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# A Variable-Decoupling- and MSR-Based Imaging Algorithm for a SAR of Curvilinear Orbit 

Peng Zhou, Mengdao Xing, Tao Xiong, Yong Wang, and Lei Zhang


#### Abstract

For a synthetic aperture radar (SAR) onboard a platform with a rectilinear track, the range history of a point target can be accurately expressed hyperbolically. The track can be curvilinear for a maneuverable SAR platform. The hyperbolic equation becomes inadequate, and an expression with high-order terms is needed. Using the method of series reversion, we derived the $2-D$ spectrum for the return signal of the curvilinear SAR. There were five independent variables in the spectrum, but available imaging algorithms could only handle three in the focusing using the spectrum. Thus, a variable-decoupling method was developed to reparameterize the initial spectrum so that only three variables remained. After the incorporation of the decoupling method into the chirp-scaling algorithm, simulations of the SAR with a curvilinear track were studied. Promising results were obtained.


Index Terms-Chirp-scaling (CS) algorithm, method of series reversion (MSR), synthetic aperture radar (SAR) of rectilinear or curvilinear track, 2-D spectrum.

## I. Introduction

ASYNTHETIC aperture radar (SAR) mounted on a maneuverable platform exhibits a curvilinear flight path. The range history of a point target cannot be expressed accurately using a hyperbolical function. An expression with higher order terms is needed. Thus, not only the history becomes complicated but also the validity of imaging algorithms based on the hyperbolical function could become questionable. To study the complicated range history, one corrects the curved flight path to a straight line [1] and then uses the hyperbolical function in the derivation of the 2-D spectrum or derives the 2-D spectrum using the complex range history [2]. The second type of approaches is popular now because of the direct use of the range history and efficiency. Thus, we focus on the second type of approaches.

Once the range history from the curvilinear flight path is expressed as a function with higher order terms, the method of series reversion (MSR) [3] can be useful in the derivation of the 2-D spectrum [4]. There are five independent

[^0]

Fig. 1. Geometry of a maneuverable SAR platform.
variables coupled in the MSR-derived 2-D spectrum. They are the range frequency, azimuth frequency, slant range, and $x$ and $y$ coordinates of a ground target. Unfortunately, current and available imaging algorithms can only handle the first three. None can be directly applied to the spectrum that is of two extra coupled variables. In the derivation of the 2-D spectrum, Yi et al. [4] replaced the $x$ and $y$ coordinates with the coordinates of a beam center. This approach works fine when the radar is narrow in beamwidth but deteriorates greatly when a wide imaging region is needed. Therefore, a variable-decoupling method was investigated. The main idea of the decoupling includes the establishment of a one-to-one relationship between the instantaneous range of a ground target at zero azimuth time and each extra variable using the concept of instantaneous Doppler frequency (IDF). After the decoupling, a new 2-D spectrum with only three variables is acquired. Thus, available imaging algorithms can be used to process the data. Simulation results show that our method focuses and performs well when the imaging region is wide in range. Details are given next.

## II. Geometry of a Maneuverable Sar Platform, and an MSR-Based 2-D Spectrum

The geometry of a maneuverable SAR platform is shown in Fig. 1. The SAR is traveling along $\overparen{A B}$ in plane $X O Z$. Let $t_{m}$ be azimuth time. Assume that, at $t_{m}=0$, the sensor is at position $O^{\prime}$ with a height of $H$. The projection of $O^{\prime}$ on $X O Y$ or ground is origin $O \cdot \vec{v}=\left(v_{x 0}, 0, v_{z 0}\right)$ and $\vec{a}=\left(\alpha_{x}, 0, \alpha_{z}\right)$ denote the velocity and acceleration vectors of the sensor at that time, respectively. Therefore, the location of the sensor at $t_{m}$ is $\left(v_{x 0} t_{m}+0.5 \alpha_{x} t_{m}^{2}, 0, H+v_{z 0} t_{m}+0.5 \alpha_{z} t_{m}^{2}\right)$. Let $P$ be a ground target with coordinates $\left(x_{P}, y_{P}, 0\right) . P_{\text {cen }}$ is a reference target located at the beam center. $R_{0}$ is the instantaneous
distance between the sensor and $P$ at $t_{m}=0$, and $R\left(t_{m}\right)$ is the range history of $P$. Thus, backscattering from $P$ can be written as

$$
\begin{align*}
s\left(\hat{t}, t_{m}\right)= & w_{r}\left(\hat{t}-\frac{2 R\left(t_{m}\right)}{c}\right) w_{a}\left(t_{m}\right) \exp \left[-j \frac{4 \pi}{\lambda} R\left(t_{m}\right)\right] \\
& \bullet \exp \left[j \pi \gamma\left(\hat{t}-\frac{2 R\left(t_{m}\right)}{c}\right)^{2}\right] \tag{1}
\end{align*}
$$

where $\hat{t}$ is the range time, $\lambda$ is the wavelength, $c$ is the speed of light, and $\gamma$ is the chirp rate. $w_{r}(\bullet)$ and $w_{a}(\bullet)$ are the range and azimuth envelopes, respectively.

To obtain the 2-D spectrum of $s\left(\hat{t}, t_{m}\right)$ analytically, we use a 2-D Fourier transform. During the derivation, the principle of stationary phase is adapted. The degree of complexity in the derivation largely depends on the expression of $R\left(t_{m}\right)$. For the SAR of rectilinear orbit

$$
\begin{equation*}
R\left(t_{m}\right)=\sqrt{R_{0}^{2}+\left(x_{P}-|\vec{v}| t_{m}\right)^{2}} \tag{2}
\end{equation*}
$$

applying (2) into (1), one can obtain its 2-D spectrum ([5, Ch. 5]). However, for the SAR of curvilinear orbit, we have (3), shown at the bottom of the page, with

$$
\left.\begin{array}{ll}
R_{0}=\sqrt{x_{p}^{2}+y_{p}^{2}+H^{2}} &  \tag{4}\\
\mu_{1}=-2\left(v_{x 0} x_{p}-v_{z 0} H\right) & \mu_{2}=v_{x 0}^{2}+v_{z 0}^{2}-a_{x} x_{p}+a_{z} H \\
\mu_{3}=v_{x 0} a_{x}+v_{z 0} a_{z} & \mu_{4}=\frac{1}{4}\left(a_{x}^{2}+a_{z}^{2}\right)
\end{array}\right\}
$$

Equation (3) is more complex than (2). This complexity causes a sixth-degree polynomial equation with respect to $t_{m}$ that needs to be solved in the derivation of the Fourier integral [6], and one cannot solve the equation algebraically. Alternatively, the MSR [3] is introduced to derive the 2-D spectrum of $s\left(\hat{t}, t_{m}\right)$.

Using the Taylor expansion, one can express (3) as

$$
\begin{equation*}
R\left(t_{m}\right) \approx R_{0}+b_{1} t_{m}+b_{2} t_{m}^{2}+b_{3} t_{m}^{3}+b_{4} t_{m}^{4} \tag{5}
\end{equation*}
$$

with

$$
\left.\begin{array}{ll}
b_{1}=\frac{\mu_{1}}{2 R_{0}} & b_{2}= \\
\frac{\mu_{2}}{2 R_{0}}-\frac{\mu_{1}^{2}}{8 R_{0}^{3}} \\
b_{3}=\frac{\mu_{3}}{2 R_{0}}-\frac{\mu_{1} \mu_{2}}{4 R_{0}^{3}}+\frac{\mu_{1}^{3}}{16 R_{0}^{5}} & b_{4}= \\
& \frac{\mu_{4}}{2 R_{0}}-\frac{2 \mu_{1} \mu_{3}+\mu_{2}^{2}}{8 R_{0}^{3}} \\
& +\frac{3 \mu_{1}^{2} \mu_{2}}{16 R_{0}^{5}}-\frac{5 \mu_{1}^{4}}{128 R_{0}^{7}}
\end{array}\right\}
$$

Then, the MSR-based 2-D spectrum [4] is

$$
\begin{equation*}
S_{\mathrm{MSR}}=w_{r}\left(f_{r}\right) w_{a}\left(f_{a}\right) \exp \left[-j \pi \frac{f_{r}^{2}}{\gamma}\right] \exp [j \Phi] \tag{7}
\end{equation*}
$$



Fig. 2. Geometry in plane $O^{\prime} O X$.
where $f_{r}$ is the range frequency and $f_{a}$ is the azimuth frequency. The second phase term

$$
\begin{align*}
\Phi= & -4 \pi \frac{R_{0}}{c}\left(f_{r}+f_{c}\right) \\
& +\pi \frac{c}{4 b_{2}} \frac{1}{\left(f_{r}+f_{c}\right)}\left(f_{a}+\left(f_{r}+f_{c}\right) \frac{2 b_{1}}{c}\right)^{2} \\
& +\pi \frac{c^{2} b_{3}}{16 b_{2}^{3}} \frac{1}{\left(f_{r}+f_{c}\right)^{2}}\left(f_{a}+\left(f_{r}+f_{c}\right) \frac{2 b_{1}}{c}\right)^{3} \\
& +\pi \frac{c^{3}\left(9 b_{3}^{2}-4 b_{2} b_{4}\right)}{256 b_{2}^{5}} \frac{1}{\left(f_{r}+f_{c}\right)^{3}}\left(f_{a}+\left(f_{r}+f_{c}\right) \frac{2 b_{1}}{c}\right)^{4} . \tag{8}
\end{align*}
$$

From (4) and (7), one finds that the MSR-based 2-D spectrum $S_{\text {MSR }}$ is a function of five independent variables, namely, $f_{r}$, $f_{a}, R_{0}, x_{P}$, and $y_{P}$. Thus, $S_{\mathrm{MSR}}$ can be generally expressed as

$$
\begin{equation*}
S_{\mathrm{MSR}}=S_{\mathrm{MSR}}\left(f_{r}, f_{a}, R_{0}, x_{P}, y_{P}\right) \tag{9}
\end{equation*}
$$

It should be noted that, for a SAR with a rectilinear orbit, the 2-D spectrum only consists of variables $f_{r}, f_{a}$, and $R_{0}$, and a typical imaging algorithm (e.g., the chirp-scaling (CS) algorithm) can be applied to process the spectrum. However, there are two extra variables $x_{P}$ and $y_{P}$ for a SAR of curvilinear track. As a result, none of the available imaging algorithms can be applied to process $S_{\mathrm{MSR}}$ directly. Thus, a variabledecoupling method is pursued next.

## III. Decoupling Method

## A. Decoupling $y_{P}$

$y_{P}$ is only in the expression of $R_{0}$, the first equation in (4). Thus, if one just treats $R_{0}$ as an independent variable (but not a function of $y_{P}$ ) and considers $y_{P}$ as an inherent component of $R_{0}, y_{P}$ is decoupled in (9). Also, $R_{0}$, as a whole, can be calculated using radar parameters so that the decoupling of $y_{P}$ from the 2-D spectrum is feasible.

$$
\begin{align*}
R\left(t_{m}\right) & =\sqrt{\left(v_{x 0} t_{m}+0.5 a_{x} t_{m}^{2}-x_{p}\right)^{2}+y_{p}^{2}+\left(H+v_{z 0} t_{m}+0.5 a_{z} t_{m}^{2}\right)^{2}} \\
& =\sqrt{R_{0}^{2}+\mu_{1} t_{m}+\mu_{2} t_{m}^{2}+\mu_{3} t_{m}^{3}+\mu_{4} t_{m}^{4}} \tag{3}
\end{align*}
$$

## B. Decoupling $x_{P}$

To decouple $x_{P}$, one needs to use the concept of IDF of target $P$ to create a new relationship between $x_{P}$ and $R_{0}$. In Fig. 1, $O^{\prime} N$ is the extended line along the moving direction of $\vec{v}$ at $t_{m}=0$. Draw line $Q N$, and let $Q N \perp O^{\prime} N$. Because $P Q \perp$ $X O Z$, then $P Q \perp O^{\prime} N$. Thus, $O^{\prime} N \perp \triangle P Q N$, and $O^{\prime} N \perp$ $P N$. After the establishment of new coordinates $X^{\prime} O^{\prime} Y^{\prime}$ on the plane where $\Delta O^{\prime} P N$ exists, one obtains the squint angle $\theta$. Therefore, the expression of the IDF of $P$ is

$$
\begin{align*}
f_{\mathrm{id}} & =\frac{2|\vec{v}|}{\lambda} \sin \theta=\frac{2|\vec{v}|}{\lambda} \sin \left(\frac{\pi}{2}-\angle P O^{\prime} N\right) \\
& =\frac{2|\vec{v}|}{\lambda} \sin \left(\frac{\pi}{2}-\arccos \left(\frac{O^{\prime} N}{R_{0}}\right)\right) \tag{10}
\end{align*}
$$

where $O^{\prime} N$ is unknown, whereas the rest of the parameters are known. To solve $O^{\prime} N$, we redraw plane $O^{\prime} O X$ as in Fig. 2. Add line $O M$ perpendicular to line $O^{\prime} X^{\prime}$ and $Q T$ perpendicular to $O M$. Thus, one has

$$
\begin{equation*}
O^{\prime} N=O^{\prime} M+M N=O^{\prime} M+T Q=H \cos \alpha+x_{P} \sin \alpha \tag{11}
\end{equation*}
$$

where $\quad \alpha=\left\{\begin{array}{ll}\pi-\tan ^{-1}\left(v_{x} / v_{z}\right), & v_{z}>0 \\ -\tan ^{-1}\left(v_{x} / v_{z}\right), & v_{z} \leq 0 .\end{array}\right.$ Substituting (11) into (10) and after algebraic manipulation, one obtains

$$
\begin{equation*}
x_{P}=\frac{\lambda f_{\mathrm{id}} R_{0}}{2|\vec{v}| \sin \alpha}-H \cot \alpha \tag{12}
\end{equation*}
$$

Thus, $x_{P}$ is a function of $f_{\text {id }}$ and $R_{0}$. If $f_{\text {id }}$ is known, $x_{P}$ becomes a function of $R_{0}$ only. $x_{P}$ is no longer an independent variable. Since $P$ and $P_{\text {cen }}$ are within the same instantaneous beam, then intuitively, we can set $f_{\text {id }}$ of $P$ being equal to the IDF of $P_{\text {cen }}$ or the Doppler centroid $\left(f_{\mathrm{dc}}\right)$ of the return signal. Substituting $f_{\text {id }}$ with $f_{\mathrm{dc}}$ in (12), one approximates

$$
\begin{equation*}
x_{P}=\frac{\lambda f_{\mathrm{dc}} R_{0}}{2|\vec{v}| \sin \alpha}-H \cot \alpha \tag{13}
\end{equation*}
$$

Thus, $x_{P}$ is approximately one-to-one related to $R_{0}$. This approximation is feasible if the beamwidth is not very wide in the azimuth direction. In such case, the difference of $f_{\text {id }}$ and $f_{\text {dc }}$ is small. Therefore, with (13) and the first equation in (4), we have decoupled $x_{P}$ and $y_{P}$ from the phase term in (9). Thus, $\Phi$ can be re-expressed as $\Phi_{\text {new }}\left(f_{r}, f_{a}, R_{0}\right)$, and the 2-D spectrum (9) can be re-expressed with only three variables as

$$
\begin{align*}
S_{\mathrm{MSR} \_ \text {new }}\left(f_{r}, f_{a}, R_{0}\right)= & w_{r}\left(f_{r}\right) w_{a}\left(f_{a}\right) \exp \left[-j \pi \frac{f_{r}^{2}}{\gamma}\right] \\
& \times \exp \left[j \Phi_{\text {new }}\left(f_{r}, f_{a}, R_{0}\right)\right] \tag{14}
\end{align*}
$$

The spectrum for a SAR onboard a maneuverable platform with a curvilinear orbit is now similar in form to the spectrum for a SAR with a rectilinear orbit but with different content. An imaging algorithm that processes the spectrum with three variables can be used to focus the radar return. Before the implementation of the re-expressed spectrum into the imaging algorithm, one needs to understand the re-expressed spectrum further.

## IV. Re-Expressed Spectrum

$\Phi_{\text {new }}$ in (14) can be expanded into a polynomial with respect to $f_{r}$. Since $S_{\mathrm{MSR} \text { _new }}$ and $S_{\mathrm{MSR}}$ are of similar form, we directly use the expansion of $S_{\mathrm{MSR}}$ in [4] to expand $\Phi_{\text {new }}$ as

$$
\begin{equation*}
\Phi_{\mathrm{new}}=\phi_{0}\left(f_{a} ; R_{0}\right)+\phi_{1}\left(f_{a} ; R_{0}\right) f_{r}+\phi_{2}\left(f_{a} ; R_{0}\right) f_{r}^{2}+\cdots \tag{15}
\end{equation*}
$$

where $\phi_{1}$ contains the range cell migration (RCM) and is

$$
\begin{align*}
& \phi_{1}\left(f_{a} ; R_{0}\right) \\
& =-4 \pi \frac{R_{0}}{c}+\pi \frac{c}{4 b_{2}}\left(4\left(\frac{b_{1}}{c}\right)^{2}-\frac{f_{a}^{2}}{f_{c}^{2}}\right) \\
& +\pi \frac{c^{2} b_{3}}{16 b_{2}^{3}}\left[8\left(\frac{b_{1}}{c}\right)^{3}-2 \frac{f_{a}^{3}}{f_{c}^{3}}-6 \frac{b_{1}}{c} \frac{f_{a}^{2}}{f_{c}^{2}}\right]+\pi \frac{c^{3}\left(9 b_{3}^{2}-4 b_{2} b_{4}\right)}{256 b_{2}^{5}} \\
& \times\left[-3 \frac{f_{a}^{4}}{f_{c}^{4}}-16 \frac{1}{f_{c}^{3}} \frac{b_{1}}{c} f_{a}^{3}-24 \frac{1}{f_{c}^{2}}\left(\frac{b_{1}}{c}\right)^{2} f_{a}^{2}+16\left(\frac{b_{1}}{c}\right)^{4}\right] \tag{16}
\end{align*}
$$

Equation (16) is generally complex but can be regrouped and expanded into another Taylor series at a reference range $R_{\text {ref }}$

$$
\begin{align*}
\phi_{1}\left(f_{a} ; R_{0}\right)= & -4 \pi \frac{R_{0}}{c}+\phi_{\Delta}\left(f_{a} ; R_{0}\right) \approx-4 \pi \frac{R_{0}}{c} \\
& +A\left(f_{a} ; R_{\mathrm{ref}}\right)+B\left(f_{a} ; R_{\mathrm{ref}}\right)\left(R_{0}-R_{\mathrm{ref}}\right) \tag{17}
\end{align*}
$$

where $A\left(f_{a} ; R_{\text {ref }}\right)$ and $B\left(f_{a} ; R_{\text {ref }}\right)$ are Taylor coefficients

$$
\left.\begin{array}{l}
A\left(f_{a} ; R_{\mathrm{ref}}\right)=\left.\phi_{\Delta}\left(f_{a} ; R_{0}\right)\right|_{R_{0}=R_{\mathrm{ref}}}  \tag{18}\\
B\left(f_{a} ; R_{\mathrm{ref}}\right)=\left.\frac{\partial \phi_{\Delta}\left(f_{a} ; R_{0}\right)}{\partial R_{0}}\right|_{R_{0}=R_{\mathrm{ref}}}
\end{array}\right\} .
$$

The instantaneous range of $P_{\text {cen }}$ at $t_{m}=0$ is usually chosen as $R_{\text {ref }}$. With (17) the RCM can be written as

$$
\begin{align*}
& R\left(f_{a}, R_{0}\right) \\
& =-\frac{c}{4 \pi} \phi_{1}\left(f_{a} ; R_{0}\right)=R_{0}-\frac{c}{4 \pi} A\left(f_{a} ; R_{\mathrm{ref}}\right) \\
& \quad-\frac{c}{4 \pi} B\left(f_{a} ; R_{\mathrm{ref}}\right)\left(R_{0}-R_{\mathrm{ref}}\right)=R_{0}+R_{\mathrm{res}}+a\left(f_{a}\right) R_{0} \tag{19}
\end{align*}
$$

with $\quad R_{\text {res }}=-(c / 4 \pi)\left[A\left(f_{a} ; R_{\text {ref }}\right)-B\left(f_{a} ; R_{\mathrm{ref}}\right) R_{\mathrm{ref}}\right] \quad$ and $a\left(f_{a}\right)=-(c / 4 \pi) B\left(f_{a} ; R_{\mathrm{ref}}\right)$. In (19), the RCM is linearly related to $R_{0}$ so that the CS algorithm [7] can be applied to it. Then, the scaled RCM is

$$
\begin{equation*}
R_{\mathrm{CS}}\left(f_{a} ; R_{0}\right)=R_{0}+R_{\mathrm{res}}+a\left(f_{a}\right) R_{\mathrm{ref}} \tag{20}
\end{equation*}
$$

This equation indicates that, after the CS operation, targets of different $R_{0}$ 's have the same RCM curve as that of the reference target with $R_{\text {ref }}$. Thus, a bulk RCM correction could be carried out in the 2-D frequency domain to eliminate $R_{\text {res }}+a\left(f_{a}\right) R_{\text {ref }}$. Next, we implement and validate the decoupling method.

## V. Implementation Into the CS Algorithm

In a typical CS algorithm, one starts with the transform of raw data in the time domain into the range-Doppler domain


Fig. 3. Flow diagram of the variable-decoupling- and MSR-based CS algorithm.

TABLE I
Simulation Parameters

| Wavelength | 0.02 m |
| :---: | :---: |
| Bandwidth | 100 MHz |
| Oversampling ratio | 1.5 |
| Pulse duration | $2 \mu \mathrm{~s}$ |
| Antenna size | 1 m |
| PRF | 8 kHz |
| H | 9 km |
| $\vec{v}$ | $(1200,0,-170) \mathrm{m} / \mathrm{s}$ |
| $\vec{a}$ | $(-10,0,5) \mathrm{m} / \mathrm{s}^{2}$ |

(Fig. 3). However, with required decoupling, we need to apply an inverse Fourier transform in range to (14) to derive the expression in the range-Doppler domain

$$
\begin{align*}
S\left(\hat{t}, f_{a}\right)= & \exp \left[j \phi_{0}\left(f_{a} ; R_{0}\right)\right] \\
& \times \exp \left[j \pi \gamma_{e}\left(f_{a} ; R_{0}\right)\left(\hat{t}-\frac{2}{c} R\left(f_{a} ; R_{0}\right)\right)^{2}\right] \tag{21}
\end{align*}
$$

with $\left(1 / \gamma_{e}\left(f_{a} ; R_{0}\right)\right)=(1 / \gamma)-\left(\phi_{2}\left(f_{a} ; R_{0}\right) / \pi\right)$ and amplitude ignored. Then, similar to the derivations in the CS algorithm [7], we obtain $H_{\mathrm{CS}}, H_{\mathrm{rg}}$, and $H_{\mathrm{az}}$ (Fig. 3) as

$$
\begin{aligned}
H_{\mathrm{CS}}=\exp & {\left[j \pi \gamma_{e}\left(f_{a} ; R_{\mathrm{ref}}\right) a\left(f_{a}\right)\right.} \\
& \left.\times\left(\hat{t}-\frac{2}{c}\left(R_{\mathrm{ref}}+a\left(f_{a}\right) R_{\mathrm{ref}}+R_{\mathrm{res}}\right)\right)^{2}\right]
\end{aligned}
$$

$$
\begin{align*}
H_{\mathrm{rg}}\left(R_{0}, f_{a}\right)= & \exp \left[j \frac{\pi}{\gamma_{e}\left(f_{a} ; R_{\mathrm{ref}}\right)\left(a\left(f_{a}\right)+1\right)} f_{r}^{2}\right]  \tag{22}\\
& \bullet \exp \left[j \frac{4 \pi}{c}\left(a\left(f_{a}\right) R_{\mathrm{ref}}+R_{\mathrm{res}}\right) f_{r}\right] \tag{23}
\end{align*}
$$

$$
\begin{equation*}
H_{\mathrm{az}}\left(R_{0}, f_{a}\right)=\exp \left[-j \phi_{0}\left(f_{a} ; R_{0}\right)\right] \exp \left[-j \Theta_{\Delta}\left(f_{a}, R_{0}\right)\right] \tag{24}
\end{equation*}
$$

with

$$
\begin{aligned}
\Theta_{\Delta}\left(f_{a}, R_{0}\right)= & \pi \frac{4}{c^{2}} \gamma_{e}\left(f_{a} ; R_{\mathrm{ref}}\right)\left(1+a\left(f_{a}\right)\right) a\left(f_{a}\right) \\
& \times\left[\left(R_{0}-R_{\mathrm{ref}}\right)^{2}-4 R_{\mathrm{ref}} R_{\mathrm{res}}\right] .
\end{aligned}
$$

Next, simulations are carried out, and the results are discussed.


Fig. 4. Five ground targets $A-E$ and their coordinates. Range contours are shown as dashed lines, and Doppler frequency contours are shown as curves at $t_{m}=0$.


Fig. 5. Contours of the impulse responses from targets $A-C$ using (a) this method and (b) the method in [4].


Fig. 6. Contours of the impulse responses from targets $D$ and $E$ using (a) this method and (b) the method in [4].

## VI. Simulations and Results

The simulation parameters are given in Table I. The range resolution is 1.33 m , and the azimuth resolution is 0.44 m after the rectangular weighting with a factor of 0.886 . Five ground targets $A-E$ with their $x$ and $y$ coordinates are shown in Fig. 4. The footprint center of $A$ is selected as the reference point $P_{\text {cen }}$ with $R_{\text {ref }}$ of 10548 m . Fig. 4 shows the range contours (dashed lines) and IDF contours (curves) at $t_{m}=0$. Targets $B$

TABLE II
IRW, PSLR, and ISLR Data for the Five Targets Using the Decoupling Method and the Method in [4]. The Values From the Method in [4] Are in Parentheses

| Targets |  | $A$ | $B$ | $C$ | $D$ | $E$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Azimuth | IRW $(\mathrm{m})$ | $0.44(0.45)$ | $0.46(0.57)$ | $0.45(0.64)$ | $0.45(0.45)$ | $0.61(0.61)$ |
|  | PSLR $(\mathrm{dB})$ | $-13.35(-13.35)$ | $-13.35(-18.61)$ | $-13.35(-23.39)$ | $-13.25(-13.07)$ | $-12.72(-12.80)$ |
|  | ISLR $(\mathrm{dB})$ | $-10.36(-10.37)$ | $-10.82(-15.67)$ | $-10.28(-18.46)$ | $-10.12(-10.00)$ | $-10.01(-10.10)$ |
| Range | IRW $(\mathrm{m})$ | $1.34(1.5)$ | $1.34(1.56)$ | $1.34(2.13)$ | $1.34(1.34)$ | $1.34(1.34)$ |
|  | PSLR $(\mathrm{dB})$ | $-13.28(-17.53)$ | $-13.26(-18.39)$ | $-13.23(-31.8)$ | $-13.28(-13.29)$ | $-13.25(-13.26)$ |
|  | ISLR $(\mathrm{dB})$ | $-9.92(-16.91)$ | $-9.85(-13.82)$ | $-10.01(-25.62)$ | $-9.92(-9.92)$ | $-9.90(-9.90)$ |

and $C$, as well as $A$, of the same IDF but at different ranges are chosen to test if the decoupling algorithm can handle a large imaging extent in the range direction. Targets $D$ and $E$, and $A$, that have the same range but differ in IDFs are selected to see whether the algorithm can process a large azimuth extent.
The impulse responses of targets $A, B$, and $C$ are shown as contours in the range and azimuth plane [Fig. 5(a)]. The responses are well focused with clear separations of the main lobes and first and subsequential sidelobes. For comparison, we also show the simulated outputs using the method in [4] in Fig. 5(b). The degree of the focusing of targets $A-C$ indicates some deterioration. There are no clear delineations between the main lobes and sidelobes for $B$ and $C$, although the delineation is clear for $A$.

The impulse responses of target $D$ [Fig. 6(a)] are well focused in the range direction. However, defocusing occurs in the azimuth direction, which is caused by the difference in IDFs of $D$ and $A$. The azimuth broadening is $\sim 4 \%$, which is acceptable in SAR data processing. The impulse responses of target $E$ might be similar to those of target $D$, but the azimuth defocusing worsens. The broadening is around $39 \%$ or unacceptable. The cause is directly related to the large distance between $E$ and $A$ as compared to the small distance of $D$ and $A$. In contrast, the impulse responses of targets $D$ and $E$ [Fig. 6(b)] using the method by Yi et al. could be similar in focusing. Finally, the values of impulse response width (IRW), peak sidelobe ratio (PSLR), and integral sidelobe ratio (ISLR) in the range and azimuth directions are further used to assess both methods. Quantitatively, our method is generally superior to the method by Yi et al. (Table II). Consequently, with the incorporation of the decoupling method into the CS algorithm, promising results are obtained for a SAR onboard a maneuverable platform.

## VII. CONCLUSION

For an echo acquired by a SAR with a curvilinear track, we have developed a variable-decoupling method to decouple the $x$ and $y$ coordinates in the 2-D spectrum of the echo on the basis of an imaging geometry of the curvilinear track and using the concept of IDF. The decoupling was done through the reparameterization of $x$ or $y$ solely as a function of the instantaneous distance between the sensor and the target at zero azimuth time $\left(R_{0}\right)$, as well as the position, attitude, and system parameters of the SAR. Equivalently, only the range frequency, azimuth frequency, and $R_{0}$ remained in the 2-D spectrum after decoupling. Then, we implemented the decoupling method into a widely used CS imaging algorithm. Promising results from targets spread widely in the range and azimuth dimensions were obtained in the simulations.

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