# Adaptive Fuzzy Integral Sliding Mode Control for Flexible Air-Breathing Hypersonic Vehicles Subject to Input Nonlinearity

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**Abstract:** This paper proposes a fuzzy integral sliding mode control strategy for flexible air-breathing hypersonic vehicles with input nonlinearity. Based on the complex nonlinear dynamics in the considered vehicles, a control-oriented model, which can retain the dominant features of the higher-fidelity model, is adopted for the control design. First, the T-S fuzzy approach is utilized to model the nonlinear dynamic of flexible air-breathing hypersonic vehicles, and the input nonlinearity model, which comprises dead-zones, sector nonlinearities, and actuator saturation, is considered. Then, based on the constructed T-S fuzzy model, a novel fuzzy integral sliding mode control method is proposed. This method can eliminate the reaching phase of the traditional sliding mode control by designing a novel sliding surface. Moreover, by the parallel-distributed compensation scheme, a sufficient condition is established to guarantee the global robust stability of the sliding mode dynamics in the specified surface in the presence of unmatched uncertainties and disturbance. The reachability of the specified sliding surface is guaranteed by an adaptive sliding mode controller under input nonlinearity. Finally, simulation results are given to show the effectiveness of the proposed control methods. **DOI:** 10.1061/(ASCE)AS.1943-5525.0000193. *© 2013 American Society of Civil Engineers.*

**CE Database subject headings:** Vehicles; Nonlinear systems; Fuzzy sets; Sliding; Aerospace engineering.

**Author keywords:** Flexible air-breathing hypersonic vehicles (FAHVs); Nonlinear dynamic system; Fuzzy integral sliding mode control (FISMC); Input nonlinearity; Actuator saturation.



## Introduction

In response to the promising prospects for reliable affordable access to space exploration and global reaching capabilities, air-breathing hypersonic vehicles (AHVs) have drawn much attention in recent years (Bertin and Cumming 2003). Such hypersonic flight technology, which may enable cost-effective vehicle systems for use in space launching, orbiting, and maneuvering, would translate to technical improvements in military interceptors and tactical and strategic reconnaissance, as well as in high-speed and orbital transport activities.

Because of the unique characteristics of AHVs dynamics, the design of guidance and control systems for this type of vehicle is a challenge (Bolender and Doman 2007). The couplings between propulsive and aerodynamic forces are very strong, because AHVs use the technology of airframe integrated with scramjet engine configuration (Curran 2001). The aerodynamic forces are significantly affected by the length, slender geometry, and flexibility of the vehicle structure (McRuer 1991; Oppenheimer et al. 2007). In addition, modeling inaccuracies and parameter uncertainties always exist, as do various other disturbances. All of these can exert strong adverse effects on the performance of AHVs control systems, so any sensible control design for AHVs must provide robustness against (possibly large) uncertainties and disturbances.

The modeling and controller design problems of AHVs have been widely studied over the last few years. Because of the dynamics' enormous complexity, only the longitudinal dynamics models of AHVs have been used for control design. In Schmidt (1992) and Chavez and Schmidt (1994), a comprehensive analytical model of hypersonic vehicles was first developed. The model is highly nonlinear, multivariable, and has strong couplings between the propulsive and aerodynamic effects (Chavez and Schmidt 1999). Robust controller design methods have been used to analyze this model, such as, for example, $H_\infty$ and $\mu$-synthesis methods (Buschek and Calise 1997), model reference adaptive control (Mooij 2001), and linear parameter-varying control (Lind 2002). Several other nonlinear control approaches were also proposed for this generic vehicle model. For example, in Marrison and Stengel (1998) and Wang and Stengel (2000), robust flight control systems with nonlinear dynamic inversion structures were synthesized via the stochastic robustness analysis approach. An adaptive sliding controller was also designed for the same model in Xu et al. (2004).

A flexible air-breathing hypersonic flight vehicle (FAHV) model, which included the flexible dynamics, was developed in Bolender and Doman (2007, 2006). Based on this model, some flight-control design problems have been studied in recent years. The equations of this model will become exceedingly complex when flexibility effects are considered, so this model can be used only for simulations or validation purposes (Oppenheimer et al. 2007; Williams et al. 2006). In Parker et al. (2007), a control-oriented model was derived for the FAHV model using curve fits calculated directly from the forces and moments included in the truth model, and an approximate feedback

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liberalization example of control design was given to derive a nonlinear controller. In Sigthorsson et al. (2008), dynamic output feedback techniques were used to provide reference robust velocity and altitude tracking control in the presence of model uncertainties and varying flight conditions, and in Groves et al. (2006) and Sigthorsson et al. (2006), linear controllers with input constraints using on-line optimization and anti-windup techniques were also proposed. More recently, a nonlinear robust adaptive control design method was presented in Fiorentini et al. (2009), and in Wilcox et al. (2010), the authors considered the modeling of aerothermoelastic effects and gave a Lyapunov-based tracking controller (Lyapunov 1992).

Although many approaches have been presented, robust control for the high nonlinear dynamics of FAHVs is still a problem.

Recently, the Takagi-Sugeno (T-S) (Tanaka and Wang 2001) fuzzy approach has been successfully applied to the control and filter designs of nonlinear systems, such as for the stabilization problem (Wu and Zheng 2009; Zhao et al. 2009; Du and Zhang 2009), the tracking problem (Zheng et al. 2002), and the filtering problem (Wu and Wang 2009; Li and Tseng 2009). The T-S modeling technique is a very good representation for a certain class of nonlinear dynamic systems (Feng 2006). Any smooth nonlinear function can be approximated by a T-S fuzzy model to any specified accuracy with linear rule consequence (Tanaka and Wang 2001). The T-S fuzzy model is represented by a set of linear models by fuzzy IF-THEN rules and the conventional linear control theories can be applied to analysis and synthesis of nonlinear systems based on the parallel-distributed compensation (PDC) scheme. In this paper, the T-S fuzzy control will be implemented to develop efficient control approaches to the tracking control of FAHVs subject to complex nonlinear and coupling.

Most of the proposed approaches for the control of FAHVs are based on the assumption that the considered systems are characterized by linear inputs, so the input dynamics can be reasonably approximated by a linear model. However, in practice, the linear model is an exception, and the actuator seems to have a nonlinear character because of its physical limitations. The presence of input nonlinearity in control may cause serious influence on FAHV stability and performance, and may even cause the FAHVs' system to reflect unpredictable results. Therefore, the effect of input nonlinearity cannot be ignored in the analysis and realization of controller design. The robust control problem of uncertain multivariable systems with nonlinear input channels has received remarkable attention because of ubiquitous input nonlinearities such as saturation, quantization, backlash, and dead-zones. However, to our knowledge, the control problem for FAHVs in containing the input nonlinearity has not been widely discussed.

Sliding mode control (SMC) is well known for its robustness to parametric uncertainties and external disturbances, as long as the uncertainties and disturbances satisfy the matched conditions (Hung et al. 1993; Yong et al. 1999). However, when the matched condition is not satisfied, the robustness cannot be guaranteed. A novel control scheme called *integral sliding mode control* (ISMC) has been studied in Chern and Wu (1991, 1992), Cheng and Liu (1999), and Choi (2007), and the robustness of the system can be guaranteed even if the matched conditions are not satisfied (Cao and Xu 2004; Castacos and Fridman 2006; Utkin and Shi 1996). In Hsu (1999), Hsu et al. (2004), Niu and Ho (2006), and Shyu et al. (2005), it has been shown that SMC can overcome the limitations of systems with multiple inputs containing both sector nonlinearities and dead-zones. Therefore, in this paper, we will investigate a novel robust control approach for FAHVs with multiple inputs containing sector nonlinearities, dead-zones, and actuator saturation.

Motivated by the preceding discussions in the literature, in this paper, we will propose a T-S model-based fuzzy integral sliding mode control (FISMC) design approach for the longitudinal model of FAHVs. The nonlinear dynamics of FAHVs are supposed to suffer from parameter uncertainties, nonlinear perturbations, and input nonlinearity. First, a T-S fuzzy model is constructed to represent the complex nonlinear longitudinal model of a FAHV. The model of input nonlinearity is considered; therefore, the model contains dead-zones, sector nonlinearities, and actuator saturation. A novel fuzzy integral-type sliding surface is developed and the robustness of the equivalent dynamics on the designed sliding surface is discussed. Based on the PDC scheme, an equivalent control is designed and a sufficient condition is given to guarantee the stabilization of the equivalent dynamics in the integral-type sliding surface. An adaptive SMC law is then synthesized to guarantee the reachability of the specified sliding surface in the presence of input nonlinearity. Finally, an illustrative example is provided to show the effectiveness of the proposed control design methods.

The rest of this paper is organized as follows: The fuzzy modeling and control problem of FAHVs is presented in Problem Statement and Preliminaries, followed by the Main Result presenting the design of robust FISMC for the T-S model, then Simulation Results and the Conclusion.

## Problem Statement and Preliminaries

### T-S Fuzzy Modeling of FAHVs

A longitudinal sketch of FAHVs is given in Fig. 1, and the nonlinear dynamics of FAHVs considered in this paper was first developed by Bolender and Doman (2007, 2006), which is shown as follows:

$$
\begin{aligned}
\dot{h} &= V\sin(\theta-\alpha),\\
\dot{V} &= \frac{1}{m}(T\cos\alpha - D) - g\sin(\theta-\alpha),\\
\alpha &= \frac{1}{mV}(-T\sin\alpha - L) + Q + \frac{1}{V}\cos(\theta-\alpha),\\
\dot{\theta} &= Q,\\
I_{yy}\dot{Q} &= M,\\
\ddot{\eta}_1 &= -2\varsigma_1\omega_1\dot{\eta}_1 - \omega_1^2\eta_1 + N_1,\\
\ddot{\eta}_2 &= -2\varsigma_2\omega_2\dot{\eta}_2 - \omega_2^2\eta_2 + N_2
\end{aligned}
\tag{1}
$$

where $T$, $L$, $D$, and $N_i$ = thrust, lift, drag, and generalized elastic forces, respectively; and $M$ = pitching moment. This nonlinear model is composed of five rigid-body state variables: $x(t) = (h, V, \alpha, \theta, Q)^T$ and four flexible states $\eta(t) = (\eta_1, \dot{\eta}_1, \eta_2, \dot{\eta}_2)^T$. The control input $u(t) = (\Phi, \delta_e)^T$ does not appear explicitly in the equations. The preceding equations are exceedingly complex, so these models should be used for simulations or validation purposes only. Using curve-fits approximations, polynomial expressions of the forces and moments $T$, $L$, $D$, $M$, $N_1$, and $N_2$ are constructed in Parker et al. (2007), which are shown as follows:

$$
\begin{aligned}
L &\approx \frac{1}{2}\rho V^2 S C_L(\alpha,\delta_e),\\
D &\approx \frac{1}{2}\rho V^2 S C_D(\alpha,\delta_e),\\
M &\approx z_T T + \frac{1}{2}\rho V^2 S\bar{c}\left[C_{M,\alpha}(\alpha) + C_{M,\delta_e}(\delta_e)\right],\\
T &\approx C_T^{\alpha^3}\alpha^3 + C_T^{\alpha^2}\alpha^2 + C_T^{\alpha}\alpha + C_T^0,\\
N_1 &\approx N_1^{\alpha^2}\alpha^2 + N_1^{\alpha}\alpha + N_1^0,\\
N_2 &\approx N_2^{\alpha^2}\alpha^2 + N_2^{\alpha}\alpha + N_2^{\delta}\delta_e + N_2^0
\end{aligned}
\tag{2}
$$

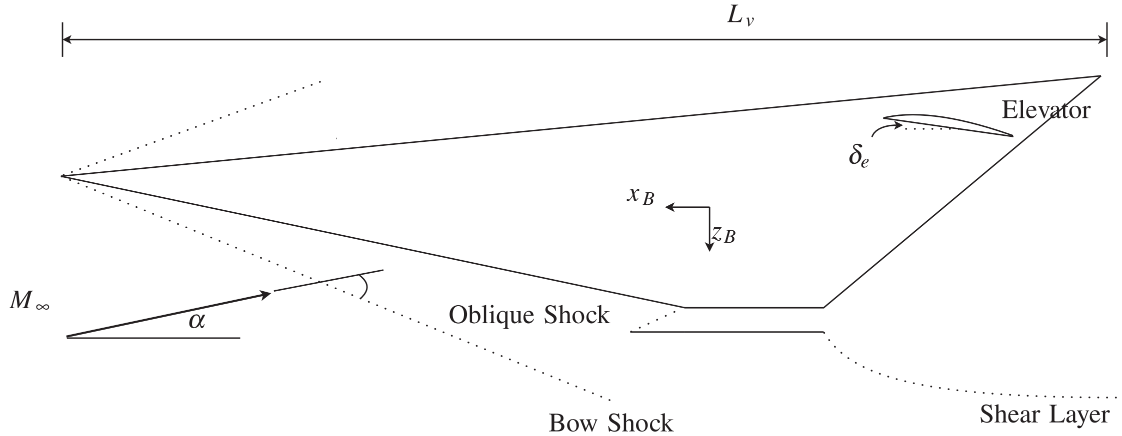


**Fig. 1.** Geometry of the flexible hypersonic vehicle model

with

$$
\begin{aligned}
&\rho = \rho_0 \exp\left[\frac{-(h-h_0)}{h_s}\right],\\
&C_L = C_L^{\alpha}\alpha + C_L^{\delta_e}\delta_e + C_L^0,\\
&C_D = C_D^{\alpha^2}\alpha^2 + C_D^{\alpha}\alpha + C_D^{\delta_e^2}\delta_e^2 + C_D^{\delta_e}\delta_e + C_D^0,\\
&C_{M,\alpha} = C_{M,\alpha}^{\alpha^2}\alpha + C_{M,\alpha}^{\alpha}\delta + C_{M,\alpha}^0, \quad C_{M,\delta_e} = c_e\delta_e,\\
&C_T^{\alpha^3} = \beta_1(h,\bar{q})\Phi + \beta_2(h,\bar{q}), \quad \bar{q} = \frac{1}{2}\rho V^2,\\
&C_T^{\alpha^2} = \beta_3(h,\bar{q})\Phi + \beta_4(h,\bar{q}),\\
&C_T^{\alpha} = \beta_5(h,\bar{q})\Phi + \beta_6(h,\bar{q}),\\
&C_T^{0} = \beta_7(h,\bar{q})\Phi + \beta_8(h,\bar{q})
\end{aligned}
\tag{3}
$$

The polynomial expressions of the forces and moments, while retaining the relevant dynamics feature of Eq. (1), simplify the control design and stability analysis of FAHVs, so in this paper, they are utilized to develop the controller.

By inserting Eqs. (2) and (3) into Eq. (1), the nonlinear equations can be rewritten in an affine nonlinear form, that is

$$
\begin{aligned}
&\dot{x}(t) = f[x(t)] + g[x(t)]u(t),\\
&y(t) = Cx(t)
\end{aligned}
\tag{4}
$$

where $x(t) = (h, V, \alpha, \theta, Q, \eta_1, \dot{\eta}_1, \eta_2, \dot{\eta}_2)^T$, $u(t) = (\Phi,\ \delta_e)^T$, and

$$
C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\tag{5}
$$

The nonlinear dynamic of FAHVs is highly complex, so it is difficult to design a controller directly. The T-S fuzzy model is said to be efficient for approaching a nonlinear system at any precision, so in this paper, we will construct a T-S fuzzy model for the nonlinear hypersonic vehicle system from Eq. (1). When establishing the fuzzy model, the T-S fuzzy modeling technique expressed in Teixeira and Zak (1999) is employed. This paper gives a novel approach for constructing a T-S fuzzy model by utilizing the mathematical approximation method. Simultaneously, we will consider the system with parameter uncertainties and disturbances. The system Eq. (4) can be represented by the following linear models:

If $z_1(t) = M_1^i$ and $z_2(t) = M_2^i \ldots z_p(t) = M_p^i$, then

$$
\begin{aligned}
&\dot{x}(t) = (A_i + \Delta A_i)x(t) + B_i u(t) + D_i\varpi(t),\\
&y(t) = Cx(t)
\end{aligned}
\tag{6}
$$

where $z(t) = [z_1(t)\, z_2(t) \ldots z_p(t)]$ = premise variables; $M_l^i, \ldots M_p^i$ = fuzzy sets; $A_i$, $B_i$, $C$, and $D_i$ = known constant matrices; $\Delta A_i$ = unknown parameter uncertainty of $A_i$; and $\varpi(t)$ = uncertain extraneous disturbance or the nonlinearity. Then, the T-S fuzzy model representing the nonlinear hypersonic vehicle model from Eq. (1) can be inferred as follows:

$$
\begin{aligned}
&\dot{x}(t) = \sum_{i=1}^{l}\mu_i(z)[(A_i + \Delta A_i)x(t) + B_i u(t) + D_i\varpi(t)],\\
&y(t) = Cx(t)
\end{aligned}
\tag{7}
$$

where $l$ = number of the fuzzy rules and

$$
\mu_i(z) = \frac{m_i(z)}{\sum_{i=1}^{l} m_i(z)}, \quad m_i(z) = \prod_{j=1}^{q} M_j^i(z)
$$

with $\mu_i(t) \geq 0$, $i = 1, \ldots, l$ and $\sum_{i=1}^{l}\mu_i(t) = 1$.

For description brevity, the preceding equation can be written as

$$
\begin{aligned}
&\dot{x}(t) = (A_\mu + \Delta A_\mu)x(t) + B_\mu u(t) + D_\mu\varpi(t),\\
&y(t) = Cx(t)
\end{aligned}
\tag{8}
$$

where

$$
\begin{aligned}
A_\mu &= \sum_{i=1}^{l}\mu_i(t)A_i,\\
B_\mu &= \sum_{i=1}^{l}\mu_i(t)B_i,\\
\Delta A_\mu &= \sum_{i=1}^{l}\mu_i(t)\Delta A_i,\\
D_\mu &= \sum_{i=1}^{l}\mu_i(t)D_i
\end{aligned}
$$



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## Input Nonlinearity Model

The input nonlinearity under consideration can be described by the following mathematical model:

$$\psi_\pi[u_\pi(t)] = \begin{cases} \psi_\pi^+[u_\pi(t)]\left[u_\pi(t) - u_\pi^+\right], & u_\pi > u_\pi^+ \\ 0, & -u_\pi^- \le u_\pi \le u_\pi^+ \\ \psi_\pi^-[u_\pi(t)]\left[u_\pi(t) + u_\pi^-\right], & u_\pi < -u_\pi^- \end{cases} \quad (9)$$

where $\psi_\pi^+[u_\pi(t)] > 0$ and $\psi_\pi^-[u_\pi(t)] > 0$ = nonlinear functions of $u_\pi(t)$; $u_\pi(t)$ = inputs of FAHVs; $\pi = \Phi$ and $\delta_e$; and $u_\pi^+ > 0$ and $u_\pi^- > 0$ = known constants.

**Assumption 1**
The input nonlinear functions $\psi_\pi^+[u_\pi(t)]$ and $\psi_\pi^-[u_\pi(t)]$ satisfy

$$\begin{aligned} &\alpha_\pi^+\left[u_\pi(t) - u_\pi^+\right] \le \psi_\pi^+[u_\pi(t)] \le \beta_\pi^+\left[u_\pi(t) - u_\pi^+\right], \quad u_\pi(t) > u_\pi^+ \\ &\beta_\pi^-\left[u_\pi(t) + u_\pi^-\right] \le \psi_\pi^+[u_\pi(t)] \le \alpha_\pi^-\left[u_\pi(t) - u_\pi^-\right], \quad u_\pi(t) < -u_\pi^- \end{aligned} \quad (10)$$

where $\alpha_\pi^+$, $\beta_\pi^+$, $\alpha_\pi^-$, and $\beta_\pi^-$ (called gain reduction tolerances) = known constants.

In addition, both of the control inputs are constrained by a saturation value, expressed by

$$-u_{\pi\,\min} \le \psi_\pi[u_\pi(t)] \le u_{\pi\,\max}$$

Each of the inputs has a separate saturation limit, but because the real value of the input $\psi_\pi[u_\pi(t)]$ is unknown in practice, it is difficult to define a saturation function directly for each component of $\psi[u(t)]$. Because the nonlinear functions of the input satisfy Eq. (10), the saturation function $\mathrm{sat}(u)$ is defined as

$$\mathrm{sat}\{\psi_\pi[u_\pi(t)]\} = \aleph_\pi[u_\pi(t)]\psi_\pi[u_\pi(t)]$$

where $0 < \aleph_\pi[u_\pi(t)] \le 1$ and

$$\aleph_\pi[u_\pi(t)] = \begin{cases} \dfrac{u_{\pi\,\max}}{\psi_\pi[u_\pi(t)]}, & \beta_\pi^+\left[u_\pi(t) - u_\pi^+\right] > u_{\pi\,\max} \\ 1, & \begin{array}{l} 0 \le \beta_\pi^+\left[u_\pi(t) - u_\pi^+\right] \le u_{\pi\,\max} \\ \text{or} - u_{\pi\,\min} \le \beta_\pi^-\left[u_\pi(t) - u_\pi^-\right] < 0 \end{array} \\ \dfrac{u_{\pi\,\min}}{\psi_\pi[u_\pi(t)]}, & \beta_\pi^-\left[u_\pi(t) - u_\pi^-\right] < -u_{\pi\,\min} \end{cases} \quad (11)$$

**Remark 1**: Obviously, the saturation functions described in Eq. (11) constitute an approximate approach. Under the unknown nonlinear input, the approach can provide a effective way to the saturation problem.

The nonlinear input function is shown in Fig. 2.

Hence, by considering the input nonlinearity problem, the fuzzy systems in Eq. (7) can be transformed into

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{l} \mu_i(z)[(A_i + \Delta A_i)x(t) + B_i \mathrm{sat}\{\psi[u(t)]\} + D_i \varpi(t)], \\ y(t) &= Cx(t) \end{aligned} \quad (12)$$

**Remark 2**: From the model presented in Eqs. (9) and (10), we can see that this kind of mathematical model can represent not only dead-zones but also sector nonlinearities. From a practical point of view, the parameters $u_\pi^+$ and $u_\pi^-$ are not required to be equivalent, so $\alpha_\pi^+$ and $\alpha_\pi^-$ are not required to be equivalent either.
**Remark 3**: In practice, the fuel/air $\Phi$ may have a sector nonlinear character, but based on the exit of friction and derived delay, the elevator deflection $\delta_e$ would suffer both from dead-zones and sector nonlinearities.

## Model Reference Control Objective

For the purpose of model reference, the reference model of the fuzzy system in Eq. (12) can be defined by

Rule $i$: If $z_1(t) = M_1^i$, $z_2(t) = M_2^i \ldots z_p(t) = M_p^i$,
then

$$\dot{x}_m(t) = A_{mi}x_m(t) + B_{mi}r(t) \quad (13)$$

where $x_m(t) \in \mathbb{R}^n$ = state of the reference model; $r(t) \in \mathbb{R}^m$ = known, piecewise continuous and bounded reference input; and $A_{mi}$ and $B_{mi}$ = known and real constant matrices with approximate dimensions. In addition, $A_{mi}$ is assumed to be a stable matrix.

The vector of the tracking error is defined as

$$e(t) = x(t) - x_m(t) \quad (14)$$

By differentiating Eq. (14) with respect to time and considering Eqs. (12) and (13), the dynamic equation of tracking error can be described by

Rule $i$: If $z_1(t) = M_1^i$, $z_2(t) = M_2^i \ldots z_p(t) = M_p^i$,
then

$$\begin{aligned} \dot{e}(t) = A_{mi}e(t) + B_i\mathrm{sat}\{\psi[u(t)]\} + (A_i + \Delta A_i - A_{mi})x(t) \\ - B_{mi}r(t) + D_i\varpi(t) \end{aligned} \quad (15)$$

The overall tracking error system is given by

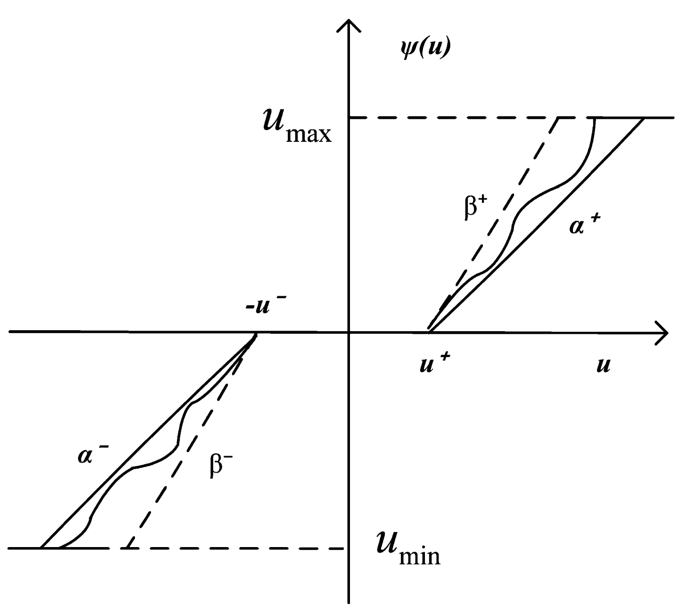


**Fig. 2.** Nonlinear input function

$$\dot{e}(t) = \sum_{i=1}^{l} \mu_i(t)[A_{mi}e(t) + B_i \mathrm{sat}\{\psi[u(t)]\} + (A_i + \Delta A_i - A_{mi})x(t) - B_{mi}r(t) + D_i \varpi(t)], \quad (16)$$
$$= \sum_{i=1}^{l} \mu_i(t)[A_{mi}e(t) + B_i \mathrm{sat}\{\psi[u(t)]\} + \chi_i(t)]$$

where $\chi_i(t) = (A_i - A_{mi})x(t) - B_{mi}r(t) + \Delta A_i x(t) + D_i \varpi(t)$.

To ensure the achievement of model reference's objective, the following assumptions are necessary.

**Assumption 2**

The pairs ($A_{mi}$, $B_i$) and ($A_i$, $B_i$) are controllable.

**Assumption 3**

Matrix $B_i$ is of full column rank.

**Assumption 4**

There exist positive scalars $\rho_0$ and $\rho_1$ such that $\|\chi_i(t)\|_{\max} \le \rho_0 + \rho_1 \|e(t)\|$ for all $i = 1, 2, \ldots, 9$.

**Remark 4**: Assumption 4 means that $\chi_i(t)$ is not necessary to satisfy the matched condition, but it satisfies $\|\chi_i(t)\|_{\max} \le \rho_0 + \rho_1 \|e(t)\|$.

The main objective of this paper is to design a fuzzy model reference controller, such that the states of Eq. (12) track those of the reference model in the asymptotic sense. In addition, the globally asymptotic stability of the system Eq. (16) should be guaranteed even with the existence of parameter uncertainties and extraneous disturbance.

## Main Result

In this paper, a FISMC technique will be utilized to stabilize the tracking error's dynamics in Eq. (16). By the FISMC, the robustness of the system can be guaranteed even if the uncertainties and disturbance do not satisfy the matched condition. To this end, in this section, a fuzzy integral sliding surface for the T-S fuzzy mode will be designed, and the robustness of this sliding surface will be guaranteed by designing a controller based on the PDC scheme.

### *Integral Sliding Mode Design and Stability Analysis*

Choose the following integral-type sliding surface:

Rule $i$: If $z_1(t) = M_1^i$, $z_2(t) = M_2^i \ldots z_p(t) = M_p^i$, then

$$s_i(t) = G_i \left\{ [e(t) - e(t_0)] - \int_{t_0}^{t} [A_{mi}e(\tau) + B_i K_i e(\tau)] d\tau \right\} \quad (17)$$

where $G_i \in R^{m \times n}$ and $K_i \in R^{n \times m}$ = constant matrices to be designed. The matrices $K_i$ and $G_i$ are to be designed such that the matrix $(A_{mi} + B_i K_i)$ is Hurwitz (Wang et al. 1994) and the matrix $G_i B_i$ is nonsingular. Then, the overall sliding surface can be described as

$$s(t) = \sum_{i=1}^{9} \mu_i(t) s_i(t) \quad (18)$$

Note that $s[x(t_0, t_0)] = 0$. The integral sliding mode Eq. (18) converges to zero at the beginning (i.e., the reaching phase is eliminated). When the state trajectories of the system enter the sliding surface, we have $s_i(t) = 0$ and $\dot{s}_i(t) = 0$; that is

$$\dot{s}_i(t) = G_i[A_{mi}e(t) + B_i \mathrm{sat}\{\psi[u(t)]\} + \chi_i(t) - [A_{mi}e(t) + B_i K_i e(t)]], \\ = G_i B_i(\mathrm{sat}\{\psi[u(t)]\} - K_i e(t)) + G_i \chi_i(t) = 0 \quad (19)$$

Then the $i$th equivalent control sat $\{\psi[u(t)]\}_{\mathrm{eqi}}$ is obtained as

$$\mathrm{sat}\{\psi[u(t)]\}_{\mathrm{eqi}} = K_i e(t) - (G_i B_i)^{-1} G_i \chi_i(t) \quad (20)$$

The $i$th sliding mode dynamics can be obtained by substituting Eq. (20) into Eq. (15), that is,

Rule $i$: If $z_1(t) = M_1^i$, $z_2(t) = M_2^i \cdots z_p(t) = M_p^i$, then

$$\dot{e}(t) = \left\{ A_{mi}e(t) + B_i K_i e(t) + \left[ I - B_i (G_i B_i)^{-1} G_i \right] \chi_i(t) \right\} \quad (21)$$

**Remark 5**: From the preceding analysis, it can be seen that the nonlinear inputs in the sliding mode dynamics possess the same property as those without. Therefore, the nonlinear inputs have no effect on sliding mode dynamics.

In the following, we will consider $\chi_i(t)$ as an unmatched perturbation, and design the fuzzy equivalent controller parameter $K_i$ to stabilize the nominal case of the preceding sliding mode dynamics. Before proceeding, the matrix $G_i$ should meet the following two constraints:

1. $G_i B_i$ is invertible; and
2. The norm of $[I - B_i (G_i B_i)^{-1} G_i] \chi_i(t)$ is minimized.

**Remark 6**: The first constraint is a necessary condition for the exit of sliding surface and the second is for minimizing the influence of $\chi_i(t)$.

**Lemma 1** (Castacos and Fridman 2006). For the sliding mode dynamics Eq. (21), $G_i = B_i^+ = (B_i^T B_i)^{-1} B_i^T$ satisfies the previously-mentioned two constraints. By choosing $G_i = B_i^+$, the norm of $[I - B_i (G_i B_i)^{-1} G_i] \chi_i(t)$ can be minimized, and its Euclidean norm is not bigger than the original one.

Based on Lemma 1, the overall sliding mode dynamics can be described by

$$\dot{e}(t) = \sum_{i=1}^{l} \sum_{i=1}^{l} \mu_i(t) \mu_j(t) \left[ A_{mi}e(t) + B_i K_j e(t) + \omega(t) \right] \quad (22)$$

where $\|\omega(t)\| = \max[\|\chi_i(t)\|, i = 1, 2, \ldots, 9]$.

In the following, we will consider the design of fuzzy equivalent controller parameter $K_i$. For a T-S fuzzy system, the PDC offers a procedure to design a fuzzy controller. In the PDC design, each control rule is designed from the corresponding rule of a T-S fuzzy model. The designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts. In this paper, our main work is to design a SMC such that the tracking errors $e(t) = x(t) - x_m(t)$ converge to zero. However, there is unmatched perturbation in Eq. (22). To this end, the following $H_\infty$ tracking performance related to the tracking error vector $e(t)$ in Eq. (22) is set as follows:

$$\int_0^\infty e^T(t) \Omega e(t)\, dt \le \rho^2 \int_0^\infty \omega^T(t) \omega(t)\, dt \quad (23)$$

or

$$\frac{\int_0^\infty e^T(t)\Omega e(t)\,dt}{\int_0^\infty \omega^T(t)\omega(t)\,dt} \le \rho^2 \tag{24}$$

where $\omega(t) \in L_2(0, \infty)$; $\Omega$ = positive definite weighting matrix; and $\rho$ = prescribed attenuation level.

The physical meaning of Eq. (23) [or Eq. (24)] is that the effect of any $\omega(t)$ on tracking error $e(t) = x(t) - x_m(t)$ must be attenuated below a desired level $\rho$ from the viewpoint of energy, no matter what $\omega(t)$ is [i.e., the gain from $\omega(t)$ to $e(t) = x(t) - x_m(t)$ must be equal to, or less than, a prescribed value $\rho$].

To design the robust controller, the following lemmas are necessary:

**Lemma 2** (Petersen 1987). Let $X$ and $Y$ = real matrices (or vectors) of appropriate dimensions; then for any scalar $\varepsilon > 0$, we have

$$X^TY + Y^TX \le \varepsilon X^TX + \varepsilon^{-1}Y^TY$$

**Lemma 3** (Tuan et al. 2001). The parameterized linear matrix inequalities,

$$\sum_{i=1}^{k}\sum_{i=1}^{k} \mu_i\mu_j M_{ij} < 0$$

are fulfilled, if the following condition holds:

$$M_{ii} < 0,$$
$$\frac{1}{k-1}M_{ii} + \frac{1}{2}\left(M_{ij} + M_{ji}\right) < 0, \quad 1 \le i \ne j \le k$$

**Theorem 1.** Consider the system in Eq. (16) with the designed sliding surface in Eq. (18) and Assumptions 2–4. If there exist matrices $X > 0$, $Y_i$, and a scalar $\rho > 0$ satisfying

$$\Theta_{ii} < 0, \quad i = 1, 2, \ldots, l \tag{25}$$

$$\frac{1}{k-1}\Theta_{ii} + \frac{1}{2}\left(\Theta_{ij} + \Theta_{ji}\right) < 0, \quad 1 \le i \ne j \le l \tag{26}$$

where

$$\Theta_{ij} = \begin{bmatrix} A_{mi}X + X^TA_{mi} + B_iY_j + Y_j^TB_i^T + \rho^{-2}I & X \\ X^T & -\Omega^{-1} \end{bmatrix} \tag{27}$$

then the sliding mode dynamics in Eq. (22) is robustly stable and the $H_\infty$ performance in Eq. (23) or Eq. (24) is guaranteed for a prescribed performance index $\rho$. Moreover, the desired fuzzy controller gain matrices $K_i$ is given by

$$K_i = Y_iX^{-1}$$

**Proof.** Defining $P = X^{-1}$, pre- and post-multiplying Eq. (27) by $\operatorname{diag}\{X^{-1} \quad I \quad I\}$ and its transpose, respectively, we have

$$\hat{\Theta}_{ij} = \begin{bmatrix} P\left(A_{mi} + B_iK_j\right) + \left(A_{mi} + B_iK_j\right)^TP + \rho^{-2}PP & I \\ I & -\Omega^{-1} \end{bmatrix}$$

So Eqs. (25) and (26) are equal to

$$\hat{\Theta}_{ii} < 0, \qquad i = 1, 2, \ldots, 9,$$
$$\frac{1}{k-1}\hat{\Theta}_{ii} + \frac{1}{2}\left(\hat{\Theta}_{ij} + \hat{\Theta}_{ji}\right) < 0, \quad 1 \le i \ne j \le 9$$

According to Lemma 3, if the preceding inequalities hold, the following inequality is guaranteed:

$$\sum_{i=1}^{l}\sum_{i=1}^{l} \mu_i(t)\mu_j(t)\begin{bmatrix} P\left(A_{mi} + B_iK_j\right) + \left(A_{mi} + B_iK_j\right)^TP + \rho^{-2}PP & I \\ I & -\Omega^{-1} \end{bmatrix} < 0$$

Using the Schur complement, the preceding inequality is equivalent to

$$\sum_{i=1}^{l}\sum_{i=1}^{l} \mu_i(t)\mu_j(t)\Big[P\left(A_{mi} + B_iK_j\right) + \left(A_{mi} + B_iK_j\right)^TP + \rho^{-2}PP + \Omega\Big] < 0 \tag{28}$$

Choose the following Lyapunov function for system Eq. (22):

$$V(t) = e^T(t)Pe(t) \tag{29}$$

and taking the time derivative of $V(t)$ along with the solution of Eq. (22) with $\omega(t) = 0$, we have

$$\begin{aligned} \dot{V}(t) &= 2e^T(t)P\dot{e}(t) \\ &= 2e^T(t)P\left\{\sum_{i=1}^{l}\sum_{i=1}^{l}\mu_i(t)\mu_j(t)\left[A_{mi}e(t) + B_iK_je(t)\right]\right\} \\ &= \sum_{i=1}^{l}\sum_{i=1}^{l}\mu_i(t)\mu_j(t)e^T(t)\Big[P\left(A_{mi} + B_iK_j\right) + \left(A_{mi} + B_iK_j\right)^TP\Big]e(t) \end{aligned} \tag{30}$$

Considering $\Omega > 0$ and $P > 0$, it follows that $\rho^{-2}PP + \Omega > 0$. Therefore, from Eq. (28) we obtain

$$\sum_{i=1}^{l}\sum_{i=1}^{l} \mu_i(t)\mu_j(t)\Big[P\left(A_{mi} + B_iK_j\right) + \left(A_{mi} + B_iK_j\right)^TP\Big] < 0$$

which means that

$$\dot{V}(t) = \sum_{i=1}^{l}\sum_{i=1}^{l}\mu_i(t)\mu_j(t)e^T(t)\Big[P\big(A_{mi}+B_iK_j\big) + \big(A_{mi}+B_iK_j\big)^TP\Big]e(t) < 0$$

Therefore, the sliding mode dynamics equation, Eq. (22), is asymptotically stable with $\omega(t)=0$.

Next, we shall establish the $H_\infty$ performance of sliding mode dynamics Eq. (22). It can be shown that for any nonzero $\omega(t)\in L_2[0,\infty)$ and $t>0$,

$$\int_0^\infty e^T(t)Qe(t)dt = e^T(0)Pe(0) - e^T(\infty)Pe(\infty) + \int_0^\infty\Big[e^T(t)\Omega e(t) + \dot{e}^T(t)Pe(t) + \frac{d}{dt}e^T(t)Pe(t)\Big]dt$$

$$\le \int_0^\infty\Big[e^T(t)\Omega e(t) + \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t)\Big]dt = \int_0^\infty\sum_{i=1}^{l}\sum_{i=1}^{l}\mu_i(t)\mu_j(t)\Big\{\big[A_{mi}e(t)+B_iK_je(t)+\omega(t)\big]^TPe(t)$$

$$+ e^T(t)P\big[A_{mi}e(t)+B_iK_je(t)+\omega(t)\big] + e^T(t)\Omega e(t)\Big\}dt$$

$$= \int_0^\infty\sum_{i=1}^{l}\sum_{i=1}^{l}\mu_i(t)\mu_j(t)\Big\{e^T(t)\big[P\big(A_{mi}+B_iK_j\big) + \big(A_{mi}+B_iK_j\big)^TP + \Omega\big]\Big\}e(t)\omega^T(t)Pe(t) + e^T(t)P\omega(t)dt$$

Using Lemma 2, we have

$$\int_0^\infty\Big[\omega^T(t)Pe(t) + e^T(t)P\omega(t)\Big]dt,$$

$$\le \rho^2\int_0^\infty\omega^T(t)\omega(t)dt + \rho^{-2}\int_0^\infty e^T(t)PPe(t)dt$$

Considering Eq. (28), we have

$$\int_0^\infty e^T(t)Qe(t)dt \le \rho^2\int_0^\infty\omega^T(t)\omega(t)dt$$

Therefore, the tracking control performance is achieved with a prescribed level $\rho$. The proof is completed.

To obtain a better tracking performance, the tracking control problem can be obtained by solving the following optimal problem:

$$\min_{X>0}\ \rho \quad \text{s.t.} \quad \text{linear matrix inequality (LMI) Eqs. (25)}-(27) \tag{31}$$

### Adaptive Controller Design

After designing the sliding surface, the next phase of the traditional SMC is to design an appropriate SMC law, such that the error dynamics will be driven onto the sliding surface and remain there. When utilizing the conventional SMC technique, to design a control law with the switching part dominating the influence of perturbations, the upper bound of $\|\chi_i(t)\|_{\max}$ should be known (Hung et al. 1993; Yong et al. 1999). For the system from Eq. (16), $\|\chi_i(t)\|_{\max} \le \rho_0 + \rho_1\|e(t)\|$, the parameters $\rho_0$ and $\rho_1$ should be known for designing the sliding mode controller. However, in practice it is difficult to obtain $\rho_0$ and $\rho_1$. In this section, an adaptive law will be designed to estimate $\rho_0$ and $\rho_1$, thus an adaptive sliding mode controller for the system from Eq. (16) will be synthesized. Because $0<\aleph_\pi[u_\pi(t)]\le 1$, there always exists a constant $\gamma_\pi$ that satisfies

$$0<\gamma_\pi\le\aleph_\pi[u_\pi(t)]\le 1$$

Choose $\gamma=\min(\gamma_\Phi,\gamma_{\delta_e})$, then $\gamma\le\aleph_\pi[u_\pi(t)]\le 1$ is always satisfied. The following designed controller will also provide an adaptive law to deal with the saturation problem.

Let $\hat{\rho}_0(t)$, $\hat{\rho}_1(t)$, and $\hat{\gamma}(t)$ represent the estimate of $\rho_0$, $\rho_1$, and $\gamma$, respectively. The adaptive SMC that drives the system dynamics onto the sliding surface can be designed as

$$u_\pi(t) = \begin{cases} -k_\pi\hat{\gamma}(t)\nu(t)\dfrac{s_\pi(t)}{\|s(t)\|} - u_\pi^-, & s_\pi>0 \\ 0, & s_\pi=0 \\ -k_\pi\hat{\gamma}(t)\nu(t)\dfrac{s_\pi(t)}{\|s(t)\|} + u_\pi^+, & s_\pi<0 \end{cases} \tag{32}$$

where $k_\pi>1/\lambda_\pi, \lambda_\pi=\min\{\alpha_\pi^+,\ \alpha_\pi^-\}$, $\nu(t)=\hat{\rho}_0(t)+\hat{\rho}_1(t)\|e(t)\|$. The adaptive laws are designed as follows:

$$\dot{\hat{\rho}}_0(t) = q_1\|s(t)\|, \quad \dot{\hat{\rho}}_1(t) = q_2\|s(t)\|\|e(t)\| \tag{33}$$

with $\hat{\rho}_0(0)=\hat{\rho}_1(0)=0$, where $q_1$ and $q_2$ = adjustable positive constants. The value $\hat{\gamma}(t)$ is the solution of the following linear differential equation:

$$\dot{\hat{\gamma}}(t) = \overline{\beta}\hat{\gamma}^3(t)\nu(t)\|s(t)\|$$

with $\hat{\gamma}(0)=\gamma_0$, where $\gamma_0$ = bounded positive initial value of $\hat{\gamma}(t)$, $\overline{\beta}>0$; the rest will be defined later.

**Theorem 2.** Consider the uncertain tracking dynamic system described by Eq. (16) with input nonlinearity in Eq. (9) under Assumptions 1–4; suppose that the integral sliding surface is given in Eq. (18), with $G_i=B_i^+$, $K_i$ satisfying LMI Eqs. (25)–(27). If the system is controlled by Eq. (32) with adaptive laws in Eqs. (33), then the reachability of the sliding surface $s(t)=0$ is guaranteed.



J. Aerosp. Eng. 2013.26:721-734.

**Proof.** Considering Eqs. (9) and (10), when $u_\pi(t) < -u_\pi^-$ for $s_\pi > 0$,

$$\left[u_\pi(t) + u_\pi^-\right]\mathrm{sat}\left\{\psi_\pi^-[u_\pi(t)]\right\} = -k_\pi\hat{\gamma}(t)\nu(t)\frac{s_\pi(t)\mathrm{sat}\left\{\psi_\pi^-[u_\pi(t)]\right\}}{\|s(t)\|} \geq \alpha_\pi^- k_\pi^2\gamma\hat{\gamma}^2(t)\nu^2(t)\frac{s_\pi^2(t)}{\|s(t)\|^2}$$

and for $u_\pi(t) > u_\pi^+$ for $s_\pi < 0$

$$\left[u_\pi(t) - u_\pi^+\right]\mathrm{sat}\left\{\psi_\pi^+[u_\pi(t)]\right\} = -k_\pi\tilde{\gamma}(t)\nu(t)\frac{s_\pi(t)\mathrm{sat}\left\{\psi_\pi^-[u_\pi(t)]\right\}}{\|s(t)\|} \geq \alpha_\pi^+ k_\pi^2\gamma\hat{\gamma}^2(t)\nu^2(t)\frac{s_\pi^2(t)}{\|s(t)\|^2}$$

Hence, we can get that for all $s_\pi(t)$

$$s_\pi(t)\mathrm{sat}\{\psi_\pi[u_\pi(t)]\} \leq -\lambda_\pi k_\pi\gamma\hat{\gamma}(t)\nu(t)\frac{s_\pi^2(t)}{\|s(t)\|}$$

where $k_\pi > 1/\lambda_\pi, \lambda_\pi = \min\{\alpha_\pi^+, \ \alpha_\pi^-\}$. Then, for the overall sliding surface

$$\begin{aligned} s^T(t)\mathrm{sat}\{\psi[u(t)]\} &= s_\Phi^T(t)\mathrm{sat}\{\psi_\Phi[u_\Phi(t)]\} + s_{\delta_e}^T(t)\mathrm{sat}\left\{\psi_{\delta_e}\left[u_{\delta_e}(t)\right]\right\} \\ &\leq -\lambda_\Phi k_\Phi\hat{\gamma}(t)\nu(t)\frac{s_\Phi^T(t)s_\Phi(t)}{\|s(t)\|} - \lambda_{\delta_e}k_{\delta_e}\hat{\gamma}(t)\nu(t)\frac{s_{\delta_e}^T(t)s_{\delta_e}(t)}{\|s(t)\|} \\ &\leq -\overline{\beta}\gamma\hat{\gamma}(t)\nu(t)\|s(t)\| \end{aligned} \tag{34}$$

where $\overline{\beta} = \min\{\lambda_\Phi k_\Phi, \lambda_{\delta_e}k_{\delta_e}\}$. Because $k_\pi > 1/\lambda_\pi, \overline{\beta} > 1$ is always satisfied.

Choose the Lyapunov function as

$$V(t) = \frac{s(t)^T s(t)}{2} + \frac{1}{2}\tilde{\gamma}^2(t) + \frac{1}{2q_1}\tilde{\rho}_0^2(t) + \frac{1}{2q_2}\tilde{\rho}_1^2(t) \tag{35}$$

where $\tilde{\rho}_0(t) = \rho_0 - \hat{\rho}_0(t), \tilde{\rho}_1(t) = \rho_1 - \hat{\rho}_1(t)$, and $\tilde{\gamma}(t) = \gamma - \hat{\gamma}^{-1}(t)$. Note that $\dot{\tilde{\rho}}_0(t) = -\dot{\hat{\rho}}_0(t)$, $\dot{\tilde{\rho}}_1(t) = -\dot{\hat{\rho}}_1(t)$, and $\dot{\tilde{\gamma}}(t) = \hat{\gamma}^{-2}(t)\dot{\hat{\gamma}}(t)$. By using Eqs. (19) and (34), we have

$$\begin{aligned} \dot{V}(t) &= s^T(t)\dot{s}(t) + \tilde{\gamma}(t)\dot{\tilde{\gamma}}(t) - \frac{1}{q_1}\tilde{\rho}_0(t)\dot{\hat{\rho}}_0(t) - \frac{1}{q_2}\tilde{\rho}_1(t)\dot{\hat{\rho}}_1(t) = s^T(t)\left\{\sum_{i=1}^{l}\mu_i(t)(\mathrm{sat}\{\psi[u(t)]\} - K_i e(t)) + \chi_i[x(t), t]\right\} \\ &+ \tilde{\gamma}(t)\dot{\tilde{\gamma}}(t) - \frac{1}{q_1}\tilde{\rho}_0(t)\dot{\hat{\rho}}_0(t) - \frac{1}{q_2}\tilde{\rho}_1(t)\dot{\hat{\rho}}_1(t) = s^T(t)\mathrm{sat}\{\psi[u(t)]\} - \sum_{i=1}^{l}\mu_i(t)\left\{s^T(t)K_i e(t) - s^T(t)\chi_i[x(t), t]\right\} + \tilde{\gamma}(t)\dot{\tilde{\gamma}}(t) \\ &- \frac{1}{q_1}\tilde{\rho}_0(t)\dot{\hat{\rho}}_0(t) - \frac{1}{q_2}\tilde{\rho}_1(t)\dot{\hat{\rho}}_1(t) \leq -\overline{\beta}\gamma\hat{\gamma}(t)\nu(t)\|s(t)\| - \sum_{i=1}^{l}\mu_i(t)\|s(t)\|\|K_i\|\|e(t)\| + \|s(t)\|(\rho_0 + \rho_1\|e(t)\|) + \tilde{\gamma}(t)\hat{\gamma}^{-2}(t)\dot{\hat{\gamma}}(t) \\ &- \frac{1}{q_1}\tilde{\rho}_0(t)\dot{\hat{\rho}}_0(t) - \frac{1}{q_2}\tilde{\rho}_1(t)\dot{\hat{\rho}}_1(t) \leq -\overline{\beta}\nu(t)\|s(t)\| - \sum_{i=1}^{l}\mu_i(t)\|s(t)\|\|K_i\|\|e(t)\| + \|s(t)\|(\rho_0 + \rho_1\|e(t)\|) \\ &- \tilde{\rho}_0(t)\|s(t)\| - \tilde{\rho}_1(t)\|s(t)\|\|e(t)\| \leq -\left(\overline{\beta} - 1\right)\left[\hat{\rho}_0(t) + \hat{\rho}_1(t)\|e(t)\|\right]\|s(t)\| - \sum_{i=1}^{l}\mu_i(t)\|s(t)\|\|K_i\|\|e(t)\| \end{aligned}$$

Note that $\hat{\rho}_0(0) = \hat{\rho}_1(0) = 0$; $\dot{\hat{\rho}}_0(t) > 0$; $\dot{\hat{\rho}}_1(t) > 0$; and for $t > 0$, $\hat{\rho}_0(t) > 0$ and $\hat{\rho}_1(t) > 0$. Then for $t > 0$

$$\dot{V}(t) < 0$$

Therefore, the adaptive control law from Eq. (32) can drive the system dynamics onto the sliding surface Eq. (18). The proof is completed.

**Remark 7**: For the sliding mode control system, chattering is a common phenomenon (Hung et al. 1993), which is usually caused by the term of $s_\pi(t)/\|s(t)\|$ in the sliding mode controller of Eq. (32). To reduce the chattering phenomenon, one simple and useful way is to replace the term $s_\pi(t)/\|s(t)\|$ by $s_\pi(t)/\|s(t)\| + \delta$, where $\delta$ is an adjustable scalar.

## Simulation Results

In this section, a numerical example is given to test the effectiveness of the proposed fuzzy adaptive FISMC. The parameter values of FAHVs model are borrowed from Parker et al. (2007). The equilibrium point of the nonlinear vehicle dynamics is listed in Table 1. When constituting the fuzzy model, the output $y = [V, h]^T$ is selected as the determinant of the premise variables, and for the application of the method expressed in Teixeira and Zak (1999), three levels are chosen for every premise variable: a lower bound, an upper bound, and an equilibrium point, which are named *small* ($S$), *big* ($B$), and *middle* ($M$), respectively. The low and high bounds of $V$ and $h$ are chosen as $V_B = 9{,}000$ ft/s, $V_s = 6{,}400$ ft/s, $h_B = 10{,}000$ ft, and

**Table 1.** Trim Point

| State | Value | State | Value | State | Value |
|---|---|---|---|---|---|
| $h$ | 85,000 ft | $\eta_1$ | 1.5122 | $\Phi$ | 0.2514 |
| $V$ | 7,702.0808 ft·s$^{-1}$ | $\dot{\eta}_1$ | 0 | $\delta_e$ | 11.4635 |
| $\alpha$ | 1.5153° | $\eta_2$ | 1.2144 | | |
| $\theta$ | 1.5153° | $\dot{\eta}_2$ | 0 | | |
| $Q$ | 0°·s$^{-1}$ | | | | |



J. Aerosp. Eng. 2013.26:721-734.

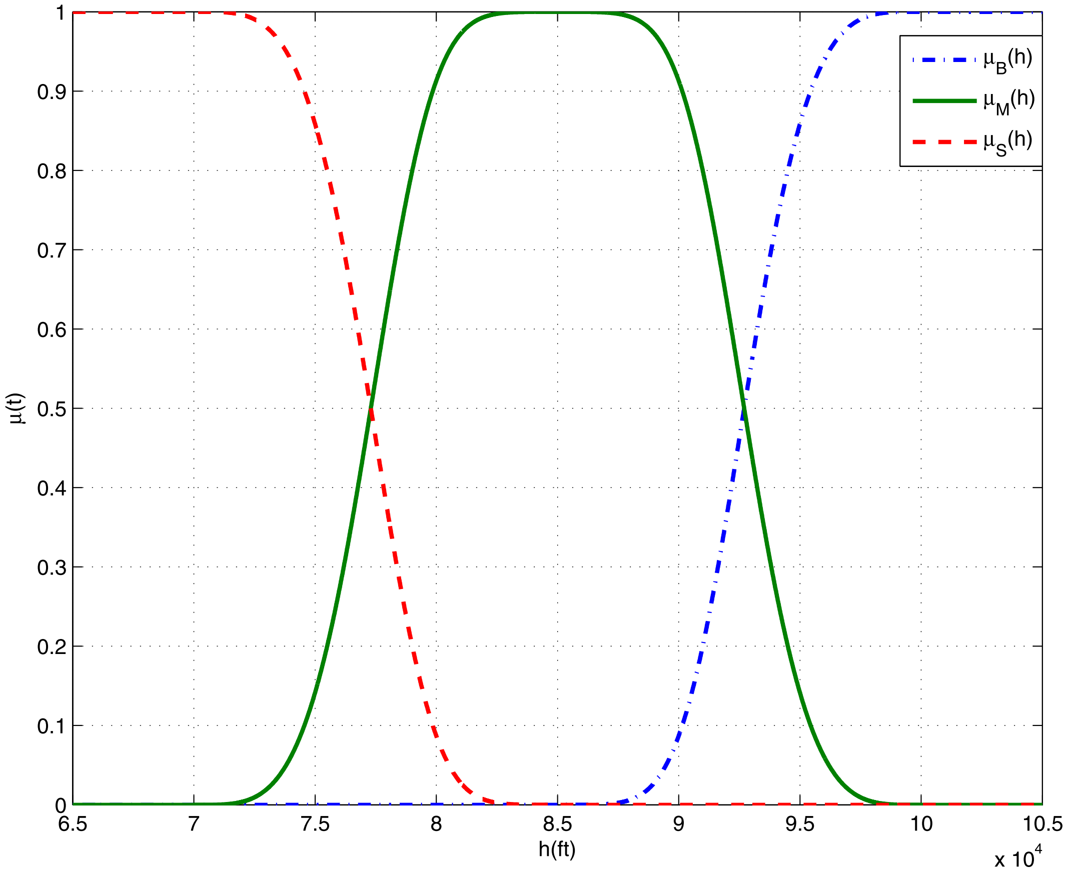


**Fig. 3.** Membership functions of $h$

$h_S = 7{,}000$ ft. Other states are chosen according to the flight envelope. Then the T-S fuzzy model can be constructed by using the modeling method described in Main Result. The fuzzy membership functions of $V$ and $h$ are defined as follows:

$$\text{if } V > V_M, \quad \begin{aligned} &\mu_S(V) = 0, \\ &\mu_M(V) = 1 - \mu_B(V), \\ &\mu_B(V) = \exp\left[-3.5 \times 10^{-12}|V(t) - V_B|^4\right] \end{aligned}$$

$$\text{if } V < V_M, \quad \begin{aligned} &\mu_S(V) = \exp\left[-3.5 \times 10^{-12}|V(t) - V_S|^4\right], \\ &\mu_M(V) = 1 - \mu_B(V), \\ &\mu_B(V) = 0 \end{aligned}$$

$$\text{if } h > h_M, \quad \begin{aligned} &\mu_S(h) = 0, \\ &\mu_M(h) = 1 - \mu_B(h), \\ &\mu_B(h) = \exp\left[-2.44 \times 10^{-16}|h(t) - h_B|^4\right] \end{aligned}$$

$$\text{if } h < h_M, \quad \begin{aligned} &\mu_S(h) = \exp\left[-2.44 \times 10^{-16}|h(t) - h_S|^4\right], \\ &\mu_M(h) = 1 - \mu_b(h), \\ &\mu_B(h) = 0 \end{aligned}$$

The membership functions of the fuzzy model are shown in Figs. 3 and 4. Fig. 3 shows the membership functions of $h$ and Fig. 4 shows that of $V$. The input nonlinearity model of a FAHV is chosen as the following:

$$\psi_\Phi[u_\Phi(t)] = \begin{cases} [u_\Phi(t) - 0.1]\{1 + 0.1 \times \sin[u_\Phi(t) - 0.1]\}, & u_\Phi(t) > 0.1 \\ 0, & -0.1 \leq u_\Phi(t) \leq 0.1 \\ [u_\Phi(t) + 0.1]\{1 + 0.1 \times \cos[u_\Phi(t) + 0.1]\}, & u_\Phi(t) < -0.1 \end{cases} \tag{36}$$

$$\psi_{\delta_e}\left[u_{\delta_e}(t)\right] = \begin{cases} \left[u_{\delta_e}(t) - 3\right]\left\{1 + 0.1 \times \sin\left[u_{\delta_e}(t) - 3\right]\right\}, & u_{\delta_e}(t) > 3 \\ 0, & -3 \leq u_{\delta_e}(t) \leq 3 \\ \left[u_{\delta_e}(t) + 3\right]\left\{1 + 0.1 \times \cos\left[u_{\delta_e}(t) + 3\right]\right\}, & u_{\delta_e}(t) < -3 \end{cases} \tag{37}$$



J. Aerosp. Eng. 2013.26:721-734.

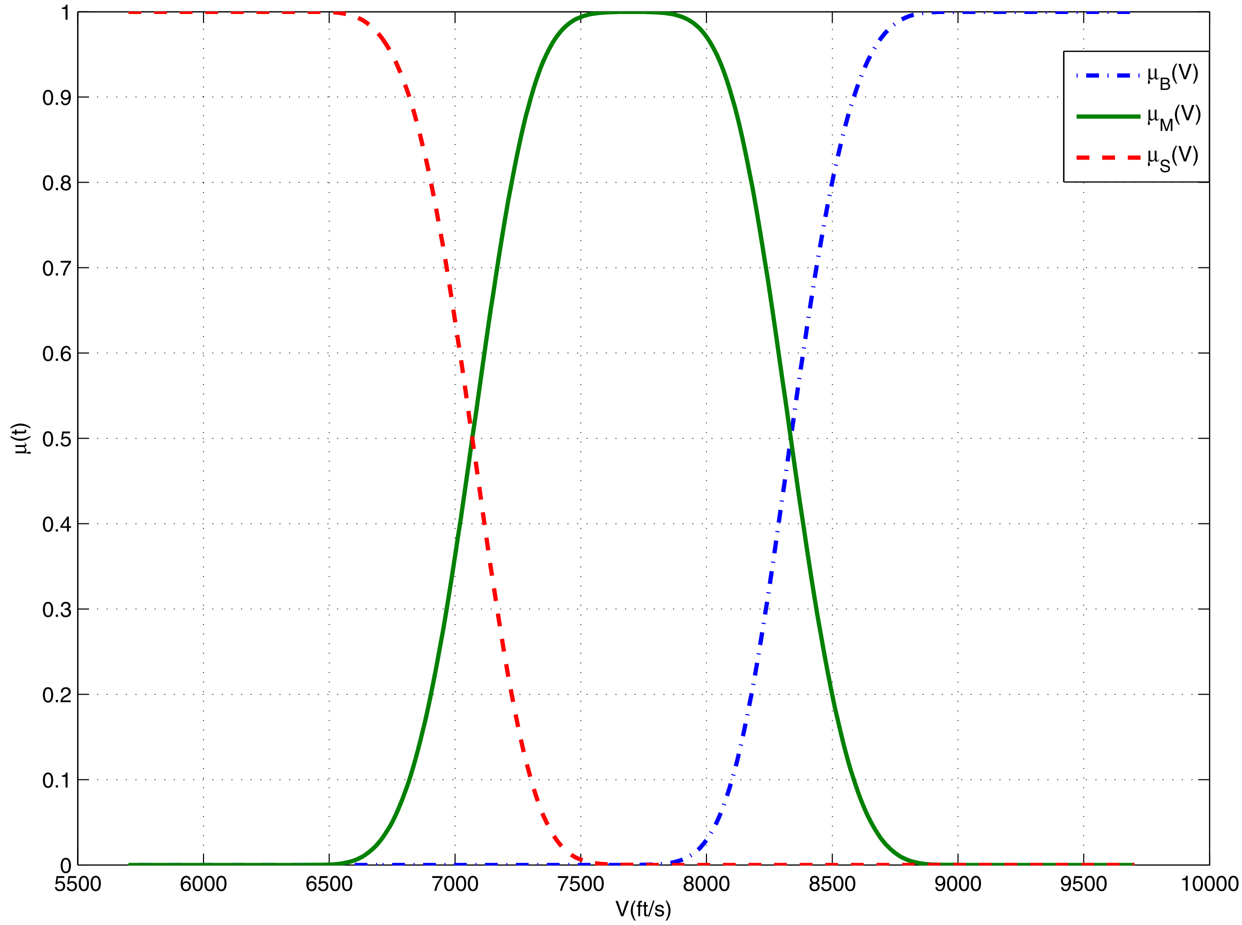


**Fig. 4.** Membership functions of $V$

Then the gain reduction tolerances of the plant, $\alpha_\pi^+$, $\beta_\pi^+$, $\alpha_\pi^-$, and $\beta_\pi^-$, can be derived, where $\alpha_\pi^+ = \alpha_\pi^- = 0.9$ and $\beta_\pi^+ = \beta_\pi^- = 1.1$. Because the T-S model is derived from regional linearization, with respect to the trim point, the actuator saturation value $u_{\Phi\max} = 0.7$ and $u_{\delta_e \max} = 9^\circ$.

The nonlinear model Eq. (1) can then be represented by a nine-rule T-S fuzzy model. The details of constructing the T-S fuzzy model for FAHVs is omitted here, which can be found in our previous work.

Rule 1. If $V$ is small ($V_S$) and $h$ is small ($h_S$), then

$$\dot{x}(t) = (A_1 + \Delta A_1)x(t) + B_1 \mathrm{sat}\{\psi[u(t)]\} + D_1 \varpi(t),$$
$$y(t) = C_1 x(t)$$

Rule 2. If $V$ is small ($V_S$) and $h$ is middle ($h_M$), then

$$\dot{x}(t) = (A_2 + \Delta A_2)x(t) + B_2 \mathrm{sat}\{\psi[u(t)]\} + D_2 \varpi(t),$$
$$y(t) = C_2 x(t)$$

Rule 3. If $V$ is small ($V_S$) and $h$ is big ($h_B$), then

$$\dot{x}(t) = (A_3 + \Delta A_3)x(t) + B_3 \mathrm{sat}\{\psi[u(t)]\} + D_3 \varpi(t),$$
$$y(t) = C_3 x(t)$$

Rule 4. If $V$ is middle ($V_M$) and $h$ is small ($h_S$), then

$$\dot{x}(t) = (A_4 + \Delta A_4)x(t) + B_4 \mathrm{sat}\{\psi[u(t)]\} + D_4 \varpi(t),$$
$$y(t) = C_4 x(t)$$

Rule 5. If $V$ is middle ($V_M$) and $h$ is middle ($h_M$), then

$$\dot{x}(t) = (A_5 + \Delta A_5)x(t) + B_5 \mathrm{sat}\{\psi[u(t)]\} + D_5 \varpi(t),$$
$$y(t) = C_5 x(t)$$

Rule 6. If $V$ is middle ($V_M$) and $h$ is big ($h_B$), then

$$\dot{x}(t) = (A_6 + \Delta A_6)x(t) + B_6 \mathrm{sat}\{\psi[u(t)]\} + D_6 \varpi(t),$$
$$y(t) = C_6 x(t)$$

Rule 7. If $V$ is big ($V_B$) and $h$ is small ($h_S$), then

$$\dot{x}(t) = (A_7 + \Delta A_7(t))x(t) + B_7 \mathrm{sat}\{\psi[u(t)]\} + D_7 \varpi(t),$$
$$y(t) = C_7 x(t)$$

Rule 8. If $V$ is big ($V_B$) and $h$ is middle ($h_M$), then

$$\dot{x}(t) = (A_8 + \Delta A_8)x(t) + B_8 \mathrm{sat}\{\psi[u(t)]\} + D_8 \varpi(t),$$
$$y(t) = C_8 x(t)$$

Rule 9. If $V$ is big ($V_B$) and $h$ is big ($h_B$), then

$$\dot{x}(t) = (A_9 + \Delta A_9)x(t) + B_9 \mathrm{sat}\{\psi[u(t)]\} + D_9 \varpi(t),$$
$$y(t) = C_9 x(t)$$

The main control objective is to track a set step with respect to a trim condition, which is a reasonable requirement for this kind of vehicle. The input reference commands are chosen as step inputs, so each command will pass through a prefilter as

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where $\zeta$ = damping ratio and $\omega_n$ = natural frequency and are assumed to be 0.9 and 0.01 rad/s, respectively. The output of the prefilter is defined as a reference input of the reference model. In the simulation, we choose $A_{m1} = A_{m2} = \cdots = A_{m9} = A_m$ and $B_{m1} = B_{m2} = \cdots = B_{m9} = B_m$. Based on the method proposed in Dong et al. (2010), the matrices $A_m$ and $B_m$ of the reference model are chosen as

$$A_m = \begin{bmatrix} 0 & 0 & -7{,}702 & 7{,}702 & 0 & 0 & 0 & 0 & 0 \\ -3.162 & -0.62 & 18{,}253.45 & -21{,}295.5 & -441.1 & 90.6 & -6.95 & -1.9 & -7.5 \\ 1.434\times10^{-4} & 1.633\times10^{-4} & 0.881 & -1.055 & 0.949 & -1.165\times10^{-4} & 4.311 & -8.329\times10^{-4} & 3.75\times10^{-5} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -0.03336 & 0.01556 & 210.91 & -236.47 & -8.23 & 0.477 & -0.0326 & -0.106 & -0.036 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 4{,}648 & 0 & 0 & -272.3 & -0.66 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2{,}598 & 0 & 0 & 0 & 0 & -400 & -0.8 \end{bmatrix}$$

$$B_m = \begin{bmatrix} 0 & 3.165 & 1.437\times10^{-4} & 0 & 0.0336 & 0 & 0 & 0 & -16.99 \\ 0 & 0.599 & -1.65\times10^{-4} & 0 & -0.0158 & 0 & 0 & 0 & 17.95 \end{bmatrix}^T$$

The modeling of parameter uncertainties is similar to Buschek and Calise (1997), and in this work, the parameters of $C_L^\alpha$, $C_L^{\delta_e}$, $C_L^0$, $C_D^{\alpha^2}$, $C_D^\alpha$, $C_D^{\delta_e^2}$, $C_D^{\delta_e}$, $C_D^0$, $C_{M,\alpha}^{\alpha^2}$, $C_{M,\alpha}^\alpha$, $C_{M,\alpha}^0$, and $C_{M,\delta_e}$ are assumed to be uncertain, and these uncertainties are assumed to lie within $\pm10\%$ of nominal values, respectively. The uncertainty of $S$ lies within $\pm5\%$ of nominal value, as does the mean aerodynamic chord

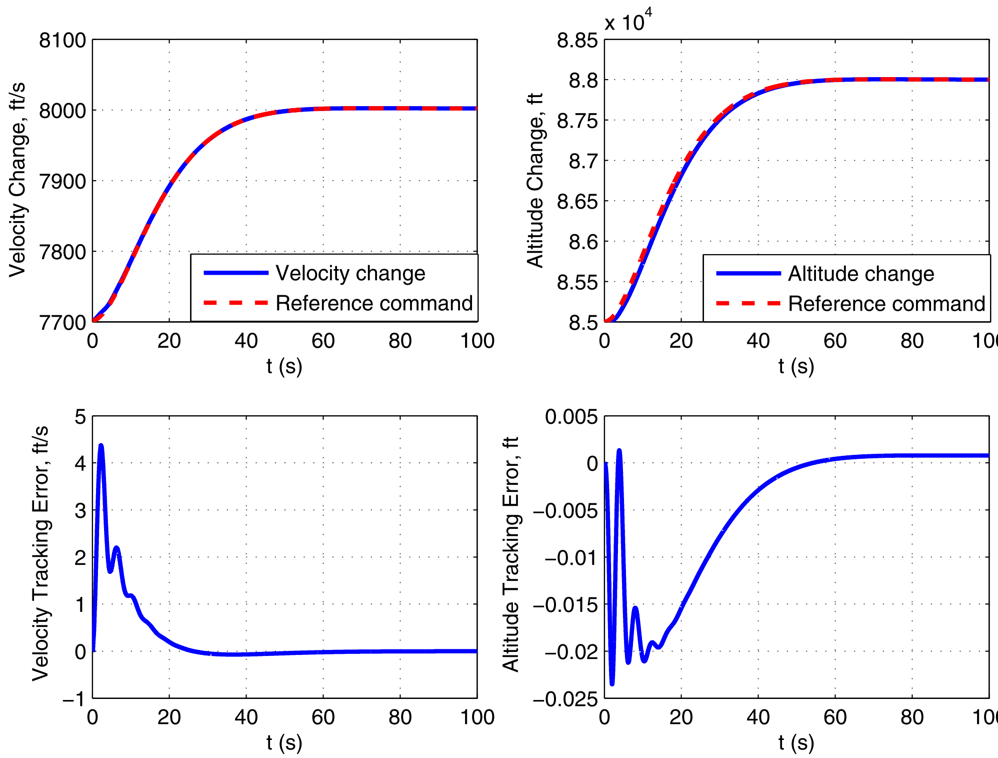


**Fig. 5.** Tracking performances of closed-loop

$\bar{c}$. According to Gibson et al. (2009), the disturbance $\varpi(t)$ is assumed to be bounded, which can be regarded as a gust of wind in aerospace. Here we choose a random disturbance, described by

$$\varpi(t) = d_{\max} N(\bullet)$$

whose maximum absolute $d_{\max} = 0.1$ and $N(\bullet)$ = normal distribution with mean zero and SD 1.

Suppose the weighting matrix $\Omega = 1 \times 10^{-6} I$. By solving LMI Eqs. (25)–(27), the controller gain matrices can be obtained. For the adaptive SMC law, the parameters are set to be $(q_1, q_2, \text{ and } \delta) = (0.001, 0.0001, \text{ and } 0.01)$.

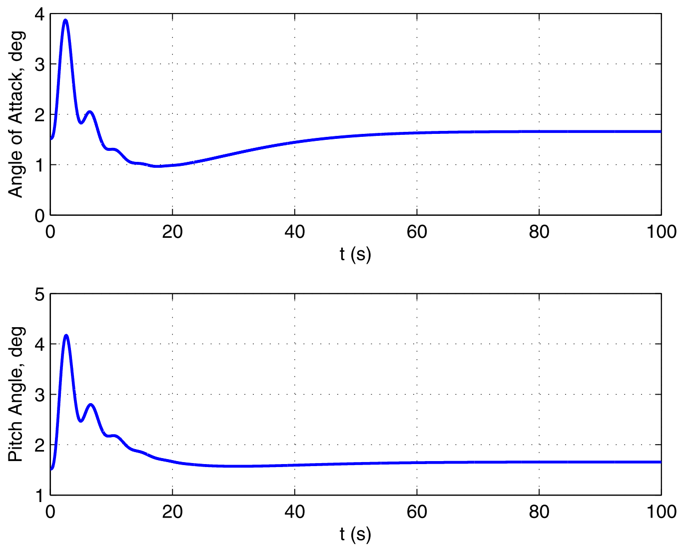


**Fig. 6.** Angle of attack and flight-path angle

In simulation, to illustrate the effectiveness of the proposed controller, we will use the original nonlinear model (not the linear model) to test the performance of the control system. The parameter uncertainties are set to be 10 or 5% of the nominal case, respectively. Here, we consider a climbing maneuver with longitudinal acceleration using separate reference commands for altitude and velocity. In the simulation, the reference commands for velocity and altitude are chosen as 300 and 3,000 ft, respectively.

The effectiveness and tracking performance of the proposed controller are shown in Figs. 5 and 6. From the figures, we can see that the controller provides stable tracking of the reference trajectories. More specifically, the tracking performance for the velocity and altitude is shown in Fig. 5. Fig. 6 shows the angle of attack and the flight-path angle, respectively. The input of the plant $\text{sat}\{\psi[u(t)]\}$ is given in Fig. 7. Moreover, in Fig. 8, we also give the trajectory of the sliding surface function.

Note from the preceding figures that the tracking performance is good and the tracking error remains remarkably small during the whole maneuver. Hence, the fuzzy model reference ISMC method can stabilize the nonlinear system in Eq. (1) and can guarantee the tracking performance.

## Conclusion

Because of the large flight envelope, the complex interactions between the rigid and flexible modes, and the intricate coupling among the engine and light dynamics, parameter uncertainties, and external disturbances, the robust control of FAHVs is highly challenging. The T-S model can approach a nonlinear system with some precision, so in this paper, the T-S modeling method has been used to represent the nonlinear longitudinal dynamics of FAHVs. A new FISMC strategy has also been presented for the model reference tracking problem of FAHVs. In the control design, the input nonlinearity, which comprises not only dead-zones but also sector nonlinearities, has been considered. Based on a PDC scheme,

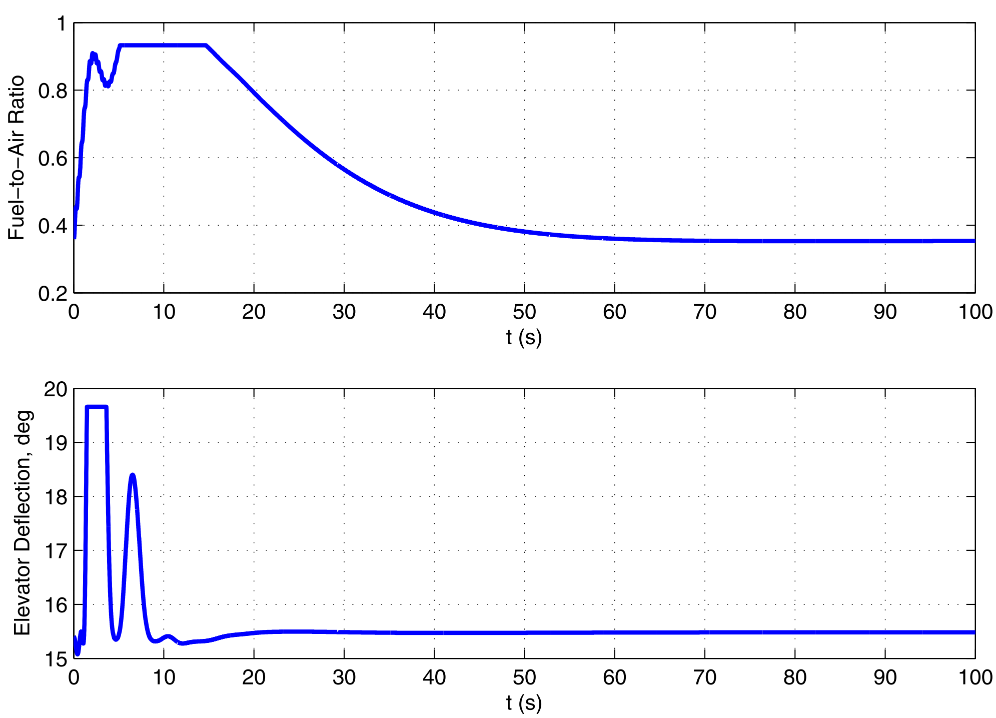


**Fig. 7.** Inputs of the plant

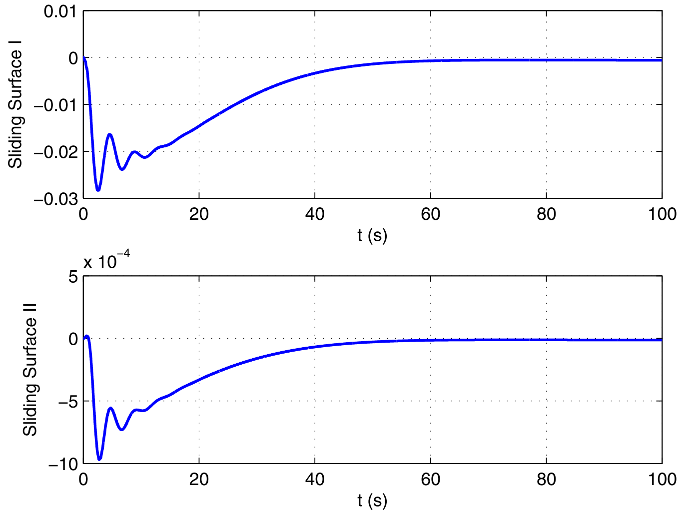


**Fig. 8.** Sliding surface

sufficient conditions for the stability of the designed fuzzy integral sliding surfaces have been proposed in terms of LMIs. An adaptive SMC law has also been synthesized such that the state trajectories of the closed-loop system are globally driven onto the specified sliding surface. Simulation results validated the effectiveness of the proposed control methods by showing excellent tracking performance.

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## Notation

*The following symbols are used in this paper:*

$C_D(\alpha, \delta_e)$ = drag coefficient;
$C_D^{\alpha_i}$ = $i$th order coefficient of $\alpha$ contribution to $C_D(\alpha, \delta_e)$;
$C_D^{\delta_e^i}$ = $i$th order coefficient of $\delta_e$ contribution to $C_D(\alpha, \delta_e)$;
$C_D^0$ = constant term in $C_D(\alpha, \delta_e)$;
$C_L(\alpha, \delta_e)$ = lift coefficient;
$C_L^{\alpha_i}$ = $i$th order coefficient of $\alpha$ contribution to $C_L(\alpha, \delta_e)$;
$C_L^{\delta_e}$ = coefficient of $\delta_e$ contribution to $C_L(\alpha, \delta_e)$;
$C_L^0$ = constant term in $C_L(\alpha, \delta_e)$;
$C_{M,Q}(\alpha, Q)$ = contribution to moment due to pitch rate;
$C_{M,\alpha}(\alpha)$ = contribution to moment due to angle of attack;
$C_{M,\delta_e}(\delta_e, \delta_c)$ = control surface contribution to moment;
$C_{M,\alpha}^{\alpha_i}$ = $i$th order coefficient of $\alpha$ contribution to $C_{M,\alpha}(\alpha)$;
$C_{M,\alpha}^0$ = constant term in $C_{M,\alpha}(\alpha)$;
$C_T^{\alpha_i}(\Phi)$ = $i$th order coefficient of $\alpha$ in $T$;
$\bar{c}$ = mean aerodynamic chord;
$c_c$ = canard coefficient in $C_{M,\delta_e}(\delta_e, \delta_c)$;
$c_e$ = elevator coefficient in $C_{M,\delta_e}(\delta_e, \delta_c)$;
$D$ = drag;
$\mathrm{diag}\{\ldots\}$ = a block-diagonal matrix;
$g$ = acceleration due to gravity;
$h$ = altitude;
$I$ = identity matrix;
$I_{yy}$ = moment of inertia;
$L$ = left;
$L_v$ = vehicle length;
$M$ = pitching moment;
$m$ = vehicle mass;
$N_i$ = $i$th generalized force;
$N_i^{\alpha_j}$ = $j$th order contribution of $\alpha$ to $N_i$;
$N_i^0$ = constant term in $N_i$;
$N_2^{\delta_e}$ = contribution of $\delta_e$ to $N_2$;
$P > 0$ = $P$ is real symmetric and positive definite;
$Q$ = pitch rate;
$\bar{q}$ = dynamic pressure;
$R^n$ = n-dimensional Euclidean space;
$S$ = reference area;
$T$ = thrust;
$T$ = matrix transposition;
$V$ = velocity;
$x$ = state of the control-oriented model;
$\alpha$ = angle of attack;
$\beta_i(h, \bar{q})$ = $i$th thrust fit parameter;
$\gamma$ = flight path angle, $\gamma = \theta - \alpha$;
$\delta_e$ = elevator angular deflection;
$\eta_i$ = $i$th generalized elastic coordinate;
$\theta$ = pitch angle;
$\lambda_i$ = inertial coupling term of $i$th elastic mode;
$\xi$ = damping ratio for the $\Phi$ dynamics;
$\xi_i$ = damping ratio for elastic mode $\eta_i$;
$\rho$ = density of air;
$\Phi$ = stoichiometrically normalized fuel/air;
$\omega$ = natural frequency for the $\Phi$ dynamics;
$\omega_i$ = natural frequency for elastic mode $\eta_i$;
$0$ = zero matrix; and
$1/h_s$ = air density decay rate.

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