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Derivation of Topological Graphs of Some Planar 4DOF Redundant Closed Mechanisms by Contracted Graphs and Arrays

Some planar redundantly closed mechanisms (RCMs) have better dexterity, less singular configuration, and higher stiffness. In this paper, the derivation of valid topology graphs (TGs) of some planar four degrees of freedom (4DOF) RCMs is studied based on the contracted graph (CG), arrays, and topology graph with digits (DTG). First, some CGs without any binary links are constructed for the planar 4DOF RCMs, some curves with only binary links are distributed over CGs, and some valid TGs of the planar 4DOF RCMs are derived. Second, a complicated derivation of TG is transformed into an easy derivation of array and DTG, and some programs are compiled in VISUAL BASIC*; all valid arrays corresponding to nonisomorphic TGs are derived, and some invalid arrays corresponding to the isomorphic TGs and invalid TGs are determined and removed by the compiled programs. Third, many valid TGs of the planar 4DOF RCMs with various basic links are derived from valid arrays and DTGs. Finally, some application examples are illustrated.* [DOI: 10.1115/1.4001735]

Keywords: redundant closed mechanism, topological graph, array

1 Introduction

Applications of redundant closed mechanisms (RCMs) can be found in textile machines, printing presses, iron-steel making equipments, petroleum pumping units, automobiles, robots, surgical tools, machine tools, and micro mechtronic manipulators. A planar four degrees of freedom (4DOF) RCM includes four actuators, in which one is a redundant actuator. Comparing with the nonredundant mechanisms, the planar 4DOF RCMs have better dexterity, less singular configuration, and higher stiffness [1–5]. Although planar 5- and 6DOF RCMs belong to the redundant mechanisms, more redundant actuators may result in a more complicated structure and are difficult to control. To create more novel RCMs with useful functions has been a significant and challenging issue. In type synthesis of mechanisms, Sohn and Freudenstein [6], Vucina and Freudenstein [7], and Tsai [8] proposed contracted graphs (CGs). Jin and Yang [9] designed the topology structure of mechanisms using a topology graph (TG). Johnson [10] derived planar associated linkage (AL) using a determining tree. Gogu [11] studied the type synthesis of parallel mechanisms (PMs) by morphological/evolutionary approaches. Lu and Leinonen [12] proposed some basic rules for deriving some acceptable CGs from AL and derived TGs from CG by visual inspection. Sardain [13] combined the type synthesis with the dimension synthesis of the mechanism. By changing the types and/or motion orientations of joints, Yan and Kang [14] studied the configuration synthesis of mechanisms. Hervé [15] proposed the Lie group for type synthesis of mechanisms. Pucheta and Cardona [16,17] synthesized planar linkages based on constrained subgraph isomorphism detection and existing mechanisms. Saxena and Ananthasuresh [18] selected the best configuration based on kinetostatic design specifications. Kong and Gosselin [19] studied the type synthesis of PMs based on the screw theory and virtual joint. Hess-Coelho [20] studied the topological synthesis of PMs based on wrist design requirements. Each of the above approaches has its merits and different foci. Currently, some simple planar RCMs have been synthesized [1–5], and many complicated RCMs with more links are applied widely. Since TG is a simple and effective tool for the type synthesis of mechanisms, some more complicated CGs should be applied to derive TGs for the type synthesis of complicated RCMs. However, when a CG includes more ternary T, quaternary Q, pentagonal links P_e, and hexagonal H links, a large number of different TGs can be derived. Meanwhile, the derivations of many valid TGs for planar 4DOF RCMs clearly become more complicated since many isomorphic TGs and invalid TGs cannot be identified easily by visual inspection. Hence, this paper focuses on the computational derivation of valid arrays and valid TGs of the planar 4DOF RCMs by means of CGs and arrays. The following problems are solved: (1) the construction of CGs of the planar 4DOF RCMs with ($2T$s, $2Q$s, $2P_e$s, $H+2T$, $4T$s, $Q+2T$, $2Q+2T$, or P_e+3T); (2) the compilation of some programs in VISUAL BASIC for finding all the arrays and identifying the isomorphic TGs and invalid TGs; (3) the computational derivation of all valid arrays from CGs; (4) the derivation of valid DTGs from valid arrays; (5) the construction of valid TG from valid DTG.

2 Some Concepts and Conditions

Let J be a point of connection with 1DOF. Some links with $3J$s or more (such as T with $3J$s, Q with $4J$s, P_e with $5J$s, and H with $6J$s) are defined as the basic links. A CG only includes the basic links with $3J$s or more [8,12]. Each of the basic links in CG is represented by a dot. These dots are connected with each other by some curves. For example, the simplest CG includes $2T$s, which are connected with each other by three curves c_1, c_2, and c_3 (see Fig. 1(a)).

A TG is similar to CG, except that each of the curves is replaced by some Bs connected in a series by some Js. For example, a TG includes $2T+7B$ and $10J$s (see Fig. 1(c)). It can be derived from CG in Fig. 1(a) by replacing (c_1,c_2,c_3) with (2,2,3) Bs connected in a series by (3,3,4) Js, respectively. In order to sim-

Contributed by the Mechanisms and Robotics Committee of ASME for publication in the JOURNAL OF MECHANISMS AND ROBOTICS. Manuscript received April 19, 2009; final manuscript received February 3, 2010; published online July 23, 2010. Assoc. Editor: Sundar Krishnamurty.

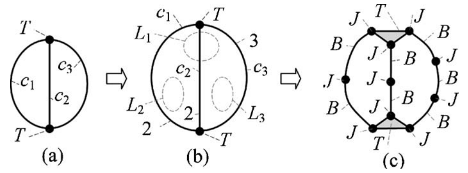

Fig. 1 (*a*) A CG with 2*T*s, (*b*) a DTG with 2*T*+7*B* represented by digits, and (*c*) a TG with 2*T*+7*B* and 10*J*s

plify the TG, a TG can be represented by a DTG, in which each of the curves is represented by a curve with a digit versus the number of B (see Fig. 1(*b*)). Many different DTGs can be derived from the same CG by distributing some Bs over the curves. Suppose that both TG x and TG y are derived from the same CG. If TG x is the same as TG y, then TG x and TG y must be the isomorphism of each other.

Let (n_2,n_3,n_4,n_5,n_6) be the number of (B,T,Q,P_e,H), respectively. A general planar closed mechanism includes a moving platform m with an end-effector, a fixed base b, and some branches with various chain structures for connecting m with b [10–12]. Several different CGs can be derived from one AL with the same group of basic links [12]. A valid TG can be derived from its valid DTG. Many different mechanisms can be synthesized from one valid TG by replacing any J with a revolute joint R or a prismatic joint P [10]. In order to derive the valid TGs of the planar 4DOF RCMs from CGs, two conditions must be satisfied as follows:

1. The number of links in any closed loop chain should be 4 or more.
2. The number of B in each of curves must be 5 or less.

Two auxiliary conditions should be satisfied as follows:

1. The number of links connected in series in each branch from m to b should be 2 or more.
2. A curve should not include three successive Ps.

The four conditions above are discussed as follows.

Since the simplest TG of 1DOF planar closed mechanism includes 4Js connected with four links [10,12], in order to avoid some local structures with DOF=0, condition 1 must be satisfied.

When a curve includes 6Bs and 7Js, it must have 4DOF and needs four actuators. Thus, other parts of the planar 4DOF RCM must be a structure with DOF=0. Hence, condition 2 must be satisfied.

If m is connected with b by one link and 2Js, it has 2DOF and moved by two actuators, the other two actuators are redundant. If m is connected with b by 1J and moved by one actuator, other three actuators are redundant. Since the planar 4DOF RCMs only have one redundant actuator, condition 3 must be satisfied.

When a curve includes three successive Ps, one of 3Ps may be redundant and a passive DOF may exist.

For example, a valid TG with $2T+7B$ in Fig. 1(*b*) includes three closed loop chains L_1, L_2, and L_3. L_1 is composed of c_1, c_3, and 2Ts. L_2 is composed of c_1, c_2, and 2Ts. L_3 is composed of c_2, c_3, and 2Ts. Since curves (c_1,c_2,c_3) have (2,2,3) Bs, respectively, the number of links in (L_1,L_2,L_3) are (7,6,7), respectively. Thus, conditions 1 and 3 are satisfied. In addition, 2Ts can be m and b, according to condition 3.

The numbers of B and some basic links in TGs have been determined in Ref. [12] for synthesizing some planar ALs with 4DOF, see Table 1.

3 CG and TG of Some Simple ALs

Generally, TGs of some simple ALs can be derived easily from their CGs. Their isomorphic TGs and invalid TGs can be identified easily.

When a planar 4DOF RCM includes 2Ts, its CG is constructed (see Fig. 2(*a*)). Based on conditions 1–3 in Sec. 2 and the AL 1 in Table 1, the six valid arrays are derived from this CG (see Table 2). Then, the six valid DTGs with $2T+7B$ are derived from the six valid arrays (see Fig. 2(*a*)). Based on conditions 1 and 3 in Sec. 2, 2Ts can be m and b at the same time only in No. 6 DTG.

When a planar 4DOF RCM includes 2Qs, its CG is constructed (see Fig. 2(*b*)). Based on conditions 1–3 in Sec. 2 and the AL 2 in Table 1, the eight valid arrays are derived from this CG (see Table 2). Then, the eight valid DTGs with 2Qs and 9Bs are derived from the eight valid arrays (see Fig. 2(*b*)). Based on conditions 1 and 3 in Sec. 2, 2Qs can be m and b at the same time only in No. 8 DTG.

When a planar 4DOF RCM includes 2P_es, its CG is constructed (see Fig. 2(*c*)). Based on conditions 1–3 in Sec. 2 and the AL 3 in Table 1, the 11 valid arrays are derived from this CG (see Table 2). Then, the 11 valid DTGs with 2P_es and 11Bs are derived from the 11 valid arrays (see Fig. 2(*c*)). Based on conditions 1 and 3 in Sec. 2, 2P_es can be m and b at the same time in No. 11 DTG.

When a planar 4DOF RCM includes $H+2T$, its CG is constructed (see Fig. 2(*d*)). Based on conditions 1–3 in Sec. 2 and the AL 4 in Table 1, the 15 valid arrays are derived from this CG. Then, the 15 valid DTGs with $H+2T+10B$ are derived from the 15 valid arrays (see Fig. 2(*d*)). Many planar 4DOF serial-parallel mechanisms can be created from these DTGs.

4 TGs and Their Arrays

When the planar 4DOF RCMs include more links, the derivations of their TGs clearly become more complicated. Generally, several different CGs versus one AL can be constructed. During the derivation of all valid TGs from each of CGs, some isomorphic TGs and some invalid TGs must be eliminated in order to avoid some identical and invalid mechanisms. However, it is a challenging issue to identify the isomorphic TGs and invalid TGs from some complicated CGs. Hence, an array approach is used for deriving all the valid different TGs from each of the CGs.

Table 1 The number of *B* and basic links in some planar ALs with 4DOF

Associated linkage	The number of B	The number of basic links			
	n_2	n_3	n_4	n_5	n_6
1	3+DOF=7	2	0	0	0
2	5+DOF=9	0	2	0	0
3	7+DOF=11	0	0	2	0
4	6+DOF=10	2	0	0	1
5	3+DOF=7	4	0	0	0
6	4+DOF=8	2	1	0	0
7	5+DOF=9	2	2	0	0
8	5+DOF=9	3	0	1	0

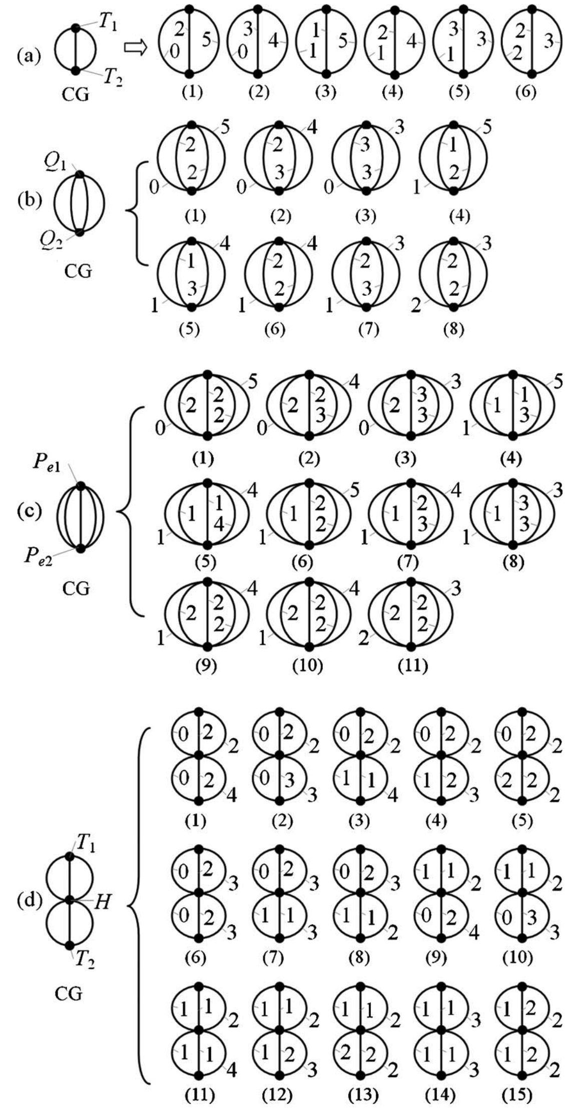

Fig. 2 Some CGs and valid DTGs versus ALs 1–4 in Table 1

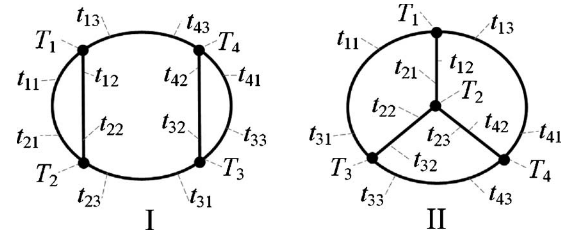

Fig. 3 CGs I and II with 4*T*s versus AL 5 in Table 1

A CG may include some ternary T_i $(i=1,\ldots,n_3)$, quaternary Q_j $(j=1,\ldots,n_4)$, pentagonal P_{ek} $(k=1,\ldots,n_5)$, and hexagonal H_l $(l=1,\ldots,n_6)$ links. Each of (T_i, Q_j, P_k, and H_l) includes (3, 4, 5, and 6) branches, respectively. Let (t_{i1},t_{i2},t_{i3}) be the number of B in three branches of T_i, $(q_{j1},q_{j2},q_{j3},q_{j4})$ be the number of B in four branches of Q_j, $(p_{k1},p_{k2},p_{k3},p_{k4},p_{k5})$ be the number of B in five branches of P_{ek}, and $(h_{l1},h_{l2},h_{l3},h_{l},h_{l5},h_{l6})$ be the number of B in six branches of H_l. For example, two different CGs I and II can be constructed from an AL with 4*T*s, and the number of B in three branches of T_i is represented (see Fig. 3).

Each of DTGs can be represented by an array (see Table 3 and Fig. 4). The isomorphic or invalid DTGs derived from the same CG can be determined by the arrays. When representing the DTG of planar 4DOF RCMs by an array, some criterions must be satisfied as follows:

(1) Each of the arrays includes several strings. Each of the strings corresponds to one basic link; the number of digits in the strings is the same as that of the branch of the basic link; the digits of the strings are the same as the number of B in the branches of the link. For example, when a CG only includes T_i $(i=1,2,3,4)$, No. 1 array {020, 020, 005, 500} with four strings is derived (see Table 3). In this array, the first string 020 corresponds to the number of B in three branches of T_1, i.e., ($t_{11}=0$, $t_{12}=2$, $t_{13}=0$); the second string 020 corresponds to the number of B in three branches of T_2, i.e., ($t_{21}=0$, $t_{22}=2$, $t_{23}=0$); the third string 005 corresponds to the number of B in three branches of T_3, i.e., ($t_{31}=0$, $t_{32}=0$, $t_{33}=5$); the fourth string 500 corresponds to the number of B in four branches of T_4. i.e., ($t_{41}=5$, $t_{42}=0$, $t_{43}=0$).
(2) The sum of digits in every array is $2n_2$.
(3) Suppose that both No. x and No. y arrays are derived from the same CG. When the digits of the strings in array x are

Table 2 Some CGs and valid arrays versus ALs 1–4 in Table 1

No.	T_1	T_2	No.	Q_1	Q_2	No.	P_{e1}	P_{e2}	No.	T_1	T_2	H
1	025	025	1	0225	0225	1	02225	02225	1	022	024	022024
2	034	034	2	0234	0234	2	02234	02234	2	022	033	022033
3	115	115	3	0333	0333	3	02333	02333	3	022	114	022114
4	124	124	4	1125	1125	4	11135	11135	4	022	123	022123
5	133	133	5	1134	1134	5	11144	11144	5	022	222	022222
6	223	223	6	1224	1224	6	11225	11225	6	023	023	023023
	$n_2=7$, $n_3=2$, Fig. 2(*a*)		7	1233	1233	7	11234	11234	7	023	113	023113
			8	2223	2223	8	11333	11333	8	023	122	023122
				$n_2=9$, $n_4=2$, Fig. 2(*b*)		9	12224	12224	9	112	024	112024
						10	12233	12233	10	112	033	112033
						11	22223	22223	11	112	114	112114
							$n_2=11$, $n_5=2$, Fig. 2(*c*)		12	112	123	112123
									13	112	222	112222
									14	113	113	113113
									15	122	122	122122
										$n_2=10$, $n_3=2$, $n_6=1$, Fig. 2(*d*)		

Table 3 The 28 valid arrays derived from CG I with 4*T*s

No.	T_1	T_2	T_3	T_4
	t_{12} , t_{11}, t_{13}	t_{22} , t_{21}, t_{23}	t_{32} , t_{31}, t_{33}	t_{42} , t_{41}, t_{43}
1	020	020	005	500
2	020	020	014	410
3	020	020	023	320
4	020	021	104	400
5	020	021	113	310
6	020	021	122	220
7	020	022	203	300
8	020	023	311	110
9	021	021	103	301
10	021	021	112	211
11	021	022	202	201
12	021	022	211	111
13	030	030	004	400
14	030	030	013	310
15	030	030	022	220
16	030	031	112	210
17	030	032	211	110
18	031	031	111	111
19	040	040	012	210
20	040	041	111	110
21	050	050	011	110
22	110	110	014	410
23	110	110	023	320
24	110	111	113	310
25	110	111	122	220
26	110	112	212	210
27	110	113	311	110
28	111	111	112	211
29	041	140	020	020

the same as that of the strings in array *y* without considering the string's and the digit's order in each of strings, DTG *x* versus No. *x* array must be an isomorphism of DTG *y* versus No. *y* array, and one of them must be eliminated [21]. For example, both No. 2 {020, 020, 014, 410} and No. 29 {041, 140, 020, 020} arrays derived from the same CG I in Table 3 include four strings. Since the digits of the (first, second, third, and fourth) strings in No. 2 array are the same as that of the (third, fourth, first, second) strings in No. 29 array without considering the string's and the digit's order, a DTG 2 versus No. 2 array and a DTG 29 versus No. 29 array are derived (see Fig. 4). Then, a TG 2 versus DTG 2 and a TG 29 versus DTG 29 are constructed, respectively. It is known that TG 29 is formed only by rotating TG 2 by 180 deg. Hence, TG 2 and TG 29 are the isomorphism each other.

When a planar 4DOF RCM includes 4*T*s, its two different CGs I and II with 4*T*s are constructed (see Fig. 3). Based on conditions

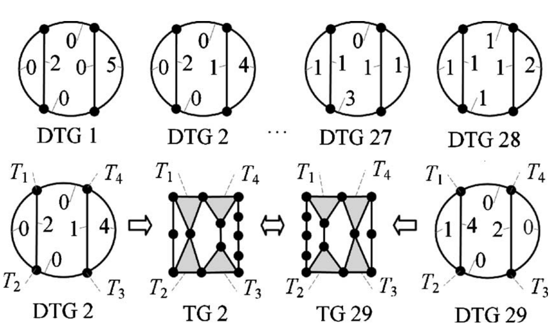

Fig. 4 The 28 valid DTGs versus the 28 valid arrays and two isomorphic TG 2 versus DTG 2 and TG 29 versus DTG 29

Table 4 The 39 valid arrays derived from CG II

No.	T_1	T_2	T_3	T_4
1	000	011	015	015
2	000	012	014	024
3	000	013	013	033
4	000	022	023	023
5	001	010	015	105
6	001	011	014	114
7	001	012	013	123
8	001	020	024	104
9	001	021	023	113
10	001	022	022	122
11	001	030	033	103
12	001	031	032	112
13	001	040	042	102
14	001	041	041	111
15	001	050	051	101
16	002	011	013	213
17	002	012	012	222
18	002	020	023	203
19	002	021	022	212
20	002	030	032	202
21	002	031	031	211
22	002	040	041	201
23	002	050	050	200
24	003	011	012	312
25	003	021	021	311
25	003	030	031	301
27	003	040	040	300
28	004	011	011	411
29	011	110	014	104
30	011	111	013	113
31	011	112	012	122
32	011	113	011	131
33	011	120	023	103
34	011	121	022	112
35	012	111	012	212
36	012	120	022	202
37	012	121	021	211
38	012	130	031	201
39	111	111	112	112

1–3 in Sec. 2, the 28 valid arrays are derived computationally from CG I (see Table 3). The 28 valid DTGs with $4T+7B$ are derived from the 28 valid arrays (see Fig. 4).

5 CGs and TGs of Complicated ALs

Similarly, the 39 valid arrays are derived from CG II by a compiled program (see Table 4).

Then, the 39 valid DTGs with $4T+7B$ are derived from the 39 valid arrays (see Fig. 5).

When a planar 4DOF RCM includes $Q+2T$, its CG can be constructed (see Fig. 6(*a*)). Similarly, the 46 valid arrays are derived from the CG with $Q+2T$ by a compiled program (see Table 5). Then, the 46 valid DTGs with $Q+2T+8B$ are derived from 46 valid arrays (see Fig. 7).

When a planar 4DOF RCM includes $2Q+2T$, the four different valid CGs are constructed from AL 7 in Table 1 (see Fig. 8).

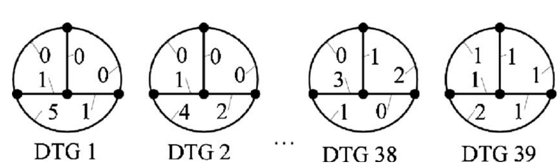

Fig. 5 The 39 valid DTGs versus the 39 valid arrays

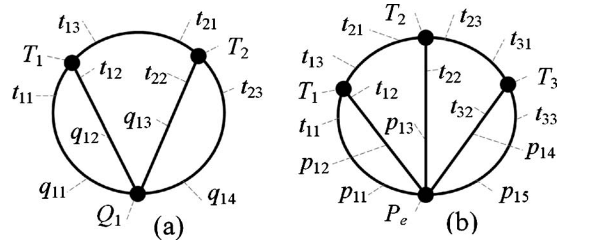

Fig. 6 (*a*) A CG with Q+2T versus AL 6; (*b*) a CGs with P_e +3T versus AL 8 in Table 1

Similarly, the 115 valid arrays are derived from the CG with $2Q+2T$ in Fig. 8(*a*) by a compiled program (see Table 6). Then, the 115 valid DTGs with $2Q+2T+9B$ can be derived from the 115 valid arrays (see Fig. 9(*a*)).

When a planar 4DOF RCM includes P_e+3T, a valid CG with P_e+3T is be constructed (see Fig. 6(*b*)). Similarly, the 237 valid arrays are derived computationally from this CG (see Table 7). Then, the 237 valid DTGs with $P_e+3T+9B$ are derived from the 237 valid arrays (see Fig. 9(*b*)).

Table 5 The 46 valid arrays from CG with Q+2T

No.	T_1	T_2	Q
1	020	015	0215
2	020	024	0224
3	020	033	0233
4	021	105	0205
5	021	114	0214
6	021	123	0223
7	022	204	0204
8	022	213	0213
9	022	222	0222
10	023	303	0203
11	023	312	0212
12	024	402	0202
13	024	411	0211
14	030	005	0305
15	030	014	0314
16	030	023	0323
17	031	104	0304
18	031	113	0313
19	031	122	0322
20	032	203	0303
21	032	212	0312
22	033	311	0311
23	040	004	0404
24	040	013	0413
25	040	022	0422
26	041	112	0412
27	042	211	0411
28	050	012	0512
29	051	111	0511
30	110	015	1115
31	110	024	1124
32	110	033	1133
33	111	114	1114
34	111	123	1123
35	112	213	1113
36	112	222	1122
37	113	312	1112
38	114	411	1111
39	120	014	1214
40	120	023	1223
41	121	113	1213
42	121	122	1222
43	122	212	1212
44	130	013	1313
45	130	022	1322
46	220	022	2222

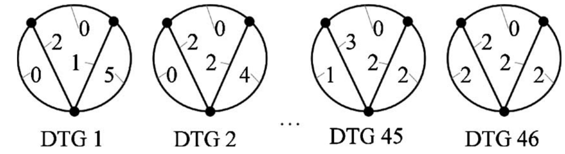

Fig. 7 The 46 valid DTGs versus the 46 valid arrays in Table 5

Thus, all valid TGs are derived from all valid DTGs in Figs. 4, 5, 7, and 9 (see Fig. 10).

It is known from Fig. 10 that conditions 1 and 2 in Sec. 2 are satisfied. In order to satisfy condition 3 in Sec. 2, some Bs with gray color and basic links with dark color in Fig. 10 can be m and b of each other. In addition, in Fig. 10, there are many other cases that satisfy condition 3.

6 Selection of TGs and Their Application Examples

In order to select a better TG from many valid TGs, some factors should be considered as follows:

(1) any valid TG may be potential one for creating novel planar 4DOF RCM
(2) a simple TG should be selected for designing a simple mechanism
(3) a complicated TG should be used for obtaining a large output of the mechanical force/torque

In order to install all actuators on the same base, a TG should include a basic link with 4Js or more, and this basic link should be a base or platform.

The lower the difference between the numbers of B in all curves is, the better of TG is.

All active joints are marked by an underline.

Example 1. A planar 4DOF 2RRR+PRR RCM with a redundant actuator can be synthesized from DTG 6 and TG 6 with $2T+7B$ in Fig. 2(*a*) (see Fig. 11(*a*)). This RCM includes a platform (T), a base (T), three rotational actuators, a translational actuator, nine revolute joints, and one prismatic joint.

Example 2. A planar 4DOF serial-parallel RCM with a redundant actuator can be synthesized from DTG 5 and TG 5 with $H+2T+10B$ in Fig. 2(*d*) (see Fig. 11(*b*)). This mechanism is composed of a 3DOF 3RRR parallel manipulator and a 1DOF Watt mechanism. It includes four rotational actuators and 16 revolute joints.

Example 3. A planar 4DOF RCM can be synthesized from DTG 42 and TG 42 with $Q+2T+8B$ in Figs. 7 and 10 (see Fig. 12(*a*)). It includes one rotational actuator, three translational actuators, two sliders (T_1, T_2), a moving platform Q, a base B, five prismatic joints P_i $(i=1,2,\ldots,5)$, and eight revolute joints $(R_j, j=1,2,\ldots,8)$.

Example 4. A planar 4DOF RCM is synthesized from DTG 218 and TG 218 with $P_e+3T+9B$ in Figs. 9(*b*) and 10 (see Fig. 12(*b*)).

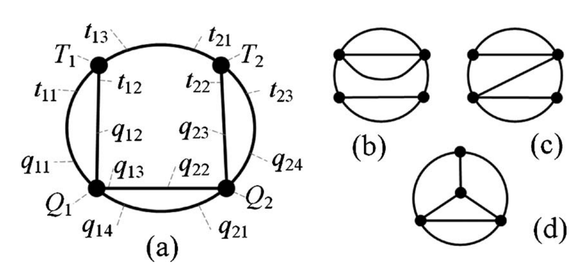

Fig. 8 Four different CGs with 2Q+2T versus AL 7 in Table 1

Table 6 The 115 valid arrays from CG with $2Q+2T$

No.	T_1	T_2	Q_1	Q_2
1	020	005	0220	0205
2	020	014	0220	0214
3	020	023	0220	0223
4	020	004	0230	0304
5	020	013	0230	0313
6	020	022	0230	0322
7	020	003	0240	0403
8	020	012	0240	0412
9	020	002	0250	0502
10	020	011	0250	0511
11	020	005	0211	1105
12	020	014	0211	1114
13	020	023	0211	1123
14	020	004	0221	1204
15	020	013	0221	1213
16	020	022	0221	1222
17	020	003	0231	1303
18	020	012	0231	1312
19	020	002	0241	1402
20	020	011	0241	1411
21	020	003	0222	2203
22	020	012	0222	2212
23	020	002	0232	2302
24	020	011	0232	2311
25	021	104	0220	0204
26	021	113	0220	0213
27	021	122	0220	0222
28	021	103	0230	0303
29	021	112	0230	0312
30	021	102	0240	0402
31	021	111	0240	0411
32	021	104	0211	1104
33	021	113	0211	1113
34	021	122	0211	1122
35	021	103	0221	1203
36	021	112	0221	1212
37	021	102	0231	1302
38	021	111	0231	1311
39	021	102	0222	2202
40	021	111	0222	2211
41	022	203	0220	0203
42	022	212	0220	0212
43	022	202	0230	0302
44	022	211	0230	0311
45	022	203	0211	1103
46	022	212	0211	1112
47	022	202	0221	1202
48	022	211	0221	1211
49	023	302	0220	0202
50	023	311	0220	0211
51	023	302	0211	1102
52	023	311	0211	1111
53	030	004	0320	0204
54	030	013	0320	0213
55	030	022	0320	0222
56	030	003	0330	0303
57	030	012	0330	0312
58	030	011	0340	0411
59	030	004	0311	1104
60	030	013	0311	1113
61	030	022	0311	1122
62	030	003	0321	1203
63	030	012	0321	1212
64	030	011	0331	1311
65	030	011	0322	2211
66	031	103	0320	0203
67	031	112	0320	0212
68	031	111	0330	0311
69	031	103	0311	1103
70	031	112	0311	1112
71	031	111	0321	1211
72	032	211	0320	0211
73	032	211	0311	1111
74	040	012	0420	0212
75	040	011	0430	0311
76	040	012	0411	1112
77	040	011	0421	1211
78	041	111	0420	0211
79	041	111	0411	1111
80	050	011	0520	0211
81	050	011	0511	1111
82	110	014	1120	0214
83	110	023	1120	0223
84	110	013	1130	0313
85	110	022	1130	0322
86	110	012	1140	0412
87	110	011	1150	0511
88	110	014	1111	1114
89	110	023	1111	1123
90	110	013	1121	1213
91	110	022	1121	1222
92	110	012	1131	1312
93	110	011	1141	1411
94	110	012	1122	2212
95	110	011	1132	2311
96	111	113	1120	0213
97	111	122	1120	0222
98	111	112	1130	0312
99	111	111	1140	0411
100	111	113	1111	1113
101	111	122	1111	1122
102	111	112	1121	1212
103	111	111	1131	1311
104	111	111	1122	2211
105	112	212	1120	0212
106	112	211	1130	0311
107	112	212	1111	1112
108	112	211	1121	1211
109	113	311	1120	0211
110	113	311	1111	1111
111	120	022	1220	0222
112	120	013	1211	1113
113	120	022	1211	1122
114	120	012	1221	1212
115	121	112	1220	0212

It includes a rotational actuator, three translational actuators, two sliders (T_1, T_3), a moving platform P_e, a base T_2, six prismatic joints P_i $(i=1,2,\ldots,6)$, and nine revolute joints $(R_j, j=1,2,\ldots,9)$.

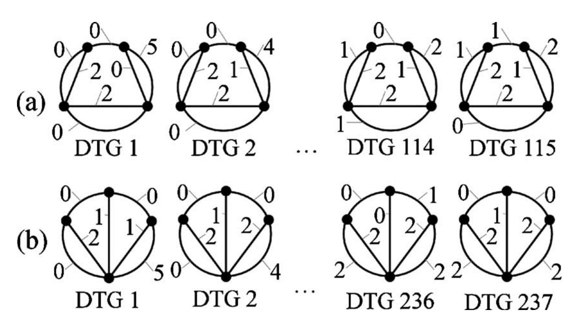

Fig. 9 (*a*) **The 115 valid DTGs versus the 115 valid arrays in Table 6;** (**b**) **the 237 valid DTGs versus the 237 valid arrays in Table 7**

Table 7 The 237 valid arrays derived from CG with P_e+3T versus AL 8 in Table 1

No.	T_1	T_2	T_3	P_e
1	020	010	015	02115
2	020	010	024	02124
3	020	010	033	02133
4	020	011	105	02105
5	020	011	114	02114
6	020	011	123	02123
7	020	012	204	02104
8	020	012	213	02113
9	020	012	222	02122
10	020	013	303	02103
11	020	013	312	02112
12	020	014	402	02102
13	020	014	411	02111
14	020	020	005	02205
15	020	020	014	02214
16	020	020	023	02223
17	020	021	104	02204
18	020	021	113	02213
19	020	021	122	02222
20	020	022	203	02203
21	020	022	212	02212
22	020	023	302	02202
23	020	023	311	02211
24	020	030	004	02304
25	020	030	013	02313
26	020	030	022	02322
27	020	031	103	02303
28	020	031	112	02312
29	020	032	211	02311
30	020	040	012	02412
31	020	041	111	02411
32	020	050	011	02511
33	021	100	015	02015
34	021	100	024	02024
35	021	100	033	02033
36	021	101	105	02005
37	021	101	114	02014
38	021	101	123	02023
39	021	102	204	02004
40	021	102	213	02013
41	021	102	222	02022
42	021	103	303	02003
43	021	103	312	02012
44	021	104	402	02002
45	021	104	411	02011
46	021	110	005	02105
47	021	110	014	02114
48	021	110	023	02123
49	021	111	104	02104
50	021	111	113	02113
51	021	111	122	02122
52	021	112	203	02103
53	021	112	212	02112
54	021	113	302	02102
55	021	113	311	02111
56	021	120	004	02204
57	021	120	013	02213
58	021	120	022	02222
59	021	121	103	02203
60	021	121	112	02212
61	021	122	202	02202
62	021	130	003	02303
63	021	131	102	02302
64	022	200	014	02014
65	022	201	104	02004
66	022	201	113	02013
67	022	202	203	02003
68	022	202	212	02012
69	022	203	302	02002
70	022	203	311	02011
71	022	210	004	02104

Table 7 (*Continued.*)

No.	T_1	T_2	T_3	P_e
72	022	210	013	02113
73	022	210	022	02122
74	022	211	103	02103
75	022	211	112	02112
76	022	212	211	02111
77	022	220	003	02203
78	022	221	111	02211
79	022	230	011	02311
80	023	300	013	02013
81	023	300	022	02022
82	023	301	103	02003
83	023	301	112	02012
84	023	302	211	02011
85	023	310	012	02112
86	023	311	111	02111
87	023	320	011	02211
88	024	400	012	02012
89	024	401	111	02011
90	024	410	011	02111
91	025	500	011	02011
92	030	010	005	03105
93	030	010	014	03114
94	030	010	023	03123
95	030	011	104	03104
96	030	011	113	03113
97	030	011	122	03122
98	030	012	203	03103
99	030	012	212	03112
100	030	013	311	03111
101	030	021	112	03212
102	030	022	211	03211
103	030	030	003	03303
104	030	030	012	03312
105	030	031	111	03311
106	030	040	011	03411
107	031	100	014	03014
108	031	100	023	03023
109	031	101	104	03004
110	031	101	113	03013
111	031	101	122	03022
112	031	102	203	03003
113	031	102	212	03012
114	031	103	311	03011
115	031	110	004	03104
116	031	110	013	03113
117	031	110	022	03122
118	031	111	103	03103
119	031	111	112	03112
120	031	112	211	03111
121	032	200	013	03013
122	032	202	211	03011
123	032	210	012	03112
124	032	211	111	03111
125	033	300	012	03012
126	033	301	111	03011
127	033	310	011	03111
128	034	400	011	03011
129	040	010	004	04104
130	040	010	013	04113
131	040	010	022	04122
132	040	011	112	04112
133	040	012	211	04111
134	040	021	111	04211
135	041	100	022	04022
136	041	101	112	04012
137	041	102	211	04011
138	041	111	111	04111
139	042	201	111	04011
140	042	210	011	04111
141	043	300	011	04011
142	050	010	012	05112
143	050	011	111	05111

Table 7 (*Continued.*)

No.	T_1	T_2	T_3	P_e
144	051	101	111	05011
145	051	110	011	05111
146	052	200	011	05011
147	110	000	025	11025
148	110	000	034	11034
149	110	001	115	11015
150	110	001	124	11024
151	110	001	133	11033
152	110	002	214	11014
153	110	002	223	11023
154	110	003	313	11013
155	110	003	322	11022
156	110	004	412	11012
157	110	005	511	11011
158	110	010	015	11115
159	110	010	024	11124
160	110	010	033	11133
161	110	011	114	11114
162	110	011	123	11123
163	110	012	213	11113
164	110	012	222	11122
165	110	013	312	11112
166	110	014	411	11111
167	110	020	014	11214
168	110	020	023	11223
169	110	021	113	11213
170	110	021	122	11222
171	110	022	212	11212
172	110	023	311	11211
173	110	030	013	11313
174	110	030	022	11322
175	110	031	112	11312
176	110	032	211	11311
177	110	040	012	11412
178	110	041	111	11411
179	110	050	011	11511
180	111	100	015	11015
181	111	100	024	11024
182	111	100	033	11033
183	111	101	114	11014
184	111	101	123	11023
185	111	102	213	11013
186	111	102	222	11022
187	111	103	312	11012
188	111	104	411	11011
189	111	110	023	11123
190	111	111	113	11113
191	111	111	122	11122
192	111	112	212	11112
193	111	113	311	11111
194	111	120	013	11213
195	111	120	022	11222
196	111	121	112	11212
197	112	200	014	11014
198	112	201	113	11013
199	112	203	311	11011
200	112	210	013	11113
201	112	210	022	11122
202	112	211	112	11112
203	112	212	211	11111
204	113	300	022	11022
205	113	301	112	11012
206	113	310	012	11112
207	114	400	012	11012
208	120	000	015	12015
209	120	000	024	12024
210	120	000	033	12033
211	120	001	114	12014
212	120	001	123	12023
213	120	003	312	12012
214	120	010	014	12114
215	120	010	023	12123
216	120	012	212	12112
217	120	020	013	12213
218	120	020	022	12222
219	120	021	112	12212
220	120	030	012	12312
221	121	100	014	12014
222	121	100	023	12023
223	121	101	113	12013
224	121	101	122	12022
225	121	110	022	12122
226	122	200	013	12013
227	130	000	014	13014
228	130	000	023	13023
229	130	001	113	13013
230	130	001	122	13022
231	130	010	013	13113
232	130	010	022	13122
233	131	100	022	13022
234	140	000	022	14022
235	220	000	023	22023
236	220	001	122	22022
237	220	010	022	22122

7 Conclusions

Some contracted graphs without any binary links can be constructed for the planar 4DOF RCMs. Many different topology graphs of the planar 4DOF RCMs can be derived from the same contracted graph.

A topology graph can be represented by a topology graph with digits. A topology graph with digits can be represented by a valid array. All valid arrays can be derived computationally from a contracted graph.

When given a valid array, a valid topology graph with digits can be derived from its contracted graph and given valid array. After that, a valid topology graph can be constructed easily from its topology graph with digits.

The isomorphic and invalid topology graphs can be determined computationally by identifying the isomorphic and invalid arrays.

A large number of different planar 4DOF RCMs can be synthesized based on valid topology graphs. Comparing with some ex-

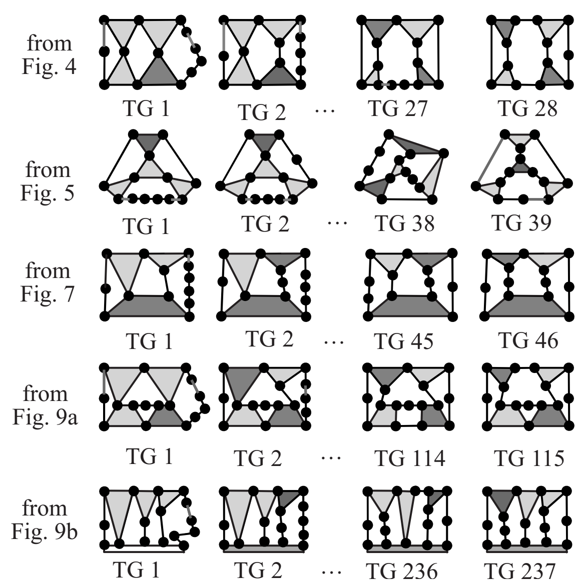

Fig. 10 All valid TGs versus all valid DTGs in Figs. 4, 5, 7, and 9

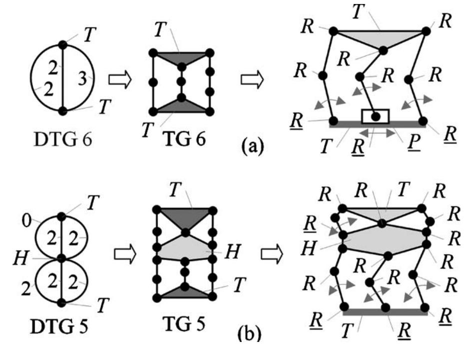

Fig. 11 Two planar 4DOF RCMs and their two DTGs and TGs

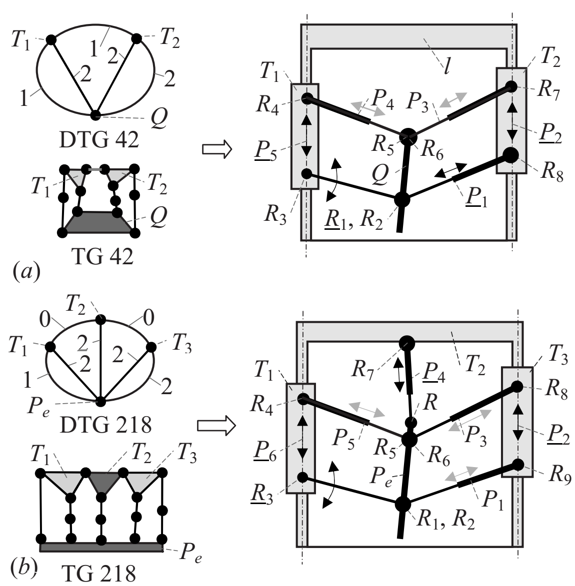

Fig. 12 Two planar 4DOF redundant parallel machine tools and their DTGs and TGs

isting planar 4DOF RCMs, there are still a large number of novel planar 4DOF RCMs that can be synthesized based on topology graphs and various chain structures of branches. They may be selected as novel planar 4DOF RCMs with better characteristics.

Acknowledgment

The authors would like to acknowledge the financial support of the National Natural Science Foundation of China (NNSFC) under Grant No. 50575198 and the financial support of Doctoral Fund from the National Education Ministry under Grant No. 20060216006.

Nomenclature

Symbol List

m = moving platform
b = fixed base,
P = prismatic joint (active P with under line)
R = revolute joint (active R with under line)
B = binary link
T = ternary link
Q = quaternary link
P_e = pentagonal link
H = hexagonal link
J = point of connection with 1DOF
c_i = the ith curve

References

[1] Özbay, U., Türker Şahin, H., and Zergeroglu, E., 2008, "Robust Tracking Control of Kinematically Redundant Robot Manipulators Subject to Multiple Self-Motion Criteria," Robotica, **26**(6), pp. 711–728.
[2] Wu, J., Wang, J. S., Wang, L. P., and Li, T. M., 2009, "Dynamics and Control of a Planar 3-DOF Parallel Manipulator With Actuation Redundancy," Mech. Mach. Theory, **44**(4), pp. 835–849.
[3] Wu, J., Wang, J. S., Li, T. M., and Wang, L. P., 2007, "Performance Analysis and Application of a Redundantly Actuated Parallel Manipulator for Milling," J. Intell. Robotic Syst., **50**(2), pp. 163–180.
[4] Wu, J., Wang, J. S., Li, T. M., and Wang, L. P., 2007, "Analysis and Application of a 2-DOF Planar Parallel Mechanism," ASME J. Mech. Des., **129**(4), pp. 434–437.
[5] Wang, J.-S., Wu, J., Li, T.-M., and Wang, L.-P., 2009, "Workspace and Singularity Analysis of a 3-DOF Planar Parallel Manipulator With Actuation Redundancy," Robotica, **27**(1), pp. 51–57.
[6] Sohn, W. J., and Freudenstein, F., 1986, "An Application of Dual Graphs to the Automatic Generation of the Kinematic Structures of Mechanisms," Proceedings of the ASME Design Engineering Technical Conference, Columbus, OH.
[7] Vucina, D., and Freudenstein, F., 1991, "Application of Graph Theory and Nonlinear Programming to the Kinematic Synthesis of Mechanisms," Mech. Mach. Theory, **26**(6), pp. 553–563.
[8] Tsai, L. W., 2001, *Mechanism Design: Enumeration of Kinematic Structures According to Function*, CRC, Boca Raton, FL.
[9] Jin, Q., and Yang, T. L., 2004, "Theory for Topology Synthesis of Parallel Manipulators and Its Application to Three-Dimension-Translation Parallel Manipulators," ASME J. Mech. Des., **126**(4), pp. 625–639.
[10] Johnson, R. C., 1987, *Mechanical Design Synthesis—Creative Design and Optimize*, Huntington, New York.
[11] Grigore, G., 2008, *Structural Synthesis of Parallel Robots: Part 1: Methodology*, Springer, New York.
[12] Lu, Y., and Leinonen, T., 2005, "Type Synthesis of Unified Planar-Spatial Mechanisms by Systematic Linkage and Topology Matrix-Graph Technique," Mech. Mach. Theory, **40**(10), pp. 1145–1163.
[13] Sardain, P., 1997, "Linkage Synthesis: Topology Selection Fixed by Dimensional Constraints, Study of an Example," Mech. Mach. Theory, **32**(1), pp. 91–102.
[14] Yan, H. S., and Kang, C. H., 2009, "Configuration Synthesis of Mechanisms With Variable Topologies," Mech. Mach. Theory, **44**(5), pp. 896–911.
[15] Hervé, J. M., 1999, "The Lie Group of Rigid Displacements, a Fundamental Tool for Mechanical Design," Mech. Mach. Theory, **34**(5), pp. 719–730.
[16] Pucheta, M., and Cardona, A., 2007, "An Automated Method for Type Synthesis of Planar Linkages Based on a Constrained Subgraph Isomorphism Detection," Multibody Syst. Dyn., **18**(2), pp. 233–258.
[17] Pucheta, M., and Cardona, A., 2008, "Synthesis of Planar Multiloop Linkages Starting From Existing Parts or Mechanisms: Enumeration and Initial Sizing," Mech. Based Des. Struct. Mach., **36**(4), pp. 364–391.
[18] Saxena, A., and Ananthasuresh, G. K., 2003, "A Computational Approach to the Number of Synthesis of Linkages," ASME J. Mech. Des., **125**(1), pp. 110–118.
[19] Kong, X. W., and Gosselin, C. M., 2004, "Type Synthesis of 3-DOF Translational Parallel Manipulators Based on Screw Theory," ASME J. Mech. Des., **126**(1), pp. 83–92.
[20] Hess-Coelho, T. A., 2006, "A. Topological Synthesis of a Parallel Wrist Mechanism," ASME J. Mech. Des., **128**(1), pp. 230–235.
[21] Lu, Y., Ding, L., and Yu, J. P., 2010, "Autoderivation of Topological Graphs for Type Synthesis of Planar 3DOF Parallel Mechanisms," ASME J. Mech. Rob., **2**(1), pp. 0110021.