Analysis and experimental demonstration of
adaptive optics based on the modal control optimization

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ABSTRACT

A modal control optimization method for adaptive optics on the tempo-spatial domain is presented. The spatial modes of the adaptive optics system can be obtained by the singular value decomposition of the response function matrix of the adaptive optics system. The number of correction modes is determined dynamically by the root mean square estimation of the residual aberration after the correction with different number of modes. A Smith compensator is designed to reduce the time delay effect on the closed-loop system. The modal optimization method is experimentally verified by compensating phase distortion produced by artificial atmospheric turbulence in laboratory. Experimental results show that the correction capability of the adaptive optics system can be greatly improved in comparison to that of the generic modal gain integrator approach with the fixed number of correction modes. The modal control optimization method is an attractive and practical alternative to adaptive optics control.

Keywords: adaptive optics; modal control; optimization

1. INTRODUCTION

The control algorithm is a vital link between the wave front sensor (WFS) and the deformable mirror (DM) in an adaptive optics (AO) system. Its function is to convert the measurements of the WFS into a set of actuator signals that are applied to the DM to satisfy the system performance criterion, such as minimizing the residual wave aberrations.

Modal control is commonly used in AO systems, where the wave front is decomposed into a linear combination of some orthogonal modes[1-4]. The primary reason is convenient for the optimization of the AO system on both the spatial and the temporal domains. In term of the spatial domain, the correction capability of the AO system can be increased by choosing different optimum numbers of the correction modes according to different incident wave fronts [5]. In the point of view of the temporal domain[6, 7], because each mode has an individual behavior characterized by its signal to noise ratio and correlation time, the correction performance can be improved by optimizing correction bandwidth of the AO system to each mode. However, for the modal control method there are still many issues which significantly affect the performance of the AO system, for example, how to determine the numbers of the correction modes, and how to reduce the time delay influence to the performance of the AO closed-loop system when optimizing the bandwidth of the system. They still remain open issues which are not yet well resolved in the open literatures.

A modal control optimization method for adaptive optics on the tempo-spatial domain is described. In section 2, a detailed analysis of an optimization method for modal control on both the spatial and the temporal domains is proposed. A method to determine the number of the correction modes is presented and a Smith compensator is designed to reduce the time delay effect on the performance of the AO closed-loop system. Section 3 shows the experimental system and the results of the modal control optimization method by compensating phase distortion produced by artificial atmospheric turbulence in laboratory. Section 4 gives the discussion of the optimization method.

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2. AN OPTIMIZATION METHOD TO MODAL CONTROL

2.1 A dynamic optimization method for determining the number of the correction modes

In an AO system, the degree of the spatial correction is primarily limited by the number of the actuators of the DM, in particular, for the membrane deformable mirror (MDM). Although the MDM is a modal device, but the deformations produced by each actuator are by no means orthogonal. A set of orthogonal bases of an AO system can be obtained by the singular value decomposition (SVD) of the response function matrix of the AO system. Assuming the AO system is a linear one, the response function matrix, denoted by F, is a $m \times n$ matrix, where n is the number of the actuators of the MDM, and m the number of the WFS measurement. The response function matrix is determined by the calibration of the AO system. For an AO system with a MDM, the nth column of F is the increments of the measurement vector of the WFS when the increments of the square of the signal applied to the nth actuator of DM are equal to unit. So the response function matrix F defines the sensitivity of the WFS to the DM and can be used to compute the actuator signals, denoted by S, from the measurement vector of the WFS, denoted by C, by a simple matrix multiplication.

$$S = F^+ C,\tag{1}$$

where the vector S ($S = [s_1^2, s_2^2, \cdots, s_n^2]^T$) is a set of the squares of n signals, and the vector C ($C = [a_1, a_2, \cdots, a_m]^T$) is wave front measurements. Each component of S would be applied to the corresponding actuator of the DM. The vector C is a representative of the surface $\varphi(x,y)$ expressed in terms of an expansion of Zernike polynomials $z_i(x,y)$ [8],

$$\varphi(x,y) = \sum_{i=1}^{m} a_i z_i(x,y).\tag{2}$$

The Matrix F^+ is the pseudo-inverse of F and can be obtained by the SVD of F [9].

$$F = U \times \Sigma \times V^T,\tag{3}$$

and

$$F^+ = V \times \Sigma^+ \times U^T,\tag{4}$$

where U is an $m \times m$ orthogonal matrix, and V a $n \times n$ orthogonal matrix. The columns of U and V are denoted by the vectors u_i, $i = 1, \cdots, m$, and v_i, $i = 1, \cdots, n$, respectively. Σ is an $m \times n$ diagonal matrix with the nonnegative singular values λ_i, $i = 1, \cdots, min(m,n)$, arranged in non-increasing order along the diagonal. U contains different aberration modes and V corresponds to control signals. Each mode, associated with a nonzero singular value, represents the surface that the DM produces perfectly. The smaller the singular value is, the less responsive the mirror and the larger the signals needed to be sent for a unit mode amplitude. Because of the limited range of signals that can be applied, some of these modes can be reproduced only within a small range. The modes corresponding to small singular values are the natural choice to be eliminated from the standpoint of practical use, considering that a few of correction modes make the mirror control more robust. So it is necessary to limit the number of modes used for the correction. If the temporal control is a simple integrator, the actuator signals for the next iteration of the control loop are given by

$$S^k = h(S^{k-1} - \mu \sum_{i=1}^{p} (u_i^T C^k)(v_i / \lambda_i)),\tag{5}$$

where p is the number of modes used for the correction, vector S^{k-1} the square of the n current set signals, C^k the current WFS measurements, and μ the integrator gain. In the following, it will be shown that μ is a very important parameter to the performance of the AO system. The limiting function $y = h(x)$ (x and y are n dimension vectors) is given by

$$y_i = \begin{cases} x_i & -x_{max} \leq x_i \leq x_{max} \\ x_{max} & x_i > x_{max} \\ -x_{max} & x_i < -x_{max} \end{cases}, i = 1, 2, \cdots, n,\tag{6}$$

where x_{max} is the maximum actuator signal permissible. The parameter p of formula (4) can be determined by a dynamic optimized method which dynamically calculates the optimal number of correction modes for every iteration by the estimation of the root mean square(RMS) of the residual aberration obtained after the correction with different

numbers of modes. Neglecting the changes of the incident wavefront in one control cycle, the measurements of the WFS after the current iteration is given by

$$C^{k+1} = C^k - FS^{k-1} + FS^k .$$
(7)

When the number of correction modes is p_0, the RMS of the residual aberration after the current iteration is given by

$$\tilde{\sigma}_{RMS}^k(p_0) = \left\| C^k - FS^{k-1} + h(S^{k-1} - \mu \sum_{i=1}^{p_0} (\boldsymbol{u_i}^T C^k)(\boldsymbol{v_i} / \lambda_i)) \right\| .$$
(8)

If

$$\tilde{\sigma}_{RMS}^k(p_1) = min(\tilde{\sigma}_{RMS}^k(1), \tilde{\sigma}_{RMS}^k(2), \cdots, \tilde{\sigma}_{RMS}^k(r)), r = min(m, n) ,$$
(9)

p_1 is the optimum number of correction modes for the current iteration.

2.2 Design and analysis of a Smith Compensator

An AO system is a multiple-input and multiple-output control system. Because the wave front can be project on to a set of non-coupled orthogonal modes defined in section 2.1, the AO system control loop can be split into n independent single-input and single-output control loops working in parallel. The temporal behavior of each channel can be independently analyzed and optimized.
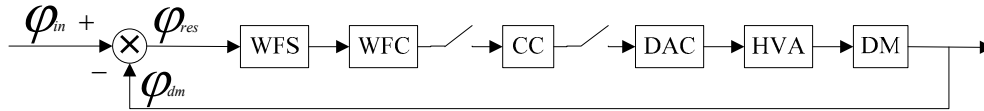

Figure 1 Block-diagram representation of an AO system

Figure 1 gives a classical block-diagram representation of an AO system. The WFS gives measurements of the residual optical phase. The associated detector of the WFS integrates the photons coming from the guide star during a time T, and then delivers an intensity measurement. The WFS measurements, derived from this analog signal, are then available only at sampling times whose period is T. The wave front computer (WFC) is used to deduce the wavefront from measurements of the WFS. The control computer (CC) is used to calculate the DM control signals from the wavefront measurements. Digital to Analog converter (DAC) is used at the output of the CC to drive the high voltage amplifier(HVA) of the DM. The main temporal characteristic of the WFC is a pure time delay due to the read-out of the detector and the computation. To study the behavior of the AO system it is assumed that the delay time is equal to that of the wavefront detection, T. All but the control calculation are determined by characteristics of components of the AO system, and the equivalent transfer function is defined as $G_0(s)$ [10]:

$$\begin{aligned} G_0(s) &= H_{WFS}(s) \cdot H_{WFC}(s) \cdot H_{DAC}(s) \cdot H_{HVA}(s) \cdot H_{DM}(s) \\ &= \frac{1 - exp(-Ts)}{Ts} \cdot exp(-Ts) \cdot \frac{1 - exp(-Ts)}{s} \cdot 1 \cdot 1 \\ &= \frac{(1 - exp(-Ts))^2}{Ts^2} exp(-Ts) \end{aligned}$$
(10)

The control model is discretized, and the Z-domain discrete modal of $G_0(s)$ is:

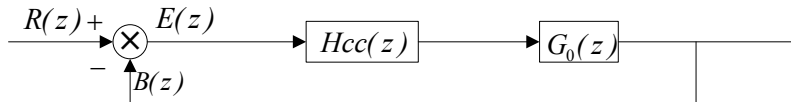$$G_0(z) = Z\left[\frac{(1 - exp(-Ts))^2}{Ts^2} exp(-Ts) \right] = z^{-2} .$$
(11)

Figure 2 Equivalent simplification block-diagram of the AO system

Therefore, the AO system can be regarded as a pure delay of two exposure cycles. Because of the delay, the phase lag which reduces the frequency bandwidth of the AO system is induced. The goal of the $H_{CC}(z)$ optimization is to

decrease this phase lag. After equivalent simplification, the block diagram of the control model is shown in Figure 2.

An integration controller, $Hcc(z) = \mu z / (z-1)$, can be used to reduce the phase lag. The bandwidth of the AO system can be improved by increasing the integrator gain μ. But the dynamic performance is mainly limited by the low gains necessary to ensure a sufficient stability. The Smith Compensator is a compensation configuration which aims to reject the time delay influence out of the closed-loop. Based on the principle of Smith Control compensation, a compensator, $G_\tau(z) = 1 - G_0(z)$, which is in parallel with $H_{CC}(z)$, is introduced, and the equivalent control target no longer contains any delay after compensation[10]. The block diagram of the control model after Smith compensation is shown as Figure 3, and the transfer function of the equivalent controller is

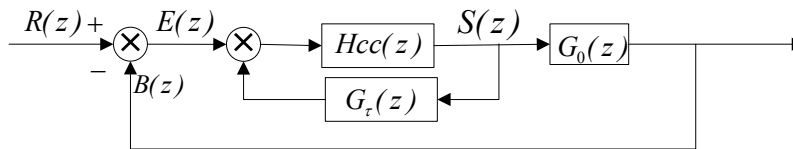$$H_{smith} = \frac{S(z)}{E(z)} = \frac{\mu z^2}{(1+\mu)z^2 - z - \mu}. \tag{12}$$

Figure 3 Block-diagram of the AO system with a Smith compensator

According to the closed-loop resonant peak M_r and the system bandwidth defined as the 0 dB closed-loop error cut-off frequency f_e of the AO system, the relationship between gain coefficient μ and the performance of the control system without and with the Smith Compensator is analyzed. Table 1 shows that the simulation results by MATLAB when the gain coefficients are given to different values, and the sample time of the system is set as 0.01 second. It can be seen from table 1 that f_e increases with the rise of μ whether the controller of the system is integrator with a Smith compensator or not. But the system without a Smith compensator becomes unstable when the gain coefficient μ is great than 0.4, because the closed-loop resonant peak M_r will exceed 3 dB. But this situation does not happen to the system with a Smith compensator. From table 1, it can obviously be seen that the Smith compensator greatly improves the dynamic characteristic of the AO system with good stability, because the bandwidth of the AO system with a Smith compensator for μ set as 10 is larger than that of the system without a Smith compensator for μ set as the optimum value 0.4.

Table 1 Simulation results of the control system with different gain coefficients

controller	Integrator without a Smith compensator					Integrator with a Smith compensator			
μ	0.1	0.2	0.3	0.4	0.5	0.1	1	5	10
M_r (dB)	0	0	0	0.37	3.01	0	0	0	0
f_e (Hz)	2.94	4.18	5.16	5.98	6.75	2.77	6.16	7.73	8.02

3. EXPERIMENT AND RESULTS

3.1 Description of experimental system

The optimization method for modal control on the tempo-spatial domain analyzed above is demonstrated in a closed-loop AO experimental system. Figure 4 shows a schematic of the experimental system comprising of a Shack-Hartmann WFS and a 32 actuator MDM. The WFS consists of a microlens array (0.15-mm-square lenslets; focal length, 3.7 mm) and a CCD camera. The wave front is measured at a maximum rate of 15 Hz, limited by the acquisition time of the camera. A beam splitter placed before the WFS allows of recording the light images simultaneously to the WFS measurements by an additional high resolution camera. The MDM, made by ADAPTICA, is composed of a thin reflective and conducting membrane which faces a close-packed circular actuators of spacing 1.75mm. The dynamic atmospheric turbulence is simulated by a rotating glass plate, which is generated with a fractal method[11].
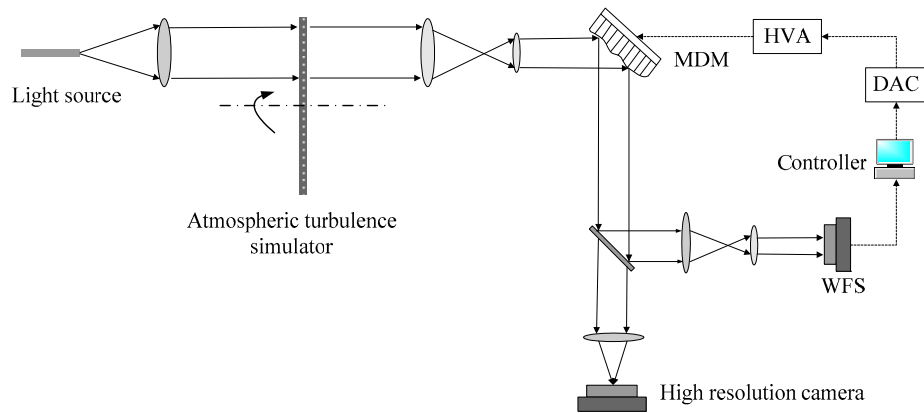
Figure 4 Schematic of the AO experimental system

3.2 Results

Figure 5 shows the effect of choosing numbers of the correction modes by the dynamic optimization method, compared with the effect with different fixed numbers of correction modes. A number of 100 iterations are evaluated to give a sufficiently good approximation. It can be seen from figure 5 that the correction results of the system can be improved by the dynamic optimization method, which makes the AO system have a smaller residual error. Figure 6 shows the effect of the dynamic optimization method with a Smith compensator, compared with the effect of a fixed number of the correction modes without a Smith compensator. The statistical results are derived from the continuous records of 90 iterations from the eleventh iteration. According to figure 6 of the mean value and the standard deviation of the RMS of the residual aberration, it can be seen that the stability and accuracy of the AO system can be greatly improved by the dynamic optimization method with a Smith compensator. Figure 7 shows an example of the real-time correction of the dynamic atmospheric turbulence. Both the evolution of the RMS of the residual aberration during the compensation process, and the image of the light source before and after correction are presented in Figure 7. The results show an intensity enhancement at the central peak of the images, which further validates that most of the atmospheric turbulence has been properly corrected.
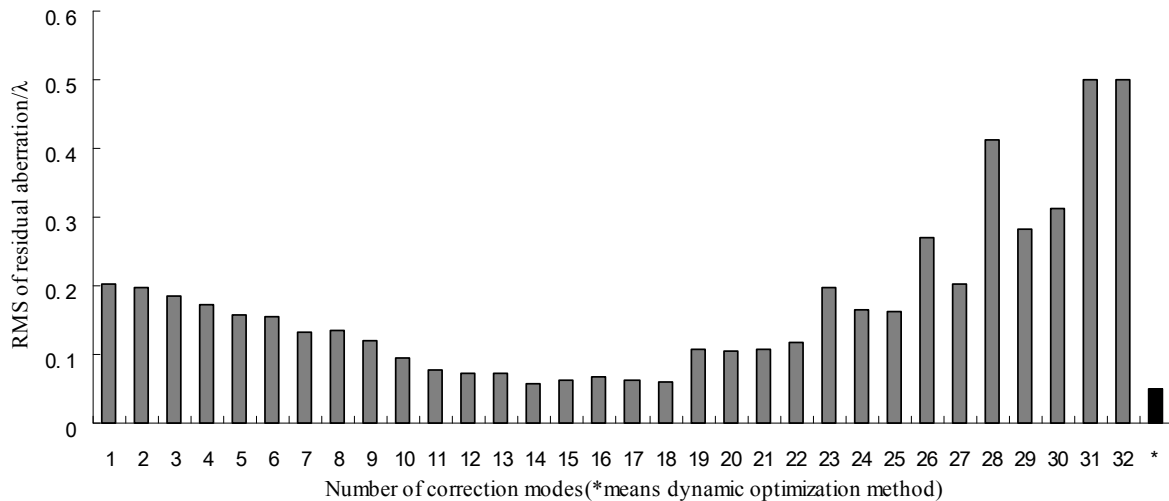

Figure 5 RMS of the residual aberration v. the number of correction modes when the experimental system corrects the static atmospheric turbulence aberration with an integrator controller
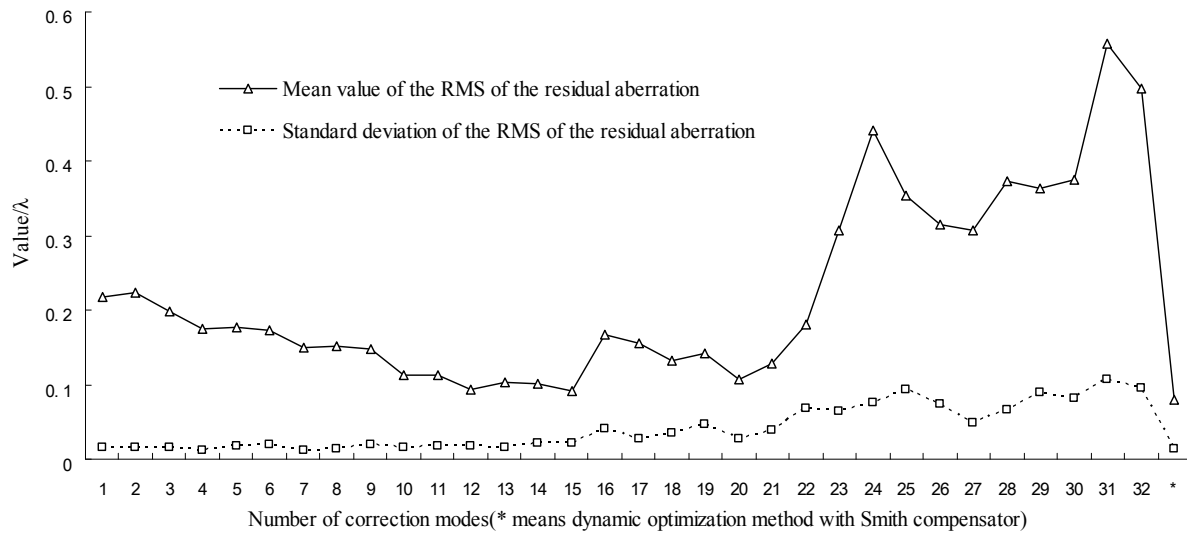
Figure 6 Mean value and standard deviation of the RMS of the residual aberration v. the number of correction modes when the experimental system corrects the dynamic atmospheric turbulence aberration with and without a Smith compensator
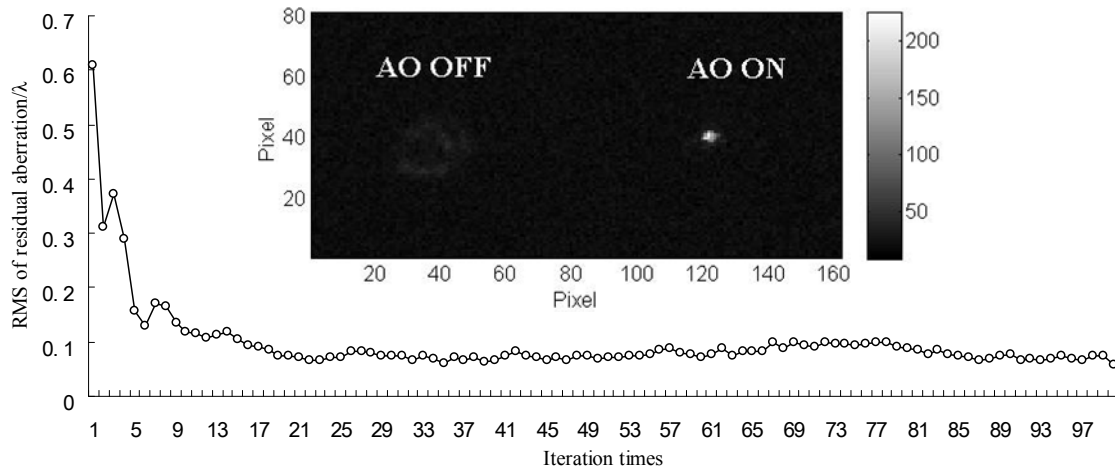

Figure 7 Evolution of the RMS of the residual aberration and the image of light source with and without AO

4. DISCUSSION

A modal control optimization method for AO system on both the spatial and the temporal domains is presented. The modal optimization method is demonstrated on an AO system comprising of a Shack-Hartmann WFS and a 32 actuator MDM by compensating phase distortion produced by artificial atmospheric turbulence in laboratory. It has been experimentally demonstrated that the accuracy and stability of the AO system can be greatly improved by a dynamic optimization method with a Smith compensator, in comparison to that of the generic modal gain integrator approach with a fixed number of correction modes. So the modal control optimization method is an attractive and practical alternative to AO system control. However, it must be pointed out that the dynamics of the incident aberration of the AO system is not mentioned during the process of the optimization. Further work in this direction is needed. By understanding the temporal characteristic of the incident aberrations, the correction bandwidth to each mode may be, respectively, optimized and a better correction would be achieved.

5. ACKNOWLEDGEMENT

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