosine of the lead angle in the biased term is to guarantee that the lead angle remains in the open interval from -90° to 90°, which is required in the development of time-to-go estimation procedure. To estimate the time-to-go, by introducing a self-convergent angle named as alfa, the closed equations of motion are transformed to a different form, which can be solved conveniently under the assumption of small lead angle. For the case of large lead angle, the time interval of time-to-go is partitioned into *n* segments, the maximum increment of lead angle is supposed to be a small angle in each segment, the transformed closed equations of motion can be expressed as function of alfa angle and solved analytically. A geometric approach is proposed to determine conservatively a suitable alfa angle to guarantee that the maximum increment of lead angle is a small angle in each segment. The time-to-go estimation procedure for the new IACG law are illustrated. Simulations are performed to verify the effectiveness of the proposed IACG law and the accuracy of the time-to-go estimation procedure.

Keywords

Guidance law, biased proportional navigation guidance, impact angle control, impact time control, time-to-go estimation

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Introduction

The impact angle and impact time are important constraints for missile's homing problem. The impact angle control is widely used to increase the lethality of warheads. Kim et al.[1] proposed a biased proportional navigation guidance (BPNG) law for impact with angular constraint. Ryoo et al.[2] suggested a generalized form of optimal impact angle control law with an arbitrary missile system order and practical time-to-go estimation methods for implementation of the proposed guidance law. They also suggested a time-to-go weighted optimal guidance law (OGL) with impact angle constraints and a time-to-go estimation method.[3] Park et al.[4] proposed an optimal impact angle control guidance law considering the seeker's field-of-view limits for missiles with strapdown seekers. The proposed optimal guidance law is composed of three types of acceleration commands. The first command is to approach the maximum look angle of the seeker during the initial guidance phase. The second is to keep the look angle constant during the mid-guidance phase. The third is the optimal impact angle control guidance command during the final guidance phase. Lee et al.[5] developed a new impact angle control guidance (IACG) law for a stationary or slowly moving target using the high-performance sliding mode control methodology. Lee et al.[6] investigated the generalized formulation of

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weighted OGL with impact angle constraint. Under the assumptions of a stationary target and a lag-free missile with a constant speed and a small flight path angle, Lee et al.[7] demonstrated the optimality of linear time-varying guidance laws for controlling impact angles as well as terminal misses. The time-to-go calculation methods were included for implementation of the guidance law. Kim et al.[8] introduced a new IACG law for a homing missile system equipped with a passive seeker against a stationary or slowly moving target. Practical time-to-go calculation method and the maximum bounds of guidance gains were given. Lee et al.[9] proposed the time-to-go polynomial guidance (TPG) laws with impact angle control and terminal acceleration constraints for a stationary or slowly moving target. A time-to-go estimation method were also given.

Guidance laws with impact time constraints can be applied to salvo attack for anti-ship missiles or cooperative missions for unmanned aerial vehicles. By contrast, the studies on impact time control guidance (ITCG) law are relatively rare. Assuming that the heading angle and the lead angle are small, Jeon et al.[10] proposed an ITCG law and a time-cooperative guidance law[11] for anti-ship missiles. Based on the studies of Jeon et al.,[10] Sang et al.[12] developed a guidance law switching logic to maintain the seeker lock-on condition. By calculating the estimated time of arrival of the existing OGL with impact angle constraint, Arita et al.[13] applied the OGL to the engagement of designated impact time control. Zhao et al.[14] proposed the centralized and distributed coordinated algorithms based on ITCG law, which can realize cooperative attack. Zou et al.[15] proposed a distributed time-cooperative guidance law using the decentralized consensus algorithms. Zhang et al.[16] proposed a time-cooperative guidance law implemented by a leader–follower strategy.

The studies on ITCG laws with impact angle constraint (ITIACG) are much more rare.[17–21] Assuming small heading angle, Jeon et al.[17] derived an OGL to control impact time and impact angle simultaneously, where the jerk was used as the guidance command. Assuming small lead angle, Zhang et al.[18] designed an ITIACG law by adding a biased item to a BPNG law, where the lateral acceleration was used as the guidance command directly. To improve the control precision of the guidance law in the study of Jeon et al.[17] when the terminal impact angle was large, Chen et al.[19] compensated the linearized error induced from the small angle assumption by nonlinear feedback. Taking the heading angle as independent variable, a mid-course ITIACG law was derived by using optimal control theory,[20] but the obtained results contains singular solutions, as pointed by the author. Assuming that the target position was known beforehand, Harl et al.[21] presented a sliding mode based ITIACG law by introducing a line-of-sight (LOS) rate shaping process, where the ITIACG parameters must be tuned by hand or by off-line iterative routine. It comprises a feedback loop and an additional control command, the first to achieve the desired impact angle with zero miss distance, and the second to control the impact time.

To implement ITCG, ITIACG, or OGL, the time-to-go must be estimated accurately. A recursive time-to-go calculation is proposed in Tahk et al.[22] This method first calculates the minimum time-to-go and then recursively compensates the time-to-go error resulting from the path curvature. For the time-to-go estimation methods presented in the studies of Ryoo et al.,[2,3] Lee et al.,[7] Kim et al.,[8] Lee et al.,[9] Arita and Ueno,[13] Lee et al.,[17] Zhang et al.,[18] and Chen et al.,[19] the time-to-go estimation errors increase as the desired impact angle increase. A time-to-go estimation algorithm is proposed using guidance command histories by Shin et al.[23] In this method, it is assumed that the missile trajectory can be given as a polynomial function of down range. Then, time-to-go is calculated using the Taylor series expansion.

For large impact angle control problem (here, the "large impact angle" means the impact angle in the scope of $(-180^\circ, 180^\circ)$), constructing the guidance law and estimating the time-to-go accurately is the key of ITIACG law, which remains to be solved. This paper will address this problem.

Problem formulation

Consider a two-dimensional homing scenario shown in Figure 1, where the missile M has a constant speed V and the target T is stationary. R, q, θ, and φ denote the range-to-go, the LOS angle, the heading angle, and the lead angle in the inertial reference frame, respectively. The designated impact angle is represented as θ_d. From Figure 1, the following equations can be obtained

$$\dot{R} = -V\cos\varphi, \quad R\dot{q} = V\sin\varphi, \quad \dot{\theta} = a_n/V, \quad q = \theta + \varphi \tag{1}$$

where a_n is the missile's lateral acceleration, i.e., the guidance command. The initial conditions are

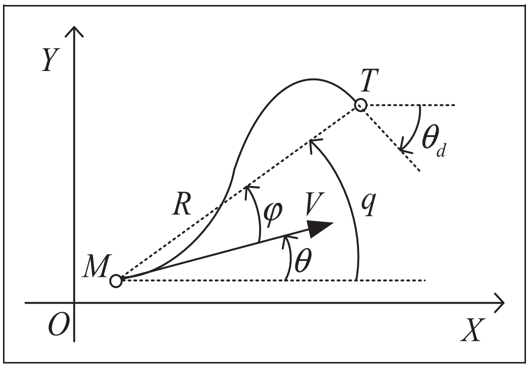

Figure 1. Homing guidance geometry.

represented as $R(t_0) = R_0$, $\varphi(t_0) = \varphi_0$, $q(t_0) = q_0$, $\theta(t_0) = \theta_0$, where t_0 is the initial time.

The true time-to-go, i.e., t_{go}, is defined as

$$t_{go} = t_f - t \tag{2}$$

where t_f represents the final time of homing, and t is the current time. t_{go} is not known in advance, its estimation is denoted as $\hat{t}_{go}$.

The objectives of this paper are to construct a new IACG law, which is suitable for large impact angle control, and present a time-to-go estimation procedure for the new IACG law, which is suitable for designing ITIACG law with large impact angle control constraint.

The guidance law

The new BPNG law with large impact angle constraint is constructed as follows

$$a_{BPNG} = NV\dot{q} - KV^2[\theta - Nq + (N-1)\theta_d]\cos\varphi/R \tag{3}$$

where the coefficients are set as $N \geqslant 3$, $K \geqslant 1$.

Note that, when φ is small, equation (3) is accordant to the result in Zhang et al.[18] Define

$$\alpha = \theta - Nq + (N-1)\theta_d, \quad \bar{\alpha} = \alpha/\alpha_0 \tag{4}$$

where $\alpha_0 = \alpha(t_0) = \theta_0 - Nq_0 + (N-1)\theta_d$.

Substituting $a_n = a_{BPNG}$ into equation (1) yields

$$dR/dt = -V\cos\varphi \tag{5a}$$

$$d\varphi/dt = -(N-1)V\sin\varphi/R + KV\alpha\cos\varphi/R \tag{5b}$$

$$d\alpha/dt = -KV\alpha\cos\varphi/R \tag{5c}$$

As is known from equation (5b), $\varphi(t)$ will always remain in the interval of $(-90^\circ, 90^\circ)$ for $\varphi_0 \in (-90^\circ, 90^\circ)$, which is required in the development of time-to-go estimation procedure. The extreme value of $\varphi(t)$ (denoted as φ_m) will satisfy $-(N-1)V\sin\varphi_m/R + KV\alpha\cos\varphi_m/R = 0$, so, $\varphi_m = \arctan(K\alpha/(N-1))$. Equation (5c) indicates that $\alpha \to 0$ monotonously from its initial value α_0 (so $\bar{\alpha}$ reduces to 0 from its initial value of 1). Equation (5b) reveals that $\varphi \to 0$ when $\alpha \to 0$. From $\alpha = \theta - Nq + (N-1)\theta_d = -N\varphi - (N-1)(\theta - \theta_d)$, it is seen that $\theta \to \theta_d$ finally. Thus, the guidance law (3) can achieve arbitrarily designated large impact angle while guaranteeing that $\varphi(t)$ remains in the interval of $(-90^\circ, 90^\circ)$ during homing.

Remark 1: If $\alpha_0 = 0$, then $\alpha \equiv 0$. The guidance law (3) reduces to the traditional PNG law. In the following discussion, it is assumed that $\alpha_0 \neq 0$.

The time-to-go estimation

The approximated analytical solution for equations (5a) to (5c)

Eliminating the time variable from equations (5b) and (5c) yields

$$d\varphi/d\alpha = B_1 \tan\varphi/\alpha - 1 \tag{6}$$

where $B_1 = (N-1)/K$.

If φ is small, then $\tan\varphi \approx \varphi$, and equation (6) reduces to a homogeneous differential equation, which can be solved easily. However, it is difficult to solve equation (6) when φ is large. Inspired by the case when φ is small, the time interval of time-to-go at $t = t_0$, i.e., $[t_0, t_f]$, is partitioned into n segments, i.e., $[t_0, t_1]$, $[t_1, t_2]$, ..., $[t_{n-2}, t_{n-1}]$, $[t_{n-1}, t_f]$, so that the maximum increment of φ in each segment is a small angle. Thus, equation (6) can be solved in each segment. Take the first segment $[t_0, t_1]$ as an example, where $t_1 = t_0 + \Delta t_1$, large $\varphi(t)$ is rewritten as

$$\varphi(t) = \varphi_0 + \Delta\varphi(t) \tag{7}$$

where $\Delta\varphi(t)$ represents the increment of $\varphi(t)$ from its initial value φ_0. Suppose that $\Delta\varphi(t)$ is a small angle for $t \in [t_0, t_0 + \Delta t_1]$, then $\Delta\varphi_0 = \Delta\varphi(t_0) = 0$. Using the first-order Taylor series expansion yields

$$\tan\varphi(t) = \tan\varphi_0 + \Delta\varphi(t)/\cos^2\varphi_0 \tag{8}$$

Differentiating equation (7) with respect to α and using equations (6) and (8) yields

$$d\Delta\varphi/d\alpha = B_1[\tan\varphi_0 + \Delta\varphi/\cos^2\varphi_0]/\alpha - 1 \tag{9}$$

To be concise, define

$$u = [\tan\varphi_0 + \Delta\varphi/\cos^2\varphi_0]/\alpha \tag{10}$$

Note that $u_0 = u(t_0) = \tan\varphi_0/\alpha_0$, equation (9) is rewritten as

$$d(\alpha u)/d\alpha = (B_1 u - 1)/\cos^2\varphi_0 \tag{11}$$

The left side of equation (11) can be expanded as

$$d(\alpha u)/d\alpha = u + \alpha du/d\alpha \tag{12}$$

From equations (11) and (12), equation (13) can be obtained

$$du/(B_2 u - B_3) = d\alpha/\alpha \tag{13}$$

where $B_2 = B_1/\cos^2\varphi_0 - 1$, $B_3 = 1/\cos^2\varphi_0$, which are guaranteed nonsingular by the cosine of the lead angle in the biased term in the guidance law (3).

To solve equation (13), two cases are investigated.

Case 1: When $B_2 = 0$, equation (13) is reduced as

$$du = -B_3 \mathrm{d}\alpha/\alpha \tag{14}$$

Integration of equation (14) on the interval of $[t_0, t] \subseteq [t_0, t_0 + \Delta t_1]$ yields

$$\int_{u_0}^{u} du = -B_3 \int_{\alpha_0}^{\alpha} \frac{1}{\alpha} \mathrm{d}\alpha \tag{15}$$

Thus

$$u = u_0 - B_3 \ln \bar{\alpha} \tag{16}$$

Substituting equation (10) into equation (16), and using the notation $\bar{\alpha} = \alpha/\alpha_0$ defined in equation (4), yields

$$\Delta\varphi(\bar{\alpha}) = B_4(\bar{\alpha} - 1) - B_5\bar{\alpha} \ln \bar{\alpha} \tag{17}$$

where $B_4 = \sin\varphi_0 \cos\varphi_0$, $B_5 = \alpha_0$.

Case 2: When $B_2 \neq 0$, equation (13) can be rewritten as

$$d(B_2u - B_3)/(B_2u - B_3) = B_2 \mathrm{d}\alpha/\alpha \tag{18}$$

Integration of equation (18) on the interval of $[t_0, t] \subseteq [t_0, t_0 + \Delta t_1]$ yields

$$\int_{B_2u_0 - B_3}^{B_2u - B_3} \frac{1}{B_2u - B_3} d(B_2u - B_3) = B_2 \int_{\alpha_0}^{\alpha} \frac{1}{\alpha} \mathrm{d}\alpha \tag{19}$$

Thus

$$u = [u_0 - B_3/B_2]\bar{\alpha}^{B_2} + B_3/B_2 \tag{20}$$

$$dt = \begin{cases} -B_6\bar{\alpha}^{1/K-1}[1 + B_7(\bar{\alpha} - 1) - B_8\bar{\alpha}\ln\bar{\alpha}]\mathrm{d}\bar{\alpha} & \text{if } B_2 = 0 \\ -B_6\bar{\alpha}^{1/K-1}[1 + B_7(\bar{\alpha}^{B_2+1} - 1) - B_8\bar{\alpha}(\bar{\alpha}^{B_2} - 1)/B_2]\mathrm{d}\bar{\alpha} & \text{if } B_2 \neq 0 \end{cases} \tag{27}$$

Substituting equation (10) into equation (20) yields

$$\Delta\varphi(\bar{\alpha}) = B_4(\bar{\alpha}^{B_2+1} - 1) - B_5\bar{\alpha}(\bar{\alpha}^{B_2} - 1)/B_2 \tag{21}$$

Hence, the solution of equation (9) can be expressed as

$$\Delta\varphi(\bar{\alpha}) = \begin{cases} B_4(\bar{\alpha} - 1) - B_5\bar{\alpha}\ln\bar{\alpha} & \text{if } B_2 = 0 \\ B_4(\bar{\alpha}^{B_2+1} - 1) - B_5\bar{\alpha}(\bar{\alpha}^{B_2} - 1)/B_2 & \text{if } B_2 \neq 0 \end{cases} \tag{22}$$

Remark 2: If $B_2 \to 0$, then $\bar{\alpha}^{B_2+1} \to \bar{\alpha}$ and $(\bar{\alpha}^{B_2} - 1)/B_2 \to \ln\bar{\alpha}$, which means that, $\Delta\varphi = \Delta\varphi(\bar{\alpha})$ is a continuous function about B_2.

Eliminating time variable from equations (5a) and (5c) yields

$$\frac{\mathrm{d}R}{R} = \frac{\mathrm{d}\alpha}{K\alpha} = \frac{\mathrm{d}\bar{\alpha}}{K\bar{\alpha}} \tag{23}$$

Integration of equation (23) on the interval of $[t_0, t] \subseteq [t_0, t_0 + \Delta t_1]$ yields

$$R(\bar{\alpha}) = R_0\bar{\alpha}^{1/K} \tag{24}$$

Equation (5c) can be rewritten as

$$\mathrm{d}t = -\frac{R}{KV\bar{\alpha}\cos\varphi}\mathrm{d}\bar{\alpha} \tag{25}$$

Using Taylor series first-order expansion at φ_0, $1/\cos\varphi$ can be expressed as

$$1/\cos\varphi \approx [1 + \tan\varphi_0\Delta\varphi]/\cos\varphi_0 \tag{26}$$

Equation (27) can be derived from equations (22), (24) to (26)

where $B_6 = R_0/[KV\cos\varphi_0]$, $B_7 = \sin^2\varphi_0$, $B_8 = \alpha_0 \tan\varphi_0$.

Integration of equation (27) on the interval of $[t_0, t_0 + \Delta t_1]$ yields

$$\int_{t_0}^{t_0+\Delta t_1} dt = \begin{cases} -B_6\int_1^{\bar{\alpha}_1}\bar{\alpha}^{1/K-1}[1 + B_7(\bar{\alpha} - 1) - B_8\bar{\alpha}\ln\bar{\alpha}]\mathrm{d}\bar{\alpha} & \text{if } B_2 = 0 \\ -B_6\int_1^{\bar{\alpha}_1}\bar{\alpha}^{1/K-1}[1 + B_7(\bar{\alpha}^{B_2+1} - 1) - B_8\bar{\alpha}(\bar{\alpha}^{B_2} - 1)/B_2]\mathrm{d}\bar{\alpha} & \text{if } B_2 \neq 0 \end{cases} \tag{28}$$

where $\bar{\alpha}_1 = \bar{\alpha}(t_1) = \bar{\alpha}(t_0 + \Delta t_1)$.

Thus

$$\Delta t_1(\bar{\alpha}_1) = \begin{cases} \begin{array}{c} B_6[K(1 - B_7)(1 - \bar{\alpha}_1^{1/K}) + KB_7(1 - \bar{\alpha}_1^{1/K+1})/(K+1) + KB_8\bar{\alpha}_1^{1/K+1}\ln\bar{\alpha}_1/(K+1) \\ +K^2B_8(1 - \bar{\alpha}_1^{1/K+1})/(K+1)^2] \end{array} & \text{if } B_2 = 0 \\ \begin{array}{c} B_6\{K(1 - B_7)(1 - \bar{\alpha}_1^{1/K}) + K(B_7 - B_8/B_2)(1 - \bar{\alpha}_1^{1/K+B_2+1})/(B_2K + K + 1) \\ +KB_8(1 - \bar{\alpha}_1^{1/K+1})/[B_2(K+1)]\} \end{array} & \text{if } B_2 \neq 0 \end{cases} \tag{29}$$

Known from equations (22), (24) and (29), $\Delta\varphi(\bar{\alpha})$ and $R(\bar{\alpha})$ are functions of $\bar{\alpha}$ which reduces to 0 from 1 monotonously, $\Delta t_1(\bar{\alpha}_1)$ is a function of $\bar{\alpha}_1$. Thus, if $\bar{\alpha}_1$ satisfies $|\Delta\varphi(\bar{\alpha})| \leqslant \Omega$, $\bar{\alpha} \in [\bar{\alpha}_1, 1]$ is determined properly, where, Ω is a given small angle, e.g., $\Omega = 10°$, then, the values of $\Delta t(\bar{\alpha}_1)$, $\Delta\varphi(\bar{\alpha}_1)$ and $R(\bar{\alpha}_1)$ can all be determined. So, a geometry approach to determine $\bar{\alpha}_1$ is given next.

A geometry approach to determine $\bar{\alpha}_1$

First, examining whether or not $\Delta\varphi = \Delta\varphi(\bar{\alpha})$ owns some extreme points except the endpoints. The first-order derivative of $\Delta\varphi$ with respect to $\bar{\alpha}$ is

$$\Delta\varphi'(\bar{\alpha}) = \begin{cases} B_4 - B_5(\ln\bar{\alpha} + 1) & \text{if } B_2 = 0 \\ (B_4 - B_5/B_2)(B_2 + 1)\bar{\alpha}^{B_2} + B_5/B_2 & \text{if } B_2 \neq 0 \end{cases} \tag{30}$$

Let $\Delta\varphi'(\bar{\alpha}) = 0$, the possible extreme point $\bar{\alpha}_m$ can be found as

$$\bar{\alpha}_m = \begin{cases} e^{B_4/B_5 - 1} & \text{if } B_2 = 0 \\ \{-B_5/[(B_4B_2 - B_5)(B_2 + 1)]\}^{1/B_2} & \text{if } B_2 \neq 0 \end{cases} \tag{31}$$

The function $\Delta\varphi = \Delta\varphi(\bar{\alpha})$ owns one extreme point for $\bar{\alpha} \in (0, 1)$ if the following condition is met

$$\begin{cases} e^{B_4/B_5 - 1} \in (0, 1) & \text{if } \quad B_2 = 0 \\ -B_5/[(B_4B_2 - B_5)(B_2 + 1)] > 0, \{-B_5/[(B_4B_2 - B_5)(B_2 + 1)]\}^{1/B_2} \in (0, 1) & \text{if } B_2 \neq 0 \end{cases} \tag{32}$$

otherwise, there is no extreme point.

Therefore, $\Delta\varphi(\bar{\alpha})$ owns at most one extreme point except the endpoints.

Second, the curve shape of $\Delta\varphi(\bar{\alpha})$ is examined. The second-order derivative of $\Delta\varphi$ with respect to $\bar{\alpha}$ is

$$\Delta\varphi''(\bar{\alpha}) = \begin{cases} -B_5\bar{\alpha}^{-1} & \text{if } B_2 = 0 \\ (B_4B_2 - B_5)(B_2 + 1)\bar{\alpha}^{B_2 - 1} & \text{if } B_2 \neq 0 \end{cases} \tag{33}$$

Equation (33) can be expressed as $\Delta\varphi''(\bar{\alpha}) = (B_4B_2 - B_5)(B_2 + 1)\bar{\alpha}^{B_2 - 1}$. The sign of $\Delta\varphi''(\bar{\alpha})$ is unchanged for $\bar{\alpha} \in (0, 1)$. This means that $\Delta\varphi'(\bar{\alpha})$ increases or decreases monotonously from its initial values, i.e., the function of $\Delta\varphi(\bar{\alpha})$ is convex. Because $\Delta\varphi(\bar{\alpha})$ owns one extreme point at most, the curve shapes of $\Delta\varphi(\bar{\alpha})$ can be classified as two cases, as shown in Figure 2 (here taking $\Delta\varphi''(\bar{\alpha}) < 0$ as example).

For the case of Figure 2(a):

(1) When $|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=\bar{\alpha}_m}| > \Omega$ (condition 1). Considering that $|\Delta\varphi'|$ reduces from its initial value $|\Delta\varphi'(\bar{\alpha})|_{\bar{\alpha}=1}|$ to 0 monotonously as $\bar{\alpha}$ goes from 1 to $\bar{\alpha}_m$, $\bar{\alpha}_1$ is conservatively chosen as $\bar{\alpha}_1 = 1 - \Omega/|\Delta\varphi'(\bar{\alpha})|_{\bar{\alpha}=1}|$, which ensures $\Omega/[1 - \bar{\alpha}_1] = |\Delta\varphi'(\bar{\alpha})|_{\bar{\alpha}=1}|$, as shown in Figure 3(a), where k_1, k_2 denote the slopes of the corresponding lines.

(2) When $|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=\bar{\alpha}_m}| \leqslant \Omega$, and $|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}| > \Omega$ (condition 2). Considering that $|\Delta\varphi'|$ increases from 0 to $|\Delta\varphi'(\bar{\alpha})|_{\bar{\alpha}=0}|$ monotonously as $\bar{\alpha}$ goes from $\bar{\alpha}_m$ to 0, $\bar{\alpha}_1$ is conservatively chosen as $\bar{\alpha}_1 = [|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}| - \Omega]\bar{\alpha}_m/|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=\bar{\alpha}_m} - \Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}|$, which ensures $[|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}| - \Omega]/\bar{\alpha}_1 = |\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}|$, as shown in Figure 3(b).

(3) When $|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=\bar{\alpha}_m}| \leqslant \Omega$, and $|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}| \leqslant \Omega$ (condition 3). Take $\bar{\alpha}_1 = 0$ directly to ensure $|\Delta\varphi(\bar{\alpha})| \leqslant \Omega$, $\bar{\alpha} \in [\bar{\alpha}_1, 1]$, as shown in Figure 3(c).

For the case of Figure 2(b):

(4) When $|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}| > \Omega$, and $|k_2| > |k_1|$, i.e., $|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}|/1 > |\Delta\varphi'(\bar{\alpha})|_{\bar{\alpha}=1}|$ (condition 4). $\bar{\alpha}_1$ is conservatively chosen as $\bar{\alpha}_1 = 1 - \Omega/|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}|$, which ensures $[|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}| - \Omega]/\bar{\alpha}_1 = |\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}|$, as shown in Figure 3(d);

(5) When $|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}| > \Omega$, and $|k_2| \leqslant |k_1|$, i.e., $|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}|/1 \leqslant |\Delta\varphi'(\bar{\alpha})|_{\bar{\alpha}=1}|$ (condition 5). $\bar{\alpha}_1$ is conservatively chosen as $\bar{\alpha}_1 = 1 - \Omega/|\Delta\varphi'(\bar{\alpha})|_{\bar{\alpha}=1}|$,

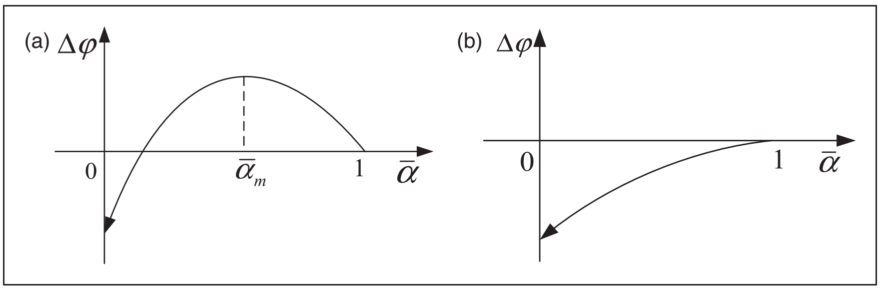

Figure 2. The curve shapes of $\Delta\varphi(\bar{\alpha})$: (a) the case with one extreme point for $\bar{\alpha} \in (0, 1)$; (b) the case with no extreme point for $\bar{\alpha} \in (0, 1)$.

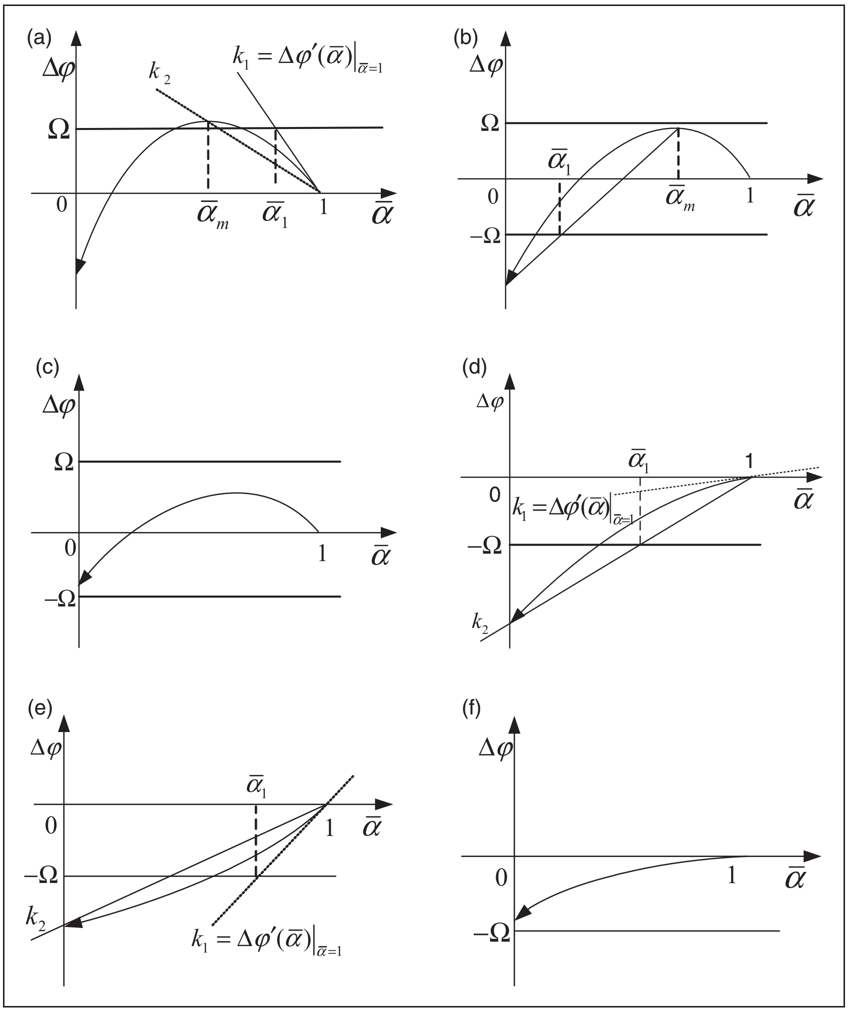

Figure 3. Geometry method for choosing $\bar{\alpha}_1$. (a) condition 1; (b) condition 2; (c) condition 3; (d) condition 4; (e) condition 5; (f) condition 6.

which ensures $\Omega/(1-\bar{\alpha}_1) = |\Delta\varphi'(\bar{\alpha})|_{\bar{\alpha}=1}|$, as shown in Figure 3(e);

(6) When $|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}| \leqslant \Omega$ (condition 6). Take $\bar{\alpha}_1 = 0$ directly to ensure $|\Delta\varphi(\bar{\alpha})| \leqslant \Omega$, $\bar{\alpha} \in [\bar{\alpha}_1, 1]$, as shown in Figure 3(f).

To summarize, $\bar{\alpha}_1$ is chosen as

$$\bar{\alpha}_1 = \begin{cases} 1-\Omega/\left|\Delta\varphi'(\bar{\alpha})|_{\bar{\alpha}=1}\right| & \text{if } \exists\, \bar{\alpha}_m, \left|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=\bar{\alpha}_m}\right| > \Omega \\ \left[\left|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}\right| - \Omega\right]\bar{\alpha}_m/\left|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=\bar{\alpha}_m} - \Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}\right| & \text{if } \exists\, \bar{\alpha}_m, \left|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=\bar{\alpha}_m}\right| \leqslant \Omega, \left|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}\right| > \Omega \\ 0 & \text{if } \exists\, \bar{\alpha}_m, \left|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=\bar{\alpha}_m}\right| \leqslant \Omega, \left|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}\right| \leqslant \Omega \\ 1-\Omega/\left|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}\right| & \text{if } \nexists\, \bar{\alpha}_m, \left|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}\right| > \Omega, \left|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}\right| > \left|\Delta\varphi'(\bar{\alpha})|_{\bar{\alpha}=1}\right| \\ 1-\Omega/\left|\Delta\varphi'(\bar{\alpha})|_{\bar{\alpha}=1}\right| & \text{if } \nexists\, \bar{\alpha}_m, \left|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}\right| > \Omega, \left|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}\right| \leqslant \left|\Delta\varphi'(\bar{\alpha})|_{\bar{\alpha}=1}\right| \\ 0 & \text{if } \nexists\, \bar{\alpha}_m, \left|\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0}\right| \leqslant \Omega \end{cases} \tag{34}$$

where $\Delta\varphi'(\bar{\alpha})|_{\bar{\alpha}=1} = B_2B_4 + B_4 - B_5$, .

$$\Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=\bar{\alpha}_m} = \begin{cases} B_4(\bar{\alpha}_m - 1) - B_5\bar{\alpha}_m \ln \bar{\alpha}_m & \text{if } B_2 = 0 \\ B_4(\bar{\alpha}_m^{B_2+1} - 1) - B_5\bar{\alpha}_m(\bar{\alpha}_m^{B_2} - 1)/B_2 & \text{if } B_2 \neq 0 \end{cases}, \quad \Delta\varphi(\bar{\alpha})|_{\bar{\alpha}=0} = -\varphi_0$$

Remark 3: If $\bar{\alpha} = \bar{\alpha}_1 = 0$, $\bar{\alpha}\ln\bar{\alpha}$ in equation (22), and $\bar{\alpha}_1^{1/K+1}\ln\bar{\alpha}_1/(K+1)$ in equation (29) must be replaced by 0 due to $\lim_{\bar{\alpha}\to 0}(\bar{\alpha}\ln\bar{\alpha}) = \lim_{\bar{\alpha}\to 0}(\ln\bar{\alpha}^{\bar{\alpha}}) = \ln 1 = 0$.

The time-to-go estimation procedure

The procedure of estimating the time-to-go at the initial time t_0 for the BPNG law (3) is illustrated as follows: for the first segment $[t_0, t_1]$, the calculated $\alpha_0 = \theta_0 - Nq_0 + (N-1)\theta_d$ and $\varphi_0 = q_0 - \theta_0$ using the measured $q(t_0) = q_0$ and $\theta(t_0) = \theta_0$ at the initial time t_0, along with $R(t_0) = R_0$ (measured at the initial time t_0), are regarded as initial conditions for solving (predicting) $\bar{\alpha}_1$ or $\alpha(t_1)$, $\Delta\varphi(\bar{\alpha}_1)$ or $\varphi(t_1)$, $R(\bar{\alpha}_1)$ or $R(t_1)$, and $\Delta t_1(\bar{\alpha}_1)$ or t_1; for the second segment $[t_1, t_2]$, the predicted $\alpha(t_1)$, $\varphi(t_1)$, and $R(t_1)$ at the predicted time t_1 are regarded as initial conditions for solving (predicting) $\alpha(t_2)$, $\varphi(t_2)$, $R(t_2)$, and $\Delta t_2 = t_2 - t_1$ or t_2; for the nth segment $[t_{n-1}, t_n]$, the predicted $\alpha(t_{n-1})$, $\varphi(t_{n-1})$ and $R(t_{n-1})$ at the predicted time t_{n-1} are regarded as initial conditions for solving (predicting) $\alpha(t_n)$, $\varphi(t_n)$, $R(t_n)$, and $\Delta t_n = t_n - t_{n-1}$ or t_n; The time-to-go estimation $\hat{t}_{go}$ at the initial time t_0 is $\hat{t}_{go} = \Delta t_1 + \Delta t_2 + \cdots + \Delta t_{n-1} + \Delta t_n$. The current time-to-go procedure is the same as the time-to-go procedure at the initial time t_0 if the current time t is regarded as the initial time t_0.

From the above time-to-go estimation procedure and the curve shapes of $\Delta\varphi(\bar{\alpha})$, it can be found that other cases for determining $\bar{\alpha}_1$ will change to the case as shown in Figure 3(c) and (f) finally with the homing going on, which guarantees the convergence of the procedure.

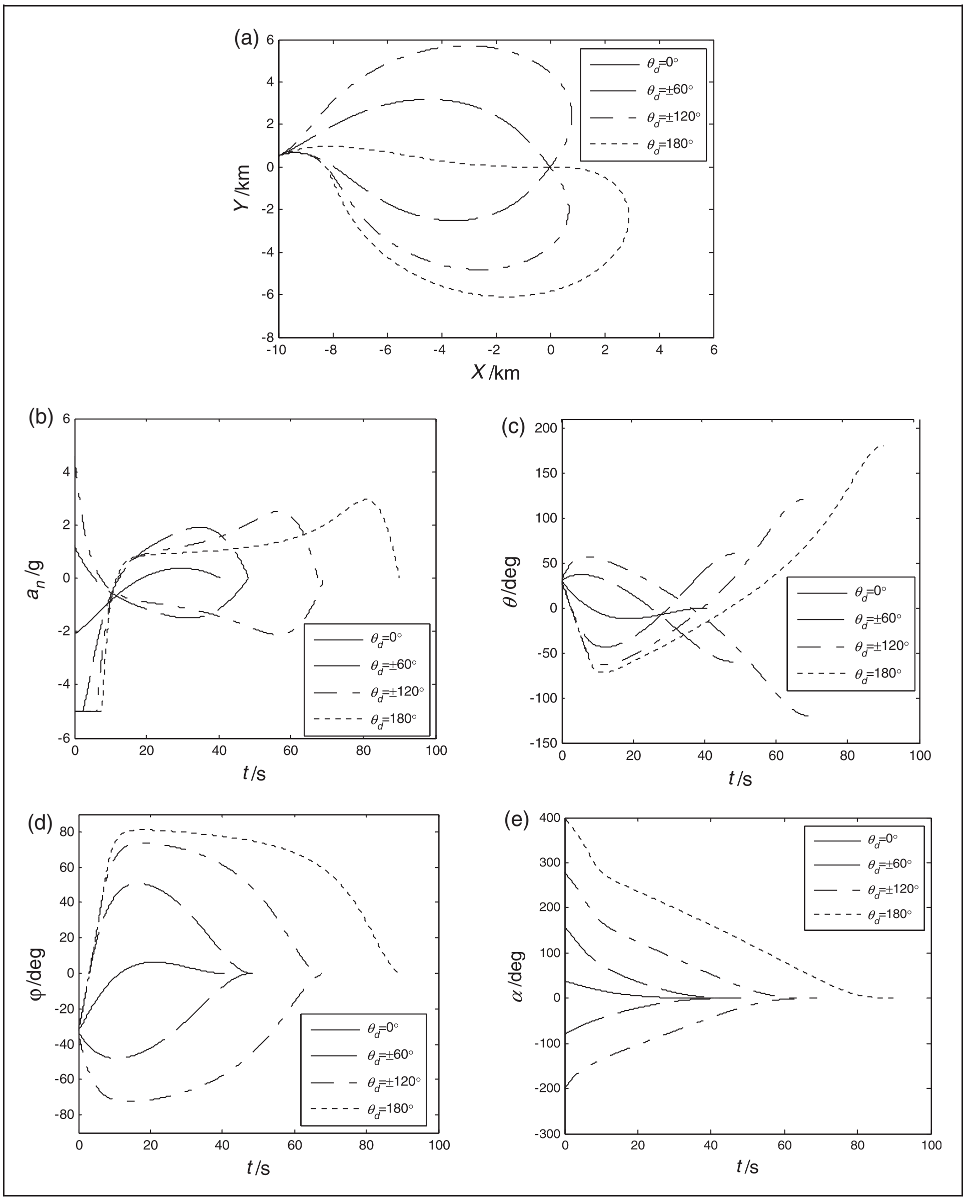

Figure 4. Simulation results for the proposed BPNG law: (a) the flight trajectories; (b) the BPNG commands; (c) the heading angles; (d) the lead angles; and (e) the α angles.

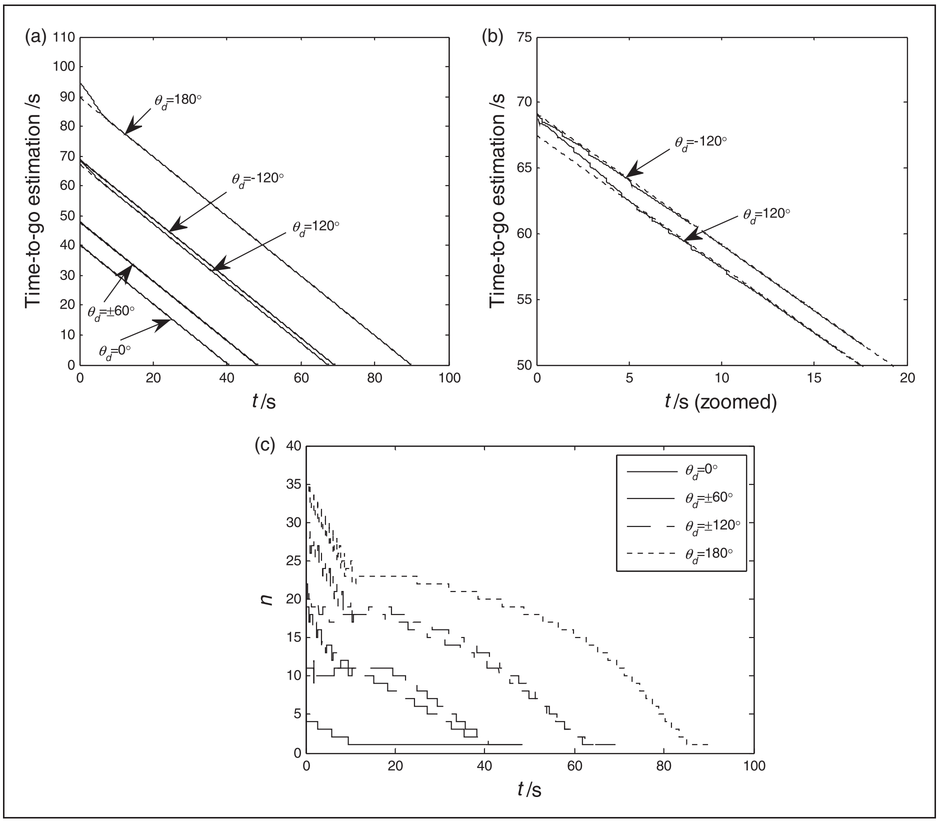

Figure 5. Calculation results of the time-to-go estimation procedure for $\Omega = 10^\circ$: (a) the time-to-go estimation; (b) the zoomed time-to-go estimation for $\theta_d = \pm 120^\circ$; (c) the number of segments during homing.

Simulation result and analysis

Simulations are performed to verify the effectiveness of the proposed BPNG law (3) and the accuracy of the time-to-go estimation procedure. In the engagement, the missile has a constant speed of 250 m/s and the target is a stationary ship. The initial positions of the missile and the target are set to be (−10, 0.5) km and (0, 0) km respectively, and the initial heading angle of the missile is 30°. The missile is guided by the proposed BPNG law (3) with a limited normal acceleration of 5g. The parameters are chosen as $N = 3$, $K = 3$. The simulation step is chosen as 0.01 s. The terminal impact angle constraints are given as 0°, ±60°, ±120°, and 180°, respectively.

The trajectories are shown in Figure 4. Simulation results show that the proposed BPNG law can satisfy the requirements for different impact angles (especially large impact angles). All the lead angles always remain in the interval of $(-90^\circ, 90^\circ)$ for $\varphi_0 \in (-90^\circ, 90^\circ)$. All the α angles approach to zero monotonously from different initial values α_0. All the heading angles approach to the desired heading angles finally.

The calculation results of the time-to-go estimation procedure for the specified constant small angle $\Omega = 10^\circ$ are shown in Figure 5. In Figure 5(c), n is the number of segments.

Figure 5(a) shows that the curves of time-to-go estimation $\hat{t}_{go}$ (the solid lines) and the curves of true time-to-go t_{go} (the dotted lines, computed from $t_f - t$) coincide nearly for the cases of $\theta_d = 0^\circ$ and $\theta_d = \pm 60^\circ$. Generally speaking, the largest time-to-go estimation error occurs at the initial time, hence, to demonstrate the accuracy of the proposed time-to-go estimation procedure, the time-to-go estimation errors (absolute value) at the initial time are given as 0.09 s, 0.23 s, 0.15 s, 1.3 s, 0.13 s, and 4.75 s for $\theta_d = 0^\circ$, $\pm 60^\circ$, $\pm 120^\circ$, and 180°, respectively. Note that, for the largest impact angle case, $\theta_d = 180^\circ$, the largest time-to-go estimation error is 4.75 s (the relative error is 5.3%), this is the worst case, the flight time is also the longest.

Figure 5(c) shows the number of segments during homing. Obviously, since the fight time is the longest for $\theta_d = 180^\circ$, the initial value of n is the maximum (less than 35 segments). For all the impact angle cases, the number of segments n go to 1 finally, which means that $\Delta\varphi$ maintains small in the last stage of homing, and the time-to-go estimations are calculated directly without division ($[t, t_f]$ as one segment finally).

Figure 6 shows the calculation results of the time-to-go estimation procedure for $\Omega = 20^\circ$ and 30°, respectively (take $\theta_d = 120^\circ$ as an example). It can be seen that the largest time-to-go estimation errors increased to 1.47 s and 2.95 s for $\Omega = 20^\circ$ and 30° respectively from 1.3 s for $\Omega = 10^\circ$, but the number of segments n decreased greatly (nearly halved for

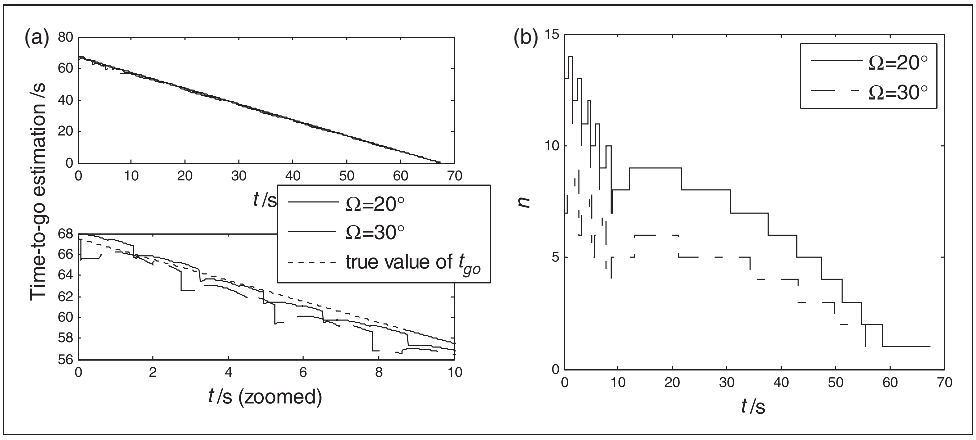

Figure 6. Calculation results of the time-to-go estimation procedure for $\Omega = 20^\circ$ and 30°, respectively ($\theta_d = 120^\circ$): (a) estimated time-to-go; and (b) the number of segments during homing.

$\Omega = 20^\circ$ relative to $\Omega = 10^\circ$), which is favorable for alleviating the burden of calculation.

Conclusion

In this paper, a new IACG law, which is suitable for large impact angle control, is constructed. The proposed IACG law is in nature a BPNG law, the cosine of the lead angle in the biased term can guarantee the lead angle remains in the interval of $(-90^\circ, 90^\circ)$, which is required in the development of time-to-go estimation procedure. The time-to-go estimation procedure for the new IACG law is illustrated. The time interval of time-to-go is partitioned into n segments to make the maximum increment of lead angle in each segment a small angle, and the transformed closed equations of motion are expressed as function of alfa angle and solved analytically. A geometric approach is proposed to determine conservatively a suitable alfa angle to guarantee that the absolute value of the maximum increment of lead angle is lesser than or equal to a specified constant small angle which makes the approximated analytical solution aforementioned reasonable. Simulation results show that the proposed IACG law can satisfy the requirements for different impact angles (especially large impact angles), and the performance of the proposed time-to-go estimation procedure is satisfactory. If the specified constant small angle doubles, the time-to-go estimation error will increase, while the number of segments n will halve nearly, which is favorable for alleviating the burden of calculation. The proposed BPNG law and the time-to-go estimation procedure can be applied to the design of ITIACG law for salvo attack, which will be the subject for a follow-up paper.

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Conflict of interest

None declared.

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Appendix

Notation

a_n	the missile's lateral acceleration, i.e., the guidance command
a_{BPNG}	the missile's lateral acceleration produced by the new BPNG law
B_i	a notation defined as $B_1 = (N-1)/K$, $B_2 = B_1/\cos^2\varphi_0 - 1$, $B_3 = 1/\cos^2\varphi_0$, $B_4 = \sin\varphi_0\cos\varphi_0$, $B_5 = \alpha_0$, $B_6 = R_0/[KV\cos\varphi_0]$, $B_7 = \sin^2\varphi_0$, $B_8 = \alpha_0\tan\varphi_0$ respectively for $i = 1, 2, 3, \ldots, 8$
k_1, k_2	the slopes of the corresponding lines in Fig.3.
K	a coefficient in the new BPNG law
n	the number of segments during homing
N	a coefficient in the new BPNG law
q	the LOS angle
R	the range-to-go, or missile-target relative distance
t	the current time
t_f	the final time of homing
t_{go}	the true time-to-go
$\hat{t}_{go}$	the estimated time-to-go
t_1	the end point of the first segment $[t_0, t_1]$, i.e., $t_1 = t_0 + \Delta t_1$
Δt_1	the length of the first segment $[t_0, t_1]$
u	a variable defined as $u = [\tan\varphi_0 + \Delta\varphi/\cos^2\varphi_0]/\alpha$
V	constant missile speed
XOY	the inertial reference frame
α	an angle defined as $\alpha = \theta - Nq + (N-1)\theta_d$
$\bar{\alpha}$	normalized α, i.e.,$\bar{\alpha} = \alpha/\alpha_0$
$\bar{\alpha}_1$	the value of $\bar{\alpha}$ at t_1, i.e.,$\bar{\alpha}_1 = \bar{\alpha}(t_1) = \bar{\alpha}(t_0 + \Delta t_1)$.
$\bar{\alpha}_m$	the value of $\bar{\alpha}$ at the extreme point of $\Delta\varphi(t)$
θ	the heading angle
θ_d	the designated impact angle
φ	the lead angle
φ_m	the extreme value of $\varphi(t)$
$\Delta\varphi(t)$	the increment of $\varphi(t)$ from its initial value φ_0 in the first segment $[t_0, t_1]$.
$\Delta\varphi'$	the first-order derivative of $\Delta\varphi$ with respect to $\bar{\alpha}$
$\Delta\varphi''$	the second-order derivative of $\Delta\varphi$ with respect to $\bar{\alpha}$
Ω	a given small angle
subscript 0	the initial value of the corresponding variable, for example, t_0: the initial time; R_0, φ_0, $q_0$and θ_0: the initial conditions for R, φ, $q$and θ respectively, i.e.,$R(t_0) = R_0$, $\varphi(t_0) = \varphi_0$, $q(t_0) = q_0$, and $\theta(t_0) = \theta_0$; α_0: initial α, i.e., $\alpha_0 = \alpha(t_0)$