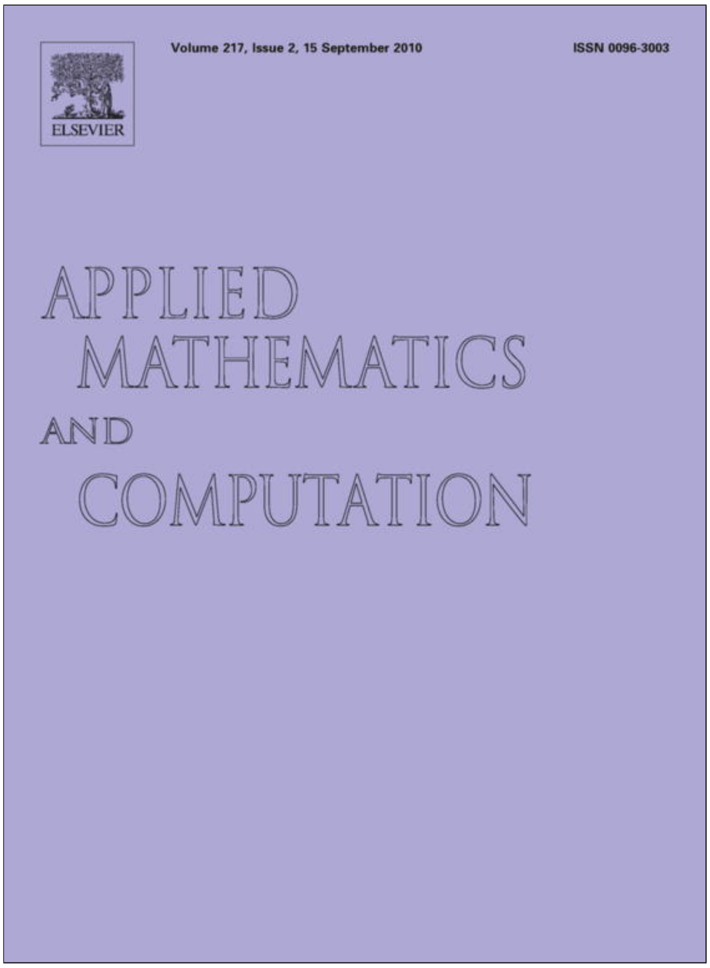
Volume 217, Issue 2, 15 September 2010
ISSN 0096-3003
ELSEVIER
APPLIED MATHEMATICS AND COMPUTATION

Applied Mathematics and Computation 217 (2010) 629–638

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A series of exact solutions for (2 + 1)-dimensional Wick-type stochastic generalized Broer–Kaup system via a modified variable-coefficient projective Riccati equation mapping method

Qing Liu [a,b,*], Jia-Cheng Lan [a], Hong-Yu Wu [a], Zi-Hua Wang [b]

[a] College of Mathematics and Physics, Lishui University, Lishui Zhejiang 323000, PR China
[b] School of Communication and Information Engineering, Shanghai University, Shanghai 200072, PR China

ARTICLE INFO

Keywords:
Exact solution
Wick-type stochastic equation
Generalized Broer–Kaup system
White noise
Projective Riccati equation mapping method

ABSTRACT

A modified variable-coefficient projective Riccati equation mapping method is applied to (2 + 1)-dimensional Wick-type stochastic generalized Broer–Kaup system. With the help of Hermit transformation, we obtain a series of new exact stochastic solutions to the stochastic Broer–Kaup system in the white noise environment.

1. Introduction

In Ref. [1], Zhang et al. researched the (2 + 1)-dimensional variable coefficient Broer–Kaup (VCBK) system:

$$u_{yt} - c(t)(u_{xxy} - 2(uu_x)_y - 2v_{xx}) = 0, \tag{1a}$$

$$v_t + c(t)(v_{xx} + 2(vu)_x) = 0, \tag{1b}$$

where the coefficient $c(t)$ is a bounded or integrable functions on $\mathbb{R}_+$. If the problem is considered in random environment, we can get a random (2 + 1)-dimensional Broer–Kaup system. In order to obtain their exact solutions, we only consider this problem in white noise environment.

In this paper, we will consider a (2 + 1)-dimensional Wick-type stochastic generalized Broer–Kaup (WSGBK) system in the following form:

$$U_{yt} - C(t) \diamondsuit (U_{xxy} - 2(U \diamondsuit U_x)_y - 2V_{xx}) = 0, \tag{2a}$$

$$V_t + C(t) \diamondsuit (V_{xx} + 2(V \diamondsuit U)_x) = 0, \tag{2b}$$

where "$\diamondsuit$" is the Wick product on the Hida distribution space $(S(\mathbb{R})^*)$, $C(t)$ is a white noise function. Eq. (2) can be seen as the perturbation of the coefficient $c(t)$ of the variable coefficient Broer–Kaup system Eq. (1) by white noise function.

In Ref. [2], Huang and Zhang investigated a invariable coefficient Broer–Kaup system by means of a variable-coefficient projective Riccati equation mapping method. Soon, the method was further improved by Liu et al. [3] and the improved approach called modified variable-coefficient projective Riccati equation mapping method was used to solve same equation in Ref. [2]. In this paper, we shall research WSGBK Eq. (2) by the aid of above improved method.

In Ref. [4], Wadati has studied for the first time the stochastic partial differential KdV equation. Recently, many authors, e.g., [4–23] and so on, have investigated more intensively the stochastic partial differential equation (SPDE). And many

* Corresponding author at: School of Communication and Information Engineering, Shanghai University, Shanghai 200072, PR China.
E-mail addresses: lsxylq@163.com, lsxylq@yahoo.com.cn (Q. Liu).

0096-3003/$ - see front matter © 2010 Elsevier Inc. All rights reserved.
doi:10.1016/j.amc.2010.05.100

methods, e.g., the homogenous balance method [13,14], the homogenous balance principle and F-expansion method [15,16], the elliptic equation mapping method [17,18], the Riccati equation mapping method [19,20], the elliptic function expansion method [21], the modified mapping method [22,23] and the like, have been continuously proposed in the investigation of the SPDEs.

In Ref. [24], Holden et al. researched stochastic partial differential equations in Wick versions on the basis of the theory of white noise function. With the help of their ideas and a modified variable-coefficient projective Riccati equation mapping method [3], we derive a series of exact solutions to the WSGBK Eq. (2).

2. Some concepts on "Wick-type" and the modified variable-coefficient projective Riccati equation mapping method

Here we outline some concepts on "Wick-type". For more details about the exchange between the Wick-type stochastic equation and the common partial differential equation, we suggest readers to see the remarkable achievement by Holden et al. [24] and the second section of Ref. [13].

The Wick product $X \diamondsuit Y$ of two elements $X = \sum_\alpha a_\alpha H_\alpha,\ Y = \sum_\alpha b_\alpha H_\alpha \in (S)^n_{-1}$ with $a_\alpha, b_\alpha \in \mathbb{R}^n$ is defined by

$$X \diamondsuit Y = \sum_{\alpha,\beta} (a_\alpha, b_\beta) H_{\alpha+\beta}.$$

For $X = \sum_\alpha a_\alpha H_\alpha \in (S)^n_{-1}$ with $a_\alpha \in \mathbb{R}^n$, the Hermite transformation of X, denoted by $\mathscr{H}(X)$ or $\widetilde{X}(z)$, is defined by

$$\mathscr{H}(X) = \widetilde{X}(z) = \sum_\alpha a_\alpha z^\alpha \in \mathbb{C}^n \quad \text{(when convergent)},$$

where $z = (z_1, z_2, \ldots) \in \mathbb{C}^n$ (the set of all sequences of complex numbers) and $z^\alpha = z_1^{\alpha_1} z_2^{\alpha_2} \cdots z_n^{\alpha_n} \cdots$ for $\alpha = (\alpha_1, \alpha_2, \ldots) \in \mathcal{J}$.

For $X, Y \in (S)^n_{-1}$, by this definition we have

$$\widetilde{X \diamondsuit Y}(z) = \widetilde{X}(z) \cdot \widetilde{Y}(z),$$

for all z such that $\widetilde{X}(z)$ and $\widetilde{Y}(z)$ exist. The product on the right-hand side of the above formula is the complex bilinear product between two elements of $\mathbb{C}^N$ defined by $(z_1^1, \ldots, z_n^1) \cdot (z_1^2, \ldots, z_n^2) = \sum_{k=1}^n z_k^1 z_k^2$, where $z_k^i \in \mathbb{C}$.

The main steps by which we get exact solutions for Wick-type stochastic equation are outlined as follows.

Step 1: Let the operators $\partial_t = \frac{\partial}{\partial t},\ \nabla_x = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \ldots, \frac{\partial}{\partial x_d}\right)$ when $x = (x_1, x_2, \ldots, x_d)$. With the help of the Hermite transformation, we transform Wick-type equation

$$A^{\diamondsuit}(t, x, \partial_t, \nabla_x, U, \omega) = 0, \tag{3}$$

into an ordinary products equation as follows

$$\widetilde{A}(t, x, \partial_t, \nabla_x, \widetilde{U}, z_1, z_2, \ldots) = 0, \tag{4}$$

which is a variable coefficient partial differential equation.

Step 2: Reduce the partial differential equation (4) to ordinary differential equation by considering the transformation $\widetilde{U}(t, x, z) = u(t, x, z) = u(\xi)$. The equation that is reduced reads

$$A(u, u_\xi, u_{\xi\xi}, \ldots) = 0. \tag{5}$$

Step 3: Seek the solutions of Eq. (5). We assume that Eq. (5) possesses the solution in the form:

$$u(\xi) = a_0 + \sum_{i=1}^n f^{i-1}(\xi)(a_i f(\xi) + b_i g(\xi)), \tag{6}$$

where $a_0 = a_0(x, y, t)$, $a_i = a_i(x, z, t)$, $b_i = b_i(x, z, t)$, $(i = 1, 2, \ldots, n)$, $\xi = \xi(x, z, t)$ are functions to be determined later, and $f(\xi)$, $g(\xi)$ satisfy

$$f'(\xi) = -f(\xi)g(\xi), \quad g'(\xi) = 1 - g^2(\xi) - rf(\xi), \tag{7a}$$

$$g^2(\xi) = R - 2rf(\xi) + \frac{(r^2 + \varepsilon)}{R} f^2(\xi), \quad R \neq 0,\ \epsilon = \pm 1, \tag{7b}$$

where R is an arbitrary constant and $'$ denotes $\frac{d}{d\xi}$. We have known that Eq. (7) possesses the following solutions:

when $\epsilon = -1$,

$$f_1(\xi) = \frac{4R}{5\cosh(\sqrt{R}\xi) + 3\sinh(\sqrt{R}\xi) + 4r}, \quad g_1(\xi) = \frac{\sqrt{R}(5\sinh(\sqrt{R}\xi) + 3\cosh(\sqrt{R}\xi))}{5\cosh(\sqrt{R}\xi) + 3\sinh(\sqrt{R}\xi) + 4r}, \tag{8}$$

$$f_2(\xi) = \frac{R}{\cosh(\sqrt{R}\xi) + r}, \quad g_2(\xi) = \frac{\sqrt{R}\sinh(\sqrt{R}\xi)}{\cosh(\sqrt{R}\xi) + r}, \tag{9}$$

when $\epsilon = 1$,

$$f_3(\xi) = \frac{R}{\sinh(\sqrt{R}\xi) + r}, \quad g_3(\xi) = \frac{\sqrt{R}\cosh(\sqrt{R}\xi)}{\sinh(\sqrt{R}\xi) + r}. \tag{10}$$

Step 4: Determine the parameter n by balancing the highest derivative term with the nonlinear terms in Eq. (5).

Step 5: Substituting Eq. (6) into Eq. (5) along with Eq. (7) and then setting all coefficients of f^ig^j ($i = 0,1,2,3,\ldots, j = 0,1$) of the resulting equation to zero, we get an over-determined system of algebraic equations with respect to a_0, a_i, b_i ($i = 1,2,\ldots,n$) and ξ.

Step 6: Solving the over-determined system of algebraic equations, we would end up with the explicit expressions for a_0, a_i, b_i and ξ.

Step 7: Substituting a_0, a_i, b_i, ξ obtained in *Step 6* into Eq. (6) along with Eqs. (8)–(10), we can deduce the solutions of Eq. (5).

Step 8: Taking the inverse Hermite transformation of $u(t,x,z)$ obtained in *Step 7*, i.e., $U(t,x) = \mathscr{H}(u(t,x,z))$, we deduce $U(t,x)$ which is the solutions of the Wick-type stochastic Eq. (3).

3. Exact solutions for stochastic Broer–Kaup system

Taking the Hermite transformation of Eq. (2), we can get the following equations:

$$\widetilde{U}_{yt} - \widetilde{C}(t,z)(\widetilde{U}_{xxy} - 2(\widetilde{U}\widetilde{U}_x)_y - 2\widetilde{V}_{xx}) = 0, \tag{11a}$$

$$\widetilde{V}_t + \widetilde{C}(t,z)(\widetilde{V}_{xx} + 2(\widetilde{V}\widetilde{U})_x) = 0, \tag{11b}$$

where $z = (z_1, z_2, \ldots) \in (\mathbb{C}^{\mathbb{N}})_c$ is a vector parameter.

For the sake of simplicity, we denote $u(t,x,y,z) = \widetilde{U}(t,x,y,z)$, $v(t,x,y,z) = \widetilde{V}(t,x,y,z)$ and $C(t,z) = \widetilde{C}(t,z)$.

We take the following Bäcklund transformations of Eq. (11)

$$u = \frac{f_x}{f} + H_0(x,t), \quad v = \frac{f_{xy}}{f} - \frac{f_x f_y}{f^2}, \tag{12}$$

which can be obtained from the standard Painlevé truncation expansion with $H_0(x,t)$, an arbitrary function of $\{x,t\}$.

It is easy to derive from Eq. (12) that

$$v = u_y. \tag{13}$$

Substituting Eq. (13) into Eq. (11), and integrating two sides of Eq. (11a) with respect to y, we can reduce Eq. (11) to a single differential equation:

$$u_t + Cu_{xx} + 2Cuu_x = Q(x,z,t), \tag{14}$$

where $Q(x,z,t)$ is an arbitrary function of the indicated variables. In what follows, we only consider $Q(x,z,t) = Q(t)$.

Suppose u have the solution in the following form:

$$u(x,y,z,t) = P_1(y,z) + P_2(t) + A(y,z)f(\xi(x,y,z,t)) + B(y,z)g(\xi(x,y,z,t)), \tag{15}$$

where $a_0(x,y,z,t) = P_1(y,z) + P_2(t)$, $a_1(x,y,z,t) = A(y,z)$, $b_1(x,y,z,t) = B(y,z)$, $\xi(x,y,z,t) = L(y,z)x + K(y,z,t)$.

Substituting Eq. (15) into Eq. (14) and letting the coefficients of f^ig^j ($i = 0,1,2,3,\ldots, j = 0,1$) of the resulting equation to zero yields an over-determined system of algebraic equations with respect to $P_1(y,z)$, $P_2(t)$, $A(y,z)$, $C(t,z)$, $B(y,z)$, $L(y,z)$, $Q(t)$ and $K(y,z,t)$, namely:

$$2(r^2 + \epsilon)ACL(L - 2B) = 0, \tag{16a}$$

$$2(r^2 + \epsilon)BCL(L - B) - 2RA^2CL = 0, \tag{16b}$$

$$3RrACL(2B - L) - (r^2 + \epsilon)B(2CL(P_1 + P_2) + K_t) = 0, \tag{16c}$$

$$-R(rBCL(L - 2B)) + A(2CL(P_1 + P_2) + K_t) = 0, \tag{16d}$$

$$R(RACL(L - 2B)) + rB(2CL(P_1 + P_2) + K_t) = 0, \tag{16e}$$

$$R(P_{2t} - Q) = 0. \tag{16f}$$

We choose $A(y,z)$ ($\neq 0$), $F_1(y,z)$ ($= -L(y,z)P_1(y,z) - C_1$), $F_2(y,z)$ and $Q(t)$ as arbitrary functions, C_1 as arbitrary constant. Solving Eq. (16), we obtain

Case 1:

$$\begin{aligned}
&P_2(t) = \int^t Q(s)ds + C_1, \quad A(y,z) = A(y,z), \quad B(y,z) = -A(y,z)\sqrt{\frac{R}{r^2+\epsilon}}, \\
&L(y,z) = -2A(y,z)\sqrt{\frac{R}{r^2+\epsilon}}, \quad P_1(y,z) = \frac{F_1(y,z)}{2A(y,z)}\sqrt{\frac{r^2+\epsilon}{R}} - C_1, \\
&K(y,z,t) = 4A(y,z)\sqrt{\frac{R}{r^2+\epsilon}}\int^t C(s,z)\int^s Q(\tau)d\tau\, ds + 2F_1(y,z)\int^t C(s,z)ds + F_2(y,z).
\end{aligned} \tag{17}$$

Case 2:

$$
\begin{aligned}
&P_2(t) = \int^t Q(s)ds + C_1, \quad A(y,z) = A(y,z), \quad B(y,z) = A(y,z)\sqrt{\frac{R}{r^2+\epsilon}},\\
&L(y,z) = 2A(y,z)\sqrt{\frac{R}{r^2+\epsilon}}, \quad P_1(y,z) = -\frac{F_1(y,z)}{2A(y,z)}\sqrt{\frac{r^2+\epsilon}{R}} - C_1,\\
&K(y,z,t) = -4A(y,z)\sqrt{\frac{R}{r^2+\epsilon}}\int^t C(s,z)\int^s Q(\tau)d\tau\,ds + 2F_1(y,z)\int^t C(s,z)ds + F_2(y,z).
\end{aligned} \tag{18}
$$

Substituting Eqs. (17) and (18) into Eqs. (15) and (13), and using Eqs. (8)–(10), we obtain some families of exact solutions of Eq. (11):

Family 1:

1. When $\varepsilon = -1$, $r^2 - 1 > 0$, $R > 0$,

$$
\begin{aligned}
u_{11}(x,y,t,z) &= \int^t Q(s)ds + \frac{F_1(y,z)}{2A(y,z)}\sqrt{\frac{r^2-1}{R}} + \frac{RA(y,z)\left(4 - \sqrt{\frac{1}{r^2-1}}(5\sinh(\sqrt{R}\xi) + 3\cosh(\sqrt{R}\xi))\right)}{5\cosh(\sqrt{R}\xi) + 3\sinh(\sqrt{R}\xi) + 4r},\\
v_{11}(x,y,t,z) &= \frac{F_{1y}(y,z)A(y,z) - F_1(y,z)A_y(y,z)}{2A^2(y,z)}\sqrt{\frac{r^2-1}{R}} + \frac{4RA_y(y,z)}{5\cosh(\sqrt{R}\xi) + 3\sinh(\sqrt{R}\xi) + 4r}\\
&\quad - \frac{R\sqrt{\frac{1}{r^2-1}}(A_y(y,z)(5\sinh(\sqrt{R}\xi) + 3\cosh(\sqrt{R}\xi)) + A(y,z)\sqrt{R}\xi_y(5\cosh(\sqrt{R}\xi) + 3\sinh(\sqrt{R}\xi)))}{5\cosh(\sqrt{R}\xi) + 3\sinh(\sqrt{R}\xi) + 4r}\\
&\quad + \frac{A(y,z)R\sqrt{R}\xi_y(5\sinh(\sqrt{R}\xi) + 3\cosh(\sqrt{R}\xi))\left(\sqrt{\frac{1}{r^2-1}}(5\sinh(\sqrt{R}\xi) + 3\cosh(\sqrt{R}\xi)) - 4\right)}{(5\cosh(\sqrt{R}\xi) + 3\sinh(\sqrt{R}\xi) + 4r)^2},
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
u_{12}(x,y,t,z) &= \int^t Q(s)ds + \frac{F_1(y,z)}{2A(y,z)}\sqrt{\frac{r^2-1}{R}} + \frac{RA(y,z)\left(1 - \sqrt{\frac{1}{r^2-1}}\sinh(\sqrt{R}\xi)\right)}{\cosh(\sqrt{R}\xi) + r},\\
v_{12}(x,y,t,z) &= \frac{F_{1y}(y,z)A(y,z) - F_1(y,z)A_y(y,z)}{2A^2(y,z)}\sqrt{\frac{r^2-1}{R}}\\
&\quad + \frac{R(A_y(y,z) - \sqrt{\frac{1}{r^2-1}}(A_y(y,z)\sinh(\sqrt{R}\xi) + A(y,z)\xi_y\cosh(\sqrt{R}\xi)))}{\cosh(\sqrt{R}\xi) + r}\\
&\quad + \frac{A(y,z)R\sqrt{R}\xi_y\sinh(\sqrt{R}\xi)\left(\sqrt{\frac{1}{r^2-1}}\sinh(\sqrt{R}\xi) - 1\right)}{(\cosh(\sqrt{R}\xi) + r)^2},
\end{aligned} \tag{20}
$$

where

$$
\xi = -2A(y,z)\sqrt{\frac{R}{r^2-1}}x + 4A(y,z)\sqrt{\frac{R}{r^2-1}}\int^t C(s,z)\int^s Q(\tau)d\tau\,ds + 2F_1(y,z)\int^t C(s,z)ds + F_2(y,z).
$$

2. When $\varepsilon = -1$, $r^2 - 1 < 0$, $R > 0$,

$$
\begin{aligned}
u_{13}(x,y,t,z) &= \int^t Q(s)ds + i\frac{F_1(y,z)}{2A(y,z)}\sqrt{\frac{r^2-1}{-R}} + \frac{RA(y,z)\left(4 - i\sqrt{\frac{-1}{r^2-1}}(5\sinh(\sqrt{R}\xi) + 3\cosh(\sqrt{R}\xi))\right)}{5\cosh(\sqrt{R}\xi) + 3\sinh(\sqrt{R}\xi) + 4r},\\
v_{13}(x,y,t,z) &= i\frac{F_{1y}(y,z)A(y,z) - F_1(y,z)A_y(y,z)}{2A^2(y,z)}\sqrt{\frac{r^2-1}{-R}} + \frac{4RA_y(y,z)}{5\cosh(\sqrt{R}\xi) + 3\sinh(\sqrt{R}\xi) + 4r}\\
&\quad - i\frac{R\sqrt{\frac{-1}{r^2-1}}(A_y(y,z)(5\sinh(\sqrt{R}\xi) + 3\cosh(\sqrt{R}\xi)) + A(y,z)\sqrt{R}\xi_y(5\cosh(\sqrt{R}\xi) + 3\sinh(\sqrt{R}\xi)))}{5\cosh(\sqrt{R}\xi) + 3\sinh(\sqrt{R}\xi) + 4r}\\
&\quad + \frac{A(y,z)R\sqrt{R}\xi_y(5\sinh(\sqrt{R}\xi) + 3\cosh(\sqrt{R}\xi))\left(i\sqrt{\frac{-1}{r^2-1}}(5\sinh(\sqrt{R}\xi) + 3\cosh(\sqrt{R}\xi)) - 4\right)}{(5\cosh(\sqrt{R}\xi) + 3\sinh(\sqrt{R}\xi) + 4r)^2},
\end{aligned} \tag{21}
$$

$$
u_{14}(x,y,t,z) = \int^t Q(s)ds + i\frac{F_1(y,z)}{2A(y,z)}\sqrt{\frac{r^2-1}{-R}} + \frac{RA(y,z)\left(1 - i\sqrt{\frac{-1}{r^2-1}}\sinh(\sqrt{R}\xi)\right)}{\cosh(\sqrt{R}\xi) + r},
$$

$$\begin{aligned} v_{14}(x,y,t,z) &= i\frac{F_{1y}(y,z)A(y,z)-F_1(y,z)A_y(y,z)}{2A^2(y,z)}\sqrt{\frac{r^2-1}{-R}} \\ &+ \frac{R(A_y(y,z)-i\sqrt{\frac{-1}{r^2-1}}(A_y(y,z)\sinh(\sqrt{R}\xi)+A(y,z)\xi_y\cosh(\sqrt{R}\xi)))}{\cosh(\sqrt{R}\xi)+r} \\ &+ \frac{A(y,z)R\sqrt{R}\xi_y\sinh(\sqrt{R}\xi)\left(i\sqrt{\frac{-1}{r^2-1}}\sinh(\sqrt{R}\xi)-1\right)}{(\cosh(\sqrt{R}\xi)+r)^2}, \end{aligned} \tag{22}$$

where

$$\xi = -i2A(y,z)\sqrt{\frac{-R}{r^2-1}}x + i4A(y,z)\sqrt{\frac{-R}{r^2-1}}\int^t C(s,z)\int^s Q(\tau)d\tau\, ds + 2F_1(y,z)\int^t C(s,z)ds + F_2(y,z).$$

3. When $\varepsilon = 1$, $R > 0$,

$$\begin{aligned} u_{15}(x,y,t,z) &= \int^t Q(s)ds + \frac{F_1(y,z)}{2A(y,z)}\sqrt{\frac{r^2+1}{R}} + \frac{RA(y,z)\left(1-\sqrt{\frac{1}{r^2+1}}\cosh(\sqrt{R}\xi)\right)}{\sinh(\sqrt{R}\xi)+r}, \\ v_{15}(x,y,t,z) &= \frac{F_{1y}(y,z)A(y,z)-F_1(y,z)A_y(y,z)}{2A^2(y,z)}\sqrt{\frac{r^2+1}{R}} \\ &+ \frac{R(A_y(y,z)-\sqrt{\frac{1}{r^2+1}}(A_y(y,z)\cosh(\sqrt{R}\xi)+A(y,z)\xi_y\sinh(\sqrt{R}\xi)))}{\sinh(\sqrt{R}\xi)+r} \\ &+ \frac{A(y,z)R\sqrt{R}\xi_y\cosh(\sqrt{R}\xi)\left(\sqrt{\frac{1}{r^2+1}}\cosh(\sqrt{R}\xi)-1\right)}{(\sinh(\sqrt{R}\xi)+r)^2}, \end{aligned} \tag{23}$$

where

$$\xi = -2A(y,z)\sqrt{\frac{R}{r^2+1}}x + 4A(y,z)\sqrt{\frac{R}{r^2+1}}\int^t C(s,z)\int^s Q(\tau)d\tau\, ds + 2F_1(y,z)\int^t C(s,z)ds + F_2(y,z).$$

Family 2:

1. When $\varepsilon = -1$, $r^2 - 1 > 0$, $R > 0$,

$$\begin{aligned} u_{21}(x,y,t,z) &= \int^t Q(s)ds - \frac{F_1(y,z)}{2A(y,z)}\sqrt{\frac{r^2-1}{R}} + \frac{RA(y,z)\left(4+\sqrt{\frac{1}{r^2-1}}(5\sinh(\sqrt{R}\xi)+3\cosh(\sqrt{R}\xi))\right)}{5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)+4r}, \\ v_{21}(x,y,t,z) &= -\frac{F_{1y}(y,z)A(y,z)-F_1(y,z)A_y(y,z)}{2A^2(y,z)}\sqrt{\frac{r^2-1}{R}} + \frac{4RA_y(y,z)}{5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)+4r} \\ &+ \frac{R\sqrt{\frac{1}{r^2-1}}(A_y(y,z)(5\sinh(\sqrt{R}\xi)+3\cosh(\sqrt{R}\xi))+A(y,z)\sqrt{R}\xi_y(5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)))}{5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)+4r} \\ &- \frac{A(y,z)R\sqrt{R}\xi_y(5\sinh(\sqrt{R}\xi)+3\cosh(\sqrt{R}\xi))\left(\sqrt{\frac{1}{r^2-1}}(5\sinh(\sqrt{R}\xi)+3\cosh(\sqrt{R}\xi))+4\right)}{(5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)+4r)^2}, \end{aligned} \tag{24}$$

$$\begin{aligned} u_{22}(x,y,t,z) &= \int^t Q(s)ds - \frac{F_1(y,z)}{2A(y,z)}\sqrt{\frac{r^2-1}{R}} + \frac{RA(y,z)\left(1+\sqrt{\frac{1}{r^2-1}}\sinh(\sqrt{R}\xi)\right)}{\cosh(\sqrt{R}\xi)+r}, \\ v_{22}(x,y,t,z) &= -\frac{F_{1y}(y,z)A(y,z)-F_1(y,z)A_y(y,z)}{2A^2(y,z)}\sqrt{\frac{r^2-1}{R}} \\ &+ \frac{R(A_y(y,z)+\sqrt{\frac{1}{r^2-1}}(A_y(y,z)\sinh(\sqrt{R}\xi)+A(y,z)\xi_y\cosh(\sqrt{R}\xi)))}{\cosh(\sqrt{R}\xi)+r} \\ &- \frac{A(y,z)R\sqrt{R}\xi_y\sinh(\sqrt{R}\xi)\left(\sqrt{\frac{1}{r^2-1}}\sinh(\sqrt{R}\xi)+1\right)}{(\cosh(\sqrt{R}\xi)+r)^2}, \end{aligned} \tag{25}$$

where

$$\xi = 2A(y,z)\sqrt{\frac{R}{r^2-1}}x - 4A(y,z)\sqrt{\frac{R}{r^2-1}}\int^t C(s,z)\int^s Q(\tau)d\tau\, ds + 2F_1(y,z)\int^t C(s,z)ds + F_2(y,z).$$

2. When $\varepsilon = -1$, $r^2 - 1 < 0$, $R > 0$,

$$u_{23}(x,y,t,z) = \int^t Q(s)ds - i\frac{F_1(y,z)}{2A(y,z)}\sqrt{\frac{r^2-1}{-R}} + \frac{RA(y,z)\left(4 + i\sqrt{\frac{-1}{r^2-1}}(5\sinh(\sqrt{R}\xi) + 3\cosh(\sqrt{R}\xi))\right)}{5\cosh(\sqrt{R}\xi) + 3\sinh(\sqrt{R}\xi) + 4r},$$

$$\begin{aligned}
v_{23}(x,y,t,z) = &-i\frac{F_{1y}(y,z)A(y,z) - F_1(y,z)A_y(y,z)}{2A^2(y,z)}\sqrt{\frac{r^2-1}{-R}} + \frac{4RA_y(y,z)}{5\cosh(\sqrt{R}\xi) + 3\sinh(\sqrt{R}\xi) + 4r}\\
&+ i\frac{R\sqrt{\frac{-1}{r^2-1}}(A_y(y,z)(5\sinh(\sqrt{R}\xi) + 3\cosh(\sqrt{R}\xi)) + A(y,z)\sqrt{R}\xi_y(5\cosh(\sqrt{R}\xi) + 3\sinh(\sqrt{R}\xi)))}{5\cosh(\sqrt{R}\xi) + 3\sinh(\sqrt{R}\xi) + 4r}\\
&- \frac{A(y,z)R\sqrt{R}\xi_y(5\sinh(\sqrt{R}\xi) + 3\cosh(\sqrt{R}\xi))\left(i\sqrt{\frac{-1}{r^2-1}}(5\sinh(\sqrt{R}\xi) + 3\cosh(\sqrt{R}\xi)) + 4\right)}{(5\cosh(\sqrt{R}\xi) + 3\sinh(\sqrt{R}\xi) + 4r)^2},
\end{aligned} \tag{26}$$

$$u_{24}(x,y,t,z) = \int^t Q(s)ds - i\frac{F_1(y,z)}{2A(y,z)}\sqrt{\frac{r^2-1}{-R}} + \frac{RA(y,z)\left(1 + i\sqrt{\frac{-1}{r^2-1}}\sinh(\sqrt{R}\xi)\right)}{\cosh(\sqrt{R}\xi) + r},$$

$$\begin{aligned}
v_{24}(x,y,t,z) = &-i\frac{F_{1y}(y,z)A(y,z) - F_1(y,z)A_y(y,z)}{2A^2(y,z)}\sqrt{\frac{r^2-1}{-R}}\\
&+ \frac{R(A_y(y,z) + i\sqrt{\frac{-1}{r^2-1}}(A_y(y,z)\sinh(\sqrt{R}\xi) + A(y,z)\xi_y\cosh(\sqrt{R}\xi)))}{\cosh(\sqrt{R}\xi) + r}\\
&- \frac{A(y,z)R\sqrt{R}\xi_y\sinh(\sqrt{R}\xi)(i\sqrt{\frac{-1}{r^2-1}}\sinh(\sqrt{R}\xi) + 1)}{(\cosh(\sqrt{R}\xi) + r)^2},
\end{aligned} \tag{27}$$

where

$$\xi = i2A(y,z)\sqrt{\frac{-R}{r^2-1}}x - i4A(y,z)\sqrt{\frac{-R}{r^2-1}}\int^t C(s,z)\int^s Q(\tau)d\tau\, ds + 2F_1(y,z)\int^t C(s,z)ds + F_2(y,z).$$

3. When $\varepsilon = 1$, $R > 0$,

$$u_{25}(x,y,t,z) = \int^t Q(s)ds - \frac{F_1(y,z)}{2A(y,z)}\sqrt{\frac{r^2+1}{R}} + \frac{RA(y,z)\left(1 + \sqrt{\frac{1}{r^2+1}}\cosh(\sqrt{R}\xi)\right)}{\sinh(\sqrt{R}\xi) + r},$$

$$\begin{aligned}
v_{25}(x,y,t,z) = &-\frac{F_{1y}(y,z)A(y,z) - F_1(y,z)A_y(y,z)}{2A^2(y,z)}\sqrt{\frac{r^2+1}{R}}\\
&+ \frac{R(A_y(y,z) + \sqrt{\frac{1}{r^2+1}}(A_y(y,z)\cosh(\sqrt{R}\xi) + A(y,z)\xi_y\sinh(\sqrt{R}\xi)))}{\sinh(\sqrt{R}\xi) + r}\\
&- \frac{A(y,z)R\sqrt{R}\xi_y\cosh(\sqrt{R}\xi)\left(\sqrt{\frac{1}{r^2+1}}\cosh(\sqrt{R}\xi) + 1\right)}{(\sinh(\sqrt{R}\xi) + r)^2},
\end{aligned} \tag{28}$$

where

$$\xi = 2A(y,z)\sqrt{\frac{R}{r^2+1}}x - 4A(y,z)\sqrt{\frac{R}{r^2+1}}\int^t C(s,z)\int^s Q(\tau)d\tau\, ds + 2F_1(y,z)\int^t C(s,z)ds + F_2(y,z).$$

Suppose $h(t)$ be integrable function on $\mathbb{R}_-$ and

$$C(t) = h(t) + bW(t),$$

where $W(t)$ is a Gaussian white noise and $B(t)$ is a Brownian motion, we know $W(t) = \dot{B}(t)$. Considering the Hermite transformation of $C(t)$, we have

$$C(t,z) = h(t) + b\widetilde{W}(t,z),$$

where $\widetilde{W}(t,z) = \sum_{k=1}^{\infty}\int_0^t \eta_k(s)ds\, z_k$.

Since $\exp^{\diamond}(B(t)) = \exp(B(t) - \frac{1}{2}t^2)$ (see Lemma 2.6.16 in Ref. [24]), set $\phi(t) = \int^t C(s)\int^s Q(\tau)d\tau\, ds$, $\psi(t) = \int^t h(s)ds + bB(t) - \frac{bt^2}{2}$. Taking the inverse Hermite transformation of Eqs. (18), (20)–(28), respectively, we can obtain the exact solutions of (2 + 1)-dimensional WSGBK Eq. (2) as follows:

Family 1:

1. When $\varepsilon = -1$, $r^2 - 1 > 0$, $R > 0$,

$$U_{11}(x,y,t) = \int^t Q(s)ds + \frac{F_1(y)}{2A(y)}\sqrt{\frac{r^2-1}{R}} + \frac{RA(y)\diamond\left(4-\sqrt{\frac{1}{r^2-1}}(5\sinh(\sqrt{R}\xi)+3\cosh(\sqrt{R}\xi))\right)}{5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)+4r},$$

$$\begin{aligned}V_{11}(x,y,t) = {}& \frac{F_{1y}(y)\diamond A(y) - F_1(y)\diamond A_y(y)}{2A^{\diamond 2}(y)}\sqrt{\frac{r^2-1}{R}} + \frac{4RA_y(y)}{5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)+4r}\\ & - \frac{R\sqrt{\frac{1}{r^2-1}}(A_y(y)\diamond(5\sinh(\sqrt{R}\xi)+3\cosh(\sqrt{R}\xi)) + \sqrt{R}\xi_y\diamond A(y)\diamond(5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)))}{5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)+4r}\\ & + \frac{R\sqrt{R}A(y)\diamond\xi_y\diamond(5\sinh(\sqrt{R}\xi)+3\cosh(\sqrt{R}\xi))\diamond\left(\sqrt{\frac{1}{r^2-1}}(5\sinh(\sqrt{R}\xi)+3\cosh(\sqrt{R}\xi))-4\right)}{(5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)+4r)^{\diamond 2}},\end{aligned} \tag{29}$$

$$U_{12}(x,y,t) = \int^t Q(s)ds + \frac{F_1(y)}{2A(y)}\sqrt{\frac{r^2-1}{R}} + \frac{RA(y)\diamond\left(1-\sqrt{\frac{1}{r^2-1}}\sinh(\sqrt{R}\xi)\right)}{\cosh(\sqrt{R}\xi)+r},$$

$$\begin{aligned}V_{12}(x,y,t) = {}& \frac{F_{1y}(y)\diamond A(y) - F_1(y)\diamond A_y(y)}{2A^{\diamond 2}(y)}\sqrt{\frac{r^2-1}{R}}\\ & + \frac{R(A_y(y) - \sqrt{\frac{1}{r^2-1}}(A_y(y)\diamond\sinh(\sqrt{R}\xi) + A(y)\diamond\xi_y\diamond\cosh(\sqrt{R}\xi)))}{\cosh(\sqrt{R}\xi)+r}\\ & + \frac{R\sqrt{R}A(y)\diamond\xi_y\diamond\sinh(\sqrt{R}\xi)\diamond\left(\sqrt{\frac{1}{r^2-1}}\sinh(\sqrt{R}\xi)-1\right)}{(\cosh(\sqrt{R}\xi)+r)^{\diamond 2}},\end{aligned} \tag{30}$$

where

$$\xi = -2A(y)\sqrt{\frac{R}{r^2-1}}x + 4\sqrt{\frac{R}{r^2-1}}A(y)\diamond\phi(t) + 2F_1(y)\diamond\psi(t) + F_2(y).$$

2. When $\varepsilon = -1$, $r^2 - 1 < 0$, $R > 0$,

$$U_{13}(x,y,t) = \int^t Q(s)ds + i\frac{F_1(y)}{2A(y)}\sqrt{\frac{r^2-1}{-R}} + \frac{RA(y)\diamond\left(4-\sqrt{i\frac{-1}{r^2-1}}(5\sinh(\sqrt{R}\xi)+3\cosh(\sqrt{R}\xi))\right)}{5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)+4r},$$

$$\begin{aligned}V_{13}(x,y,t) = {}& i\frac{F_{1y}(y)\diamond A(y) - F_1(y)\diamond A_y(y)}{2A^{\diamond 2}(y)}\sqrt{\frac{r^2-1}{-R}} + \frac{4RA_y(y)}{5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)+4r}\\ & - i\frac{R\sqrt{\frac{-1}{r^2-1}}(A_y(y)\diamond(5\sinh(\sqrt{R}\xi)+3\cosh(\sqrt{R}\xi)) + \sqrt{R}\xi_y\diamond A(y)\diamond(5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)))}{5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)+4r}\\ & + \frac{R\sqrt{R}A(y)\diamond\xi_y\diamond(5\sinh(\sqrt{R}\xi)+3\cosh(\sqrt{R}\xi))\diamond(i\sqrt{\frac{-1}{r^2-1}}(5\sinh(\sqrt{R}\xi)+3\cosh(\sqrt{R}\xi))-4)}{(5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)+4r)^{\diamond 2}},\end{aligned} \tag{31}$$

$$U_{14}(x,y,t) = \int^t Q(s)ds + i\frac{F_1(y)}{2A(y)}\sqrt{\frac{r^2-1}{-R}} + \frac{RA(y)\diamond\left(1-i\sqrt{\frac{-1}{r^2-1}}\sinh(\sqrt{R}\xi)\right)}{\cosh(\sqrt{R}\xi)+r},$$

$$\begin{aligned}V_{14}(x,y,t) = {}& i\frac{F_{1y}(y)\diamond A(y) - F_1(y)\diamond A_y(y)}{2A^{\diamond 2}(y)}\sqrt{\frac{r^2-1}{-R}}\\ & + \frac{R(A_y(y) - i\sqrt{\frac{-1}{r^2-1}}(A_y(y)\diamond\sinh(\sqrt{R}\xi) + A(y)\diamond\xi_y\diamond\cosh(\sqrt{R}\xi)))}{\cosh(\sqrt{R}\xi)+r}\\ & + \frac{R\sqrt{R}A(y)\diamond\xi_y\diamond\sinh(\sqrt{R}\xi)\diamond\left(i\sqrt{\frac{-1}{r^2-1}}\sinh(\sqrt{R}\xi)-1\right)}{(\cosh(\sqrt{R}\xi)+r)^{\diamond 2}},\end{aligned} \tag{32}$$

where

$$\xi=-i2A(y)\sqrt{\frac{-R}{r^2-1}}x+i4\sqrt{\frac{-R}{r^2-1}}A(y)\diamondsuit\phi(t)+2F_1(y)\diamondsuit\psi(t)+F_2(y).$$

3. When $\varepsilon = 1$, $R > 0$,

$$\begin{aligned}
U_{15}(x,y,t)&=\int^t Q(s)ds+\frac{F_1(y)}{2A(y)}\sqrt{\frac{r^2+1}{R}}+\frac{RA(y)\diamondsuit\left(1-\sqrt{\frac{1}{r^2+1}}\cosh(\sqrt{R}\xi)\right)}{\sinh(\sqrt{R}\xi)+r},\\
V_{15}(x,y,t)&=\frac{F_{1y}(y)\diamondsuit A(y,z)-F_1(y)\diamondsuit A_y(y)}{2A^{\diamondsuit 2}(y)}\sqrt{\frac{r^2+1}{R}}\\
&\quad+\frac{R(A_y(y)-\sqrt{\frac{1}{r^2+1}}(A_y(y)\diamondsuit\ \cosh(\sqrt{R}\xi)+A(y)\diamondsuit\xi_y\diamondsuit\ \sinh(\sqrt{R}\xi)))}{\sinh(\sqrt{R}\xi)+r}\\
&\quad+\frac{R\sqrt{R}A(y)\diamondsuit\xi_y\diamondsuit\ \cosh(\sqrt{R}\xi)\diamondsuit\left(\sqrt{\frac{1}{r^2+1}}\cosh(\sqrt{R}\xi)-1\right)}{(\sinh(\sqrt{R}\xi)+r)^{\diamondsuit 2}},
\end{aligned}\tag{33}$$

where

$$\xi=-2A(y)\sqrt{\frac{R}{r^2+1}}x+4\sqrt{\frac{R}{r^2+1}}A(y)\diamondsuit\phi(t)+2F_1(y)\diamondsuit\psi(t)+F_2(y).$$

Family 2:

1. When $\varepsilon = -1$, $r^2-1>0$, $R > 0$,

$$\begin{aligned}
U_{21}(x,y,t)&=\int^t Q(s)ds-\frac{F_1(y)}{2A(y)}\sqrt{\frac{r^2-1}{R}}+\frac{RA(y)\diamondsuit\left(4+\sqrt{\frac{1}{r^2-1}}(5\ \sinh(\sqrt{R}\xi)+3\ \cosh(\sqrt{R}\xi))\right)}{5\ \cosh(\sqrt{R}\xi)+3\ \sinh(\sqrt{R}\xi)+4r},\\
V_{21}(x,y,t)&=-\frac{F_{1y}(y)\diamondsuit A(y)-F_1(y)\diamondsuit A_y(y)}{2A^{\diamondsuit 2}(y)}\sqrt{\frac{r^2-1}{R}}+\frac{4RA_y(y)}{5\ \cosh(\sqrt{R}\xi)+3\ \sinh(\sqrt{R}\xi)+4r}\\
&\quad+\frac{R\sqrt{\frac{1}{r^2-1}}(A_y(y)\diamondsuit(5\ \sinh(\sqrt{R}\xi)+3\ \cosh(\sqrt{R}\xi))+\sqrt{R}A(y)\diamondsuit\xi_y\diamondsuit(5\ \cosh(\sqrt{R}\xi)+3\ \sinh(\sqrt{R}\xi)))}{5\ \cosh(\sqrt{R}\xi)+3\ \sinh(\sqrt{R}\xi)+4r}\\
&\quad-\frac{R\sqrt{R}A(y)\diamondsuit\xi_y\diamondsuit(5\ \sinh(\sqrt{R}\xi)+3\ \cosh(\sqrt{R}\xi))\diamondsuit\left(\sqrt{\frac{1}{r^2-1}}(5\ \sinh(\sqrt{R}\xi)+3\ \cosh(\sqrt{R}\xi))+4\right)}{(5\ \cosh(\sqrt{R}\xi)+3\ \sinh(\sqrt{R}\xi)+4r)^{\diamondsuit 2}},
\end{aligned}\tag{34}$$

$$\begin{aligned}
U_{22}(x,y,t)&=\int^t Q(s)ds-\frac{F_1(y)}{2A(y)}\sqrt{\frac{r^2-1}{R}}+\frac{RA(y)\diamondsuit\left(1+\sqrt{\frac{1}{r^2-1}}\sinh(\sqrt{R}\xi)\right)}{\cosh(\sqrt{R}\xi)+r},\\
V_{22}(x,y,t)&=-\frac{F_{1y}(y)\diamondsuit A(y)-F_1(y)\diamondsuit A_y(y)}{2A^{\diamondsuit 2}(y)}\sqrt{\frac{r^2-1}{R}}\\
&\quad+\frac{R(A_y(y)+\sqrt{\frac{1}{r^2-1}}(A_y(y)\diamondsuit\ \sinh(\sqrt{R}\xi)+A(y)\diamondsuit\xi_y\diamondsuit\ \cosh(\sqrt{R}\xi)))}{\cosh(\sqrt{R}\xi)+r}\\
&\quad-\frac{R\sqrt{R}A(y)\diamondsuit\xi_y\diamondsuit\ \sinh(\sqrt{R}\xi)v\left(\sqrt{\frac{1}{r^2-1}}\sinh(\sqrt{R}\xi)+1\right)}{(\cosh(\sqrt{R}\xi)+r)^{\diamondsuit 2}},
\end{aligned}\tag{35}$$

where

$$\xi=2A(y)\sqrt{\frac{R}{r^2-1}}x-4\sqrt{\frac{R}{r^2-1}}A(y)\diamondsuit\phi(t)+2F_1(y)\diamondsuit\psi(t)+F_2(y).$$

2. When $\varepsilon = -1$, $r^2-1<0$, $R > 0$,

$$U_{23}(x,y,t)=\int^t Q(s)ds-i\frac{F_1(y)}{2A(y)}\sqrt{\frac{r^2-1}{-R}}+\frac{RA(y)\diamondsuit\left(4+i\sqrt{\frac{-1}{r^2-1}}(5\ \sinh(\sqrt{R}\xi)+3\ \cosh(\sqrt{R}\xi))\right)}{5\ \cosh(\sqrt{R}\xi)+3\ \sinh(\sqrt{R}\xi)+4r},$$

$$V_{23}(x,y,t) = -i\frac{F_{1y}(y)\diamondsuit A(y) - F_1(y)\diamondsuit A_y(y)}{2A^{\diamondsuit 2}(y)}\sqrt{\frac{r^2-1}{-R}} + \frac{4RA_y(y)}{5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)+4r}$$
$$+i\frac{R\sqrt{\frac{-1}{r^2-1}}(A_y(y)\diamondsuit(5\sinh(\sqrt{R}\xi)+3\cosh(\sqrt{R}\xi))+\sqrt{R}A(y)\diamondsuit\xi_y\diamondsuit(5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)))}{5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)+4r}$$
$$-\frac{R\sqrt{R}A(y)\diamondsuit\xi_y\diamondsuit(5\sinh(\sqrt{R}\xi)+3\cosh(\sqrt{R}\xi))\diamondsuit\left(i\sqrt{\frac{-1}{r^2-1}}(5\sinh(\sqrt{R}\xi)+3\cosh(\sqrt{R}\xi))+4\right)}{(5\cosh(\sqrt{R}\xi)+3\sinh(\sqrt{R}\xi)+4r)^{\diamondsuit 2}}, \quad (36)$$

$$U_{24}(x,y,t) = \int^t Q(s)ds - i\frac{F_1(y)}{2A(y)}\sqrt{\frac{r^2-1}{-R}} + \frac{RA(y)\diamondsuit\left(1+i\sqrt{\frac{-1}{r^2-1}}\sinh(\sqrt{R}\xi)\right)}{\cosh(\sqrt{R}\xi)+r},$$
$$V_{24}(x,y,t) = -i\frac{F_{1y}(y)\diamondsuit A(y) - F_1(y)\diamondsuit A_y(y)}{2A^{\diamondsuit 2}(y)}\sqrt{\frac{r^2-1}{-R}}$$
$$+\frac{R(A_y(y)+i\sqrt{\frac{-1}{r^2-1}}(A_y(y)\diamondsuit\sinh(\sqrt{R}\xi)+A(y)\diamondsuit\xi_y\diamondsuit\cosh(\sqrt{R}\xi)))}{\cosh(\sqrt{R}\xi)+r}$$
$$-\frac{R\sqrt{R}A(y)\diamondsuit\xi_y\diamondsuit\sinh(\sqrt{R}\xi)\diamondsuit\left(i\sqrt{\frac{-1}{r^2-1}}\sinh(\sqrt{R}\xi)+1\right)}{(\cosh(\sqrt{R}\xi)+r)^{\diamondsuit 2}}, \quad (37)$$

where

$$\xi = i2A(y)\sqrt{\frac{-R}{r^2-1}}x - i4\sqrt{\frac{-R}{r^2-1}}A(y)\diamondsuit\phi(t) + 2F_1(y)\diamondsuit\psi(t) + F_2(y).$$

3. When $\varepsilon = 1$, $R > 0$,

$$U_{25}(x,y,t) = \int^t Q(s)ds - \frac{F_1(y)}{2A(y)}\sqrt{\frac{r^2+1}{R}} + \frac{RA(y)\diamondsuit\left(1+\sqrt{\frac{1}{r^2+1}}\cosh(\sqrt{R}\xi)\right)}{\sinh(\sqrt{R}\xi)+r},$$
$$V_{25}(x,y,t) = -\frac{F_{1y}(y)\diamondsuit A(y) - F_1(y)\diamondsuit A_y(y)}{2A^{\diamondsuit 2}(y)}\sqrt{\frac{r^2+1}{R}}$$
$$+\frac{R(A_y(y)+\sqrt{\frac{1}{r^2+1}}(A_y(y)\diamondsuit\cosh(\sqrt{R}\xi)+A(y)\diamondsuit\xi_y\diamondsuit\sinh(\sqrt{R}\xi)))}{\sinh(\sqrt{R}\xi)+r}$$
$$-\frac{R\sqrt{R}A(y)\diamondsuit\xi_y\diamondsuit\cosh(\sqrt{R}\xi)\diamondsuit\left(\sqrt{\frac{1}{r^2+1}}\cosh(\sqrt{R}\xi)+1\right)}{(\sinh(\sqrt{R}\xi)+r)^{\diamondsuit 2}}, \quad (38)$$

where

$$\xi = 2A(y)\sqrt{\frac{R}{r^2+1}}x - 4\sqrt{\frac{R}{r^2+1}}A(y)\diamondsuit\phi(t) + 2F_1(y)\diamondsuit\psi(t) + F_2(y).$$

Remark. We have discussed the exact solutions of Eq. (2), when $\varepsilon = -1$, $r^2 - 1 > 0$, $R > 0$, when $\varepsilon = -1$, $r^2 - 1 < 0$, $R > 0$, and when $\varepsilon = 1$, $R > 0$. For other cases, similar to the above, e.g. when $\varepsilon = -1$, $r^2 - 1 < 0$, $R < 0$, when $\varepsilon = -1$, $r^2 - 1 > 0$, $R < 0$, when $\varepsilon = 1$, $R < 0$, we can obtain the corresponding solutions.

4. Summary and discussion

We have discussed the solutions of SPDEs driven by Gaussian white noise. There is a unitary mapping between the Gaussian white noise space and the Poisson white noise space. This connection was given by Benth and Gjerde [25]. We can see in Section 4.9 of Ref. [24] as well. Hence, by the aid of the connection, we can derive some stochastic exact soliton solutions if the coefficients $C(t)$ are Poisson white noise functions in Eq. (2).

References

[1] J.L. Zhang, Y.M. Wang, M.L. Wang, Z.D. Fang, Exact solutions to the (2 + 1)-dimensional Broer–Kaup equation with variable coefficients, J. Atom. Mol. Phys. 20 (1) (2003) 92–94.
[2] D.J. Huang, H.Q. Zhang, Variable-coefficient projective Riccati equation method and its application to a new (2 + 1)-dimensional simplified generalized Broer–Kaup system, Chaos Solitons Fract. 23 (2005) 601–607.
[3] Q. Liu, J.M. Zhu, B.H. Hong, A modified variable-coefficient projective Riccati equation method and its application to (2 + 1)-dimensional simplified generalized Broer–Kaup system, Chaos Solitons Fract. 37 (2008) 1383–1390.
[4] M. Wadati, Stochastic Korteweg–de Vries equation, J. Phys. Soc. Jpn. 52 (1983) 2642–2648.
[5] M. Wadati, Y. Akutsu, Stochastic Korteweg–de Vries equation with and without damping, J. Phys. Soc. Jpn. 53 (1984) 3342–3350.

[6] M. Wadati, Deformation of solitons in random media, J. Phys. Soc. Jpn. 59 (1990) 4201–4203.
[7] A. de Bouard, A. Debussche, On the stochastic Korteweg–de Vries equation, J. Funct. Anal. 154 (1998) 215–251.
[8] A. de Bouard, A. Debussche, White noise driven Korteweg–de Vries equation, J. Funct. Anal. 169 (1999) 258–532.
[9] A. Debussche, J. Printems, Numerical simulation of the stochastic Korteweg–de Vries equation, Physica D 134 (1999) 200–226.
[10] A. Debussche, J. Printems, Effect of a localized random forcing term on the Korteweg–de Vries equation, J. Comput. Anal. Appl. 3 (2001) 183–206.
[11] V.V. Konotop, L. Vázquez, Nonlinear Random Waves, World Scientific, Singapore, 1994.
[12] J. Printems, The stochastic Korteweg–de Vries equationin L(R2), J. Differ. Equations 153 (1999) 338–373.
[13] Y.C. Xie, Exact solutions for stochastic KdV equations, Phys. Lett. A 310 (2003) 161–167.
[14] Y.C. Xie, Exact solutions for stochastic mKdV equations, Chaos Solitons Fract. 19 (2004) 509–513.
[15] B. Chen, Y.C. Xie, White noise functional solutions of Wick-type stochastic generalized Hirota–Satsuma coupled KdV equations, J. Comput. Appl. Math. 197 (2006) 345–354.
[16] B. Chen, Y.C. Xie, Periodic-like solutions of variable coefficient and Wick-type stochastic NLS equations, J. Comput. Appl. Math. 203 (2007) 249–263.
[17] Q. Liu, J.M. Zhu, H.Y. Wu, The elliptic equation mapping and its application to stochastic Korteweg–de Vries equation, J. Phys. Soc. Jpn. 75 (2006) 014002.
[18] Q. Liu, J.M. Zhu, B.H. Hong, New exact Jacobian elliptic function solutions for Wick-type stochastic KdV equation, Chaos Solitons Fract. 31 (2007) 205–210.
[19] Q. Liu, Some exact solutions for stochastic mKdV equation, Chaos Solitons Fract. 32 (2007) 1224–1230.
[20] Q. Liu, D.L. Jia, Z.H. Wang, Three types of exact solutions to Wick-type generalized stochastic Korteweg–de Vries equation, Appl. Math. Comput. 215 (2010) 3495–3500.
[21] Q. Liu, A modified Jacobi elliptic function expansion method and its application to Wick-type stochastic KdV equation, Chaos Solitons Fract. 32 (2007) 1215–1223.
[22] Q. Liu, Various types of exact solutions for stochastic mKdV equation via a modified mapping method, Europhys. Lett. 74 (2006) 377–383.
[23] Q. Liu, Uniformly constructing a series of exact solutions for (2 + 1)-dimensional stochastic Broer–Kaup system, Chaos Solitons Fract. 36 (2008) 1037–1043.
[24] H. Holden, B. Øsendal, J. Ubøe, T. Zhang, Stochastic Partial Differential Equations, BirhKäuser, Boston, 1996.
[25] E. Benth, J. Gjerde, A remark on the equivalence between Poisson and Gaussian stochastic partial differential equations, Potential Anal. 8 (2) (1998) 179–193.