have $\gamma_{i-1} < \gamma_i, 2 \leq i \leq N$. This implies that the diagonal elements of $\check{\mathbf{R}}$ are nondecreasing (more precisely, stay the same inside a block, but increase from a block to the next).

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Downlink Throughput Maximization for OFDMA Systems With Feedback Channel Capacity Constraints

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Abstract—In this correspondence, we study the downlink throughput maximization for orthogonal frequency-division multiple-access (OFDMA) systems in the presence of feedback channel capacity constraints. We establish an information-theoretic lower bound on the capacity of feedback channel and build the corresponding test channel that achieves this lower bound. Based on the derived test channel, we formulate two optimization problems that maximize the downlink throughput with quantized channel state information (CSI): i) one problem where throughput is defined with the ergodic throughput and ii) the other problem with the outage throughput, from which we can see the performance limit for given limit feedback channels. We solve the throughput maximization problem through an iterative approach, which achieves the optimal ergodic throughput and the near-optimal outage throughput. Numerical results show that the downlink throughput with a limited feedback of CSI can be close to that with perfect CSI by exploiting correlation properties of downlink CSI for quantization.

Index Terms—Limited feedback, orthogonal frequency-division multiple access (OFDMA), quantized channel information, rate distortion, resource allocation.

I. INTRODUCTION

Orthogonal frequency-division multiple access (OFDMA) is a promising multiple-access technique that provides high spectral efficiency for next-generation broadband wireless systems. In the downlink of a cellular OFDMA system, the base station (BS) communicates with users over a set of subcarriers. For systems that employ frequency-division duplexing (FDD), the BS obtains the downlink channel state information (CSI) from users through feedback channels. If perfect CSI is available at the BS, flexible resource allocation schemes can considerably improve system performance. However, conveying perfect CSI requires infinite CSI-feedback rate. In practical communication systems, since the capacity of the feedback channel is limited, only quantized feedback CSI can be fed back to the BS. As a result, the performance of resource allocation schemes is degraded due to imperfect CSI. Analyzing the effect of finite feedback rate in OFDMA systems turns to be a crucial problem.

In this correspondence, we investigate the performance limit imposed on the downlink throughput of an OFDMA system with quantized feedback. The main contributions of our correspondence include the following.

- We use the rate-distortion theory to derive the minimum quantization distortion under a rate constraint of feedback channel. Then, we evaluate the downlink throughput with the quantized CSI, by which we can characterize the maximum achievable downlink throughput for a given capacity of feedback channel. To the best of our knowledge, maximizing the OFDMA downlink throughput

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from the rate-distortion theory point of view has not been studied yet.

- We evaluate the downlink throughput based on two information-theoretic metrics, the ergodic throughput and the outage throughput (under the assumption that an ideal channel code is used). For delay-insensitive services, the ergodic throughput is a suitable performance measure [1], while for real-time applications that cannot tolerate long delays, it is more appropriate to consider the outage throughput [2]. However, to the best of our knowledge, no work has considered outage throughput maximization for OFDMA systems.
- We propose the iterative methods to maximize the ergodic throughput and the outage throughput, respectively. For ergodic throughput maximization, the iterative method can find the optimal solution; for outage throughput maximization, the iterative method can achieve a near-optimal solution.

Notations: Vectors and matrices are represented by bold. $\mathbf{0}_N$ denotes an $N \times 1$ column vector whose elements are all zeros, $\mathbf{I}_N$ denotes an $N \times N$ identity matrix, and $\mathbf{B}^H$ denotes the conjugate transpose of $\mathbf{B}$. $E[\cdot]$ denotes the statistical expectation, and in particular $E_X[\cdot]$ denotes that with respect to X.

A. Overview

We continue the introduction with a short review of related work in SubSection I-B. Section II outlines the downlink channel model, and derives the RDF for the downlink CSI. Section III presents the expressions of the ergodic throughput and the outage throughput, and proposes resource allocation algorithms to maximize the ergodic throughput and the outage throughput, respectively. Numerical results are presented in Section IV, and conclusions are drawn in Section V.

B. Related Works

In OFDMA systems using FDD, due to the limited capacity of feedback channel, the transmitter can obtain only some level of downlink CSI from the receivers. Thus, resource allocation with quantized CSI becomes one of the most critical research topics in OFDM systems. Quantized feedback in single-user OFDM systems with on-off power allocations was the focus of [3] and [4]. Adaptive subcarrier selection, power allocation, and modulation selection using only one bit per subcarrier were investigated by [5]. Later, an approximate waterfilling method using order information of the subcarrier channel gains was proposed by [6]. Limited feedback schemes with multiple quantization regions of downlink CSI in OFDMA systems were extensively studied in [7] and [8]. In both works, the design parameters related to quantized CSI, such as quantization levels and feedback period, were optimized to reduce the feedback overhead with a guaranteed system performance for OFDMA systems. In [9], the authors derived close-form expressions for the ergodic throughput in an OFDMA system assuming equal power distribution over all subcarriers. However, existing research works for OFDMA systems have focused on simple but suboptimal quantization methods. Thus, these results could not show the best achievable performance when the quantization for CSI feedback is optimized in terms of the rate-distortion theory point of view. As mentioned earlier, we address the performance limit of OFDMA systems with quantized CSI using the rate-distortion theory in this correspondence.

II. System Model

Consider an OFDMA system with N subcarriers that will be shared by K users. We assume that this system employs FDD, and thereby, the BS obtains the downlink CSI from users' feedback. With the knowledge of CSI, the BS can perform the subcarrier and power allocation.

A. Downlink Channel Model

The downlink channel is assumed to be a multipath fading channel. The baseband channel gain from the BS to the kth user on the nth subcarrier can be written as

$$H_{k,n} = \sum_{l=1}^{L_k} a_{k,l} e^{-j2\pi\tau_{k,l}(n-\frac{N+1}{2})\Delta f} \tag{1}$$

where L_k is the number of multipath taps, Δf is the subcarrier spacing, and $a_{k,l}$ and $\tau_{k,l}$ denote the attenuation factor and the propagation delay of the lth multipath tap at the kth user's channel, respectively. The multipath channel taps at the kth user $(a_{k,1}, \ldots, a_{k,L})^T$ can be modeled as a zero-mean circularly symmetric complex Gaussian (ZMCSCG) vector with independent entries $a_{k,l} \sim \mathcal{CN}(0, \sigma^2_{a_{k,l}})$. Then, $\mathbf{H}_k = (H_{k,1}, \ldots, H_{k,N})^T$ satisfies $\mathbf{H}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}_{\mathbf{H}_k})$, where the (n_1, n_2)th entry of $\mathbf{\Sigma}_{\mathbf{H}_k}$ is

$$[\mathbf{\Sigma}_{\mathbf{H}_k}]_{n_1,n_2} = \sum_{l=1}^{L_k} \sigma^2_{a_{k,l}} e^{-j2\pi\tau_{k,l}(n_1-n_2)\Delta f}. \tag{2}$$

B. Feedback of Downlink CSI

Now we consider the quantization of downlink CSI and determine the capacity of feedback channel required to deliver the quantized CSI using the rate-distortion theory. From this, we can characterize the minimum distortion of the quantized CSI for a given capacity of the feedback channel.

The user k describes his/her knowledge of downlink CSI $\mathbf{H}_k$ by an index $I_k \in \mathcal{I}_k = \{1, 2, \ldots, 2^{C_k}\}$, and sends the index I_k to the BS. The BS then reproduces the channel gain $\hat{\mathbf{H}}_k$ from the index I_k, where $\hat{\mathbf{H}}_k = (\hat{H}_{k,1}, \ldots, \hat{H}_{k,N})^T$ is the quantized description of $\mathbf{H}_k$.

We introduce the distortion measure function with the following criterion:

$$d(\mathbf{H}_k, \hat{\mathbf{H}}_k) = \sum_{n=1}^{N} |H_{k,n} - \hat{H}_{k,n}|^2.$$

Then, we can define the information RDF of $\mathbf{H}_k$ as [10]

$$R_k(D_k) = \inf_{E[d(\mathbf{H}_k, \hat{\mathbf{H}}_k)] \le D_k} I(\mathbf{H}_k; \hat{\mathbf{H}}_k)$$

where D_k denotes an upper bound on the quantization error, and $I(\mathbf{H}_k; \hat{\mathbf{H}}_k)$ denotes the mutual information between $\mathbf{H}_k$ and $\hat{\mathbf{H}}_k$.

By the source-channel separation theorem [10], the quantization error D_k is achievable if and only if the feedback channel's capacity of the user k satisfies $C_k > R_k(D_k)$. Thus, to characterize the feedback channel's capacity C_k, we need to find the RDF for $\mathbf{H}_k$. We have the following result.

Theorem 1: Suppose that we are given a ZMCSCG vector $\mathbf{H}_k$ with the autocorrelation given in (2). Let the eigenvalue decomposition of $\mathbf{\Sigma}_{\mathbf{H}_k}$ be

$$\mathbf{\Sigma}_{\mathbf{H}_k} = \mathbf{U}_k \mathbf{\Lambda}_k \mathbf{U}^H \tag{3}$$

where $\mathbf{U}_k$ is the $N \times N$ matrix with orthogonal column vectors and $\mathbf{\Lambda}_k$ is the $N \times N$ diagonal matrix with $[\mathbf{\Lambda}_k]_{n,n} = \lambda_{k,n}$. Then,

1) The RDF of $\mathbf{H}_k$ is given by

$$R_k(D_k) = \sum_{n=1}^{N} \log \max\left\{\frac{\lambda_{k,n}}{\theta_k}, 1\right\}, \tag{4}$$

where

$$D_k = \sum_{n=1}^{N} \min\{\theta_k, \lambda_{k,n}\}. \tag{5}$$

Here, θ_k is the Lagrangian multiplier which can be decided for given D_k to satisfy (5).

2) The test channel that achieves the RDF is given by

$$\mathbf{H}_k = \hat{\mathbf{H}}_k + \mathbf{U}_k\mathbf{Z}_k, \quad \hat{\mathbf{H}}_k = \mathbf{U}_k\hat{\mathbf{Y}}_k \tag{6}$$

where $\hat{\mathbf{Y}}_k = (\hat{Y}_{k,1}, \ldots, \hat{Y}_{k,N})^T$ and $\mathbf{Z}_k = (Z_{k,1}, \ldots, Z_{k,N})^T$ are mutually independent ZMCSCG vectors with uncorrelated components, and $\hat{Y}_{k,n} \sim \mathcal{CN}(0, \max\{\lambda_{k,n} - \theta_k, 0\})$ and $Z_{k,n} \sim \mathcal{CN}(0, \min\{\lambda_{k,n}, \theta_k\})$.

Sketch of Proof: We prove the theorem by transforming the correlated ZMCSCG vector $\mathbf{H}_k$ to an uncorrelated ZMCSCG vector $\mathbf{Y}_k = (Y_{k,1}, \ldots Y_{k,N})^T$ by setting $\mathbf{Y}_k = \mathbf{U}_k^H\mathbf{H}_k$. From (3), we have $\mathbf{Y}_k \sim \mathcal{CN}(\mathbf{0}_N, \mathbf{\Lambda}_k)$. Let $\hat{\mathbf{Y}}_k = \mathbf{U}_k^H\hat{\mathbf{H}}_k$. Since $\mathbf{U}_k$ is unitary and invertible, we have $I(\mathbf{Y}_k; \hat{\mathbf{Y}}_k) = I(\mathbf{H}_k; \hat{\mathbf{H}}_k)$ and $d(\mathbf{Y}_k, \hat{\mathbf{Y}}_k) = d(\mathbf{H}_k, \hat{\mathbf{H}}_k)$. Now the problem reduces to finding the RDF for the uncorrelated ZMCSCG vector $\mathbf{Y}_k$. Following the similar approach which derives the RDF of a parallel Gaussian source in [10, Theorem 10.3.3], we can obtain the result in Theorem 1. ■

By the second part of Theorem 1, the probability distributions of $\mathbf{U}_k\mathbf{Z}_k$ and $\hat{\mathbf{H}}_k$ are

$$\mathbf{U}_k\mathbf{Z}_k \sim \mathcal{CN}(\mathbf{0}_N, \mathbf{\Sigma}_k), \quad \hat{\mathbf{H}}_k \sim \mathcal{CN}(\mathbf{0}_N, \mathbf{\Sigma}_{\mathbf{H}_k} - \mathbf{\Sigma}_k) \tag{7}$$

where $\mathbf{\Sigma}_k = \mathbf{U}_k\mathbf{\Sigma}_{\mathbf{Z}_k}\mathbf{U}_k^H$ and $\mathbf{\Sigma}_{\mathbf{Z}_k} = \mathrm{diag}(\min\{\lambda_{k,1}, \theta_k\}, \ldots, \min\{\lambda_{k,N}, \theta_k\})$. The nth diagonal element of $\mathbf{\Sigma}_k$, denoted by $\sigma_{k,n}^2$, can be regarded as the variance of quantization error for $H_{k,n}$. From (6) and (7), the conditional pdf of $H_{k,n}$ for a given $\hat{H}_{k,n}$ is $H_{k,n} \mid \hat{H}_{k,n} \sim \mathcal{CN}(\hat{H}_{k,n}, \sigma_{k,n}^2)$. Thus, the actual power gain $\alpha_{k,n} = |H_{k,n}|^2$ conditioned on $\hat{\alpha}_{k,n} = |\hat{H}_{k,n}|^2$ is

$$f(\alpha_{k,n}|\hat{\alpha}_{k,n}) = \frac{1}{\sigma_{k,n}^2} e^{-\frac{\hat{\alpha}_{k,n}+\alpha_{k,n}}{\sigma_{k,n}^2}} I_0\left(\frac{2}{\sigma_{k,n}^2}\sqrt{\hat{\alpha}_{k,n}\alpha_{k,n}}\right) \tag{8}$$

where $I_0(\cdot)$ is the 0th-order modified Bessel function of the first kind. Note that although we only consider the distortion due to the quantization in this correspondence, it is straightforward to take into account feedback delay through the prediction error as shown in [11, eq. (7)].

III. Downlink Throughput Maximization With Quantized CSI

For a given capacity of the feedback channel, we have characterized the distortion in Section II-B. With the quantized downlink CSI, the resource allocation can be carried out for a given performance measure. Here, we consider maximizing the ergodic throughput and the outage throughput.

A. Definition of the Performance Measures

1) Ergodic Throughput: The ergodic throughput is defined as the average data rate over all possible fading states. For a given quantized power gain, $\hat{\alpha}_{k,n}$, the ergodic throughput of the nth subcarrier, provided that this subcarrier is assigned to the kth user, is expressed as

$$T_{k,n}^e(\gamma_n, \hat{\alpha}_{k,n}) = E_{\alpha_{k,n} \mid \hat{\alpha}_{k,n}}[\log_2(1 + \alpha_{k,n}\gamma_n) \mid \hat{\alpha}_{k,n}] \tag{9}$$

where γ_n denotes the input signal-to-noise ratio (SNR) of the nth subcarrier, which is proportional to the transmit power on the nth subcarrier. In the following, the terms "input SNR" and "transmit power" are used without distinction.

2) Outage Throughput: Given $\hat{\alpha}_{k,n}$, the outage probability on the nth subcarrier to the kth user is

$$\epsilon = \Pr(\log_2(1 + \alpha_{k,n}\gamma_n) < R \mid \hat{\alpha}_{k,n}) \tag{10}$$

where R is the transmission rate. It can be shown that the maximum transmission rate R that can maintain the outage probability ϵ is

$$R(\gamma_n, \hat{\alpha}_{k,n}, \epsilon) = \log_2(1 + \gamma_n F_{\alpha_{k,n} \mid \hat{\alpha}_{k,n}}^{-1}(\epsilon))$$

where $F_{\alpha_{k,n} \mid \hat{\alpha}_{k,n}}^{-1}(\epsilon)$ is the inverse function of $F_{\alpha_{k,n} \mid \hat{\alpha}_{k,n}}(x) = \Pr(\alpha_{k,n} < x \mid \hat{\alpha}_{k,n})$ which denotes the cumulative distribution function (cdf) of $\alpha_{k,n}$ conditioned on $\hat{\alpha}_{k,n}$. Here, we define the outage throughput as the maximum expected information successfully delivered to users,

$$T_{k,n}^o(\gamma_n, \hat{\alpha}_{k,n}) = \max_{\epsilon}(1 - \epsilon)R(\gamma_n, \hat{\alpha}_{k,n}, \epsilon). \tag{11}$$

Setting $\alpha_0 = F_{\alpha_{k,n} \mid \hat{\alpha}_{k,n}}^{-1}(\epsilon)$ and substituting (8) into (11) yield

$$T_{k,n}^o(\gamma_n, \hat{\alpha}_{k,n}) = \max_{\alpha_0} \tilde{T}_{k,n}^o(\gamma_n, \hat{\alpha}_{k,n}, \alpha_0) \tag{12}$$

where

$$\tilde{T}_{k,n}^o(\gamma_n, \hat{\alpha}_{k,n}, \alpha_0) = Q\left(\sqrt{\frac{2\hat{\alpha}_{k,n}}{\sigma_{k,n}^2}}, \sqrt{\frac{2\alpha_0}{\sigma_{k,n}^2}}\right)\log_2(1+\alpha_0\gamma_n) \tag{13}$$

where $Q(a, b) = \int_b^{+\infty} x e^{-(x^2+a^2)/2} I_0(ax)dx$. The following theorem shows that the outage throughput $T_{k,n}^o(\gamma_n, \hat{\alpha}_{k,n})$ is well defined.

Theorem 2: Given $\gamma_n \geq 0$ and $\hat{\alpha}_{k,n} \geq 0$, there exists a unique globally optimal α_0 that maximizes $\tilde{T}_{k,n}^o(\gamma_n, \hat{\alpha}_{k,n}, \alpha_0)$.

Proof: It has been proved in [12, Theorem 2.7] that $\ln Q(\sqrt{a}, \sqrt{b})$ is concave in b for $b > 0, a \geq 0$. Thus, it is easy to see that $\ln \tilde{T}_{k,n}^o(\gamma_n, \hat{\alpha}_{k,n}, \alpha_0) = \ln\log_2(1 + \alpha_0\gamma_n) + \ln Q(\sqrt{2\hat{\alpha}_{k,n}/\sigma_{k,n}^2}, \sqrt{2\alpha_0/\sigma_{k,n}^2})$ is concave in α_0. In addition, we have $\lim_{\alpha\to 0} \tilde{T}_{k,n}^o(\gamma_n, \hat{\alpha}_{k,n}, \alpha_0) = 0$, and

$$\begin{aligned}
0 &\leq \lim_{\alpha_0\to+\infty} \tilde{T}_{k,n}^o(\gamma_n, \hat{\alpha}_{k,n}, \alpha_0) \\
&\leq \lim_{\alpha_0\to+\infty} \frac{\log_2(1 + \alpha_0\gamma_n)}{\exp\left(\left(\sqrt{\frac{2\alpha_0}{\sigma_{k,n}^2}} - \sqrt{\frac{2\hat{\alpha}_{k,n}}{\sigma_{k,n}^2}}\right)^2 /2\right)} \\
&= \frac{1}{\ln 2}\lim_{\alpha_0\to+\infty} \frac{\gamma_n}{(1 + \alpha_0\gamma_n)\left(1 - \sqrt{\frac{\hat{\alpha}_{k,n}}{\alpha_0}}\right)/\sigma_{k,n}^2} \\
&\quad \times \frac{1}{\exp\left(\left(\sqrt{\frac{2\alpha_0}{\sigma_{k,n}^2}} - \sqrt{\frac{2\hat{\alpha}_{k,n}}{\sigma_{k,n}^2}}\right)^2 \Big/ 2\right)} \\
&= 0
\end{aligned} \tag{14}$$

where we have used L'Hospital's rule and the upper bound $Q(a, b) \leq \exp(-(b - a)^2/2)$ [13]. Thus, there exists a unique and globally optimal α_0 that maximizes $\tilde{T}_{k,n}^o(\gamma_n, \hat{\alpha}_{k,n}, \alpha_0)$. ■

B. Problem Formulation

Now we can formulate the downlink throughput maximization problem as follows:

$$\begin{aligned}
&\max_{\rho_{k,n}, \gamma_n} \quad \sum_{k=1}^{K}\sum_{n=1}^{N} w_k\rho_{k,n}T_{k,n}(\gamma_n, \hat{\alpha}_{k,n}) \\
&\text{subject to} \quad \begin{cases} \sum_k \rho_{k,n} = 1, \ \forall n, \\ \sum_n \gamma_n \leq \gamma_T, \\ \rho_{k,n} \in \{0, 1\}, \gamma_n \geq 0 \ \forall k, n \end{cases}
\end{aligned} \tag{15}$$

where $T_{k,n}$ can be either $T_{k,n}^e$ for ergodic throughput maximization or $T_{k,n}^o$ for outage throughput maximization, $\rho_{k,n}$ denotes the subcarrier allocation indicator, w_k is the positive constraint such that $\sum_k w_k = 1$.[1] Here, the first constraint ensures that each subcarrier is assigned to one user exclusively, and the second constraint is for total transmit power, denoted by γ_T.

Note that in (15), we only need $\alpha_{k,n} = |H_{k,n}|^2$ rather than $H_{k,n}$. Thus, it could be more efficient to feed back $\hat{\alpha}_{k,n}$ than $\hat{H}_{k,n}$ to minimize the CSI-feedback rate for a fixed distortion or to minimize the distortion for a fixed CSI-feedback rate. However, as shown above, the feedback of the channel gains allows the information-theoretic approach to relate the actual channel gain to the feedback channel gain for a given fixed CSI-feedback rate.

C. Joint Subcarrier and Power Allocation Algorithm

When the transmit power γ_n is fixed, the original problem (15) is decomposed into N independent subproblems:

$$\begin{aligned}\max_{\rho_{k,n}} \quad & \sum_k w_k \rho_{k,n} T_{k,n}(\gamma_n, \hat{\alpha}_{k,n}) \\ \text{subject to} \quad & \sum_k \rho_{k,n} = 1, \quad \rho_{k,n} \in \{0,1\}. \end{aligned} \tag{16}$$

Thus, the optimal subcarrier allocation is given by

$$\rho_{k,n} = \begin{cases} 1 & \text{if } k = \arg\max_k w_k T_{k,n}(\gamma_n, \hat{\alpha}_{k,n}) \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

When the subcarrier assignment $\rho_{k,n}$ is fixed, the problem in (15) becomes

$$\begin{aligned}\max_{\gamma_n} \quad & \sum_n w_{k_n} T_{k_n,n}(\gamma_n, \hat{\alpha}_{k_n,n}) \\ \text{subject to} \quad & \sum_n \gamma_n \le \gamma_T, \ \gamma_n \ge 0 \end{aligned} \tag{18}$$

where $k_n = \arg\max_k \rho_{k,n}$ denotes the assigned user on the nth subcarrier. We solve the power allocation problem through the dual approach. The dual of (18) is

$$\min_{\mu \ge 0} g(\mu) \tag{19}$$

where

$$\begin{aligned} g(\mu) &= \max_{\gamma_1,\ldots,\gamma_N \ge 0} \sum_n w_{k_n} T_{k_n,n}(\gamma_n, \hat{\alpha}_{k_n,n}) - \mu \left(\sum_n \gamma_n - \gamma_T \right) \\ &= \sum_n \max_{\gamma_n \ge 0} (w_{k_n} T_{k_n,n}(\gamma_n, \hat{\alpha}_{k_n,n}) - \mu \gamma_n) + \mu \gamma_T \end{aligned}$$

where μ is the Lagrangian multiplier of the first constraint in (18). Given μ, the optimal power allocation on the nth subcarrier is

$$\gamma_n = \arg\max_{\gamma} w_{k_n} T_{k_n,n}(\gamma, \hat{\alpha}_{k_n,n}) - \mu\gamma. \tag{20}$$

To find the optimal global optimal multiplier μ, we can use the bisection method [15]. Note that for $T_{k,n} = T^e_{k,n}$, the problem in (18) is a convex optimization problem, and the dual approach yields the water-filling solution, which achieves the optimum of (18) [16]; for $T_{k,n} = T^e_{k,n}$, since the problem in (18) is nonconvex, using the dual approach we can only obtain an upper bound on the optimum of (18), and the final power allocation γ_n^* may not satisfy $\sum_n \gamma_n^* \le \gamma_T$. In order to project the power allocation to the feasible region, we multiply the final power allocation on each subcarrier γ_n^* by $\gamma_T / \sum_n \gamma_n^*$.

[1]In this correspondence, we assume that the w_k's are given by quality of service (QoS) constraints or priorities of users [14, eq. (3.1), p. 32].

TABLE I
JOINT SUBCARRIER AND POWER ALLOCATION ALGORITHMS

1: Set $\gamma_n = \gamma_T/N$, $m = 0$, $T^{(m)} = 0$, $T^{(m-1)} = -\infty$
2: **while** $T^{(m)} - T^{(m-1)} < \epsilon$ and $m < maxLoopCount$ **do**
3: Determine subcarrier allocation $\rho_{k,n}$ using (17)
4: Use dual approach to determine the power allocation γ_n
5: Update $m = m + 1$ and $T^{(m)} = \sum_k \sum_n w_k \rho_{k,n} T_{k,n}(\gamma_n, \hat{\alpha}_{k,n})$
6: **end while**

TABLE II
COMPARISONS OF THE SUBOPTIMAL OUTAGE THROUGHPUT AND THE UPPER BOUND ON THE OPTIMUM. (a) $\gamma_T/N = 10$ dB, $(w_1, w_2) = (0.45, 0.55), D_2 = 6$, (b) $\gamma_T/N = 10$ dB, $D_1 = 2, D_2 = 6$, and (c) $(w_1, w_2) = (0.45, 0.55)$, $D_1 = 2, D_2 = 6$

(a)

D_1	1	3	5
Suboptimum (bps/Hz)	8.6135	7.6485	7.2152
Upper-bound (bps/Hz)	8.6267	7.6630	7.2279
Relative Gap (%)	0.15	0.19	0.18

(b)

w_1	0.500	0.550	0.600
Suboptimum (bps/Hz)	8.1716	8.5055	9.0062
Upper-bound (bps/Hz)	8.1892	8.5200	9.0227
Relative Gap (%)	0.22	0.17	0.18

(c)

γ_T/N (dB)	10	30	50
Suboptimum (bps/Hz)	8.0173	28.7760	53.6244
Upper-bound (bps/Hz)	8.0354	28.8047	53.6545
Relative Gap (%)	0.23	0.1	0.06

When both subcarrier assignment and power allocation can be adjusted, we can iteratively allocate subcarriers and power, which is described in Table I. Each iteration of the algorithm contains two steps. First, we evaluate the achievable throughput of all users on all subcarriers with the power allocation obtained in the previous iteration, and perform subcarrier allocation using (17). Then, we update the transmit power p_n on all subcarriers using the proposed dual approach given by (20). Note that for $T_{k,n} = T^e_{k,n}$, the optimal subcarrier and power allocation must simultaneously satisfy (17) and the water-filling solution given by the dual approach. Thus, similar to the discussion in [17, Sec. IV-B], the iterative method gives the global maximum of ergodic throughput. For $T_{k,n} = T^o_{k,n}$, since the dual approach in Step 4 is suboptimal, the iterative method yields a suboptimal solution.

IV. NUMERICAL RESULTS

We present several numerical results to demonstrate the performance of OFDMA systems under rate-distortion limit using the proposed algorithms. Our simulation is based on the COST259 channel model for a typical urban environment with $E[|h_{k,n}|^2] = 1$ [18]. To simulate imperfect CSI, we generate IID realizations of $\mathbf{Y}_k$ and $\mathbf{Z}_k$ according to Theorem 1. Then, we can generate $\hat{\mathbf{H}}_k$ and $\mathbf{H}_k$ under rate-distortion limit using (6).

First, we compare the suboptimal outage throughput given by the proposed iterative method with the upper bound on the optimum in Table II. To obtain this upper bound, we consider all possible subcarrier allocations and assign the transmit power using the dual approach without projecting the final power allocation back to the feasible region. We assume $\mathbf{\Sigma}_{\mathbf{H}_k} = \mathbf{I}_N$, $N = 8$ subcarriers, and $K = 2$ users. We can see that the performance gap between the suboptimum and the optimum is less than 0.01, and thus, the iterative method for the outage throughput maximization is near-optimal.

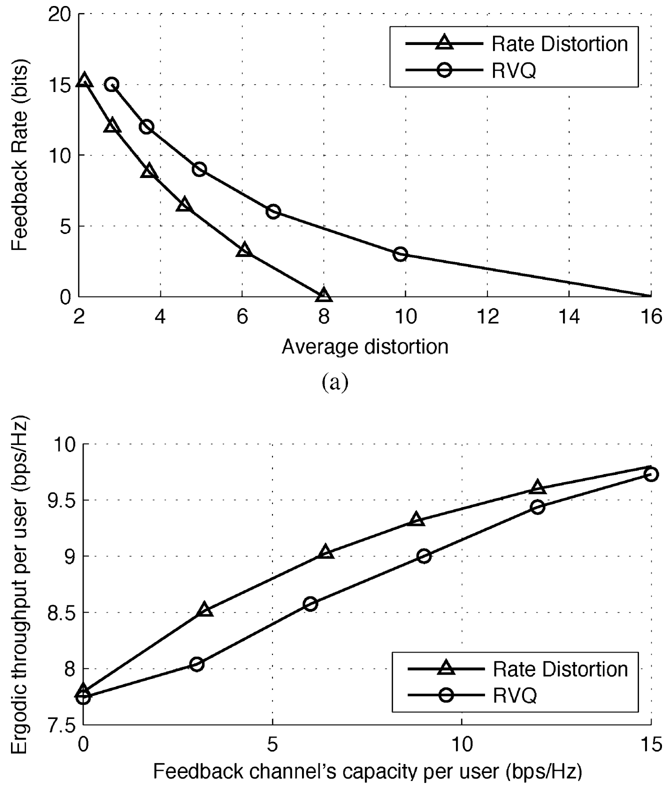

Fig. 1. Comparisons with RVQ: (a) RDF versus mean quantization error; (b) ergodic throughput versus capacity of feedback channel.

Next, assuming that there are $N = 8$ subcarriers with $\mathbf{\Sigma}_{\mathbf{H}_k} = \mathbf{I}_N$ and $K = 6$ users with $w_k = 1/K$, we compare the system performance under the rate-distortion limit with an extension of the random vector quantization (RVQ) method [19]. In this method, the codebook $\mathcal{C}_k = \{\mathbf{W}_{k,1}, \ldots, \mathbf{W}_{k,2^{C_k}}\}$ at each user k consists of 2^{C_k} independently chosen vectors with $\mathbf{W}_{k,i} \sim \mathcal{CN}(\mathbf{0}_N, \mathbf{I}_N)$. Each user quantizes its channel to the quantization vector closest to its channel vector, i.e., $I_k = \arg\min_i \sum_n |H_{k,n} - W_{k,i,n}|^2$, where $W_{k,i,n}$ is the nth element of $\mathbf{W}_{k,i}$. The BS determines subcarrier and power allocation based on the decoded quantized channel vector. In Fig. 1(a), when the capacity of feedback channel is $C_k = 0$, the quantization error of the RVQ method is $\sum_n E[|H_{k,n} - W_{k,1,n}|^2] = \sum_n E[|H_{k,n}|^2] + E[|W_{k,1,n}|^2] = 2N$; however, as C_k increases, the performance of the RVQ method approaches the rate-distortion limit. In Fig. 1(b), at $C_k = 15$ b/s/Hz, the RVQ achieves 99% of the rate-distortion limit.

Finally, in Fig. 2, we simulate the system performance in the rate-distortion limit under different frequency-selective channels. We also consider the case in which the BS has perfect CSI. In this simulation, we assume that there are $N = 32$ subcarriers with transmit power $\gamma_T/N = 20$ dB, and $K = 8$ users with equal weights $w_k = 1/K$. In Fig. 2(a), for a given quantization error, the required capacity of feedback channel decreases with the subcarrier spacing. This is due to the fact that the correlation between adjacent subcarriers becomes weaker as subcarriers are more widely separated. Thus, we can see in Fig. 2(b) and (c) that both throughput increase as the subcarrier spacing decreases for a given feedback channel's capacity. At $\Delta f = 15$ kHz, the outage throughput can achieve 99% of that with perfect CSI when the feedback capacity is $C_k/N = 1.81$ b/s/Hz, while the ergodic throughput can achieve 99% of that with perfect CSI when $C_k/N = 0.43$ b/s/Hz. Thus, we can see that although the feedback channel's capacity is finite, a near ideal performance can be achieved.

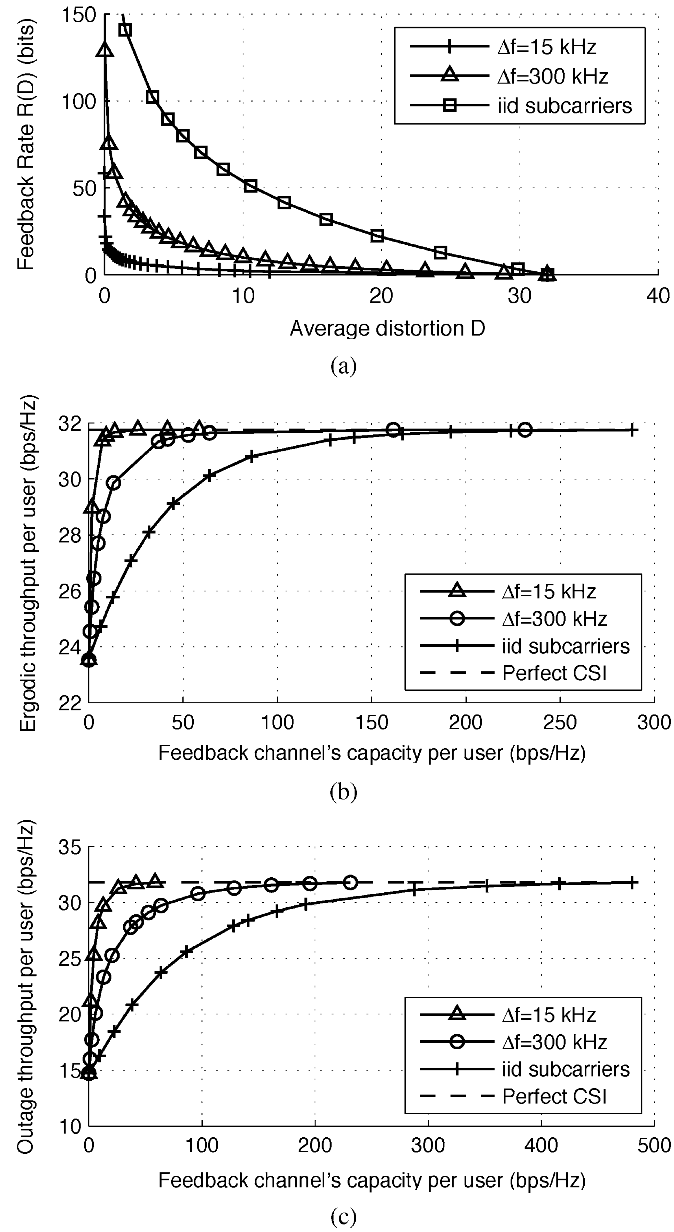

Fig. 2. Performance comparison under different frequency-selective fadings: (a) RDF versus mean quantization error; (b) ergodic throughput versus capacity of feedback channel; and (c) outage throughput versus capacity of feedback channel.

V. Conclusion

In this correspondence, we investigated the downlink throughput maximization for an OFDMA system with finite feedback rate. First, assuming that the ZMCSCG channel information is fed back to the BS, we derived the RDF for the CSI. According to the rate-distortion theory, the RDF can give a lower bound on the capacity of feedback channel. We also derived the test channel that achieves this RDF. This derived test channel enables us to formulate the resource allocation problems that maximize the ergodic throughput and outage throughput with a rate constraint on feedback channel. Then, we proposed an iterative method to solve the two throughput maximization problems, and showed that the proposed method achieves the optimum for ergodic throughput and a near-optimum for outage throughput. Through numerical results, we found that by exploiting the correlations between subcarriers, the ergodic throughput and outage throughput with a limited feedback rate can approach that with perfect CSI.

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Robust Transceiver Optimization for Downlink Multiuser MIMO Systems

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***Abstract*—This correspondence addresses the joint transceiver design for downlink multiuser multiple-input multiple-output (MIMO) systems, with imperfect channel state information (CSI) at the base station (BS) and mobile stations (MSs). By incorporating antenna correlation at both ends of the channel and taking channel estimation errors into account, we solve two robust design problems: minimization of the weighted sum mean-square-error (MSE) and minimization of the maximum weighted MSE. These problems are solved as follows: first, we establish three kinds of MSE uplink-downlink duality by transforming only the power allocation matrices from uplink channel to downlink channel and vice versa. Second, in the uplink channel, we formulate the power allocation part of each problem ensuring global optimality. Finally, based on the solution of the uplink power allocation and the MSE duality results, for each problem, we propose an iterative algorithm that performs optimization alternatively between the uplink and downlink channels. Computer simulations verify the robustness of the proposed design compared to the nonrobust/naive design.**

***Index Terms*—Downlink, duality, mean-square error, multiple-input multiple-output (MIMO), multiuser, Robust Transceiver design.**

I. Introduction

In a multiuser network the uplink-downlink duality approach for solving the downlink optimization problems has received a lot of attention. The achievable sum rate of the broadcast channel (BC) obtained by dirty paper precoding technique has been characterized for multiple-input single-output (MISO) systems [1]. The latter work has been extended in [2] for multiple-input multiple-output (MIMO) systems. These papers analyze the sum rate region of the BC channel by exploiting the duality between BC and multiple access channels (MAC). In [3], the dirty paper rate region has shown to be the capacity region of the Gaussian MIMO BC channel. In [4] and [5], mean-square-error (MSE) based uplink-downlink duality have been exploited. The latter two papers utilize their duality results to solve MSE-based design problems. All of the aforementioned duality are established by assuming that perfect channel state information (CSI) is available at the base station (BS) and mobile stations (MSs). However, due to the inevitability of channel estimation error, CSI can never be perfect. This motivates [6] to establish the MSE duality under imperfect CSI for MISO systems. The latter work is extended in [7] for MIMO case. None of [6] and [7] incorporates antenna correlation in their channel model

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