International Journal of Bifurcation and Chaos, Vol. 20, No. 7 (2010) 2203–2211

DOI: 10.1142/S0218127410027039



# MULTISCROLL CHAOTIC ATTRACTORS FROM A MODIFIED COLPITTS OSCILLATOR MODEL

BOCHENG BAO
*Department of Electronic Engineering,*
*Nanjing University of Science and Technology,*
*Nanjing 210094, P. R. China*
*School of Electrical and Information Engineering,*
*Jiangsu Teachers University of Technology,*
*Changzhou 213001, P. R. China*
*mervinbao@126.com*

GUOHUA ZHOU* and JIANPING XU†
*School of Electrical Engineering,*
*Southwest Jiaotong University,*
*Chengdu 610031, P. R. China*
**yichi331@163.com*
*†jpxu-swjtu@163.com*

ZHONG LIU
*Department of Electronic Engineering,*
*Nanjing University of Science and Technology,*
*Nanjing 210094, P. R. China*
*eezliu@mail.njust.edu.cn*

Received July 7, 2009; Revised December 4, 2009

A simple approach for generating $(2N+1)$-scroll chaotic attractor from a modified Colpitts oscillator model is proposed in this paper. The key strategy is to increase the number of index-2 equilibrium points by introducing a triangle function to directly replace the nonlinearity term of Colpitts oscillator model. The dynamical characteristics of the new multiscroll chaotic system are studied comprehensively. A circuit realization structure is introduced and the experimental results demonstrate that $(2N+1)$-scroll chaotic attractors can be obtained in practical circuit.

*Keywords*: Colpitts oscillator model; equilibrium point; multiscroll chaotic system; triangle function.

## 1. Introduction

Due to the practical applications of scroll chaotic system as broadband signal generator and true pseudorandom number generator for various chaos-based communication engineering [Lü & Chen, 2006; Yalcin, 2007], the generation of multiscroll chaotic attractors is of much interest both theoretically and practically.

Multiscroll chaotic attractor was first observed by Suykens and Vandewalle in the early 1990s [Suykens & Vandewalle, 1993]. Afterwards, Yalcin *et al.* introduced a family of one-directional $n$-scroll, two-directional $n \times m$-grid scroll and three-directional $n \times m \times l$-grid scroll chaotic attractors by using step functions [Yalcin *et al.*, 2002]. Lü *et al.* presented a switching manifold technique for creating chaotic attractors with multiple-merged basins of attraction [Lü *et al.*, 2003] and proposed saturated function series [Lü *et al.*, 2004a], hysteresis function series [Lü *et al.*, 2004b] and threshold control [Lü *et al.*, 2008] methods for generating multidirectional $n$-scroll, $n\times m$-grid scroll and $n\times m\times l$-grid scroll chaotic attractors with rigorous mathematical proofs and experimental verifications [Lü *et al.*, 2006]. Yu *et al.* proposed a new approach for generating three-directional multiscroll chaotic attractors by constructing a family of triangular function series [Yu *et al.*, 2005] and successfully obtained multiwing attractors by modifying the cross product terms of a Lorenz-like system with a multisegment function [Yu *et al.*, 2005, 2008, 2010]. Both Radwan and Granhhi obtained multiscroll chaotic attractors by using MOS realization technique respectively [Radwan *et al.*, 2007; Gandhi & Roska, 2009]. Recently, some other research results have also been reported for generating multiscroll chaotic attractors by using novel methods [Elwakil & Özoguz, 2006, 2008; Yalcin & Özoguz, 2007; Zhang *et al.*, 2009]. In 2006, Lü and Chen surveyed the theories, approaches, and applications of multiscroll chaos generation over the last two decades [Lü & Chen, 2006]. In 2007, Yu *et al.* proposed systematic methods for theoretical design and circuit implementation of multidirectional multi-torus chaotic attractors [Yu *et al.*, 2007]. Moreover, Deng and Lü proposed saturated function series [Deng & Lü, 2006] and hysteresis function series [Deng & Lü, 2007] for generating multidirectional multiscroll from fractional differential systems, respectively. Ahmad introduced a step function method for $n$-scroll chaotic attractors from fractional order systems [Ahmad, 2005].

As mentioned above, majority generation of multiscroll attractors are under the framework of Chua circuit, or Jerk equation, or Lorenz-like systems. Multiscroll attractors have a larger number of scrolls ($n > 2, n$ is the number of scroll) and have mostly been implemented by introducing additional saddle points of index 2 in terms of added breakpoints in the model system. By extending this concept of the construction of a multiscroll chaotic system, a new $(2N+1)$-scroll chaotic system evolving from the classic Colpitts oscillator model [Maggio *et al.*, 1999] through modifying its nonlinear term by a triangular function is proposed in this paper. The main idea of the proposed approach is to increase the number of index-2 equilibrium points of the modified Colpitts oscillator model, such that their distribution is extended horizontally along a particular axis. The dynamic behaviors of the multiscroll chaotic system is further investigated via the theoretical analysis and verified by numerical simulations and circuit experiments.

## 2. Generation of *n*-Scroll Attractors Under the Framework of Colpitts Oscillator

For generating a multiscroll attractor, the Colpitts oscillator model is regarded as the basic framework in this paper.

### 2.1. *Colpitts oscillator model*

With dimensionless state variables and normalized parameters, the state equations of Colpitts oscillator model can be written in the form [Maggio *et al.*, 1999].

$$\begin{cases} \dot{x} = \dfrac{g}{Q(1-k)}[-n(y)+z], \\[2ex] \dot{y} = \dfrac{g}{Qk}z, \\[2ex] \dot{z} = -\dfrac{Qk(1-k)}{g}[x+y] - \dfrac{1}{Q}z, \end{cases} \tag{1}$$

where $Q$ and $g$ are real positive constants, $k = 0.5$, and

$$n(y) = \exp(-y) - 1. \tag{2}$$

When the parameters in [Maggio *et al.*, 1999] is set as $Q = 1.4158, g = 3.1623$, system (1) is chaotic and shows a spiral attractor.

It is noted that in system (1), only the first equation contains the nonlinear term $n(y)$ which, in turn, depends only on one of the state variables, $y$. Also, it should be noted that the dynamic behavior of the system (1) depends only on two parameters: the loop gain of the oscillator $g$ and the quality factor of the tank circuit $Q$, while $k$ only has a scaling effect on the state variables.

### 2.2. *Modified Colpitts oscillator model*

In order to generate $n$-scroll chaotic attractors, it is found to be essential to design multiple index-2 equilibrium points. System (1) is therefore modified as

$$\begin{cases} \dot{x} = a[-f(y) + z], \\ \dot{y} = az, \\ \dot{z} = -\dfrac{1}{2a}(x + y) - bz, \end{cases} \tag{3}$$

where $a = 2g/Q$ and $b = 1/Q$ for simplicity. The nonlinear term in the first equation of model (1) is modified by a triangular function defined by

$$f(y) = \sum_{\substack{n=-N \\ n\neq 0}}^{N} \frac{p}{2q} \left\{ \begin{array}{l} \left| y - p\left(2n - \dfrac{|n|}{n}\right) + q \right| \\ -\left| y - p\left(2n - \dfrac{|n|}{n}\right) - q \right| \end{array} \right\} - y, \tag{4}$$

where $K$ is a positive integer, $p > 0$ and $q \in (0, p)$. An example of $f(y)$ with $N = 3$ is given in Fig. 1, from which it can be observed that the characteristics of each segment, including its slope and zero, can be easily determined.

When parameters $a = 4.5, b = 1, p = 1$ and $N = 2$ are kept constant while parameter $q$ is varying, the Lyapunov exponent spectra of system (3) and its corresponding bifurcation diagram of state variable $y$ with initial value $(0.1, 0.1, 0)$ are shown in Fig. 2. As $q$ increases, a reverse Hopf bifurcation route to chaos exists with a smaller parameter value $q$. For $q \in (0, 0.32]$, the multiscroll system is chaotic and can display a 5-scroll chaotic attractor. Furthermore, it can be observed that the smaller the value of $q$, the more uniform is the distribution of scrolls. Thus, in the following, we usually select $q = 0.02p$.

For $a = 4.5, b = 1, p = 1, q = 0.02$ and $N = 2$, system (3) is chaotic and displays a 5-scroll chaotic attractor, as shown in Fig. 3. Its Lyapunov exponents are $\mathrm{LE}_1 = 0.2027$, $\mathrm{LE}_2 = 0$, $\mathrm{LE}_3 = -1.2001$, and the Lyapunov dimension is $d_L = 2.1689$ for initial values (0.1, 0.1, 0).

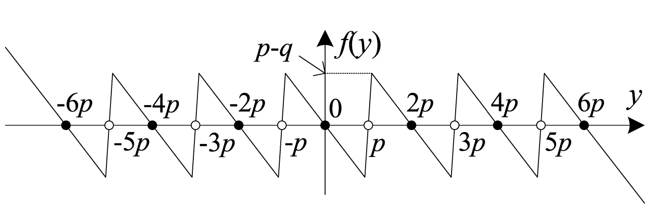



Fig. 1. Triangular function $f(y)$ with $N = 3$.

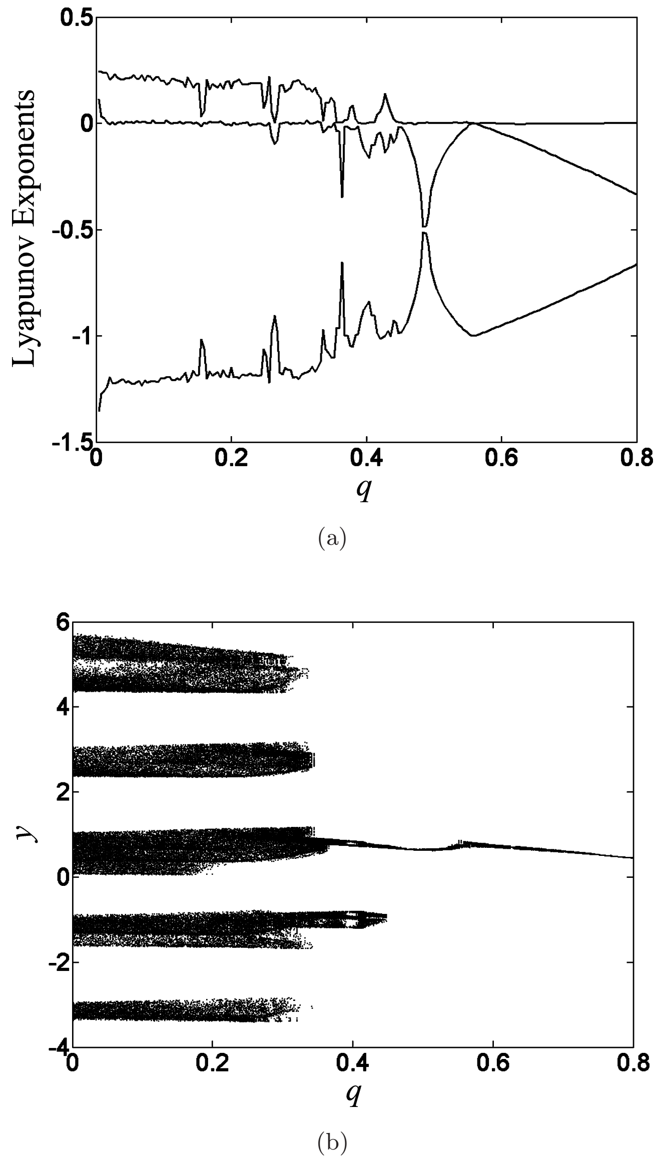
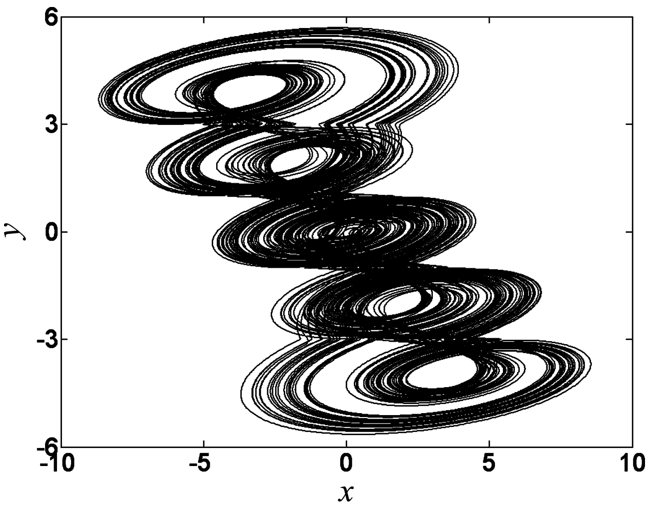


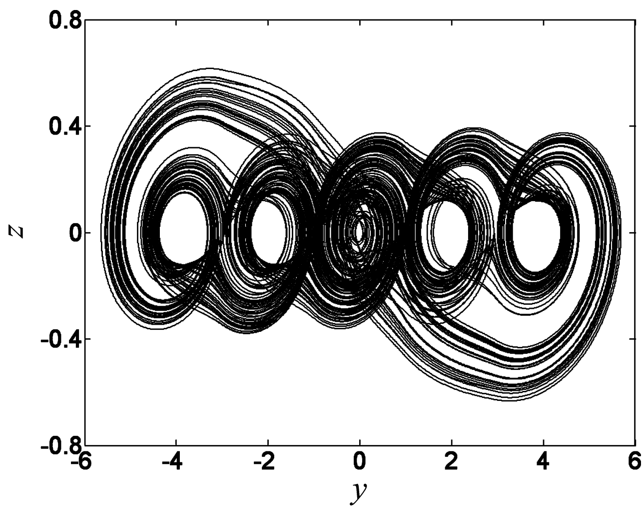

Fig. 2. (a) Lyapunov exponent spectra. (b) Bifurcation diagram of $y$ with increasing $q$.

### 2.3. *Generation mechanism of multiscroll attractor*

In order to obtain the desired $n$-scroll chaotic attractors, it is essential to distribute the index-2 equilibrium points of the system along a certain axis. Let $\dot{x} = \dot{y} = \dot{z} = 0$, then one has $z = 0, f(y) = 0, x + y = 0$. Clearly, system (3) has $(4N + 1)$ equilibrium points, in which $(2N + 1)$ equilibrium points are located on $[x, y] = [-2Np, 2Np], \ldots, [-2p, 2p], [0, 0], [2p, -2p], \ldots, [2Np, -2Np]$ with $z = 0$, called an even equilibrium point ${}^{E}S$; the other $2N$ equilibrium points are located on $[x, y] = [-(2N-1)p, (2N-1)p] \cdots [-p, p], [p, -p] \cdots [(2N-1)p, -(2K-1)p]$ with $z = 0$, called as odd




(a)




(b)

Fig. 3. The phase portraits of a 5-scroll chaotic attractor. (a) Viewed on $x$–$y$ plane, (b) viewed on $y$–$z$ plane.

equilibrium point ${}^{O}S$. As an example, the equilibrium points on $x$–$y$ plane with $N = 2, p = 1$, and $q = 0.02$ are marked in Fig. 4, where five even equilibrium points ${}^{E}S$ are marked with "•" and four odd equilibrium points ${}^{O}S$ are marked with "∘".

The Jacobian matrix of system (3) at the equilibrium point $S^* = (x^*, y^*, z^*)$ is given by

$$\mathbf{J}_{S*} = \begin{bmatrix} 0 & -af^J(y^*) & a \\ 0 & 0 & a \\ -\dfrac{0.5}{a} & -\dfrac{0.5}{a} & -b \end{bmatrix}, \tag{5}$$

where $f^J(y^*)$ is the derivative of (4) at $y = y^*$, as

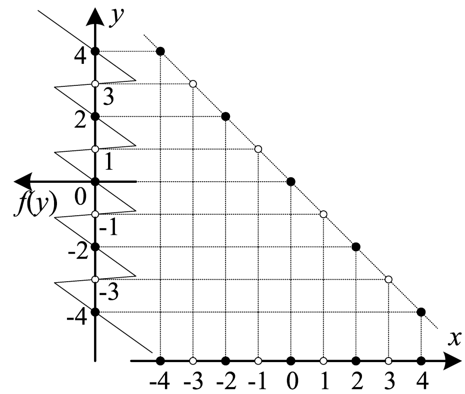



Fig. 4. Equilibrium points on $x$–$y$ plane with $N = 2, p = 1$ and $q = 0.02$.

$$f^J(y^*) = \left.\frac{\mathrm{d}f(y)}{\mathrm{d}y}\right|_{y^*}$$

$$= \sum_{\substack{n=-N \\ n\neq 0}}^{N} \frac{p}{2q} \left\{ \begin{matrix} \mathrm{sgn}\left[y^* - p\left(2n - \dfrac{|n|}{n}\right) + q\right] \\ -\mathrm{sgn}\left[y^* - p\left(2n - \dfrac{|n|}{n}\right) - q\right] \end{matrix} \right\} - 1. \tag{6}$$

If $y^* = 2np (n = 0, \pm 1, \pm 2, \ldots, \pm N)$, then $f^J(y^*) = -1$; else if $y^* = (2n - |n|/n)p (n = \pm 1, \pm 2, \ldots, \pm N)$, then $f^J(y^*) = p/q - 1$. The corresponding characteristic equation can be obtained from (5) as follows

$$g(\lambda) = \lambda^3 + b\lambda^2 + \lambda - 0.5af^J(y^*) = 0. \tag{7}$$

For even equilibrium point ${}^{E}S$, one has $f^J(y^*) = -1$, the above expression becomes

$$g(\lambda) = \lambda^3 + b\lambda^2 + \lambda + 0.5a = 0. \tag{8}$$

Note that the coefficients of this cubic polynomial are all positive for $a, b > 0$, thus $g(\lambda) > 0$ for all $\lambda > 0$. It is noted that instability appears ($\mathrm{Re}(\lambda) > 0$) only if there are two complex conjugate zeros of $g$. In fact, by Routh–Hurwitz criterion, Eq. (8) has one negative real root and two complex conjugate roots with positive real part for $a > 2b$. Hopf bifurcations emerge from the value of $a = 2b$. Here, all even equilibrium points ${}^{E}S$ are saddle points of index 2, from which a $(2N + 1)$-scroll attractor can be evolved.

For odd equilibrium point ${}^{O}S, f^J(y^*) = p/q - 1$, Eq. (7) becomes

$$g(\lambda) = \lambda^3 + b\lambda^2 + \lambda - 0.5a\left(\frac{p}{q} - 1\right) = 0. \tag{9}$$

For $p > q$ and $a, b > 0$, Eq. (9) has one positive real root and two negative real roots or two complex conjugate roots with negative real part. This means that odd equilibrium points ${}^{O}S$ are unstable saddle points of index 1.

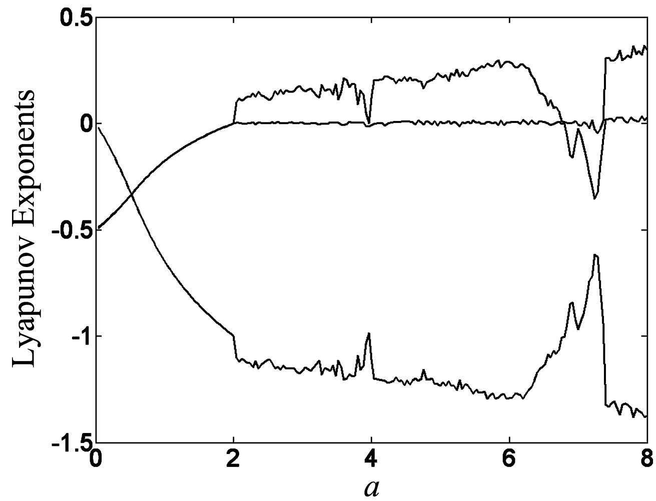
When $a = 4.5, b = 1, N = 2, p = 1$ and $q = 0.02$, system (3) has nine equilibrium points totally, of which five equilibrium points are ${}^{E}S$, the other four equilibrium points are ${}^{O}S$. Three roots of ${}^{E}S$ are $\lambda_1 = 0.208 + i1.2433, \lambda_2 = 0.208 - i1.2433, \lambda_3 = -1.416$, which implies that ${}^{E}S$ of system (3) have index 2 and a 5-scroll chaotic attractor is evolved along the $x$ axis or $y$ axis. While three roots of ${}^{O}S$ are $\lambda_1 = 4.4192, \lambda_2 = -2.7096 + i4.196, \lambda_3 = -2.7096 - i4.196$, which implies that ${}^{O}S$ of system (3) are unstable saddle points of index 1.

The above results on the analysis show that system (3) with triangular function of (4) may generate chaotic behavior under the condition of $a, b > 0$ and $a > 2b$. From the numerical simulation given in the following section, we can clearly see that system (3) displays a $(2N+1)$-scroll attractor located on three planes, where each scroll chaotic attractor is corresponding to an even equilibrium point, i.e. an index-2 equilibrium point.

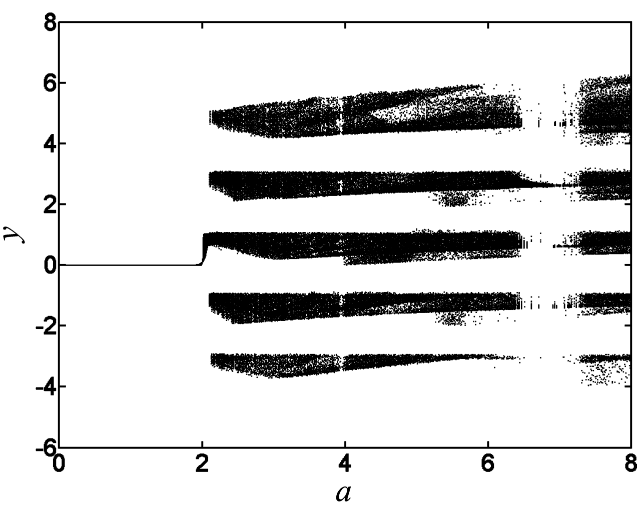
## 3. Dynamics of the Modified Colpitts Oscillator

When $b = 1, p = 1, q = 0.02$ and $N = 2$ while parameter $a$ is varying, the Lyapunov exponent spectra of system (3) and its corresponding bifurcation diagram of state variable $y$ with initial values (0.1, 0.1, 0) are shown in Fig. 5. It is clear that the bifurcation diagram well coincides with the Lyapunov exponent spectra.

When $a > 2b = 2$, in a wide parameter variation range, system (3) is chaotic with a positive Lyapunov exponent, a zero valued exponent along with a negative Lyapunov exponent observed from Fig. 5(a), and an undergoing chaotic route observed from Fig. 5(b). Obviously, the system comprises sinks for $0 < a \leq 2$ and chaos for $a > 2$. In the vicinity of $a = 3.95$, there is a narrow window of periodicity within the band of chaotic behaviors. In particular, a wider periodic window occurs in the region $6.8 \leq a \leq 7.35$. The periodic window plays an important role in the evolution of dynamic behaviors of a chaotic system. As $a$ increases in the parameter variation range, a Hopf bifurcation route to chaos exists in the system at $a = 2$,




(a)



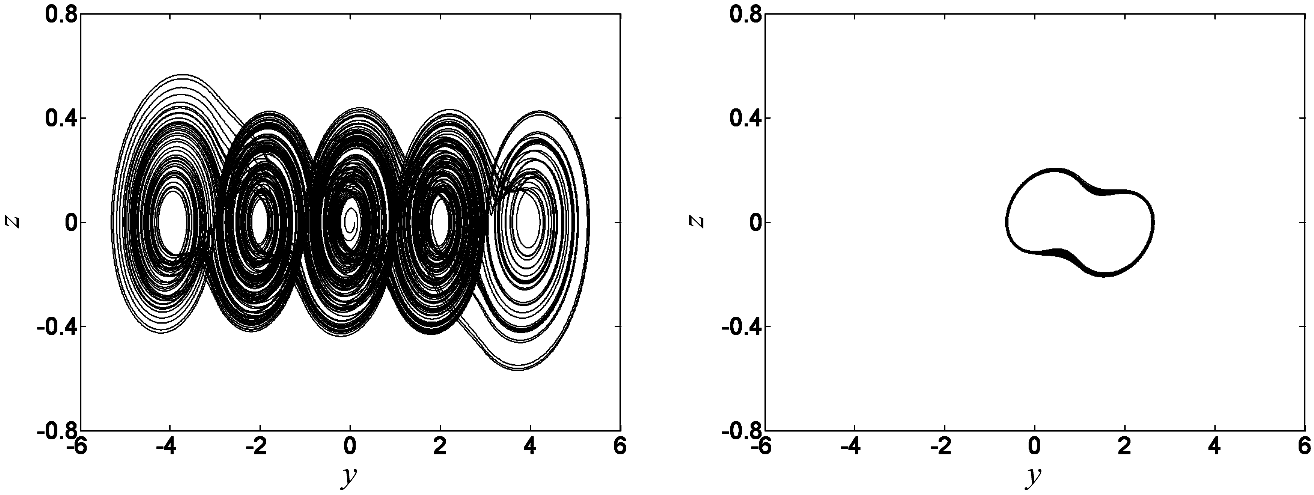

(b)

Fig. 5. (a) Lyapunov exponent spectra. (b) Bifurcation diagram of $y$ with increasing $a$.

which well coincides with the above theoretical analysis.

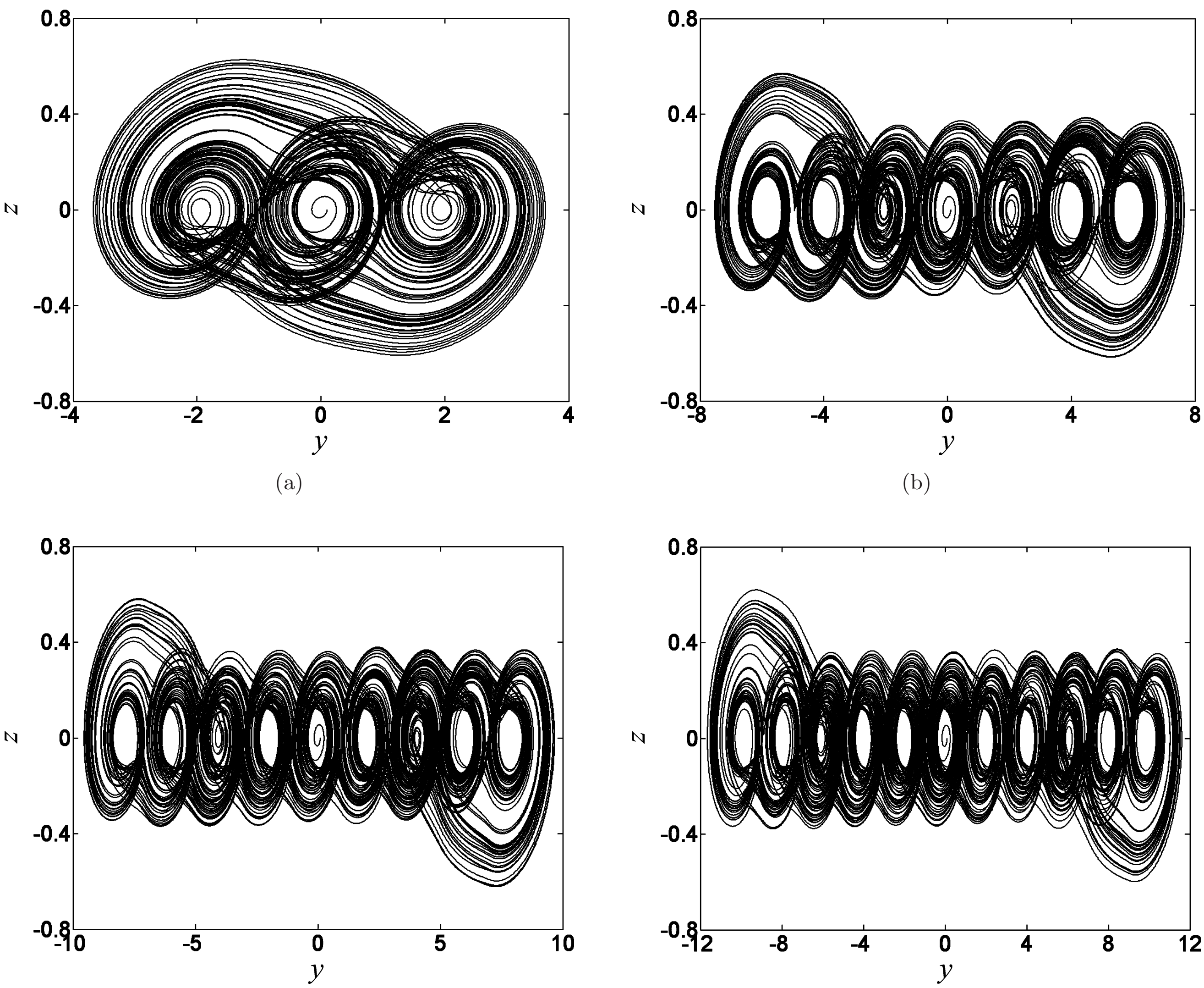
Two typical chaotic and periodic orbits of system (3) are obtained from numerical simulations as shown in Fig. 6. Figure 6(a) shows another portrait phase of chaotic region for $a = 3$, the corresponding Lyapunov exponent spectra are $\mathrm{LE}_1 = 0.163$, $\mathrm{LE}_2 = 0$, $\mathrm{LE}_3 = -1.1626$. Figure 6(b) illustrates the typical periodic orbit for $a = 7$, the corresponding Lyapunov exponent spectra are $\mathrm{LE}_1 = 0$, $\mathrm{LE}_2 = -0.0258$, $\mathrm{LE}_3 = -0.9645$.

Figure 7 shows other four multiscroll chaotic attractors with different parameters $N$, where $a = 4.5, b = 1, p = 1, q = 0.02$. When $N = 1, 3, 4$, and 5, the system has $(2N+1)$ equilibrium points with




(a) (b)

Fig. 6. Two typical orbits on $y$–$z$ plane. (a) Periodic orbit ($a = 7$), (b) Chaotic orbit ($a = 3$).




(a) (b)

(c) (d)

Fig. 7. Four $(2N + 1)$-scroll chaotic attractors on $y$–$z$ plane. (a) 3-scroll attractor ($N = 1$), (b) 7-scroll attractor ($N = 3$), (c) 9-scroll attractor ($N = 4$), (d) 11-scroll attractor ($N = 5$).

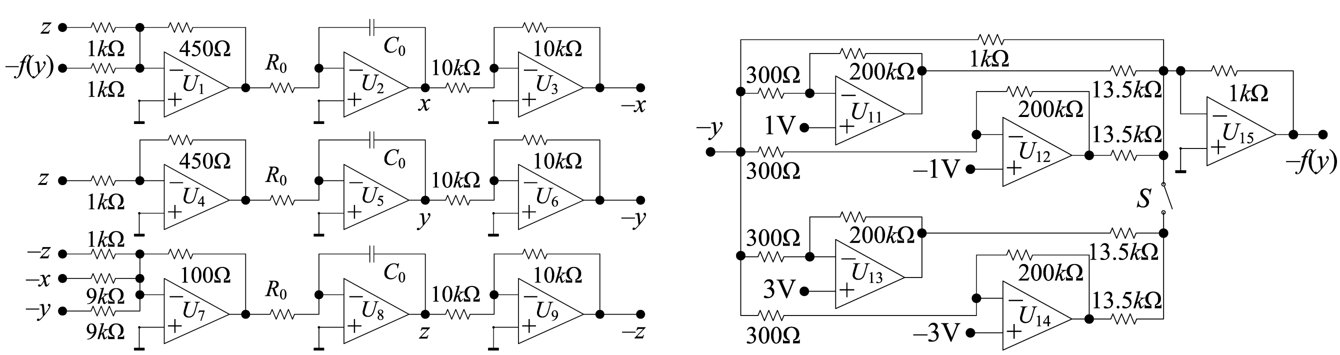
index-2 and generates 3-scroll, 7-scroll, 9-scroll and 11-scroll chaotic attractors, respectively.

As described above, this new multiscroll system has been presented by modifying the nonlinear term in the first equation with a triangular function. The proposed system is very simple, and can generate chaotic attractors with different numbers of scroll.

## 4. Circuit Realization and Experimental Results

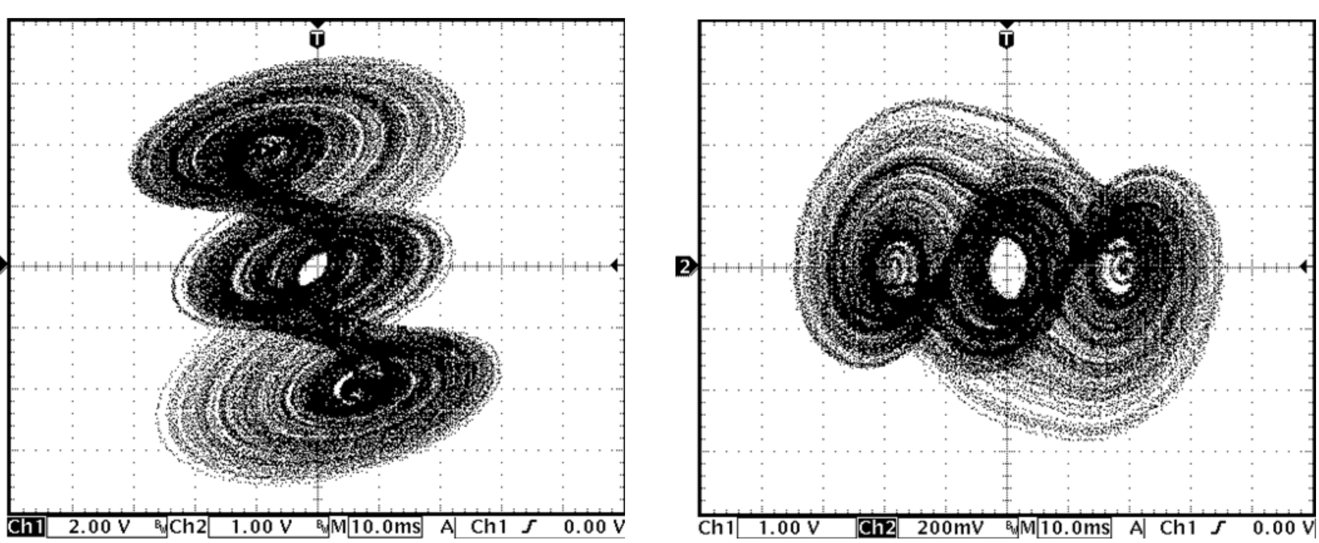
As examples, let $a = 4.5, b = 1, p = 1, q = 0.02$, and $N = 2$, an analog electronic circuit as shown in Fig. 8 has been built to physically realize the modified Colpitts oscillator model for generating $n$-scroll chaotic attractor [Lü *et al.*, 2006; Yu *et al.*, 2008, 2010]. The design is based on a direct implementation of the integrator using operational amplifier. It is constructed by two parts: one part is a basic three-dimensional system as shown in Fig. 8(a); the other part is the proposed circuit for generating triangular function $f(y)$ as shown in Fig. 8(b), where the switch $S$ provides the necessary flexibility for different values of $N$, if $S$ is opened, $N = 1$, otherwise, $N = 2$. The time constant of the integrator is determined by $R_0C_0$, where $R_0 = 2\,\text{k}\Omega, C_0 = 33\,\text{nF}$ are used in our experiment. The operational amplifier in use is TL082 with saturated voltage of $V_{\text{sat}} = \pm 13.5\,\text{V}$, assuming the dual voltage sources of $\pm 15\,\text{V}$.

The observations by experimental circuit with $N = 1$ and $N = 2$ are shown in Fig. 9, in which Figs. 9(a) and 9(b) corresponding to 3-scroll chaotic attractor on the $x$–$y$ and $y$–$z$ planes, Figs. 9(c) and



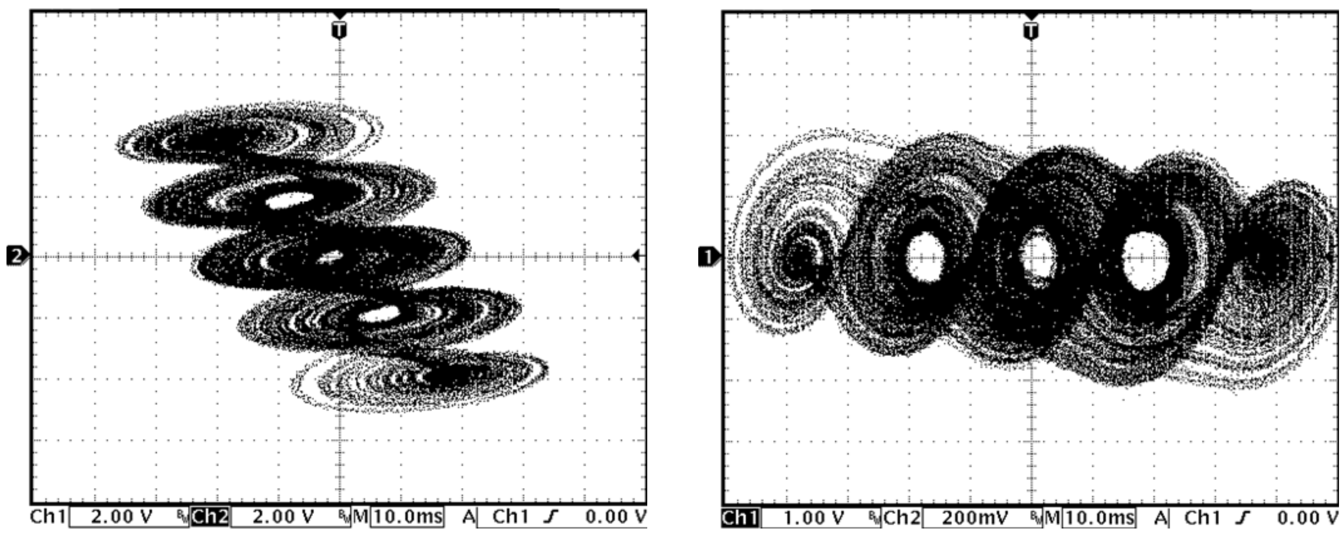

(a) (b)

Fig. 8. Electronic circuit for the modified Colpitts oscillator. (a) Basic circuit, (b) Circuit of triangular function $f(y)$.




(a) (b)

Fig. 9. Multiscroll attractors from experimental circuit of modified Colpitts oscillator. (a) 3-scroll attractor on $x$–$y$ plane, (b) 3-scroll attractor on $y$–$z$ plane, (c) 5-scroll attractor on $x$–$y$ plane, (d) 5-scroll attractor on $y$–$z$ plane.




(c) (d)

Fig. 9. (*Continued*)



9(d) are correspond to 5-scroll chaotic attractor on the $x$–$y$ and $y$–$z$ planes, respectively. Comparing Fig. 9 with Figs. 3 and 7(a), one can observe that there is a good agreement between the numerical simulations and the experimental measurement results.

## 5. Conclusions

This paper has presented a novel approach for generating the multiscroll chaotic attractor from the modified Colpitts oscillator model by modifying the nonlinear term in the first equation with a triangular function. The proposed system is very simple, and can generate multiscroll chaotic attractor with up to $(2N+1)$-scroll, which corresponds to the numbers of index-2 equilibrium points of the system. Through the analysis on even and odd equilibrium points as well as their corresponding characteristic equations, the dynamical characteristics of the multiscroll chaotic system are comprehensively studied. Furthermore, an electronic circuit is realized and multiscroll chaotic attractors are obtained by an experimental circuit.

## Acknowledgments

This work is supported by grants from the National Natural Science Foundations of China (Grant Nos. 60971090, 50677056) and the Natural Science Foundations of Jiangsu Province (Grant No. BK2009105).

## References

Ahmad, W. [2005] "Generation and control of multi-scroll chaotic attractors in fractional order systems," *Chaos Solit. Fract.* **25**, 727–735.

Deng, W. & Lü, J. H. [2006] "Design of multidirectional multiscroll chaotic attractors based on fractional differential systems via switching control," *Chaos* **16**, 043120.

Deng, W. & Lü, J. H. [2007] "Generating multi-directional multi-scroll chaotic attractors via a fractional differential hysteresis system," *Phys. Lett. A* **369**, 438–443.

Elwakil, A. S. & Özoguz, S. [2006] "Multiscroll chaotic oscillators: The non-autonomous approach," *IEEE Trans. Circuits Syst.-II* **53**, 862–866.

Elwakil, A. S. & Özoguz, S. [2008] "A system and circuit for generating 'multi-butterflies'," *Int. J. Bifurcation and Chaos* **18**, 841–844.

Gandhi, G. & Roska, T. [2009] "MOS-integrable circuitry for multi-scroll chaotic grid realization: A SPICE-assisted proof," *Int. J. Circuit Th. Appl.* **37**, 473–483.

Lü, J. H., Yu, X. H. & Chen, G. R. [2003] "Generating chaotic attractors with multiple merged basin: A switching piecewise linear control approach," *IEEE Trans. Circuits Syst.-I* **50**, 198–207.

Lü, J. H., Chen, G. R., Yu, X. H. & Leung, H. [2004a] "Design and analysis of multiscroll chaotic attractors from saturated function series," *IEEE Trans. Circuits Syst.-I* **51**, 2476–2490.

Lü, J. H., Han, F. L., Yu, X. H. & Chen, G. R. [2004b] "Generating 3-D multiscroll chaotic attractors: A hysteresis series switching method," *Automatica* **40**, 1677–1687.

Lü, J. H. & Chen, G. R. [2006] "Generating multiscroll chaotic attractors: Theories, methods and applications," *Int. J. Bifurcation and Chaos* **16**, 775–858.

Lü, J. H., Yu, S. M., Leung, H. & Chen, G. R. [2006] "Experimental verification of multidirectional multiscroll chaotic attractors," *IEEE Trans. Circuits Syst.-I* **53**, 149–165.

Lü, J. H., Murali, K., Sinha, S., Leung, H. & Aziz-Alaoui, M. A. [2008] "Generating multi-scroll chaotic attractors by thresholding," *Phys. Lett. A* **372**, 3234–3239.

Maggio, G. M., Feo, O. D. & Kennedy, M. P. [1999] "Nonlinear analysis of the Colpitts oscillator and applications to design," *IEEE Trans. Circuits Syst.-I* **46**, 1118–1130.

Radwan, A., Soliman, A. & Elwakil, A. [2007] "1-D digitally-controlled multi-scroll chaos generator," *Int. J. Bifurcation and Chaos* **17**, 227–242.

Suykens, J. A. K. & Vandewalle, J. [1993] "Generation of n-double scrolls ($n = 1, 2, 3, 4, \ldots$)," *IEEE Trans. Circuits Syst.-I* **40**, 861–867.

Yalcin, M. E., Suykens, J., Vandewalle, J. & Ozoguz, S. [2002] "Families of scroll grid attractors," *Int. J. Bifurcation and Chaos* **12**, 23–41.

Yalcin, M. E. [2007] "Increasing the entropy of a random number generator using $n$-scroll chaotic attractors," *Int. J. Bifurcation and Chaos* **17**, 4471–4479.

Yalcin, M. E. & Özoguz, S. [2007] "$n$-scroll chaotic attractors from a first-order time-delay differential equation," *Chaos* **17**, 033112.

Yu, S. M., Lü, J. H., Leung, H. & Chen, G. R. [2005] "Design and implementation of $n$-scroll chaotic attractors from a general Jerk circuit," *IEEE Trans. Circuits Syst.-I* **52**, 1459–1476.

Yu, S. M., Lü, J. H., Tang, W. K. S. & Chen, G. R. [2006] "A general multiscroll Lorenz system family and its realization via digital signal processors," *Chaos* **16**, 033126.

Yu, S. M., Lü, J. H. & Chen, G. R. [2007] "Theoretical design and circuit implementation of multi-directional multi-torus chaotic attractors," *IEEE Trans. Circuits Syst.-I* **54**, 2087–2098.

Yu, S. M., Tang, W. K. S., Lü, J. H. & Chen, G. R. [2008] "Generation of $n \times m$-wing Lorenz-like attractors from a modified Shimizu–Morioka model," *IEEE Trans. Circuits Syst.-II* **55**, 1168–1172.

Yu, S. M., Tang, W. K. S., Lü, J. H. & Chen, G. R. [2010] "Generating 2n-wing attractors from Lorenz-like systems," *Int. J. Circuit Th. Appl.* **38**, 243–258.

Zhang, C. X., Tang, W. K. S. & Yu, S. M. [2009] "A new chaotic system based on multiple-angle sinusoidal function: Design and implementation," *Int. J. Bifurcation and Chaos* **19**, 2073–2084.