
Volume 51, Issue 8, December 2011
ISSN 0041-624X
ELSEVIER
Ultrasonics
http://www.elsevier.com/locate/ultras
Available online at www.sciencedirect.com
ScienceDirect

Ultrasonics 51 (2011) 853–856

Contents lists available at ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

Short Communication

Dispersion characteristics of transverse surface waves in piezoelectric coupled solid media with hard metal interlayer

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A R T I C L E   I N F O

Article history:
Received 20 April 2011
Accepted 9 June 2011
Available online 22 June 2011

Keywords:
Transverse surface wave
Piezoelectric coupled solids
Hard metal interlayer
Dispersion relation

A B S T R A C T

The propagation of transverse surface waves in a piezoelectric layer/metal substrate system with one or multiple hard metal interlayer(s) is investigated analytically. The general dispersion equations for the existence of the waves are obtained in a simple mathematic form for class 6 mm piezoelectric materials. The presence of a hard metal interlayer can not only get rid of the undesired mode appearing in the case without an interlayer but shorten the existence range of the phase velocity within which a nonleaky but dispersive mode exists. The effects of the hard interlayer on the phase velocity can be used to manipulate the behavior of the waves and has implications in acoustic wave devices.

1. Introduction

Wave propagation and vibration in a pure piezoelectric plate have received considerable attention previously as exhibited by the work of Mindlin [1], Tiersten [2] and Bleustein [3]. Nowadays, the study of piezoelectric devices over the last three decades spans from a simple plate model to piezoelectric coupled structure model [4–6]. Transverse surface waves (such as the Love wave and the Bleustein–Gulyaev (B–G) wave) in piezoelectric coupled materials and structures are attractive for designing signal-processing devices due to their high performance and simple particle motion [7]. More information on transverse surface waves can refer to our previous work [8] and the references therein.

Much earlier, Curtis and Redwood [9] carried out a theoretical study on the propagation of transverse surface waves in a piezoelectric material carrying a metal layer of finite thickness. Such a wave is related both to the B–G wave and the Love wave, and hence would have great importance in practical applications. We studied previously the transverse surface waves in metal materials coated with a piezoelectric layer of finite thickness [8]. One interesting mode of wave propagation appears when the bulk-shear-wave velocity in the piezoelectric layer is greater than that in the metal substrate. In this paper, we carry out the propagation of transverse surface waves in a piezoelectric layer/metal substrate system with a hard metal interlayer of finite-thickness, with the aim to get rid of the undesired mode appearing in our previous work [8] and to provide a new parameter for designing acoustic wave devices. Furthermore, general expressions for the dispersion relations of transverse surface waves in piezoelectric layer/metal substrate with multiple metal interlayers are given, which can be used to conduct further research work in this field and has implications in the design of acoustic wave devices.

2. Statement of the problem

Consider a piezoelectric layer/metal substrate system with a hard metal interlayer of finite-thickness h_1, occupying the half-space $x > h_0$, as shown in Fig. 1. Suppose the piezoelectric layer of uniform thickness h_0 is deposited perfectly on the hard metal interlayer, which results in a surface at $x = -h_0$ free of external forces. Here the piezoelectric material is taken to be of class 6 mm (or ∞ m), with its polar axis oriented along the z direction of Cartesian coordinates (x, y, z). It is assumed that the waves propagate in the positive direction of the y-axis, such that the nonzero field quantities representing the motion are only functions of (x, y) and time t.

Let w and φ denote separately the mechanical displacement and electrical potential function in the piezoelectric layer. Following Bleustein [3], the coupled field equations are given by

$$\left.\begin{aligned} &\nabla^2 w - (1/c_p^2)\ddot{w} = 0 \\ &\nabla^2[\varphi - (e_{15}/\varepsilon_{11})w] = 0 \end{aligned}\right\} \tag{1}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, and $c_p[(c_{44} + e_{15}^2/\varepsilon_{11})/\rho]^{1/2}$ is the bulk-shear-wave velocity in the piezoelectric material, with c_{44}, e_{15}, ε_{11} and ρ representing the elastic, piezoelectric, dielectric constants and mass density, respectively.

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doi:10.1016/j.ultras.2011.06.005

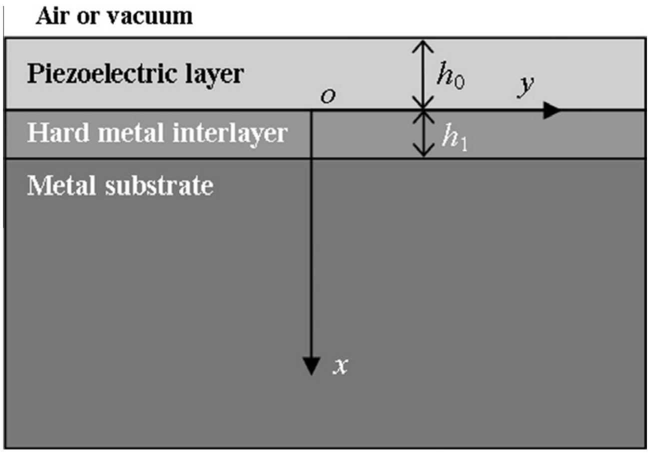

Fig. 1. A piezoelectric layered structure with one hard metal interlayer.

For the hard metal interlayer, let w_1 denote its mechanical displacement in the z direction. The governing field equation is

$$\nabla^2 w_1 - (1/c_{m1}^2)\ddot{w}_1 = 0 \tag{2}$$

where $c_{m1} = (\mu_1/\rho_1)^{1/2}$ is the bulk-shear-wave velocity in the interlayer, with μ_1 and ρ_1 representing separately the shear modulus and mass density.

Similarly, for the metal substrate, let w_2 denote its mechanical displacement in the z direction. The governing field equation is

$$\nabla^2 w_2 - (1/c_{m2}^2)\ddot{w}_2 = 0 \tag{3}$$

where $c_{m2} = (\mu_2/\rho_2)^{1/2}$ is the bulk-shear-wave velocity in the metal substrate, with μ_2 and ρ_2 representing separately the shear modulus and mass density.

The wave propagation problem specified by (1)–(3) should satisfy the following boundary and continuity conditions: (1) $\sigma_{zx} = 0$ at $x = -h_0$; (2) $w = w_1$, $\sigma_{zx} = \sigma_{zx}^1$, $\varphi = 0$ at $x = 0$; (3) $w_1 = w_2$, $\sigma_{zx}^1 = \sigma_{zx}^2$, at $x = h_1$; (4) $w_2 \to 0$ as $x \to +\infty$. The electrical conditions at the free surface can be classified into two categories, i.e., (5) electrically open circuit: $D_x = 0$ at $x = -h_0$ and (6) electrically short circuit (or metalized surface): $\varphi = 0$ at $x = -h_0$, based on the fact that the space above the piezoelectric layer is vacuum or air and its permittivity is much less than that of the piezoelectric material.

3. Dispersion relations

Built upon a previous work [8], we consider the following transverse waves satisfying attenuation condition (4):

$$\left.\begin{aligned} w(x,y,t) &= (A_1 e^{-bkx} + A_2 e^{bkx})\exp[ik(y-ct)] \\ \varphi(x,y,t) &= \left[A_3 e^{-kx} + A_4 e^{kx} + \tfrac{e_{15}}{\varepsilon_{11}}(A_1 e^{-bkx} + A_2 e^{bkx})\right]\exp[ik(y-ct)] \end{aligned}\right\}, \quad -h_0 \leqslant x \leqslant 0 \tag{4}$$

$$w_1 = (A_5 e^{-b_1 kx} + A_6 e^{b_1 kx})\exp[ik(y-ct)], \quad 0 \leqslant x \leqslant h_1 \tag{5}$$

$$w_2 = A_7 e^{-b_2 kx}\exp[ik(y-ct)], \quad x \geqslant h_1 \tag{6}$$

where $A_1, A_2, A_3, A_4, A_5, A_6$ and A_7 are arbitrary constants, $k(=2\pi/\lambda)$ is the wave number, λ is the wavelength, $i = \sqrt{-1}$, and c is the phase velocity. (4)–(6) satisfy separately Eqs. (1)–(3) if:

$$b^2 = 1 - c^2/c_p^2, \quad b_1^2 = 1 - c^2/c_{m1}^2, \quad b_2^2 = 1 - c^2/c_{m2}^2 \tag{7}$$

Substitution of (4)–(6) and the corresponding stress components into the remaining boundary and continuity conditions (1), (2), (3) and (5) or (6) yields seven linear, homogeneous algebraic equations for coefficients A_1 through A_7. The existence condition of nontrivial solutions of these coefficients leads to the following dispersion relations of the transverse surface waves described by (4)–(6)

$$k_p^2\tanh(2\pi H) - b\tanh(2\pi Hb) - \frac{\mu_1 b_1}{\bar{c}_{44}}\frac{\mu_1 b_1\tanh(2\pi Hb_1 h_r) + \mu_2 b_2}{\mu_1 b_1 + \mu_2 b_2\tanh(2\pi Hb_1 h_r)} = 0 \tag{8}$$

for the case of electrically open circuit, and

$$\begin{aligned} &(k_p^4 + b^2)\tanh(2\pi H)\tanh(2\pi Hb) \\ &\quad + 2k_p^2 b\left[\frac{1}{\cosh(2\pi H)\cosh(2\pi Hb)} - 1\right] \\ &\quad + \frac{\mu_1 b_1}{\bar{c}_{44}}\frac{\mu_1 b_1\tanh(2\pi Hb_1 h_r) + \mu_2 b_2}{\mu_1 b_1 + \mu_2 b_2\tanh(2\pi Hb_1 h_r)}[b\tanh(2\pi H) \\ &\quad - k_p^2\tanh(2\pi Hb)] = 0 \end{aligned} \tag{9}$$

for the case of electrically short circuit, respectively.

In Eqs. (8) and (9), $H = h_0/\lambda, h_r = h_1/h_0$; and $k_p^2 = e_{15}^2/\varepsilon_{11}\bar{c}_{44}$ is the piezoelectric coupling factor in the piezoelectric material with $\bar{c}_{44} = c_{44} + e_{15}^2/\varepsilon_{11}$ being the piezoelectrically stiffened elastic constant [8,9].

One observation can be made that Eqs. (8) and (9) degenerate exactly to the results obtained in our previous work [8] when the interlayer does not exist, i.e., $h_r = 0$. This serves to demonstrate the validity of the present mathematic formulation to some extent. And for the case that there are two interlayers between the piezoelectric layer and the substrate, as shown in Fig. 2. The dispersion relations can be obtained readily by replacing $\mu_2 b_2$ in Eqs. (8) and (9) with the expression defined as

$$\mu_2 b_2 \frac{\mu_2 b_2\tanh(b_2 kh_2) + \mu_3 b_3}{\mu_2 b_2 + \mu_3 b_3\tanh(b_2 kh_2)} \tag{10}$$

where μ_2, b_2 and h_2 denote the corresponding quantities of the second interlayer, and μ_3, b_3 denote those of the substrate. The detailed process to obtain Eq. (10) is omitted here for brevity. It is thus concluded that the dispersion relations for multiple-interlayer case can be deduced by analog. In order to study the effects of the hard metal interlayer on the transverse surface waves, we only discuss the case when only one interlayer exists in the following numerical examples for the sake of simplicity.

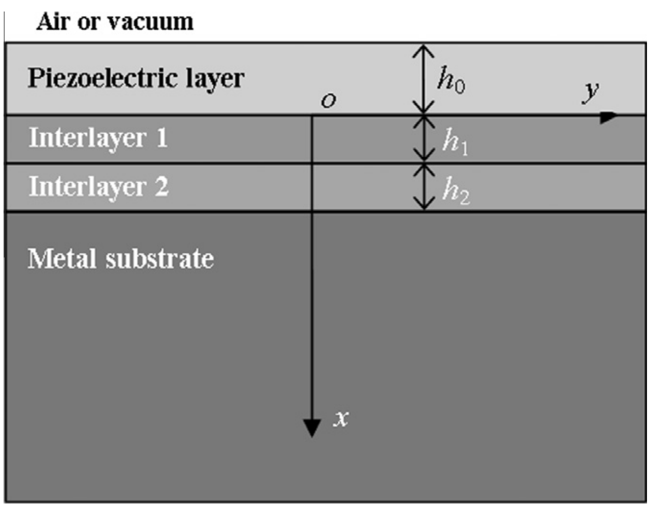

Fig. 2. A piezoelectric layered structure with two hard metal interlayers.

4. Numerical examples

Based on the previous work [8], two physical situations are possible for transverse surface waves in the piezoelectric layer/metal substrate system without the interlayer, i.e., $h_r = 0$: (1) Type 1: $c_{m2} > c_p > c_{BG}$, b real or imaginary, (2) Type 2: $c_p > c_{m2} > c_{BG}$, b always real. Here, $c_{BG} = c_p(1 - k_p^4)^{1/2}$ is the phase velocity of the B–G wave on the surface of a piezoelectric substrate coated with an infinitely thin layer of conducting material. The Type 1 wave is a kind of Love wave, which has been well known. We thus focus on the Type 2 wave and study the effects of the appearance of the hard metal interlayer on the wave propagation behavior.

The dispersion curves plotted for selected values of thickness ratio h_r are illustrated in Figs. 3–5, corresponding to a PZT-4/steel/zinc system for the cases of both electrically open circuit and electrically short circuit, respectively. In order to clearly show the effect of the hard metal interlayer on the second mode in the case of electrically short circuit, we individually plot in Fig. 5 the second mode for selected values of thickness ratio h_r different from those in Figs. 3 and 4. The material parameters for both the metal and the piezoelectric material are taken from our previous work [8]. For comparison, in each plot, the dispersion curves in the case without interlayer (i.e., $h_r = 0$) are included. It is readily seen from the three plots that the appearance of the hard metal interlayer has a significant effect on the dispersion curves, regardless of the electrically boundary conditions.

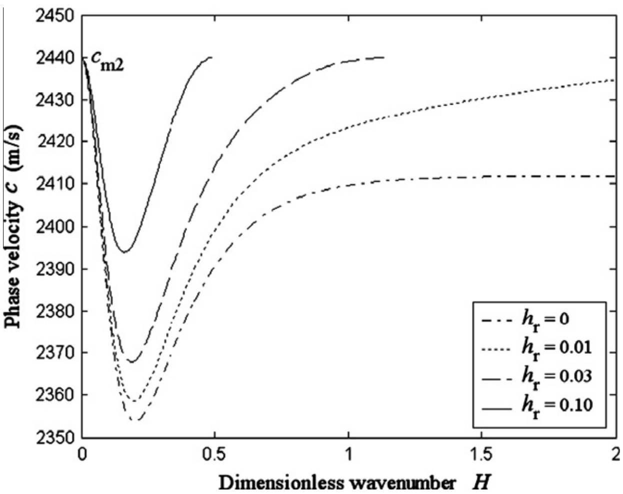

Fig. 3. Phase velocity c of the transverse surface waves in a PZT-4/steel/zinc system as a function of $H = h/\lambda$ for selected values of thickness ratio h_r in electrically open case; c_{m1} = 3281 m/s, c_{m2} = 2440 m/s, c_p = 2597 m/s, c_{BG} = 2258 m/s.

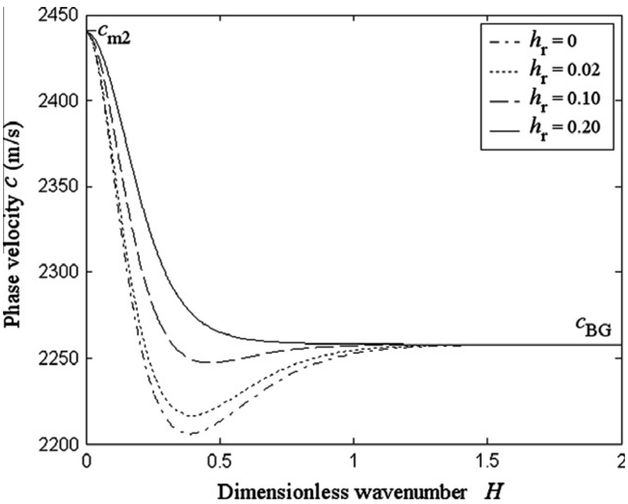

Fig. 4. Phase velocity c of the transverse surface waves in a PZT-4/steel/zinc system as a function of $H = h/\lambda$ for selected values of thickness ratio h_r for the first mode in electrically short case; c_{m1} = 3281 m/s, c_{m2} = 2440 m/s, c_p = 2597 m/s, c_{BG} = 2258 m/s.

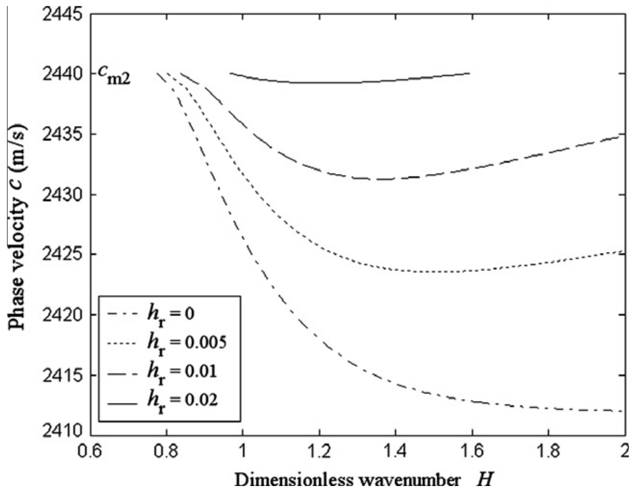

Fig. 5. Phase velocity c of the transverse surface waves in a PZT-4/steel/zinc system as a function of $H = h/\lambda$ for selected values of thickness ratio h_r for the second mode in electrically short case; c_{m1} = 3281 m/s, c_{m2} = 2440 m/s, c_p = 2597 m/s, c_{BG} = 2258 m/s.

In the case of electrically open circuit, it can be seen from Fig. 3 that the main changes induced by the appearance of the hard metal interlayer consist in the minimum velocity value and the wavenumber range for the existence of the mode. For the case without the hard metal interlayer (i.e., $h_r = 0$), the phase velocity tends to the B–G wave velocity under electrically open circuit as the dimensionless wavenumber approaches infinity, which means that the mode exists for all the wavenumber values. With the increase in the thickness ratio h_r, the wavenumber range for the existence of the mode is gradually shortened to a limited interval with its starting and end values corresponding to the bulk-shear-wave velocity in the metal substrate. However, the wavenumber corresponding to the minimum velocity almost does not change at all with the increase in the thickness ratio, which is an interesting phenomenon and appears new. That feature has the potential application to increase the operating frequency of acoustic devices at some circumstance. It will be noted that the total change in phase velocity produced in the case of electrically open circuit is relatively small, compared with that in the case of electrically short circuit. And the appearance of the hard metal interlayer makes the difference more obvious.

In the case of electrically short circuit, it can be seen from Figs. 4 and 5 that the thickness ratio affects the two modes in different ways. For the first mode, the main influence induced by the appearance of the hard metal interlayer is the dispersive type, i.e., the mode is changed gradually from partly normal dispersion [10] plus partly anomalous dispersion [10] to totally normal dispersion, as shown in Fig. 4. When the thickness ratio $h_r \geqslant 0.2$, the local minimum phase velocity in the low frequency range of the first mode disappears, i.e., the mode starts from the bulk-shear-wave velocity c_m (2440 m/s) of the metal substrate and decreases monotonically to the B–G wave velocity (2260 m/s) under electrically short circuit. For the second mode which does not have a counterpart in the case of electrically open circuit, however, it is clearly seen from Fig. 5 that even very small thickness ratio affects the mode significantly. The wavenumber range for the existence of the second mode is quickly shortened by the appearance of the hard metal interlayer. This feature can be used to remove the second mode (undesired in practical applications) in the case of electrically short circuit by using a very thin hard metal interlayer, for example, one can set the thickness ratio $h_r = 0.03$.

5. Conclusion

In summary, the theory of the propagation of transverse surface waves in a piezoelectric layer/metal substrate system with one or multiple hard metal interlayer(s) has been developed. The presence of the hard metal interlayer manipulates the wavenumber range for the existence of the mode in the case of electrically open circuit and the minimum phase velocity. In the case of electrically short circuit, the appearance of the hard metal interlayer changes the dispersive curve of the first mode from partly normal dispersion plus partly anomalous dispersion into totally normal dispersion. And the most promising feature consists in the fact that even a very thin hard metal interlayer can remove the undesired high-order mode which does not have a counterpart in the case of electrically open circuit. The study extends the regime of transverse surface waves and may provide possibilities for potential applications to acoustic wave devices.

Acknowledgment

The author (Z.H. Qian) gratefully acknowledges the support by the Global COE Program at Tokyo Institute of Technology. One of the authors (F. Jin) gratefully acknowledges the support by the National Natural Science Foundation of China (No. 10972171) and the Program for New Century Excellent Talents in Universities (No. NCET-08-0429).

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