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Fault detection of switched linear systems with its application to turntable systems

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Abstract: This paper is concerned with the H_∞ fault detection for continuous-time linear switched systems with its application to turntable systems. The solvability condition for a desired filter is established based on the proposed sufficient condition. Based on the double channel scheme of the turntable control system, the turntable system can be modeled as a switched system. Finally, by taking the turntable system as a numerical example, the effectiveness of the proposed theory is well validated.

Keywords: fault detection, H_∞ performance, exponential stability, switched system, turntable system.

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1. Introduction

Recently, the study of switched systems has received growing attention [1]. The switched system has numerous applications in the control of mechanical systems, aircraft control, air traffic control, and many other fields [2]. The central issue in the study of switched systems is the stability and stabilization problems. A number of works in this direction [1, 3, 4] have been proposed, and other problems [5–7] have also been well studied.

On the other hand, the fault detection problem is a critical task for autonomous operation of systems and many other electromechanical system, such as spacecraft, planetary rovers, industrial robot [8, 9]. It has received much attention in recent years [10, 11].

In practical applications, turntable is a kind of complex high-precision angular measuring systems and simulation equipment. The double channel scheme has been well studied in the design of a turntable control system [12]. The coarse channel resolution is adopted to stabilize the turntable when the position error is large. The fine channel is worked in the narrow scope precision angle. Unfortunately, the proposed double channels scheme is difficult to implement in practical design of control systems due to lack of a switching theorem for such system. Moreover, expected or unexpected faults may occur during the operation of the angular measuring system. How to effectively detect the faults and make system run well is also very important [13].

Motivated by the above observations, in this paper, we are interested in investigating the H_∞ fault detection for continuous-time switched hybrid systems. By using the average dwell time approach and linear matrix inequality (LMI) technique, a sufficient condition is proposed to guarantee the exponential stability and a weighted H_∞ performance for the fault detection system. Then, the corresponding solvability condition for a desired filter is established, and the filter design is cast into a convex optimization problem. The result is applied to turntable systems, and the effectiveness of the proposed theory is well validated by the numerical example.

2. Problem formulation and preliminaries

Consider a class of nonlinear switched stochastic systems of the form (Π)

$$(\Pi):\begin{cases}\dot{x}(t)=A(\beta)x(t)+B(\beta)u(t)+\\ \qquad B_0(\beta)\omega(t)+B_1(\beta)f(t)\\ y(t)=C(\beta)x(t)+D(\beta)u(t)+\\ \qquad D_0(\beta)\omega(t)+D_1(\beta)f(t)\end{cases}\tag{1}$$

where $x(t)\in R^n$ is the state vector, $\omega(t)\in R^p$ is the disturbance which belongs to $\mathscr{L}_2\,[0,\infty)$, $u(t)\in R^m$ is the input vector, $f(t)\in R^q$ is the fault vector. Without loss of generality, we assume that the $\mathscr{L}_2\,\{[0,\infty),[0,\infty)\}$ norms of $u(t)$ and $f(t)$ exist and are bounded; $y(t)\in R^l$ is the measured output. In system (Π), $\{(A(\beta),B(\beta),C(\beta),D(\beta),B_0(\beta),B_1(\beta),D_0(\beta),D_1(\beta):\beta\in\mathscr{S})\}$ is a family of matrices parameterized by an index set $\mathscr{S}=\{1,2,\ldots,S\}$ and $\beta:R\rightarrow\mathscr{S}$ is a piecewise constant

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function of time t called a switching signal. At a given time t, the value of $\beta(t)$, denoted by β for simplicity, might depend on t or $x(t)$, or both. We assume that the value of $\beta(t)$ is unknown, but its instantaneous value is available in real time.

For each value $\beta(t) = i$ ($i \in \mathscr{S}$), we will denote the system matrices associated with mode i by

$$A(i) = A(\beta), \quad B(i) = B(\beta), \quad C(i) = C(\beta)$$
$$D(i) = D(\beta), \quad B_0(i) = B_0(\beta), \quad B_1(i) = B_1(\beta)$$
$$D_0(i) = D_0(\beta), \quad D_1(i) = D_1(\beta)$$

where $A(i)$, $B(i)$, $C(i)$, $D(i)$, $B_0(i)$, $B_1(i)$, $D_0(i)$ and $D_1(i)$ are constant matrices. Corresponding to the switching signal β, we have the switching sequence $\{(i_0, 0), (i_1, t_1), \ldots, (i_k, t_k), \ldots, | i_k \in \mathscr{S}, k = 0, 1, \ldots\}$, which means that the i_kth subsystem is activated when $t \in [t_k, t_{k+1})$.

Typically, one key step of the fault detection is the generation of a residual signal, which must be sensitive to faults. This signal is then processed to decide whether or not a fault has occurred in the system. Therefore, a typical fault detection system consists of a residual generator and a residual evaluation stage including an evaluation function and a prescribed threshold.

For the switched system $(\varPi)$ in (1), we are interested in designing a fault detection filter of the following form

$$(\hat{\varPi}) : \begin{cases} \dot{\hat{x}}(t) = \hat{A}(\beta)\hat{x}(t) + \hat{B}(\beta)y(t) \\ r(t) = \hat{C}(\beta)\hat{x}(t) \end{cases} \tag{2}$$

where $\hat{x}(t) \in R^{n_f} (n_f \leqslant n)$ is the state vector of the fault detection filter, $r_k \in R^q$ is the so-called residual signal, $\hat{A}(\beta)$, $\hat{B}(\beta)$ and $\hat{C}(\beta)$ are filter matrices to be determined.

To improve the performance of the fault detection system, we add a weighting matrix function into the fault $f(t)$, that is, $\hat{f}(s) = W(s)f(s)$, where $f(s)$ and $\hat{f}(s)$ denote respectively the Laplace transforms of $f(t)$ and $\hat{f}(t)$. Here, $W(s)$ is given a priori, the choice of $W(s)$ is to impose frequency weighting on the spectrum of the fault signal for detection. One state space realization of $\hat{f}(s) = W(s)f(s)$ can be

$$(\breve{\varPi}) : \begin{cases} \dot{\breve{x}}(t) = \breve{A}\breve{x}(t) + \breve{B}f(t) \\ \hat{f}(t) = \breve{C}\breve{x}(t) \end{cases} \tag{3}$$

where $\breve{x}(t) \in R^{n_W}$ is the state vector and matrices $\breve{A}$, $\breve{B}$ and $\breve{C}$ are known a priori.

Denoting $e(t) \triangleq r(t) - \hat{f}(t)$, and augmenting the model of $(\varPi)$ to include the states of $(\hat{\varPi})$ and $(\breve{\varPi})$, then the overall dynamics of the fault detection system is governed by

$$(\tilde{\varPi}) : \begin{cases} \dot{\xi}(t) = \bar{A}(\beta)\xi(t) + \bar{B}(\beta)\upsilon(t) \\ e(t) = \bar{C}(\beta)\xi(t) \end{cases} \tag{4}$$

where $\xi(t) \triangleq [x^{\mathrm{T}}(t) \quad \hat{x}^{\mathrm{T}}(t) \quad \breve{x}^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\upsilon(t) \triangleq [u^{\mathrm{T}}(t) \quad \omega^{\mathrm{T}}(t) \quad f^{\mathrm{T}}(t)]^{\mathrm{T}}$, $e(t) \triangleq r(t) - \hat{f}(t)$ and

$$\begin{aligned}
\bar{A}(\beta) &\triangleq \left[\begin{array}{c|c} \tilde{A}(\beta) & 0 \\ \hline 0 & \breve{A} \end{array}\right] \\
\bar{B}(\beta) &\triangleq \left[\begin{array}{c|c} \tilde{B}_1(\beta) & \tilde{B}_2(\beta) \\ \hline 0 & \breve{B} \end{array}\right] \\
\bar{C}(\beta) &\triangleq \left[\begin{array}{c|c} \tilde{C}(\beta) & -\breve{C} \end{array}\right] \\
\tilde{A}(\beta) &\triangleq \begin{bmatrix} A(\beta) & 0 \\ \hat{B}(\beta)C(\beta) & \hat{A}(\beta) \end{bmatrix} \\
\tilde{B}_1(\beta) &\triangleq \begin{bmatrix} B(\beta) & B_0(\beta) \\ \hat{B}(\beta)D(\beta) & \hat{B}(\beta)D_0(\beta) \end{bmatrix} \\
\tilde{B}_2(\beta) &\triangleq \begin{bmatrix} B_1(\beta) \\ \hat{B}(\beta)D_1(\beta) \end{bmatrix}, \quad \tilde{C}(\beta) \triangleq \begin{bmatrix} 0 & \hat{C}(\beta) \end{bmatrix}
\end{aligned} \tag{5}$$

Before presenting the main objective of this paper, we first introduce the following definitions for the fault detection system $(\tilde{\varPi})$ in (4), which will be essential for our derivation subsequently.

Definition 1 [14] The equilibrium $\xi^* = 0$ of the fault detection system $(\tilde{\varPi})$ in (4) with $\upsilon(t) = 0$ is said to be exponentially stable under $\beta(t)$ if its solution $\xi(t)$ satisfies

$$\|\xi(t)\| \leqslant \rho \, \|\xi(t_0)\| \, \mathrm{e}^{-\lambda(t-t_0)}, \quad \forall t \geqslant t_0$$

for constants $\rho \geqslant 1$ and $\lambda > 0$.

Definition 2 [2] For any $T_2 > T_1 \geqslant 0$, let $N_\beta(T_1, T_2)$ denote the number of switchings of $\beta(t)$ over (T_1, T_2). If $N_\beta(T_1, T_2) \leqslant N_0 + (T_2 - T_1)/T_a$ holds for $T_a > 0$, $N_0 \geqslant 0$. Then, T_a is called the average dwell time.

Definition 3 [14] For scalars $\alpha > 0$ and $\gamma > 0$, the fault detection system $(\tilde{\varPi})$ in (4) is said to be exponentially stable with a weighted H_∞ performance level γ, if under $\beta(t)$ it is exponentially stable with $\upsilon(t) = 0$, and under zero initial condition, that is, $\xi(0) = 0$, it holds for all nonzero $\upsilon(t) \in \mathscr{L}_2[0, \infty)$ that

$$\int_0^\infty \mathrm{e}^{-\alpha s} e^{\mathrm{T}}(s)e(s)\mathrm{d}s \leqslant \gamma^2 \int_0^\infty \upsilon^{\mathrm{T}}(s)\upsilon(s)\mathrm{d}s \tag{6}$$

Therefore, the fault detection problem to be addressed in this paper can be stated as the following two steps:

Step 1 Generate a residual signal. For the switched system $(\varPi)$ in (1), develop a filter in the form of (2) to generate a residual signal $r(t)$. Meanwhile, the filter is designed to assure that the resulting fault detection system $(\tilde{\varPi})$ in (4) is exponentially stable with a H_∞ disturbance attenuation level γ.

Step 2 Set up a fault detection measure. After the residual signal being constructed, a residual evaluation value will be computed through a prescribed evaluation function, and it will be compared with a predefined threshold. When the evaluation value is larger than the threshold, an alarm of fault is generated. In this work, a residual evaluation function $J(r)$ and a threshold $J_{\rm th}$ are selected as

$$J(r)\triangleq\left\{\int_{k_0}^{k_0+L} r^{\rm T}(t)r(t){\rm d}t\right\}^{\frac{1}{2}},\quad J\triangleq\sup_{0\neq\omega,0\neq u,f=0} J(r)$$

where k_0 denotes the initial evaluation time instant and L denotes the evaluation time. Based on this, the occurrence of faults can be detected by comparing $J(r)$ and $J_{\rm th}$ according to the following test

$$\begin{aligned} &J(r)>J_{\rm th} \;\Rightarrow\; \text{with faults} \;\Rightarrow\; \text{alarm}\\ &J(r)\leqslant J_{\rm th} \;\Rightarrow\; \text{no faults}\end{aligned}$$

3. Main results

The following lemma gives a sufficient condition which guarantees the fault detection system in (4) exponentially stable with a weighted H_∞ performance.

Lemma 1 [14] Given scalars $\alpha > 0$ and $\gamma > 0$, suppose there exists matrix $P(i) > 0$ such that for $i \in \mathscr{S}$

$$\begin{bmatrix} P(i)\bar{A}(i)+\bar{A}^{\rm T}(i)P(i)+\alpha P(i) & P(i)\bar{B}(i) & \bar{C}^{\rm T}(i)\\ * & -\gamma^2 I & 0\\ * & * & -I\end{bmatrix} < 0 \tag{7}$$

Then, the fault detection system $(\tilde{\Pi})$ in (4) is exponentially stable with a weighted H_∞ performance level γ for any switching signal with average dwell time satisfying $T_a > (\ln\mu)/\alpha$, with $\mu \geqslant 1$ and satisfying

$$P(i) \leqslant \mu P(j),\quad \forall i,j\in\mathscr{S} \tag{8}$$

Moreover, an estimate of the state decay is given by

$$\|\xi(t)\| \leqslant \rho{\rm e}^{-\lambda t}\,\|\xi(0)\| \tag{9}$$

where

$$\begin{gathered}\lambda = \frac{1}{2}\left(\alpha - \frac{\ln\mu}{T_a}\right) > 0,\quad a = \min_{i\in\mathscr{S}}\lambda_{\min}\left(P(i)\right)\\ \rho=\sqrt{\frac{b}{a}}\geqslant 1,\quad b=\max_{i\in\mathscr{S}}\lambda_{\max}\left(P(i)\right)\end{gathered} \tag{10}$$

Now, we are in a position to present a solution to the fault detection filter design problem according to Lemma 1.

Theorem 1 Consider the switched linear system (Π) in (1). For given scalars $\alpha > 0$ and $\gamma > 0$, suppose there exist matrices $\mathscr{P}(i) > 0$, $\mathscr{Q}(i) > 0$, $\breve{P} > 0$, $\mathscr{A}(i)$, $\mathscr{B}(i)$ and $\mathscr{C}(i)$ such that for $i \in \mathscr{S}$

$$\begin{bmatrix} \Pi_{11}(i) & \Pi_{12}(i) & 0 & \Pi_{14}(i) & \Pi_{15}(i) & \Pi_{16}(i) & 0\\ * & \Pi_{22}(i) & 0 & \Pi_{24}(i) & \Pi_{25}(i) & \Pi_{26}(i) & \mathscr{C}^{\rm T}(i)\\ * & * & \Pi_{33}(i) & 0 & 0 & \breve{P}\breve{B} & -\breve{C}^{\rm T}\\ * & * & * & -\gamma^2 I & 0 & 0 & 0\\ * & * & * & * & -\gamma^2 I & 0 & 0\\ * & * & * & * & * & -\gamma^2 I & 0\\ * & * & * & * & * & * & -I \end{bmatrix}<0 \tag{11}$$

where

$$\Pi_{11}(i) \triangleq \mathscr{P}(i)A(i) + A^{\rm T}(i)\mathscr{P}(i) + \alpha\mathscr{P}(i)$$

$$\Pi_{12}(i) \triangleq \mathscr{A}(i) + A^{\rm T}(i)\mathscr{Q}(i) + C^{\rm T}(i)\mathscr{B}^{\rm T}(i) + \alpha\mathscr{Q}(i)$$

$$\Pi_{22}(i) \triangleq \mathscr{A}(i) + \mathscr{A}^{\rm T}(i) + \alpha\mathscr{Q}(i)$$

$$\Pi_{33}(i) \triangleq \breve{P}\breve{A} + \breve{A}^{\rm T}\breve{P}^{\rm T} + \alpha\breve{P}$$

$$\Pi_{14}(i) \triangleq \mathscr{P}(i)B(i) + \mathscr{B}(i)D(i)$$

$$\Pi_{15}(i) \triangleq \mathscr{P}(i)B_0(i) + \mathscr{B}(i)D_0(i)$$

$$\Pi_{16}(i) \triangleq \mathscr{P}(i)B_1(i) + \mathscr{B}(i)D_1(i)$$

$$\Pi_{24}(i) \triangleq \mathscr{Q}(i)B(i) + \mathscr{B}(i)D(i)$$

$$\Pi_{25}(i) \triangleq \mathscr{Q}(i)B_0(i) + \mathscr{B}(i)D_0(i)$$

$$\Pi_{26}(i) \triangleq \mathscr{Q}(i)B_1(i) + \mathscr{B}(i)D_1(i)$$

Then, there exists a fault detection filter $(\hat{\Pi})$ in the form of (2) such that the fault detection system $(\tilde{\Pi})$ in (4) is exponentially stable with a weighted H_∞ performance level γ for any switching signal with average dwell time satisfying $T_a > (\ln\mu)/\alpha$, where $\mu \geqslant 1$ and satisfies

$$\begin{bmatrix}\mathscr{P}(i) & \mathscr{Q}(i)\\ * & \mathscr{Q}(i)\end{bmatrix} \leqslant \mu\begin{bmatrix}\mathscr{P}(j) & \mathscr{Q}(j)\\ * & \mathscr{Q}(j)\end{bmatrix},\quad \forall i,j\in\mathscr{S} \tag{12}$$

Moreover, if the above LMI conditions are feasible for $\left(\mathscr{P}(i),\mathscr{Q}(i),\breve{P},\mathscr{A}(i),\mathscr{B}(i),\mathscr{C}(i)\right)$, then a desired H_∞ fault detection filter realization is given by

$$\begin{bmatrix}\hat{A}(i) & \hat{B}(i)\\ \hat{C}(i) & 0\end{bmatrix} = \begin{bmatrix}\mathscr{Q}^{-1}(i) & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}\mathscr{A}(i) & \mathscr{B}(i)\\ \mathscr{C}(i) & 0\end{bmatrix} \tag{13}$$

Proof By Lemma 1, let $P(i) \triangleq {\rm diag}\{M(i),\breve{P}\} > 0$ in (7) and (8), where $M(i) \in R^{(n+n_f)\times(n+n_f)}$, $\breve{P} \in R^{n_W\times n_W}$, we get a new result. Specially, given scalars $\alpha > 0$ and $\gamma > 0$, suppose there exist matrices $M(i) > 0$, $\breve{P} > 0$ such that for $i \in \mathscr{S}$

$$\begin{bmatrix} M(i)\tilde{A}(i)+\tilde{A}^{\mathrm{T}}(i)M(i)+\alpha M(i) & 0 & M(i)\tilde{B}_1(i) & M(i)\tilde{B}_2(i) & \tilde{C}^{\mathrm{T}}(i) \\ * & \breve{P}\breve{A}+\breve{A}^{\mathrm{T}}\breve{P}+\alpha\breve{P} & 0 & \breve{P}\breve{B} & -\breve{C}^{\mathrm{T}} \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{14}$$

where $\tilde{A}(i)$, $\tilde{B}_1(i)$, $\tilde{B}_2(i)$ and $\tilde{C}(i)$ are defined in (5). Then, the fault detection system $(\tilde{\varPi})$ in (4) is exponentially stable with a weighted H_∞ performance level γ for any switching signal with average dwell time satisfying $T_a > (\ln\mu)/\alpha$, with $\mu \geqslant 1$ and satisfying

$$M(i) \leqslant \mu M(j), \quad \forall i,j \in \mathscr{S} \tag{15}$$

Moreover, an estimate of the state decay is given by (9).

Let the matrix $M(i)$ be partitioned as

$$M(i) \triangleq \begin{bmatrix} M_1(i) & M_2(i) \\ * & M_3(i) \end{bmatrix} > 0 \tag{16}$$

By invoking a small perturbation if necessary, we can assume that $M_2(i)$ and $M_3(i)$ are nonsingular.

Define the following matrices

$$\mathscr{J}(i) \triangleq \begin{bmatrix} I & 0 \\ 0 & M_3^{-1}(i)M_2^{\mathrm{T}}(i) \end{bmatrix}$$
$$\mathscr{P}(i) \triangleq M_1(i), \quad \mathscr{Q}(i) \triangleq M_2(i)M_3^{-1}(i)M_2^{\mathrm{T}}(i) \tag{17}$$

and

$$\begin{bmatrix} \mathscr{A}(i) & \mathscr{B}(i) \\ \mathscr{C}(i) & 0 \end{bmatrix} \triangleq \begin{bmatrix} M_2(i) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{A}(i) & \hat{B}(i) \\ \hat{C}(i) & 0 \end{bmatrix} \times$$
$$\begin{bmatrix} M_3^{-1}(i)M_2^{\mathrm{T}}(i) & 0 \\ 0 & I \end{bmatrix} \tag{18}$$

Performing a congruence transformation to (14) and (15) by $\mathrm{diag}\{\mathscr{J}(i),I,I,I,I\}$ and $\mathscr{J}(i)$, respectively, and noting that

$$\mathscr{J}^{\mathrm{T}}(i)M(i)\tilde{A}(i)\mathscr{J}(i) = \begin{bmatrix} \mathscr{P}(i)A(i) & \mathscr{A}(i) \\ \mathscr{Q}(i)A(i)+\mathscr{B}(i)C(i) & \mathscr{A}(i) \end{bmatrix}$$

$$\mathscr{J}^{\mathrm{T}}(i)M(i)\tilde{B}_1(i) =$$
$$\begin{bmatrix} \mathscr{P}(i)B(i)+\mathscr{B}(i)D(i) & \mathscr{P}(i)B_0(i)+\mathscr{B}(i)D_0(i) \\ \mathscr{Q}(i)B(i)+\mathscr{B}(i)D(i) & \mathscr{Q}(i)B_0(i)+\mathscr{B}(i)D_0(i) \end{bmatrix}$$

$$\mathscr{J}^{\mathrm{T}}(i)M(i)\tilde{B}_2(i) = \begin{bmatrix} \mathscr{P}(i)B_1(i)+\mathscr{B}(i)D_1(i) \\ \mathscr{Q}(i)B_1(i)+\mathscr{B}(i)D_1(i) \end{bmatrix}$$

$$\mathscr{J}^{\mathrm{T}}(i)\tilde{C}^{\mathrm{T}}(i) = \begin{bmatrix} 0 \\ \mathscr{C}^{\mathrm{T}}(i) \end{bmatrix}$$

$$\mathscr{J}^{\mathrm{T}}(i)M(i)\mathscr{J}(i) = \begin{bmatrix} \mathscr{P}(i) & \mathscr{Q}(i) \\ \mathscr{Q}(i) & \mathscr{Q}(i) \end{bmatrix} \tag{19}$$

we can readily obtain LMIs (11) and (12).

Moreover, notice that (18) is equivalent to

$$\begin{bmatrix} \hat{A}(i) & \hat{B}(i) \\ \hat{C}(i) & 0 \end{bmatrix} = \begin{bmatrix} M_2^{-1}(i) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \mathscr{A}(i) & \mathscr{B}(i) \\ \mathscr{C}(i) & 0 \end{bmatrix} \times$$
$$\begin{bmatrix} M_2^{-T}(i)M_3(i) & 0 \\ 0 & I \end{bmatrix} =$$
$$\begin{bmatrix} \left(M_2^{-T}(i)M_3(i)\right)^{-1}\mathscr{Q}^{-1}(i) & 0 \\ 0 & I \end{bmatrix} \times$$
$$\begin{bmatrix} \mathscr{A}(i) & \mathscr{B}(i) \\ \mathscr{C}(i) & 0 \end{bmatrix} \begin{bmatrix} M_2^{-T}(i)M_3(i) & 0 \\ 0 & I \end{bmatrix} \tag{20}$$

Note that the filter matrices $\hat{A}(i)$, $\hat{B}(i)$ and $\hat{C}(i)$ in (2) can be written as (20), which implies that $M_2^{-T}(i)M_3(i)$ can be viewed as a similarity transformation on the state-space realization of the filter and, as such, has no effect on the filter mapping from $y(t)$ to $r(t)$. Without loss of generality, we may set $M_2^{-T}(i)M_3(i) = I$, thus obtain (13). Therefore, the filter $(\hat{\varPi})$ in (2) can be constructed by (13). □

Remark 1 Theorem 1 provides a sufficient condition for solvability of H_∞ fault detection filter for the switched linear system $(\varPi)$ in (1). Since the obtained conditions are all in LMI form, a desired filter can be determined by solving the following convex optimization problem

$$\min_{\substack{\mathscr{P}(i)>0,\mathscr{Q}(i)>0 \\ \breve{P}>0,\mathscr{A}(i) \\ \mathscr{B}(i),\mathscr{C}(i)}} \text{s.t. } (11)-(12) \text{ with } \delta=\gamma^2 \tag{21}$$

4. Illustrative example

As the demands of higher precision, quicker response and better robustness in the turntable system, a double channel scheme is often adopted in the design of the turntable control system. The system model of turntable has been well studied [12]. Fig. 1 is the block diagram of an uncorrected system, where K_T is the servo rigidity which is represented by disturbance torque and offset angular signal, J is the moment of inertia and T_d is the disturbance torque. The transfer function of the uncorrected system is described by

$$\frac{\theta(s)}{T_d(s)} = \frac{1}{Js^2+K_T} \tag{22}$$

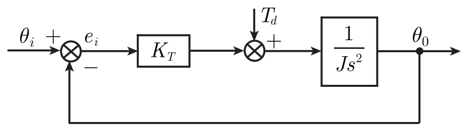

Fig. 1 Block diagram of uncorrected turntable system

Considering the measurement of angle position θ_0 and angular rate $\dot{\theta}_0$. Define $x_1 = \theta_0$, $x_2 = \dot{\theta}_0$, then the state equation of turntable can be described as

$$\dot{x}(t) = A'x(t) + B'K_u(t) + \omega'(t) \tag{23}$$

where $K_u(t)$ is the output of controller, $A' = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B' = [0 \quad K_T/J]^{\mathrm{T}}$, and $\omega' = [0 \quad 1/J]^{\mathrm{T}} \times T_d$.

Previous work has presented some different methods to generate different $K_u(t)$ [12,13]. In this paper, the output of the controller $K_u(t)$ is designed by linear state-feedback in the form of $K_u(t) = -K^{\mathrm{T}}x(t)$. To obtain the smaller error and higher precision, the state feedback gain matrix K in two channels is selected different. Considering a test turntable with the moment of inertia $J = 120\ \mathrm{kg \cdot m^2}$, servo rigidity $K_T = 20\ \mathrm{N \cdot M/V}$, the state feedback gain matrix can be selected as $K = [75 \quad 30]^{\mathrm{T}}$ in coarse channel, and $K = [144 \quad 150]^{\mathrm{T}}$ in fine channel. Then, the switched systems model for turntable system can be described in the form of (Π) with the following specifications.

Subsystem 1 (coarse channel)

$$\begin{gathered} A(1) = A' - B'K^{\mathrm{T}}(1) = \begin{bmatrix} 0 & 1 \\ -12.5 & -5 \end{bmatrix} \\ B_1(1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_0(1) = [0 \quad 0.174]^{\mathrm{T}} \\ B(1) = [1 \quad 1]^{\mathrm{T}}, \quad C(1) = [1 \quad 0] \\ D_0(1) = D(1) = 0, \quad D_1(1) = 0.1 \end{gathered} \tag{24}$$

Subsystem 2 (fine channels)

$$\begin{gathered} A(2) = A' - B'K^{\mathrm{T}}(2) = \begin{bmatrix} 0 & 1 \\ -24 & -25 \end{bmatrix}, \quad B_1(2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ B_0(2) = [0 \quad 0.174]^{\mathrm{T}}, \quad B(2) = [0.027\,78 \quad 0.027\,78]^{\mathrm{T}} \\ C(2) = [1 \quad 0], \quad D_0(2) = D(2) = 0, \quad D_1(2) = 0.1 \end{gathered} \tag{25}$$

where the parameter $B_1(1) \neq B_1(2)$ means that the different fault in different channels. The coefficient of the disturbance can be normalized to $B_0(1) = B_0(2) \approx B' + [0 \quad 1/J] = [0 \ 0.174]^{\mathrm{T}}$ in (2). Meanwhile, the unknown disturbance $\omega(t)$ can be normalized to be random noise. Besides, the reason why parameters $B(2) = [0.027\,78 \quad 0.027\,78]^{\mathrm{T}}$ can be states as follows:

(i) The coarse channel is work when the position error is large ($[0,\ 360°]$). The fine channel is switched when the position error is small ($[0,\ 10°]$).

(ii) The parameters $B(2)$ will be selected as $B(2) = [1/36 \quad 1/36]^{\mathrm{T}} = [0.027\,78 \quad 0.027\,78]^{\mathrm{T}}$, if normalized the parameter $B(1) = [1 \quad 1]^{\mathrm{T}}$.

Given $\alpha = 0.5$, it can be checked that the above switched linear system is exponentially stable with a weighted H_∞ performance when the average dwell time $T_a > (\ln \mu)/\alpha = 0.097\,5$ (in this case, setting $\mu = 1.05$). The switching signal with the average dwell time $T_a > 0.097\,5$ is shown in Fig. 2; here '1' and '2' represent the first and the second subsystem, respectively. We can see from Fig. 2 that the average dwell time $T_a > 0.1$.

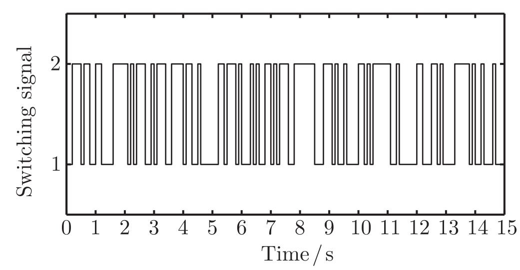

Fig. 2 Switching signal with the average dwell time $T_a > 0.097\,5$

By solving LMIs (11) and (12) in Theorem 1, we have that the minimized γ is $\gamma^* = 2.475$, and

$$\begin{gathered} \hat{A}(1) = \begin{bmatrix} -12.749\,2 & 0.484\,9 \\ -11.849\,5 & -9.578\,3 \end{bmatrix} \\ \hat{B}(1) = \begin{bmatrix} 8.337\,2 \\ -10.390\,9 \end{bmatrix}, \quad \hat{C}(1) = \begin{bmatrix} -0.003\,037 \\ -0.002\,37 \end{bmatrix}^{\mathrm{T}} \end{gathered} \tag{26}$$

$$\begin{gathered} \hat{A}(2) = \begin{bmatrix} -0.887\,1 & 1.378\,6 \\ -89.973\,4 & -84.486\,8 \end{bmatrix} \\ \hat{B}(2) = \begin{bmatrix} -0.386\,8 \\ -7.541\,2 \end{bmatrix}, \quad \hat{C}(2) = \begin{bmatrix} -0.020\,35 \\ -0.011\,1 \end{bmatrix}^{\mathrm{T}} \end{gathered} \tag{27}$$

where the weighting matrix function is given with the following specifications

$$\begin{gathered} \breve{A} = \begin{bmatrix} -2 & 1 \\ 2 & 5 \end{bmatrix}, \quad \breve{B} = \begin{bmatrix} 0.1 \\ 2 \end{bmatrix} \\ \breve{C} = [4 \quad 0] \end{gathered} \tag{28}$$

In the following, we shall further show the effectiveness of the designed robust H_∞ fault detection filter of (2) through simulation. Let the initial condition be $x(0) = [1.0 \quad 0]^{\mathrm{T}}$. The known input is given as $u(t) = 0.5\sin t$ $(0 \leqslant t \leqslant 15)$, the fault signal is set up as

$$f(t) = \begin{cases} 1, & 4 \leqslant t \leqslant 7.5 \\ 0, & \text{otherwise} \end{cases}$$

Thus, the weighting fault signal $\hat{f}(t)$ is shown in Fig. 3. The other simulation results are given in Figs. 4–6. Among them, Fig. 4 shows the states of the designed H_∞ fault detection filter, Fig. 5 depicts the generated residual signal $r(t)$, Fig. 6 presents the evaluation function of $J(r)$ for both the fault case (solid line) and fault-free case (dot line).

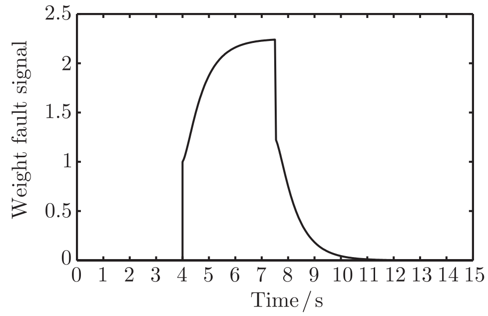

Fig. 3 Weighting fault signal $\hat{f}(t)$

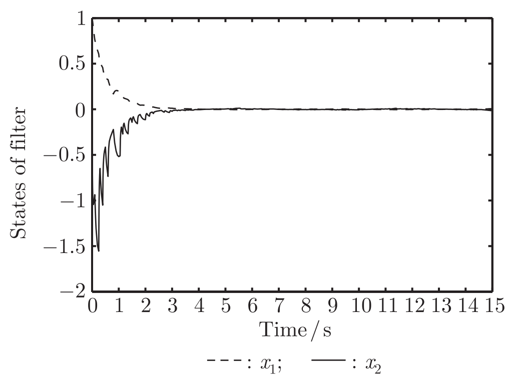

Fig. 4 States of the fault detection filter of (26) and (27)

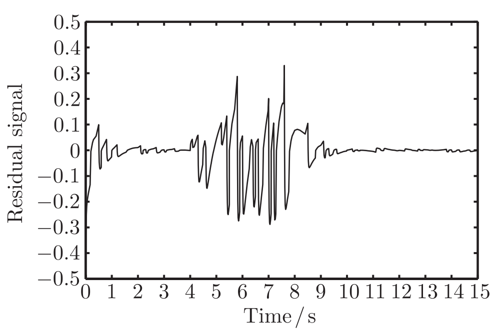

Fig. 5 Residual signal $r(t)$

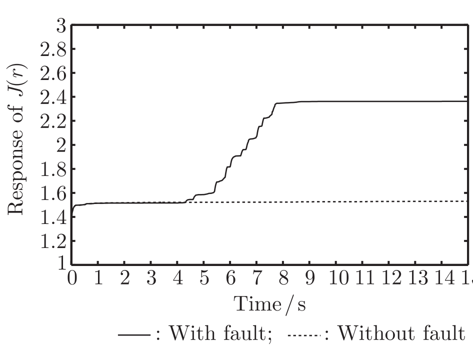

Fig. 6 Evaluation function of $J(r)$

When the residual signal is generated, the next step is to set up the fault detection measure. Choose a threshold

$$J_{\mathrm{th}} = \sup_{\omega\neq 0, u\neq 0, f=0} \left(\int_0^{15} r^{\mathrm{T}}(t) r(t)\mathrm{d}t\right)^{1/2} = 1.530\,2$$

the simulation results show that

$$\left(\int_0^{4.4} r^{\mathrm{T}}(t) r(t)\mathrm{d}t\right)^{1/2} = 1.544\,2 > J_{\mathrm{th}}$$

Thus, the appeared fault can be detected after 0.4 s. It means that any unexpected fault in the turntable system can be detected rapidly by the proposed fault detection filter.

5. Conclusion

In this paper, we have studied the problem of H_∞ fault detection filter design for a continuous-time linear switched system with its application to turntable systems. Based on the LMI technique, the solvability condition for a desired filter has been established by using the average dwell time approach. The sufficient condition has been proposed to guarantee the exponential stability with a weighted H_∞ performance for the fault detection system. Since the obtained conditions are all in LMI form, the desired filter can also be determined by solving the convex optimization problem. Moreover, the double channel scheme of the turntable control system has been modeled as a switched system. Each channel can be regard as a different subsystem of the switched system. Finally, the effectiveness of the proposed theory has been well validated through its application to turntable systems.

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