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ORIGINAL PAPER

Modelling experts' attitudes in group decision making

I. Palomares · J. Liu · Y. Xu · L. Martínez

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Abstract Nowadays, important decisions that have a significant impact either in societies or in organizations are commonly made by a group rather than a single decision maker, which might require more than a majority rule to obtain a real acceptance. Consensus-reaching processes provide a way to drive group decisions which are more accepted and appreciated by people affected by such a decision. These processes care about different consensus measures to determine the agreement in the group. The correct choice of a consensus measure that reflects the attitude of decision makers is a key issue for improving and optimizing consensus-reaching processes, which still requires further research. This paper studies the concept of group's attitude towards consensus, and presents a consensus model that integrates it in the measurement of consensus, through an extension of OWA aggregation operators, the so-called Attitude-OWA. The approach is applied to the solution of a real-like group decision making problem with the definition of different attitudes, and the results are analysed.

I. Palomares (✉) · L. Martínez
Department of Computer Science, University of Jaén,
23071 Jaén, Spain
e-mail: ivanp@ujaen.es

L. Martínez
e-mail: martin@ujaen.es

J. Liu
School of Computing and Mathematics, University of Ulster,
Newtownabbey BT37 0QB, Northern Ireland, UK

Y. Xu
School of Mathematics, Southwest Jiaotong University,
Chengdu 610031, Sichuan, China

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1 Introduction

Group decision making (GDM) problems are required throughout most companies and organizations nowadays, in order to guarantee a right development in them. They can be defined as decision situations where two or more decision makers or experts try to achieve a common solution to a decision problem, consisting of two or more possible solutions or alternatives (Kacprzyk 1986).

In real-world GDM problems, a range of situations including collaboration and competitiveness among individuals, compatible approaches or incompatible proposals might occur. Some guiding rules have been proposed to support decision making in such situations, for example the majority rule, minority rule and unanimity (Butler and Rothstein 2006). In democratic political systems, for instance, the majority rule is the most usual rule for dealing with GDM problems (Tocqueville 1840). However, in many real-world GDM problems that can affect groups or societies (civil rights, political or religious issues), the agreed solutions are highly appreciated. Therefore, the necessity of making decisions under consensus has become increasingly common in these contexts.

Consensus-reaching processes (CRPs) (Butler and Rothstein 2006; Saint and Lawson 1994) seek an experts' agreement about the problem before making the decision, thus yielding a more accepted solution by the whole group. CRPs are normally coordinated by a human figure, a so-called moderator, responsible for guiding experts throughout the overall discussion process. Different authors have proposed distinct approaches to handle CRPs, where Kacprzyk's *soft consensus* approach stands out (Kacprzyk 1986). In this approach, the concept of fuzzy linguistic majority is used to measure consensus between individuals in a flexible way. Later on, major achievements have been reached with the development of different consensus models, aimed to help decision makers to deal with CRPs. Some of these consensus models address aspects such as the use of different preference structures (Herrera-Viedma et al. 2002), management of incomplete preferences (Herrera-Viedma et al. 2007a, b), their extension to multi-criteria GDM problems (Parreiras et al. 2010; Pedrycz et al. 2011; Xu and Wu 2011) or even the introduction of adaptive consensus models based on the process performance (Mata et al. 2009).

However, some crucial aspects in CRPs still require a further study, for instance the idea of considering the group's attitude towards consensus, i.e. the experts' capacity to modify their own preferences during the CRP. Currently, consensus models found in the literature do not address the fact that if experts are reluctant to improve their attitude, the overall CRP might imply more time and cost. Additionally, it is important to consider the application of GDM problems with a large number of experts, because although real-world CRPs usually involve many experts, most developed models provide examples of performance with a small number of experts only (Herrera-Viedma et al. 2007a; Mata et al. 2009). This aspect may be addressed by designing consensus models with a high degree of automation, in which no human moderator is required to supervise experts' behaviour during the CRP.

In this paper, we develop the concept of group's attitude towards consensus and its application to CRPs, and present a consensus model that integrates it. Our goal consists in introducing such an attitude in the aggregation of information conducted during the CRP to measure the level of agreement in the group (Kuncheva and Krishnapuram 1995). To do so, we present the Attitude-OWA operator that extends the OWA operator (Yager 1988), so that it easily lets us reflect the group's attitude towards consensus. The model presented is applied to solve a real GDM problem where a large number of experts are involved, thus showing the importance and effects of integrating different attitudes.

This paper is organized as follows. In Sect. 2, some preliminaries related to consensus processes in GDM and OWA operators are reviewed. In Sect. 3, we develop in detail an approach to reflect the group's attitudes by means of the Attitude-OWA operator, and a consensus model based on such approach is defined and presented in Sect. 4. An application of the model to solve a real GDM problem by using different Attitude-OWA operators reflecting distinct attitudes is shown in Sect. 5. Finally, in Sect. 6, the main conclusions and some future works are drawn.

2 Preliminaries

In this section, we revise GDM problems and CRPs. We then briefly review OWA operators and linguistic quantifiers, which are the basis for our proposal.

2.1 Group decision making (GDM)

GDM problems are characterized by the participation of two or more experts in a decision problem, where a set of alternatives or possible solutions to the problem are presented (Butler and Rothstein 2006; Kacprzyk 1986). Formally, the main elements found in any GDM problem are:

- A set $X = \{x_1, \ldots, x_q\}, (q \geq 2)$ of possible *alternatives* to choose as possible solutions to the problem.
- A set $E = \{e_1, \ldots, e_m\}, (m \geq 2)$ of individuals or *experts*, who express their judgements or opinions on the alternatives in X.

Each expert $e_i, i \in \{1, \ldots, m\}$, provides his/her opinions over alternatives in X by means of a preference structure. One of the most usual preference structures, which also has been especially effective when dealing with uncertainty, is the so-called fuzzy preference relation.

Definition 1 (Bryson 1996; Herrera-Viedma et al. 2002) Given an expert $e_i \in E, i \in \{1, \ldots, m\}$ and two different alternatives $x_l, x_k \in X; l, k \in \{1, \ldots, q\} (l \neq k)$, a fuzzy preference relation's *assessment* on the pair (x_l, x_k), denoted as $p_i^{lk} \in [0, 1]$, represents the degree of preference of alternative x_l with respect to alternative x_k assessed by expert e_i, so that $p_i^{lk} > 1/2$ indicates that x_l is preferred to x_k, $p_i^{lk} < 1/2$ indicates that x_k is preferred to x_l, and $p_i^{lk} = 1/2$ indicates indifference between x_l and x_k.

Definition 2 (Herrera-Viedma et al. 2002) A *fuzzy preference relation* P_i associated with an expert $e_i, i \in \{1, \ldots, m\}$, on a set of alternatives X is a fuzzy set on $X \times X$, which is characterized by the membership function $\mu_{P_i} : X \times X \longrightarrow [0, 1]$. When the number of alternatives q is finite, P_i is represented by a $q \times q$ matrix of assessments $p_i^{lk} = \mu_{Pi}(x_l, x_k)$ as follows:

$$P_i = \begin{pmatrix} - & \ldots & p_i^{1q} \\ \vdots & \ddots & \vdots \\ p_i^{q1} & \ldots & - \end{pmatrix}$$

Notice here that assessments $p_i^{ll}, l \in \{1, \ldots, q\}$, situated in the diagonal of the matrix, are not defined, since an alternative x_l is not assessed with respect to itself.

In order to provide a better understanding of these definitions, a brief example is given below.

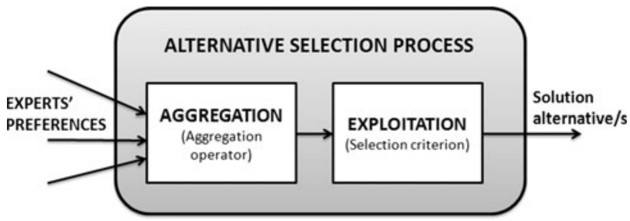


Fig. 1 Selection process in GDM problems

Example 1 Given $E = \{e_1, e_2, e_3\}$, $X = \{x_1, x_2, x_3, x_4\}$, let P_3 be the fuzzy preference relation on X expressed by e_3:

$$P_3 = \begin{pmatrix} - & 0.2 & 0.25 & 0 \\ 0.8 & - & 0.75 & 0.3 \\ 0.75 & 0.25 & - & 0 \\ 1 & 0.7 & 1 & - \end{pmatrix}$$

where we can see, for instance, that $p_3^{21} = 0.8$ indicates that x_2 is strongly preferred against x_1 by e_3, $p_3^{14} = 0$ indicates that x_1 is absolutely rejected with respect to x_4, and $p_3^{43} = 1$ indicates that x_4 is absolutely preferred against x_3.

The solution to a GDM problem may be obtained either by a direct approach, where the solution is immediately obtained from the experts' preferences, or by an indirect approach, where a social opinion is computed to determine the chosen alternative/s (Herrera et al. 1995). Regardless of the approach considered, it is necessary to apply a selection process to solve the GDM problem, which usually consists of two main phases (Fig. 1) (Roubens 1997): (1) an aggregation phase, where experts' preferences are combined and (2) an exploitation phase, which consists in obtaining an alternative or subset of alternatives as the solution to the problem.

2.2 Consensus-reaching processes (CRPs)

One of the main shortcomings found in classic GDM rules, such as the majority rule or minority rule, is the possible disagreement shown by one or more experts with the achieved solution, because they might consider that their opinions have not been taken into account sufficiently. Given the importance of obtaining an accepted solution by the whole group, CRPs as part of the decision process have attained great attention. *Consensus* can be understood as a state of mutual agreement among members of a group (Butler and Rothstein 2006; Saint and Lawson 1994), where the decision made satisfies all of them. Reaching a consensus usually requires that experts modify their initial opinions in a discussion process, making them closer to each other and towards a collective opinion which must be satisfactory for all of them.

The notion of consensus can be interpreted in different ways, ranging from consensus as total agreement to a more flexible approach (Herrera-Viedma et al. 2011). The strict notion of consensus assumes its existence only if all experts have achieved a mutual agreement in all their opinions (Tocqueville 1840). This may be quite difficult or even impossible to achieve in practice, and in the cases it could be achieved, the cost derived from the CRP would be unacceptable. Also, it might sometimes have been achieved through a normative point of view, through intimidation and other social strategies (Yager 2001). Subsequently, more flexible notions of consensus have been proposed to soften the strict view of consensus as unanimity (Elzinga et al. 2011; Herrera-Viedma et al. 2011; Kacprzyk and Fedrizzi 1988). These flexible approaches, more feasible in practice, consider different degrees of partial agreement to decide about the existence of consensus. Such degrees usually indicate how far a group of experts is from ideal consensus or unanimity.

One of the most widely accepted approaches for a flexible measurement of consensus is the so-called notion of *soft consensus*, proposed by Kacprzyk (1986). This approach introduces the concept of fuzzy linguistic majority, which establishes that there exists consensus if most experts participating in a problem agree with the most important alternatives. Soft consensus-based approaches have been used in different GDM problems, providing satisfactory results (Fedrizzi et al. 1999; Herrera et al. 1996; Kacprzyk and Zadrozny 2010; Zadrozny and Kacprzyk 2003). Consensus measures based on soft consensus are more human consistent and ideal for reflecting human perceptions of the meaning of consensus in practice (Kacprzyk and Fedrizzi 1989). The aforementioned concept of fuzzy linguistic majority has been captured by using linguistic quantifiers (Zadeh 1983).

The process to reach consensus in GDM problems is a dynamic and iterative discussion process (Saint and Lawson 1994), frequently coordinated by a human figure known as moderator, who plays a key role in CRPs (Martínez and Montero 2007). The moderator's main responsibilities are:

- Evaluate the degree of agreement achieved in each round of discussion, and decide whether it is enough to accept or not the existence of consensus.
- Identify those alternatives that hamper reaching a consensus.
- Give feedback to experts regarding changes they should make in their opinions on the previously identified alternatives, in order to increase the level of agreement in the next few rounds.

A general scheme of the phases required for conducting CRPs, depicted in Fig. 2, is briefly described below:

- *Gather preferences*: Each expert provides the moderator a preference structure with his/her opinion on the existing alternatives.

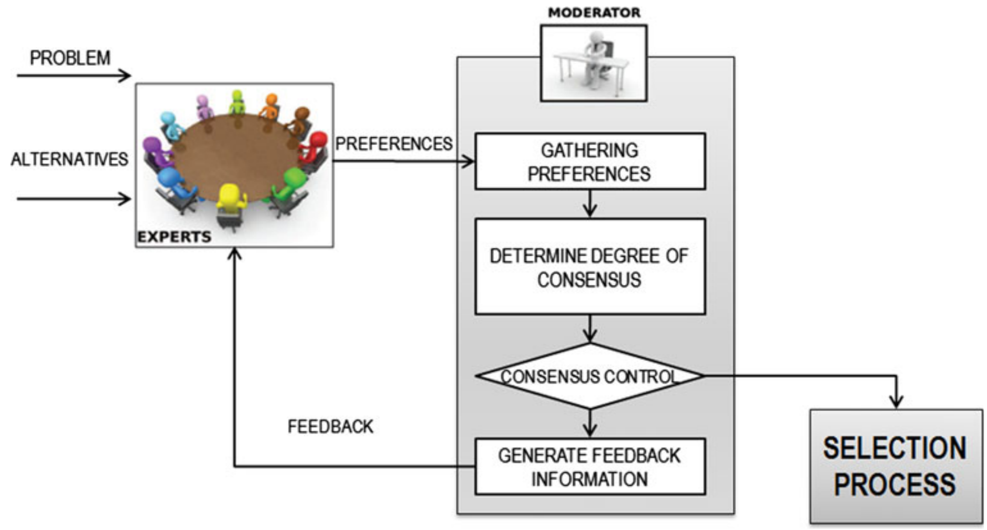


Fig. 2 General consensus process scheme in GDM problems

- *Determine degree of consensus*: The moderator computes the level of agreement in the group by means of a *consensus measure* (Herrera-Viedma et al. 2011), usually based on different *similarity measures* and *aggregation operators* (Beliakov et al. 2007).
- *Consensus control*: The consensus degree is compared with a threshold level of agreement desired by the group. If such degree is sufficient, the group moves on to the selection process; otherwise, more discussion rounds are required.
- *Generate feedback information*: The moderator identifies the farthest preferences from consensus and gives experts some pieces of advice, suggesting how to modify their opinions and make them closer to agreement. Afterwards, a new round of discussion begins with the gathering preferences phase.

2.3 OWA operators: weights computation

One of the most widely applied families of weighted aggregation operators (Beliakov et al. 2007) in different GDM approaches in the literature are the so-called ordered weighted averaging (OWA) operators, introduced by Yager:

Definition 3 (Yager 1988) Let $A = \{a_1, \ldots, a_n\}, a_i \in R$, be a set of n values to aggregate. An *OWA operator* is a mapping $F : R^n \to R$, with an associated weighting vector $W = [w_1 \ldots w_n]^\top$ ($w_i \in [0, 1], \sum_i w_i = 1$):

$$F(a_1, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j \qquad (2.1)$$

where b_j is the jth largest of a_i values.

Note that a weight w_i is associated with a particular ordered position instead of a particular element, i.e. w_i is associated with the ith largest element in $a_1, \ldots, a_n$. OWA operators are idempotent, continuous, monotone, neutral and compensative (Grabisch et al. 1998).

OWA operators are averaging aggregation functions, i.e. they lie between minimum and maximum functions, and therefore can be classified according to their optimism degree, by means of a measure, the so-called orness, associated with W. This measure provides the attitudinal character of aggregation, by determining how close the operator is to the maximum (OR) function, and is defined as (Beliakov et al. 2007):

$$\text{orness}(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i \qquad (2.2)$$

While optimistic or OR-LIKE OWA operators are those whose $\text{orness}(W) > 0.5$, in pessimistic or AND-LIKE operators we have $\text{orness}(W) < 0.5$ (Yager 1988, 1993).

Another measure, the *dispersion* (Shannon and Weaver 1949), can be used to let a further distinction amongst different OWA operators with an equal degree of optimism:

$$\text{Disp}(W) = -\sum_{i=1}^{n} w_i \ln w_i \qquad (2.3)$$

This measure can be used as an indicator of the degree to which information contained in values $a_1, \ldots, a_n$ is really used in the aggregation process.

Several approaches have been proposed to compute OWA weights (Grabisch et al. 1998), for instance by using linguistic quantifiers (Yager 1996), as considered in this paper. Linguistic quantifiers were introduced by Zadeh (1983). They can be used to semantically express aggregation policies and actually capture Kacprzyk's notion of soft consensus in consensus models (Kacprzyk 1986; Kacprzyk and Fedrizzi 1989). This paper focuses on using a particular type of relative linguistic quantifiers, the so-called regular increasing monotone (RIM) quantifiers (Liu and Han 2008; Yager 1996), defined as a fuzzy subset Q of the unit interval (Klir and Yuan 1995; Yager and Filev 1994) where for a given proportion $r \in [0,1], Q(r)$ indicates the extent to which this proportion satisfies the semantics defined in Q. RIM quantifiers are characterized by the following properties: (1) $Q(0) = 0$, (2) $Q(1) = 1$ and (3) if $r_1 > r_2$ then $Q(r_1) \geq Q(r_2)$.

Yager (1988) proposed the following method to compute OWA weights with the use of RIM quantifiers:

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), \quad i = 1, \ldots, n \tag{2.4}$$

where the linear membership function of a RIM quantifier $Q(r)$ is defined by the use of two parameters $\alpha, \beta \in [0,1]$ as

$$Q(r) = \begin{cases} 0 & \text{if } r \leq \alpha, \\ \frac{r-\alpha}{\beta-\alpha} & \text{if } \alpha < r \leq \beta, \\ 1 & \text{if } r > \beta. \end{cases} \tag{2.5}$$

OWA operators based on linguistic quantifiers have been widely applied in the literature, with multiple purposes (Reformat et al. 2011).

3 Integrating experts' attitude in consensus-reaching processes

The aim of this paper is to introduce and manage the concept of group's attitude towards consensus in CRPs, by means of a new aggregation operator based on the OWA operator that allows managing this concept in the measurement of consensus, and defines a consensus model upon it. In this section, we develop such a concept and show in detail how to implement attitude-based OWA operators. Furthermore, we will introduce in the coming sections a new attitude-based consensus model, as well as a complete study of its performance.

The concept of *attitude* refers to the importance that experts give to reach a consensus, compared to modifying their own preferences, and can be roughly classified into two types:

- *Optimistic attitude*: Achieving an agreement is more important than experts' own preferences; therefore, those positions in the group whose level of agreement is higher are given more importance in the aggregation process.
- *Pessimistic attitude*: Experts' own preferences are considered more important than achieving an agreement; therefore, those positions in the group where the level of agreement is lower attain more importance in aggregation.

The choice of an attitude depends on the prospects considered by experts in the group and the nature of the decision problem to be addressed.

Our proposal begins introducing the attitudinal parameters used by the group to reflect their attitude towards consensus, and then the Attitude-OWA operator is defined to capture such an attitude in the CRP. Attitude-OWA shall be applied to aggregate similarities between experts in the phase of computing consensus degree, as will be further detailed in Sect. 4.

3.1 Attitudinal parameters and Attitude-OWA operator

In Sect. 2.3, we reviewed RIM quantifiers and stated the membership function for a linear RIM quantifier upon two parameters α, β. Note that $[\alpha, \beta] \subseteq [0, 1]$ $(\alpha < \beta)$ defines the range of proportions r where the membership function $Q(r)$ increases, i.e. the slope of the RIM quantifier. Therefore, we have either $Q(r) = 0$ or $Q(r) = 1$ for any r situated to the left or to the right side of the slope, respectively. For a slope $[\alpha, \beta]$, its amplitude d is defined as $d = \beta - \alpha$.

When computing OWA weights from $Q(r)$ using Eq. (2.4), non-null weights w_i are assigned to elements b_i whose $r = i/n$ is situated inside the quantifier's slope, i.e. $r \in [\alpha, \beta]$. As we can see, d indicates the amount of values considered in the aggregation. In addition, orness(W) indicates how optimistic this aggregation is. These two elements let us define the *attitudinal parameters* used by the decision group to reflect an attitude towards consensus.

- $\vartheta = \text{orness}(W) \in [0, 1]$ represents the group's attitude to be taken into account in the aggregation process (see Sect. 2.3). This attitude can be either optimistic if $\vartheta > 0.5$, pessimistic if $\vartheta < 0.5$ or neutral if $\vartheta = 0.5$.
- $\varphi = d \in [0, 1]$ indicates the amount of values which are given non-null weight and therefore are considered in the aggregation. The higher the d, the wider is the range of ranked values given non-null weight and the higher is the dispersion in the corresponding Attitude-OWA operator.

We can now define an extension of OWA operators so-called Attitude-OWA for reflecting specific aggregation attitudes as follows:

Definition 4 An *Attitude-OWA operator* of dimension n on a set $A = \{a_1, \ldots, a_n\}$ of values to be aggregated, is an OWA operator based on two attitudinal parameters ϑ, φ given by a decision group to indicate their attitude towards consensus,

$$\text{Attitude-OWA}_W(A,\vartheta,\varphi)=\sum_{j=1}^{n} w_j b_j \tag{3.1}$$

where b_j is the jth largest of the a_i values, $\vartheta,\varphi\in[0,1]$ are two input attitudinal parameters, and the set of weights, W, is computed by using a RIM quantifier, as shown in Eq. (2.4).

The attitude ϑ of an Attitude-OWA operator can be determined given the associated RIM quantifier Q, when the number of elements to aggregate n is sufficiently large, as follows:

Theorem 1 *Let ϑ be the attitude of an Attitude-OWA operator based on an RIM quantifier with a differentiable membership function $Q(r)$. Then for $n\to\infty,\vartheta\in[0,1]$ is determined as follows*

$$\vartheta=\int_0^1 Q(r)\mathrm{d}r \tag{3.2}$$

The detailed analytical proof to obtain this expression is given as follows:

Proof Based on Eq. (2.2) and Eq. (2.4), we have

$$\begin{aligned}\vartheta(n)&=\text{orness}(W)(n)\\&=\frac{1}{n-1}\sum_{i=1}^{n}(n-i)\left[Q\left(\frac{i}{n}\right)-Q\left(\frac{i-1}{n}\right)\right]\end{aligned}$$

To calculate ϑ when n is sufficiently large, $n\to\infty$,

$$\begin{aligned}\vartheta&=\lim_{n\to\infty}\vartheta(n)\\&=\lim_{n\to\infty}\frac{1}{n-1}\sum_{i=1}^{n}(n-i)\left[Q\left(\frac{i}{n}\right)-Q\left(\frac{i-1}{n}\right)\right]\\&=\lim_{n\to\infty}\frac{1}{n-1}\sum_{i=1}^{n-1}(n-i)\left[Q\left(\frac{i}{n}\right)-Q\left(\frac{i-1}{n}\right)\right]\end{aligned}$$

If we consider $P=\sum_{i=1}^{n-1}(n-i)\left[Q\left(\frac{i}{n}\right)-Q\left(\frac{i-1}{n}\right)\right]$, then we have

$$\begin{aligned}P&=\sum_{i=1}^{n-1}\left[n\left[Q\left(\frac{i}{n}\right)-Q\left(\frac{i-1}{n}\right)\right]-i\left[Q\left(\frac{i}{n}\right)-Q\left(\frac{i-1}{n}\right)\right]\right]\\&=\sum_{i=1}^{n-1}n\left[Q\left(\frac{i}{n}\right)-Q\left(\frac{i-1}{n}\right)\right]-\sum_{i=1}^{n-1}i\left[Q\left(\frac{i}{n}\right)-Q\left(\frac{i-1}{n}\right)\right]\\&=n\sum_{i=1}^{n-1}\left[Q\left(\frac{i}{n}\right)-Q\left(\frac{i-1}{n}\right)\right]-\sum_{i=1}^{n-1}i\left[Q\left(\frac{i}{n}\right)-Q\left(\frac{i-1}{n}\right)\right]\end{aligned}$$

where, expanding it into the sum form, some terms are mutually deleted and finally we have

$$\begin{aligned}P&=nQ\left(\frac{n-1}{n}\right)-\left[-\left[\sum_{i=1}^{n-2}Q\left(\frac{i}{n}\right)\right]+(n-1)Q\left(\frac{n-1}{n}\right)\right]\\&=nQ\left(\frac{n-1}{n}\right)-(n-1)Q\left(\frac{n-1}{n}\right)+\sum_{i=1}^{n-2}Q\left(\frac{i}{n}\right)\\&=Q\left(\frac{n-1}{n}\right)+\sum_{i=1}^{n-2}Q\left(\frac{i}{n}\right)=\sum_{i=1}^{n-1}Q\left(\frac{i}{n}\right)\end{aligned}$$

Therefore,

$$\begin{aligned}\vartheta&=\lim_{n\to\infty}\frac{1}{n-1}\sum_{i=1}^{n}(n-i)\left[Q\left(\frac{i}{n}\right)-Q\left(\frac{i-1}{n}\right)\right]\\&=\lim_{n\to\infty}\frac{1}{n-1}\sum_{i=1}^{n-1}Q\left(\frac{i}{n}\right)\end{aligned}$$

When $n\to\infty$, it follows from the limit definition of definite integral that (Yager 1996)

$$\vartheta=\lim_{n\to\infty}\frac{1}{n-1}\sum_{i=1}^{n-1}Q\left(\frac{i}{n}\right)=\int_0^1 Q(r)\mathrm{d}r$$

where $r=i/n$. □

Since this statement is true for any function $Q(r)$ differentiable in [0,1], it can be easily extended to different types of RIM quantifiers, as shown below.

Corollary 1 *Given an RIM quantifier Q with a linear membership function $Q(r)$ as shown in Eq. (2.5), when the number of elements to aggregate n is sufficiently large, it is possible to compute the optimism degree ϑ of the Attitude-OWA operator based on Q as follows,*

$$\vartheta=1-\alpha-\frac{\varphi}{2} \tag{3.3}$$

Proof Based on the previous theorem and Eq. (2.5), we have

$$\begin{aligned}\vartheta&=\int_0^1 Q(r)\mathrm{d}r=\int_\alpha^1 Q(r)\mathrm{d}r=\text{Area}(Q)\\&=\frac{1}{2}(\beta-\alpha)+[1-\beta]=\frac{1}{2}\varphi+1-(\alpha+\varphi)=1-\alpha-\frac{\varphi}{2}\end{aligned}$$

Notice that the interval [α,1] defines the *support* of the quantifier and $\beta-\alpha=\varphi$. The meaning of the integral states that ϑ is equal to the area under the membership function $Q(r)$ (Liu and Han 2008; Yager 1996), as shown in Fig. 3. □

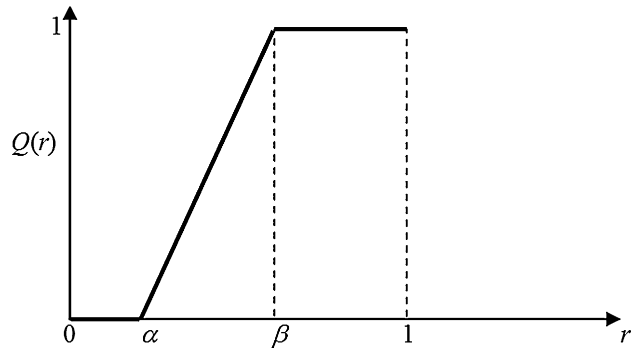


Fig. 3 Membership function in RIM quantifiers considered

Therefore, when using linear RIM quantifiers, ϑ may closely approximate to the result of Eq. (3.3) when measuring consensus in large groups where a high number of agreement values must be aggregated to measure consensus, i.e. when $n \to \infty$. As a result, since we are interested in integrating a group's attitude towards consensus by means of ϑ and φ, we use Eq. (3.3) to determine the value of α, necessary to define the RIM quantifier and compute Attitude-OWA weights, as follows:

$$\alpha = 1 - \vartheta - \frac{\varphi}{2} \tag{3.4}$$

3.2 Relations and restrictions between attitudinal parameters

Attitudinal parameters' values are related to each other, so it is convenient to clarify some existing relations and restrictions between them. As stated earlier, α and φ are used to univocally define a linear RIM quantifier Q, but the following condition must be fulfilled to define a valid RIM quantifier and therefore integrate a valid attitude in the process:

Theorem 2 *Given* $\alpha, \varphi \in [0, 1]$, *a valid attitude given by* ϑ *can be guaranteed only if* $\alpha + \varphi \leq 1$.

Proof Let us suppose $\alpha + \varphi > 1$. Considering that $\varphi = \beta - \alpha$, Eq. (3.3) leads to

$$\vartheta = 1 - \alpha - \frac{\varphi}{2} = 1 - \frac{\alpha + \beta}{2} \tag{3.5}$$

where $\frac{\alpha+\beta}{2}$ is the central value of the quantifier's slope, so that

$$\alpha \leq \frac{\alpha + \beta}{2} \leq \beta$$
$$1 - \alpha \geq 1 - \frac{\alpha + \beta}{2} \geq 1 - \beta$$
$$1 - \alpha \geq \vartheta \geq 1 - (\alpha + \varphi)$$

where $\beta = \alpha + \varphi$. Notice here that if $\alpha + \varphi > 1$ as we supposed, then ϑ can be negative; therefore, $\alpha + \varphi$ must be equal or less than one to ensure a valid attitude is defined. □

In order to avoid expressing invalid attitudinal parameters, we present the restrictions to be considered by the decision group when providing them.

Corollary 2 *The following condition must be fulfilled when the group provides a value of* ϑ:

$$\frac{\varphi}{2} \leq \vartheta \leq 1 - \frac{\varphi}{2} \tag{3.6}$$

Proof According to Eq. (3.4), α is negative if $(\vartheta + \varphi/2) > 1$. We need $\alpha \geq 0$, i.e.

$$1 - \vartheta - \frac{\varphi}{2} \geq 0$$
$$\vartheta + \frac{\varphi}{2} \leq 1$$
$$\vartheta \leq 1 - \frac{\varphi}{2}$$

However, according to Theorem 2, it is also necessary to guarantee $\alpha + \varphi \leq 1$. Based on Eq. (3.4) we have,

$$\alpha + \varphi = 1 - \vartheta - \frac{\varphi}{2} + \varphi \leq 1$$
$$1 - \vartheta + \frac{\varphi}{2} \leq 1$$
$$\vartheta \geq \frac{\varphi}{2}$$

The fulfillment of both inequalities leads to the aforementioned restriction. □

As a result, the higher the proportion of values to consider in aggregation (given by φ), the narrower range of possible attitudes or optimism degrees (given by ϑ) can be considered.

Corollary 3 *The following condition must be fulfilled when the group provides a value of* φ:

$$\varphi \leq 1 - |2\vartheta - 1| \tag{3.7}$$

Proof Based on the previous proof in Corollary 1, $\alpha \geq 0$ requires

$$\vartheta + \frac{\varphi}{2} \leq 1 \quad \text{i.e.,}$$
$$\varphi \leq 2(1 - \vartheta)$$

which is valid for $\vartheta \in [0.5, 1]$, but may give rise to $\varphi > 1$ and fail to fulfill Theorem 2 when $\vartheta < 0.5$. Let us consider Theorem 2 and Eq. (3.4). We then have

$$\alpha + \varphi = 1 - \vartheta - \frac{\varphi}{2} + \varphi \leq 1$$
$$1 - \vartheta + \frac{\varphi}{2} \leq 1$$
$$\varphi \leq 2\vartheta$$

which is valid for $\vartheta \in [0, 0.5]$, but $\varphi > 1$ may still be possible when $\vartheta > 0.5$; hence, a valid quantifier can be defined only if these restrictions are satisfied,

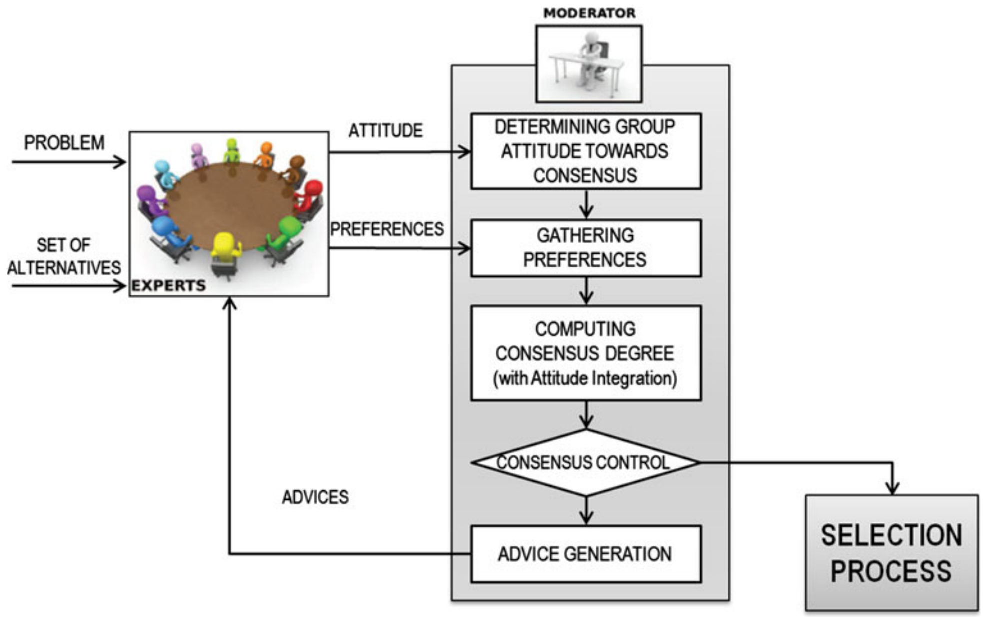


Fig. 4 Attitude-based consensus model scheme

$$\varphi \leq 2\vartheta \qquad \text{if } \vartheta \in [0, 0.5]$$
$$\varphi \leq 2(1-\vartheta) \quad \text{if } \vartheta \in [0.5, 1]$$

We finally proceed to find a single expression which considers both restrictions. On the one hand, we have

$$2\vartheta = 1 - (-2\vartheta + 1)$$

where, when $\vartheta \in [0, 0.5]$, the term $(-2\vartheta + 1) \geq 0$. On the other hand,

$$2(1-\vartheta) = 1 - (2\vartheta - 1)$$

where, when $\vartheta \in [0.5, 1]$, the term $(2\vartheta - 1) \geq 0$. This means we can consider the absolute value of the term $(2\vartheta - 1)$ to integrate both restrictions as

$$\varphi \leq 1 - |2\vartheta - 1|$$

This restriction can be interpreted as the fact that the closer ϑ is to a neutral attitude (0.5), the wider the range of possible degrees for φ that can be considered. □

If restrictions pointed out in Eqs. (3.6) and (3.7) are taken into account when expressing any two values for input attitudinal parameters (ϑ, φ), then a valid RIM quantifier is always defined, thus guaranteeing a valid Attitude-OWA operator.

4 Attitude-based consensus model

Once presented the concept of attitude towards consensus and the main features of the Attitude-OWA operator used to reflect it, in this section we present the consensus model designed to integrate such an attitude in CRPs. The model extends the main ideas of some models presented in (Herrera-Viedma et al. 2002; Mata et al. 2009) and its design allows to automate all the human moderator tasks, thus removing his/her inherent subjective biasness towards experts and facilitating the resolution of GDM problems with large groups of experts computationally.

Figure 4 shows the five phases conducted in the model, which are described in the following subsections:

4.1 Determining group attitude towards consensus

This phase is carried out at the beginning of the CRP, as part of a *pre-consensus process* (Saint and Lawson 1994). The moderator is responsible for reflecting the group's attitude towards consensus, by assigning a value to attitudinal parameters ϑ and φ, considering both the context and characteristics of the decision problem to solve, and the experts' individual concerns. Figure 5 shows the procedure to determine a group's attitude towards the achievement of consensus and integrate it in the CRP, defining the corresponding Attitude-OWA operator used in a later phase to measure consensus.

4.2 Gathering preferences

Each expert e_i provides his/her preference on alternatives in X to the moderator, by means of a fuzzy preference relation P_i, consisting of a $q \times q$ matrix of assessments p_i^{lk} on each pair of alternatives $(x_l, x_k), l, k \in \{1, \ldots, q\}$. It is advisable that experts provide consistent opinions that could be easier to achieve if they provide reciprocal assessments, i.e. if $p_i^{lk} = x, x \in [0,1], l \neq k$, then $p_i^{kl} = 1 - x$.

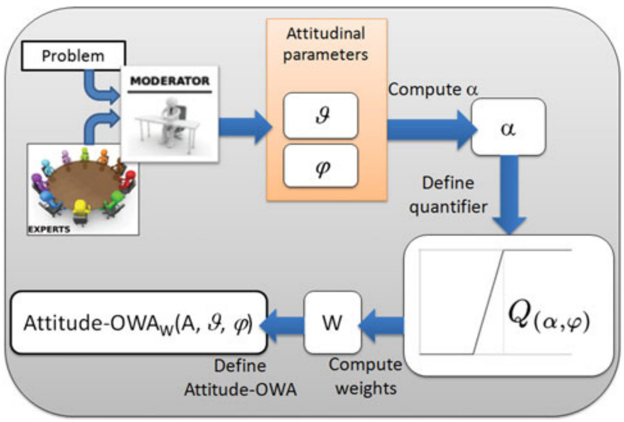


Fig. 5 Process to determine the Attitude-OWA operator used to measure consensus based on the group's attitudinal parameters ϑ and φ

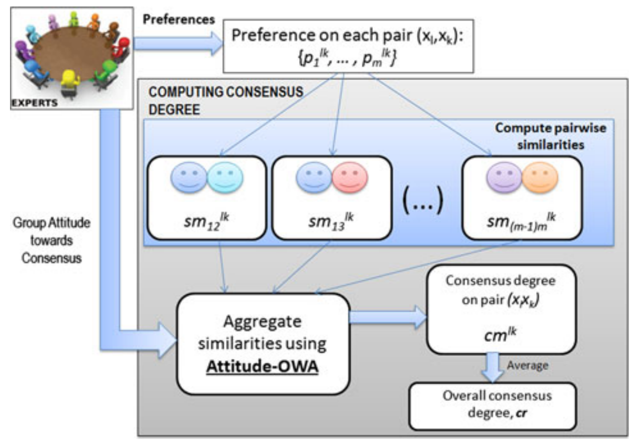


Fig. 6 Procedure to compute consensus degree based on the group's attitude

4.3 Computing consensus degree

The moderator computes the level of agreement between experts, by means of the following steps (see Fig. 6):

1. For each pair of experts e_i, e_j, $(i \neq j)$, a similarity matrix SM, defined by

$$SM_{ij} = \begin{pmatrix} - & \ldots & sm_{ij}^{1q} \\ \vdots & \ddots & \vdots \\ sm_{ij}^{q1} & \ldots & - \end{pmatrix},$$

   is computed as follows (Herrera-Viedma et al. 2005):

$$sm_{ij}^{lk} = 1 - |(p_i^{lk} - p_j^{lk})| \tag{4.8}$$

   where $sm_{ij}^{lk} \in [0,1]$ is the similarity degree between experts e_i and e_j in their assessments p_i^{lk}, p_j^{lk}.
2. A consensus matrix CM of dimension $q \times q$, defined by

$$CM = \begin{pmatrix} -\ldots & cm^{1q} & \\ \vdots & \ddots & \vdots \\ cm^{q1} & \ldots & - \end{pmatrix},$$

   is computed, taking into account the group's attitude by aggregation of similarity matrices. Each element cm^{lk}, $l \neq k$, is computed as:

$$cm^{lk} = \text{Attitude-OWA}_W(SIM^{lk}, \vartheta, \varphi) \tag{4.9}$$

   where $SIM^{lk} = \{sm_{12}^{lk}, \ldots, sm_{1m}^{lk}, sm_{23}^{lk}, \ldots, sm_{2m}^{lk}, \ldots, sm_{(m-1)m}^{lk}\}$ is the set of all pairs of experts' similarities in their opinion on (x_l, x_k). Attitude-OWA operator is used here to integrate the group's attitude towards consensus, previously gathered by means of ϑ and φ.
3. Consensus degree is computed at three different levels:
   (a) Level of pairs of alternatives (cp^{lk}): obtained from CM as $cp^{lk} = cm^{lk}$, $l, k \in \{1, \ldots, q\}$, $l \neq k$.
   (b) Level of alternatives (ca^l): the level of agreement on each alternative $x_l \in X$ is computed as:

$$ca^l = \frac{\sum_{k=1,k\neq l}^{q} cp^{lk}}{q-1} \tag{4.10}$$

   (c) Level of preference relation (overall consensus degree, cr): it is computed as:

$$cr = \frac{\sum_{l=1}^{q} ca^l}{q} \tag{4.11}$$

4.4 Consensus control

The overall consensus degree cr is compared with a consensus threshold $\mu \in [0,1]$ established a priori. If $cr \geq \mu$, then the CRP ends and the group moves on to the selection process; otherwise, more discussion rounds are required. A parameter *Maxrounds* can be used to limit the number of discussion rounds conducted in the cases that consensus cannot be achieved.

4.5 Advice generation

If $cr < \mu$, the moderator advises experts to modify their preferences in order to increase the level of agreement in the following rounds. Three steps are considered in this phase:

1. Compute a collective preference and proximity matrices for experts: A collective preference P_c is computed

for each pair of alternatives by aggregating experts' preference relations:

$$p_c^{lk} = \phi(p_1^{lk}, \ldots, p_m^{lk}) \tag{4.12}$$

where ϕ is the aggregation operator considered. Afterwards, a proximity matrix $PP_i = (pp_i^{lk})$ between each expert's preference relation and P_c is obtained:

$$PP_i = \begin{pmatrix} - & \ldots & pp_i^{1q} \\ \vdots & \ddots & \vdots \\ pp_i^{q1} & \ldots & - \end{pmatrix}$$

Proximity values pp_i^{lk} are obtained for each pair (x_l, x_k) as follows:

$$pp_i^{lk} = 1 - |(p_i^{lk} - p_c^{lk})| \tag{4.13}$$

Proximity values are used to identify the farthest preferences from the collective opinion, which should be modified by some experts.

2. Identify preferences to be changed (CC): pairs of alternatives (x_l, x_k), whose consensus degrees ca^l and cp^{lk} are not sufficient, are identified:

$$CC = \{(x_l, x_k) | ca^l < cr \wedge cp^{lk} < cr\} \tag{4.14}$$

Afterwards, the model identifies experts who should change their opinion on each of these pairs, i.e. those experts e_i whose preference p_i^{lk} on the pair $(x_l, x_k) \in CC$ is farthest to p_c^{lk}. An average proximity $\overline{pp}^{lk}$ is calculated to identify them, as follows:

$$\overline{pp}^{lk} = \phi(pp_1^{lk}, \ldots, pp_m^{lk}) \tag{4.15}$$

As a result, experts e_i whose $pp_i^{lk} < \overline{pp}^{lk}$ are advised to modify their assessment on the pair (x_l, x_k).

3. Establish change directions: several direction rules are applied to suggest the direction of changes proposed to experts, in order to increase the level of agreement in the following rounds (Mata et al, 2009).

- DIR.1: If $(p_i^{lk} - p_c^{lk}) < 0$, then expert e_i should *increase* his/her assessment on the pair of alternatives (x_l, x_k).
- DIR.2: If $(p_i^{lk} - p_c^{lk}) > 0$, then expert e_i should *decrease* his/her assessment on the pair of alternatives (x_l, x_k).
- DIR.3: If $(p_i^{lk} - p_c^{lk}) = 0$, then expert e_i should not modify his/her assessment on the pair of alternatives (x_l, x_k).

5 Experimental simulation

In this section, we use a multi-agent based consensus support system to simulate the resolution of a real GDM problem defined under uncertainty, with different instances of Attitude-OWA operator based on different group attitudes towards consensus, having a considerable number of experts in the group. Our main hypothesis focuses mainly on the effect of using different attitudes towards consensus in the process, and states that optimism, given by OR-LIKE operators, may favour a greater convergence towards consensus with a lower number of rounds; whereas pessimism, given by AND-LIKE operators, may favour a lower convergence towards consensus and, therefore, more rounds of discussion are required.

Table 1 Attitudinal parameters and RIM quantifiers used

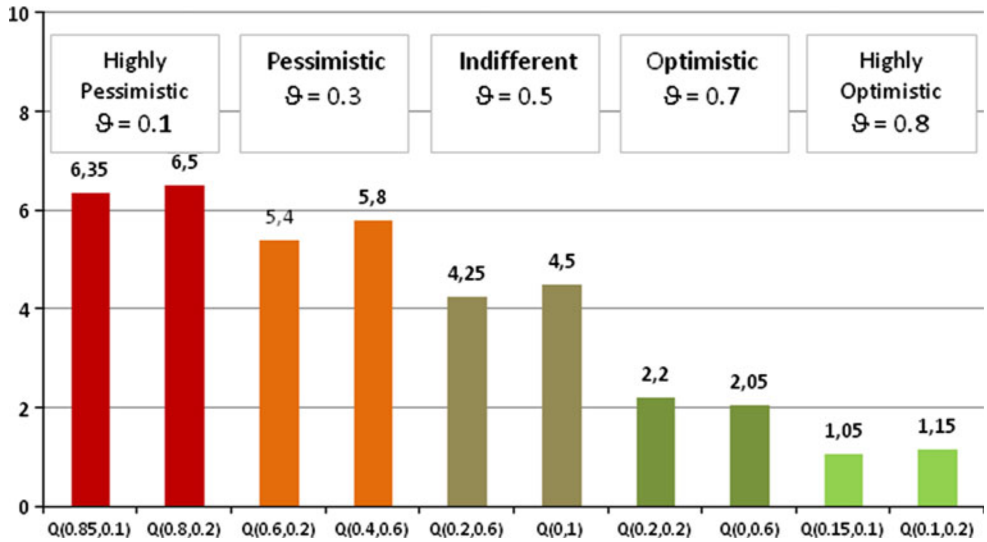
Attitude	ϑ	φ	α	$Q_{(\alpha,\varphi)}$
Highly pessimistic	0.1	0.1	0.85	$Q_{(0.85,0.1)}$
	0.1	0.2	0.8	$Q_{(0.8,0.2)}$
Pessimistic	0.3	0.2	0.6	$Q_{(0.6,0.2)}$
	0.3	0.6	0.4	$Q_{(0.4,0.6)}$
Indifferent	0.5	0.6	0.2	$Q_{(0.2,0.6)}$
	0.5	1	0	$Q_{(0,1)}$
Optimistic	0.7	0.2	0.2	$Q_{(0.2,0.2)}$
	0.7	0.6	0	$Q_{(0,0.6)}$
Highly Optimistic	0.8	0.1	0.15	$Q_{(0.15,0.1)}$
	0.8	0.2	0.1	$Q_{(0.1,0.2)}$

The presented attitude-based consensus model has been applied to simulate a real-life problem, whose formulation is as follows: let us suppose that a conference scientific committee compound by 50 scientists, $E = \{e_1, \ldots, e_{50}\}$, must grant a best Ph.D. student paper award to one out of four possible candidate papers, $X = \{x_1 =$ John's paper, $x_2 =$ Wang's paper, $x_3 =$ Sue's paper, $x_4 =$ Michael's paper$\}$. The committee must achieve a minimum level of agreement of $\mu = 0.85$ before making a decision.

The experiments consisted in defining a total of five different attitudes towards consensus, ϑ, where both optimistic, indifferent and pessimistic attitudes are reflected, and applying a CRP based on the model presented in Sect. 4. For each attitude, two different degrees of the amount of information used, given by φ, have been considered (taking into account the restrictions pointed out in Sect. 3.2). Table 1 shows the different group attitudes used in simulations, the obtained value of α [as stated in Eq. (3.4)] and the subsequent definition of ten different RIM quantifiers (denoted as $Q_{(\alpha,\varphi)}$) used in experiments. For each instance of Attitude-OWA, 20 experiments have been run.

Results from experiments include the convergence to consensus achieved, i.e. the average number of rounds of discussion required to reach a consensus for each Attitude-OWA operator defined upon an RIM quantifier. These results, which are shown in Fig. 7, allow us to confirm our hypothesis that the use of Attitude-OWA operator based on

Fig. 7 The average number of required rounds of discussion for RIM quantifier-based Attitude-OWA operators with different attitudinal parameters given by ϑ and φ



an optimistic attitude favours a greater convergence towards consensus, whereas the use of Attitude-OWA operator based on a pessimistic attitude favours a lower convergence and a further discussion process, regardless of the proportion of values considered, φ.

It can be concluded that the main advantage of integrating the group's attitude in the CRP is the fact that it lets us adapt and optimize such a process, according to the specific needs of decision makers for each GDM problem to be addressed. For instance, if decision makers' priority is achieving a consensus in a fast discussion process and they do not care about considering the highest agreement positions, they would adopt an optimistic attitude. On the other hand, if they consider that the problem requires further discussion and they want to ensure that even the most discrepant experts finally reach an agreement, they would rather consider a pessimistic attitude.

6 Conclusions and future works

In this paper, we have studied the concept of group's attitude towards consensus by means of an extension of OWA operators, the so-called Attitude-OWA, and presented a consensus model which allows to integrate it in the consensus-reaching process. The attitudinal parameters involved in the defined operator have been thoroughly studied. In addition, the performance of the proposed approach has been analysed through a simulation to solve a real group decision making problem with many experts in an automatic consensus support system. Having shown the effect of using optimistic/pessimistic attitudes in the number of discussion rounds necessary to achieve an agreement (the more optimistic the attitude, the higher is the convergence towards consensus, and vice versa), we conclude that the integration of the group's attitude provides the advantage that the consensus-reaching process can be easily adapted and optimized according to the group's needs, by choosing the appropriate values for attitudinal parameters.

Our future works are currently focused on a further analysis of the proposed attitudinal parameters, as well as introduction of the possibility that experts can express their desired attitudes in a linguistic background, thus giving them an even more natural way to provide attitudinal information. We also aim to extend Attitude-OWA operator to apply it to consensus processes where different types of quantifiers with diverse membership functions can be used, and extend the consensus model to make it adaptive, under the assumption that the group's attitude might change during the discussion process

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