Discord under the influence of a quantum phase transition*

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This paper studies the discord of a bipartite two-level system coupling to an XY spin-chain environment in a transverse field and investigates the relationship between the discord property and the environment's quantum phase transition. The results show that the quantum discord is also able to characterize the quantum phase transitions. We also discuss the difference between discord and entanglement, and show that quantum discord may reveal more general information than quantum entanglement for characterizing the environment's quantum phase transition.

Keywords: quantum discord, quantum phase transition, XY spin-chain

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1. Introduction

The investigation of the fundamental principles of quantum physics has brought us not only a deeper understanding of the microcosmic world, but has also laid the new foundations for future technologies. Among the features of quantum physics, the nonlocal property of a quantum system is a very interesting phenomenon, which has no classical correspondence and has attracted the attention of many physicists. Entanglement is such a kind of quantum nonlocal correlation that has been deeply studied over the past years.[1−4] However, quantum correlations are more general than quantum entanglement and other kinds of quantum correlations have also been described, such as quantum discord.[5,6] Quantum discord measures the quantum correlations of a more general type of entanglement and there are also separable mixed states with nonzero quantum discord.[7,8]

In the classical system, the Hamiltonian $\mathscr{H}(X) = -\sum_x p_{X=x}\log p_{X=x}$ is used to quantify the information in a random variable X, which contains state x with probability $p_{X=x}$ and the joint entropy is defined as $\mathscr{H}(X,Y) = -\sum_{x,y} p_{X=x,Y=y}\log p_{X=x,Y=y}$ with $p_{X=x,Y=y}$ being the probability of $X=x$ and $Y=y$, which measures the total uncertainty of a pair of random variables X and Y. The mutual information measures the correlation between two random variables A and B, which can be expressed as

$$I_{\rm c}(A,B) = \mathscr{H}(A) + \mathscr{H}(B) - \mathscr{H}(A,B),$$

whose quantum version can be written as

$$\mathscr{I}_{\rm q}(\rho_{AB}) = \mathscr{H}(\rho_A) + \mathscr{H}(\rho_B) - \mathscr{H}(\rho_{AB}),$$

where $\mathscr{H}(\rho) = -\mathtt{Tr}[\rho\,\mathrm{lb}\,\rho]$ is the von Neumann entropy of ρ, and $\rho_{A(B)}$ is the reduced density matrix of ρ_{AB}. For classical probability distributions, by using Bayes's rule, $\mathscr{H}(A|B) = \mathscr{H}(A,B) - \mathscr{H}(B)$, the mutual information can also be expressed as

$$J_{\rm c}(A,B) = \mathscr{H}(A) - \mathscr{H}(A|B),$$

and $J_{\rm c}(A,B) = I_{\rm c}(A,B)$.

However, in quantum physics, there is no corresponding Bayes's rule, thus

$$\mathscr{J}_{\rm q}(\rho_{AB}) = \mathscr{H}(\rho_A) - \mathscr{H}(\rho_A|\rho_B) \tag{1}$$

is not necessarily equal to $\mathscr{I}_{\rm q}(\rho_{AB})$. Olliver and Zurek investigated this problem and introduced the concept quantum discord.[5,6] They considered the quantum version of conditional entropy. Since the correlation depends on the measurement of the other quantum system, one can introduce a complete set of projectors $\{\Pi_i\}$ corresponding to the outcome i, which make $\rho_{A|i} = \mathtt{Tr}(\Pi_i\rho_{AB}\Pi_i)/p_i$, with $p_i = \mathtt{Tr}_{AB}(\Pi_i\rho_{AB}\Pi_i)$. Then one can define the quantum conditional entropy

$$\mathscr{H}_{\Pi_i}(A|B) = \sum_i p_i \mathscr{H}(\rho_{A|i}), \tag{2}$$

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and the difference has been defined as quantum discord

$$D(\rho_{AB}) = \mathscr{I}_{\mathrm{q}}(\rho_{AB}) - \mathscr{J}_{\mathrm{q}}(\rho_{AB}), \tag{3}$$

where

$$\mathscr{J}_{\mathrm{q}}(\rho_{AB}) = \max_{\{\Pi_k\}}[S(\rho_A) - S(\rho_{AB}|\{\Pi_k\})] \tag{4}$$

is the classical correlation and the maximum is taken over the set of projective measurements $\{\Pi_k\}$ on subsystem B.

From the above definition of quantum discord, we know that quantum discord is a measure of the difference between the classical correlation and quantum correlation, and can be seen as a characteristic identification of the existence of a quantum property. This quantum correlation also provides a faster way to perform some tasks than the best known classical counterpart, which has been shown theoretically[7] and experimentally[8] in a nonuniversal model of quantum computation. These studies also suggest that there are nonclassical correlations which are more general and possibly even more fundamental than entanglement. Therefore, such a nonclassical correlation might play a significant role in quantum information protocols.

Quantum discord has been used to study the correlations of many kind of quantum system, including the relevance of quantum phase transitions,[9−12] and has also been generalized to multipartite states.[13] The effects of the environment and finite temperature have also been considered,[14−20] which shows that decoherence may occur without entanglement between the system and the environment. Quantum discord will also provide an opportunity for its application in quantum information theory[17] and the classical and quantum correlations in the Grover search have been discussed in Ref. [21]. The environment can affect the system and deprive it of its quantum feature, while the feature of the environment can also be shown in the system. If the environment is a certain kind of quantum system, the environment's certain feature would also be characterized by the system, such as the decoherence,[22] entanglement[23,24] and Landau–Zener transition.[25] Quantum entanglement has been used to study quantum systems, such as quantum phase transition, in many ways over past years and is also used as a witness of the environment's quantum phase transition.[23,24,26] Since quantum discord is different from entanglement in many ways, considering the property of quantum discord in such an environment is also interesting.

In this paper, we will study the quantum discord of a bipartite system coupling to a spin 1/2 XY spin-chain and study the properties of the quantum discord under the quantum phase transition of the spin-chain.[23] We will show that quantum discord can also be used as a witness of the environment's quantum phase transition and can be used more generally than quantum entanglement. This paper is organized as follows. In Section 2, we will give the solution of the model. We will study the discord and compare it with the entanglement in Section 3, and give conclusions in Section 4.

2. Solution of the system

The model which we considered is a bipartite two-level system coupling to an XY spin 1/2 chain environment in a transverse field, it can be described by the Hamiltonian $H = H_{\mathrm{S}} + H_{\mathrm{E}} + H_{\mathrm{I}}$, where

$$\begin{aligned} H_{\mathrm{S}} &= \frac{\hbar\omega_1}{2}\sigma_1^z + \frac{\hbar\omega_2}{2}\sigma_2^z, \\ H_{\mathrm{E}} &= -\sum_{l=1}^{N}\left(\frac{1+\gamma}{2}\sigma_l^x\sigma_{l+1}^x + \frac{1-\gamma}{2}\sigma_l^y\sigma_{l+1}^y + \lambda\sigma_l^z\right), \\ H_{\mathrm{I}} &= \sum_{l=1}^{N}(g_1\sigma_1^z\sigma_l^z + g_2\sigma_2^z\sigma_l^z). \end{aligned} \tag{5}$$

Here, H_{E} describes the environment spin chain, γ describes the anisotropic property of the spin chain and λ is the strength of the transverse field, while g_1 and g_2 are the coupling strengths between the system and the environment. We only consider the periodic boundary condition $\sigma_{N+1} = \sigma_1$ for the spin chain in subsequent studies.

This system can be exactly solved.[23] We can first write the Hamiltonian in the bases of the bipartite system, i.e., $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$, and the evolution operator of the whole system may be expanded in the form $U(t) = \sum_{ij} U_{ij}|ij\rangle\langle ij|$, where $|ij\rangle = |i\rangle_1 \otimes |j\rangle_2$, and

$$\begin{aligned} & H_{ij} \\ & = -\sum_{i=1}^{N}\left(\frac{\gamma+1}{2}\sigma_l^x\sigma_{l+1}^x + \frac{1-\gamma}{2}\sigma_l^y\sigma_{l+1} + \Lambda_{ij}\sigma_l^z\right), \end{aligned}$$

where $\Lambda_{ij} = \lambda + (-1)^{i+1}g_1 + (-1)^{j+1}g_2$, $i, j = 0, 1$. The Hamiltonian H_{ij} can be diagonalized as

$$H_{ij} = \sum_k \xi_{ijk}\left(b_{ijk}^{\dagger}b_{ijk} - \frac{1}{2}\right)$$

by the standard procedure, i.e., the Jordan–Wigner transformation

$$c_l = \Big(\prod_{m<l} \sigma_m^z\Big)(\sigma_l^x + \mathrm{i}\sigma_l^y)/2,$$

the Fourier transformation

$$d_k = (1/\sqrt{N}) \sum_l c_l \exp(-2\mathrm{i}\pi lk/N)$$

and Bogoliubov transformation $b_{ijk} = d_k \cos(\theta_{ijk}/2) - \mathrm{i}d_{-k}^\dagger \sin(\theta_{ijk}/2)$, with ξ_{ijk} and θ_{ijk} defined as

$$\xi_{ijk} = 2\sqrt{\left(\Lambda_{ij} - \cos\frac{2\pi k}{N}\right)^2 + \gamma^2 \sin^2\frac{2\pi k}{N}},$$
$$\tan\theta_{ijk} = \frac{\gamma \sin\dfrac{2\pi k}{N}}{\Lambda_{ij} - \cos\dfrac{2\pi k}{N}}. \quad (6)$$

The Hamiltonian H_{E} can also be diagonalized by the same procedure, only by changing Λ_{ij} by λ, and then the other functions can be obtained correspondingly. For the simplicity of the following discussion, we will use the similar functions only by omitting the subscript ij in the above discussions when we use the expressions of diagonalized Hamiltonian H_{E}.

Assuming that initially the state of the bipartite system can be expressed as $\rho_{12}(0)$ and the spin chain is in its ground state, then the evolution of the bipartite system can be worked out as

$$\rho_{12}(t) = \sum_{ij;mn} \rho_{ij;mn}(0)\Gamma_{ij;mn}(t)|ij\rangle\langle mn|, \quad (7)$$

with

$$\begin{aligned}
&\Gamma_{ij;mn}(t)\\
&= \prod_k \mathrm{e}^{\mathrm{i}(\xi_{ij,k}-\xi_{mn,k})t/2\hbar}\Big[1 - (1 - \mathrm{e}^{-\mathrm{i}\xi_{ij,k}t/\hbar})\\
&\times \sin^2\frac{\theta_k - \theta_{ij,k}}{2} - (1 - \mathrm{e}^{\mathrm{i}\xi_{mn,k}t/\hbar})\sin^2\frac{\theta_k - \theta_{mn,k}}{2}\\
&+ \sin\frac{\theta_k - \theta_{mn,k}}{2}\sin\frac{\theta_k - \theta_{mn,k}}{2}\cos\frac{\theta_{ij,k} - \theta_{mn,k}}{2}\Big]\\
&\times (1 - \mathrm{e}^{-\mathrm{i}\xi_{ij,k}t/\hbar})(1 - \mathrm{e}^{\mathrm{i}\xi_{mn,k}t/\hbar}), \quad (8)
\end{aligned}$$

and

$$\rho_{ij;mn}(0) = \langle ij|\rho_{12}(0)|mn\rangle.$$

With the system's evolution, the correlation between the bipartite two-level system will be changed through interaction with the common environment. We will therefore study the quantum discord of the bipartite system in the following and also compare it with quantum entanglement.

3. Discord and entanglement

Now, we can study the quantum discord to detect the quantum correlation of the bipartite system and compare it with the quantum entanglement measured by Wootter's concurrence.[27]

In order to measure the quantum discord, a complete set of orthogonal projectors is needed and we choose it to be $\Pi_j = |\vartheta_j\rangle\langle\vartheta_j|, j = 1, 2$, with

$$\begin{aligned}
|\vartheta_1\rangle &= \cos\theta|0\rangle + \mathrm{e}^{\mathrm{i}\varphi}\sin\theta|1\rangle,\\
|\vartheta_2\rangle &= \mathrm{e}^{-\mathrm{i}\varphi}\sin\theta|0\rangle - \cos\theta|1\rangle. \quad (9)
\end{aligned}$$

Wootter's concurrence is defined as $\mathscr{C}(\rho_{AB}) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$, where λ_i $(i = 1, 2, 3, 4)$ are the square root of the eigenvalues of $\rho_{AB}(\sigma_A^y \otimes \sigma_B^y)\rho_{AB}^*(\sigma_A^y \otimes \sigma_B^y)^\dagger$ in decreasing order, $\sigma_{A(B)}^y$ is the y-component of the spin-1/2 Pauli operator for the partition $A(B)$.

Substituting Eqs. (7) and (9) into the former formula (3) and Wootter's concurrence $\mathscr{C}(\rho_{AB})$, we find that the quantum discord and quantum entanglement can now be worked out. In the following, we will study the quantum discord property and at the same time compare it with the quantum entanglement.

3.1. The direct product pure initial state

Assuming that the initial state of the bipartite system is a pure state, i.e., $\rho_{12}(0) = |\phi_{12}(0)\rangle\langle\phi_{12}(0)|$, with $|\phi_{12}(0)\rangle = (1/\sqrt{2})(|1\rangle_1 + |0\rangle_1) \otimes (1/\sqrt{2})(|1\rangle_2 + |0\rangle_2)$, the environment is in its ground state $|G\rangle$ and, initially, there is no correlation between the system and the environment. Then, substituting $\rho_{12}(0)$ into the discord and concurrence equations, we will obtain the exact result with the evolution of the bipartite system under the influence of the spin chain environment.

Now we turn to numerical analysis of the dynamics of the quantum discord, which has been shown in Figs. 1 and 2, where we have plotted the quantum discord and the concurrence versus time for different λ. In these numerical studies, we have set $N = 400$, $g_1 = g_2 = 0.05$ and chosen $\gamma = 0.1, 1$ for Figs. 1 and 2, respectively.

These figures show that the quantum discord property is very similar to quantum entanglement in showing the environment's quantum phase transition. It is shown that in the vicinity of the critical point $\lambda = \pm 1$, the discord changes dramatically, this clearly shows the different phases between $0 \le |\lambda| \le 1$ and

$|\lambda| > 1$. Thus, both entanglement and quantum discord can detect quantum phase transitions. However, there is also some difference between them and this can be easily found in the case of $\gamma = 0.1$ (see Fig. 1) and we may predict that for the XX environment spin chain, which corresponds to $\gamma = 0$ and has a criticality region along the line between $\lambda = -1$ and $\lambda = 1$, the criticality is reflected in the discord of the bipartite system, just as it is reflected in the entanglement of the bipartite system. In Fig. 1(a), it is clearly shown that in the case of $\gamma = 0.1$ and in the region of $-1 \leq \lambda \leq 1$, the discord remains while entanglement will quickly disappear. This property may also be typical of an XX spin chain. This phenomenon also tells us that quantum discord can exist without entanglement, which is coincidental with former studies, showing that the quantum discord is more general than quantum entanglement in describing quantum correlations.

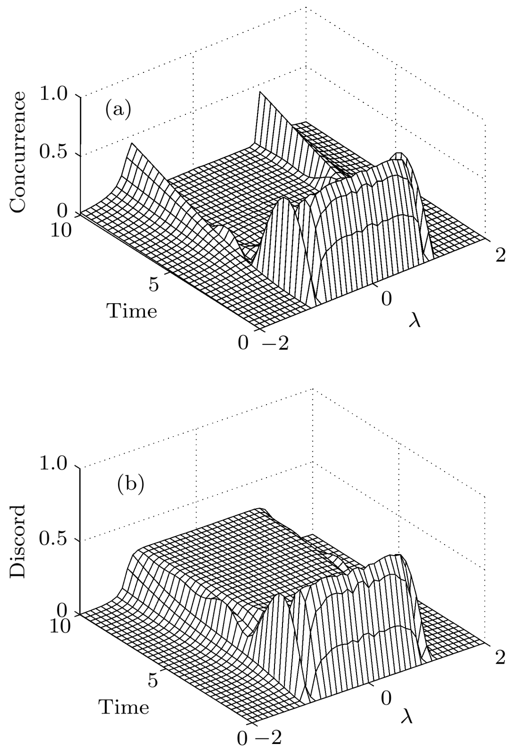

Fig. 1. Entanglement (a) and discord (b) versus time and strength of the transverse field, using $\gamma = 0.1$, $N = 401$, $g_1 = g_2 = 0.05$ in the numerical calculation.

In the case of Ising spin chain $\gamma = 1$, the criticality of the spin chain can also be reflected in the bipartite system's discord. In this case, both the entanglement and the quantum discord behave similarly. However, we can easily find from the figures that quantum discord may exist in wider regions than entanglement.

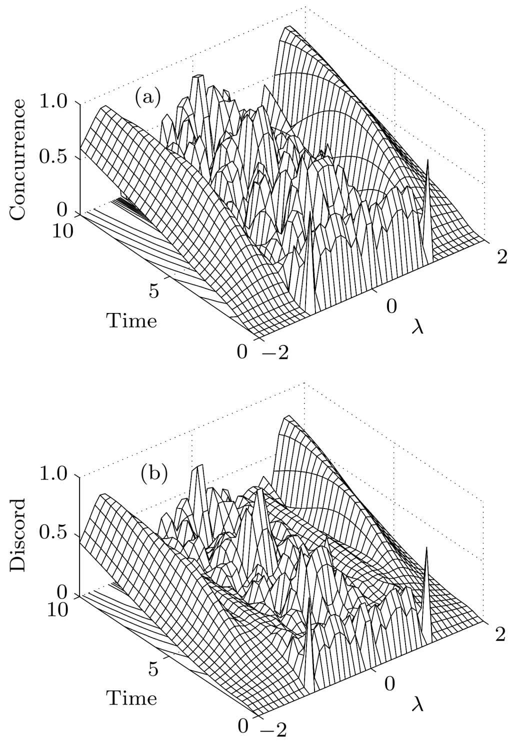

Fig. 2. Entanglement (a) and discord (b) versus time and the strength of the transverse field, using $\gamma = 1$ (Ising chain), $N = 401$, $g_1 = g_2 = 0.05$ in the numerical calculation.

3.2. The Werner mixed initial state

We now study the dynamics of mixed state quantum discord and assume that the bipartite system is initially in a Werner state. It has been discussed that the Werner mixed state is,[28]

$$\rho_{12} = P|\psi\rangle\langle\psi| + \frac{1-P}{4}I, \qquad (10)$$

where $|\psi\rangle$ is the Bell state, which is a maximal entangled state and only entangled where $P > 1/3$. This is easy to check by Wootter's concurrence. However, the discord of the Werner state is not zero, implying that the quantum correlation can exist without entanglement.

Supposing that the initial state of the system is also in a direct product form, $\rho_t = \rho_{12} \otimes |G\rangle\langle G|$, we can also work out the quantum discord property under the influence of the spin chain.

Firstly, we consider the case of $P > 1/3$, where entanglement can also exist. The quantum entanglement property and quantum discord have been shown in Fig. 3, where we have set $P = 1/2$. We can find that both the entanglement and the quantum discord can pick up the properties of the quantum critical phenomenon. Entanglement shows sudden death in many regions while quantum discord can not.

Then, we consider the case of $P < 1/3$, where there is no entanglement but quantum discord is not zero. We can obtain the quantum discord property under the influence of the spin chain. In Fig. 4, we have studied the quantum discord property in the case of $\gamma = 0.1$ (a) and $\gamma = 1$ (Ising spin chain) (b). It is shown that, in the case of $\gamma = 0.1$, the discord will disappear quickly in the region of $|\lambda| < 1$, while it will not in the case of $\gamma = 1$, showing clearly the different phase regions. In the case of $\gamma = 1$ (Ising spin chain) the different phase regions are also clearly shown by the quantum discord, especially in the case of $|\lambda| < 1$, i.e., the quantum discord oscillates with time when $|\lambda| \to 0$, which is consistent with the property of the ground state fidelity of the Ising spin chain[22] as well as the quantum entanglement.[23,24] In the case of $\gamma \to 0$ (XX spin chain), the quantum discord decays quickly where $|\lambda| \leq 1$, while it exists in the case of $|\lambda| \geq 1$, which also clearly gives us the different region of different phases of the spin chain. Although in this case the entanglement is always zero and can no longer witness the environment's property, it can be compensated by the quantum discord. Thus, the quantum discord is a more general quantum correlation than entanglement for detecting the environments' quantum phase transitions.

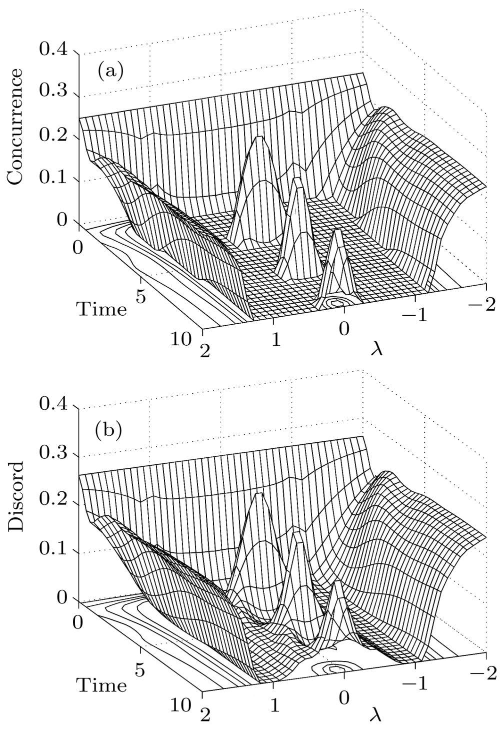

Fig. 3. Entanglement (a) and discord (b) versus time and transverse field strength in the case of $P = 1/2$ Werner states, using $\gamma = 1$ (Ising chain), $N = 401$, $g_1 = g_2 = 0.05$ in the numerical calculation.

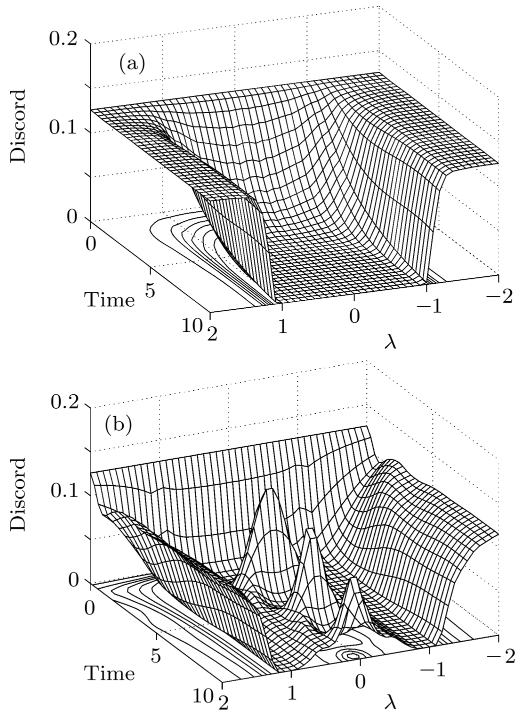

Fig. 4. Discord versus time and the transverse field strength in the case of $P = 1/3$ Werner states, using $\gamma = 0.1$, $N = 401$, $g_1 = g_2 = 0.05$ in the numerical calculation: (a)$\gamma = 0.1$; (b)$\gamma = 1$ (Ising chain).

3.3. Maximally mixed marginal initial state

Now, we will consider another kind of more general mixed state, the maximally mixed marginal initial state or Bell- diagonal states, which can be expressed as[14,29]

$$\rho_{AB} = \frac{1}{4}\left(I_{AB} + \sum_{i=x,y,z} c_i \sigma_i^A \sigma_i^B\right), \tag{11}$$

where c_i is a real number so that $0 \leq |c_i| \leq 1$ for $i = x, y, z$ and I_{AB} is the identity operator of the total system. This state will reduce to the Werner states in the case of $|c_1| = |c_2| = |c_3| = c$ and the Bell states in the case of $|c_1| = |c_2| = |c_3| = 1$. It has been shown in Ref. [29] that with this initial state, the system will emerge from a quantum to a classical correlation sudden transition in the presence of nondissipative decoherence. In the following study, we will show that, under the influence of the quantum spin chains in transverse fields, the system will also show interesting features that can be reflected by quantum discord but cannot be reflected by quantum entanglement.

As is shown in Figs. 5 and 6, where the parameters have been set as $c_1(0) = 1$, $c_2(0) = -c_3 = -0.6$

and other parameters have been set to be the same as in the former subsection, it is interesting to find that the quantum discord shows different characteristics in comparison with quantum entanglement. The quantum discord shows that the correlation will firstly increase and then decrease over time. This property will not be expressed by the quantum entanglement, which only decreases with time. In the case of $\gamma = 0.1$ and $\gamma = 1$, this characteristic is always kept on. This property tells us that the quantum correlation between the spins is very critical with time and it is not the property that entanglement displays.

Furthermore, we can also find that when the quantum spin chain is nearly at its critical points, the quantum discord also shows an increasing property and then decreases with the strength of the transverse field. Although the critical property will be shown both by the quantum discord and the quantum entanglement, we can see more information about the properties from the quantum discord.

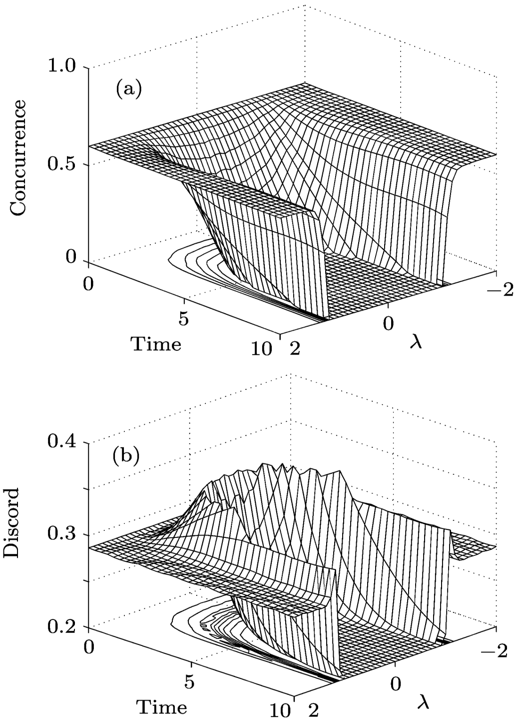

Fig. 5. Concurrence (a) and discord (b) versus time and the strength of the transverse field in the case of a maximally mixed marginal initial state, using $c_1(0) = 1$, $c_2(0) = -c_3$ and $c_3 = 0.6$, $\gamma = 0.1$, $N = 401$ and $g_1 = g_2 = 0.05$.

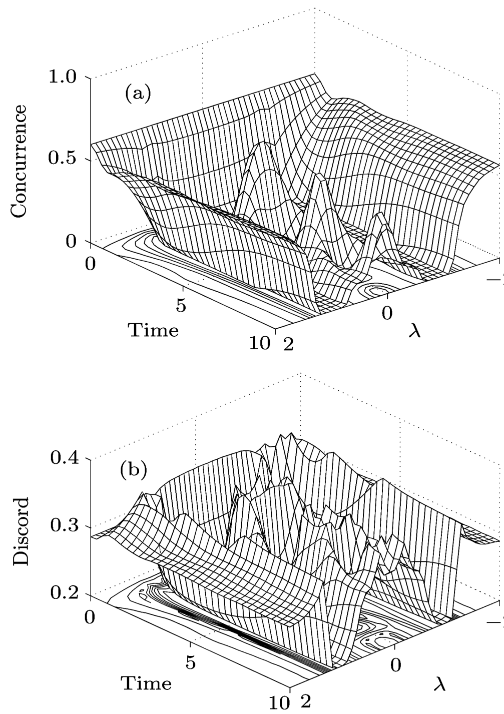

Fig. 6. Concurrence (a) and discord (b) versus time and the strength of the transverse field in the case of a maximally mixed marginal initial state, using $c_1(0) = 1$, $c_2(0) = -c_3$, $c_3 = 0.6$, $\gamma = 1$, $N = 401$ and $g_1 = g_2 = 0.05$.

4. Conclusion

In summary, we have studied the quantum discord of a bipaSrtite system under the environment of an XY spin-chain and discussed the property of the quantum discord with a quantum phase transition. Our studies show that quantum discord is different from quantum entanglement in many ways, and describes more general information relating to quantum correlation. Generally, quantum discord may reveal more information than entanglement. In the case of the Werner mixed initial quantum state, quantum discord can still give us an idea of the environment's quantum phase transition while quantum entanglement can not. In the case of a maximally mixed marginal initial state, a more interesting property may be revealed by the quantum discord. This study is an extended study of the relationship between the quantum system and its environmental properties.

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