Adaptive Fuzzy Tracking Control of a Class of Stochastic Nonlinear Systems with Unknown Dead-Zone Input

Jianjiang Yu, Kanjian Zhang, and Shumin Fei

Abstract

In this paper, a direct adaptive fuzzy tracking control scheme is presented for a class of stochastic uncertain nonlinear systems with unknown dead-zone input. A direct adaptive fuzzy tracking controller is developed by using the backstepping approach. It is proved that the design scheme ensures that all the error variables are bounded in probability while the mean square tracking error becomes semiglobally uniformly ultimately bounded (SGUUB) in an arbitrarily small area around the origin. Simulation results show the effectiveness of the control scheme.

Keywords: Stochastic system; Adaptive control; Fuzzy control; Dead-zone input; backstepping

1. Introduction

Recently, the adaptive control of uncertainty nonlinear system has been extensively studied. As a breakthrough in nonlinear control area, a recursive design procedure, backstepping approach, was presented to obtain global stability and asymptotic tracking for a large class of nonlinear system, mostly the strict-feedback system [1,2]. In some backstepping-based control laws ([3]-[8]) for stochastic strict-feedback systems that include a Wiener process have been developed to guarantee stability, known as stability in probability. Combined problem of the control of stochastic strict-feedback nonlinear system with nonlinear uncertainties is firstly studied in [9].

Nonsmooth nonlinear characteristics such as dead-zone, backlash, hysteresis are common in actuator and sensors such as mechanical connections, hydraulic actuators and electric servomotors. Dead-zone is one of the most important non-smooth nonlinearities in many industrial processes. Its presence severely limits system performance, and its study has been drawing much interest in the control community for a long time ([2] and the reference therein).

In this paper, we consider adaptive fuzzy tracking controller for a class of stochastic uncertain nonlinear systems with dead-zone input. As in [2], the dead-zone output is represented as a simple linear system with a static time-varying gain and bounded disturbance by introducing characteristic function. The first type fuzzy system is employed to approximate the unknown nonlinear system. Extensive stability analysis proves that all the error variables are bounded in probability while the mean square tracking error becomes semiglobally uniformly ultimately bounded in an arbitrarily small area around the origin.

This paper is organized as follows. The preliminaries and problem formulation are presented in Section II. In Section III, a systematic procedure for the synthesis of the adaptive fuzzy tracking controller is developed. In Section IV, simulation example is used to demonstrate the effectiveness of the proposed scheme. Finally, the conclusion is given in Section V.

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Manuscript accepted 15 March 2008.

2. Preliminaries and Problem Formulation

The following notation will be used throughout the paper. For a given vector or matrix X, X^T denotes its transpose, $Tr\{X\}$ denotes its trace when X is square, and $|X|$ denotes the Euclidean norm of a vector X. C^i denotes the set or all functions with continuous i th partial derivatives. K denotes the set of all functions: $R_+ \to R_+$, which are continuous, strictly increasing and vanishing at zero, K_∞ denotes the set of all functions which are of class K and unbounded; KL denotes the set of all functions $\beta(s,t)$: $R_+ \times R_+ \to R_+$ which is of class K for each fixed t, and decreases to zero as $t \to \infty$ for fixed s.

Consider the following stochastic nonlinear system:

$$dx = f(x)dt + h(x)d\omega, \quad \forall x \in R^n \qquad (1)$$

where $x \in R^n$ is the state of the system, ω is $r-$dimensional independent standard Wiener process defined on a probability space (Ω, F, P). $f(\cdot)$, $h(\cdot)$ are locally Lipschitz in x.

Definition 1 For any given $V(x)\in C^2$, associated with stochastic system (1), we define the differential operator L as follows:

$$LV(x)=\frac{\partial V}{\partial x}f(x)+\frac{1}{2}Tr\{h^T(x)\frac{\partial^2 V}{\partial x^2}h(x)\} \quad (2)$$

Definition 2 For the stochastic system (stochastic) with $f(0)=h(0)=0$, the equilibrium $x(t)=0$ is

(i) globally stable in probability if for any $0<\varepsilon<1$, there exists a class K function $\gamma(\cdot)$ such that

$P\{|x(t)|<\gamma(|x_0|)\}\ge 1-\varepsilon, \forall t\ge 0, \forall x_0\in R^n-\{0\}$;

(ii) globally asymptotically stable in probability if $\forall\varepsilon>0$, there exists a class KL function $\beta(\cdot,\cdot)$ such that

$P\{|x(t)|<\beta(|x_0|,t)\ge 1-\varepsilon, \forall t\ge 0, \forall x_0\in R^n-\{0\}$.

Lemma1[9] Consider the stochastic nonlinear system (1). If there exists a positive definite, radially unbounded, twice continuously differentiable Lyapunov function $V:R^n\to R$, and constants $c_1>0$, $c_2>0$ such that

$$LV(x)=-c_1V(x)+c_2 \quad (3)$$

then (i) the system has a unique solution almost surely and (ii) the system is bounded in probability.

Lemma2[10] Let $f(x)$ be a continuous function defined on a compact set Ω. Then for any constant $\varepsilon>0$, there exists a fuzzy logic system such that

$$\sup_{x\in\Omega}|f(x)-\theta^T\xi(x)|\le\varepsilon. \quad (4)$$

Define the ideal estimation parameter as

$$\theta^*=\arg\min_{\theta\in\Theta}[\sup_{x\in\Omega}|f(x)-\theta^T\xi(x)|], \quad (5)$$

where Θ and Ω denote the sets of suitable bounds on θ and x.

Consider a SISO stochastic nonlinear systems in the following form:

Plant:

$$\begin{cases} dx_1=x_2dt \\ dx_2=x_3dt \\ \vdots \\ dx_{n-1}=x_ndt \\ dx_n=[f(x)+g(x)u]dt+h^T(x)d\omega \\ y=x_1 \end{cases} \quad (6)$$

Dead-zone:

$$u=D(v)=\begin{cases} g_r(v) & if \quad v\ge b_r, \\ 0 & if \quad b_l<v<b_r, \\ g_l(v) & if \quad v\le b_l, \end{cases} \quad (7)$$

where $x=(x_1,\cdots,x_n)^T$, $y\in R$ are state variables and output, respectively. ω is $r-$ dimensional independent standard Wiener process defined on a probability space (Ω,F,P). $f(x),g(x)$ are unknown smooth functions. $u\in R$ is the output of the dead-zone. $v(t)\in R$ is the input to the dead-zone, b_l and b_r are the unknown parameters of the dead-zone. The reference signal is $y_d(t)$, and $y_d(t),y_d^{(1)}(t),\cdots,y_d^{(n)}(t)$ are smooth and bounded.

Assumption 1 There exist positive constants g_0,g_1 such that $0<g_0\le g(x)\le g_1, \forall x\in\Omega_x\subset R^n$, where Ω_x is a compact set.

Remark 1 In the paper [9], the $g(x)$ must be independent from x_n. In this paper, we removed this restriction.

For the unknown dead-zone input, we make the following assumptions.

Assumption 2 The dead-zone output, u, is not available.

Assumption 3 The dead-zone parameters, b_r and b_l, are unknown bounded constants, but their signs are known, i.e., $b_r>0$ and $b_l<0$.

Assumption 4 The functions, $g_l(v)$ and $g_r(v)$, are smooth, and there exist unknown positive constants, k_{l0},k_{l1},k_{r0}, and k_{r1} such that

$$0 < k_{l0} \le g_l'(v) \le k_{l1}, \quad \forall v \in (-\infty, b_l]$$
$$0 < k_{r0} \le g_r'(v) \le k_{r1}, \quad \forall v \in [b_r, +\infty)$$

$\beta_0 \le \min\{k_{l0}, k_{r0}\}$ is a known positive constant, where $g_l'(v) = \frac{dg_l(z)}{dz}|_{z=v}$ and $g_r'(v) = \frac{dg_r(z)}{dz}|_{z=v}$.

Based on Assumption 4, the dead-zone (5) can be rewritten as follows as shown [2]:

$$u = D(v) = K^T(t)\Phi(t)v + d(v), \tag{8}$$

where

$$\Phi(t) = [\varphi_r(t), \varphi_l(t)]^T,$$
$$\varphi_r(t) = \begin{cases} 1 & if \quad v(t) > b_l, \\ 0 & if \quad v(t) \le b_l, \end{cases}$$
$$\varphi_l(t) = \begin{cases} 1 & if \quad v(t) > b_r, \\ 0 & if \quad v(t) \le b_r, \end{cases}$$
$$K(t) = [g_r'(\xi_r(v(t))),\ g_l'(\xi_l(v(t)))]^T,$$
$$d(v) = \begin{cases} -g_r'(\xi_r(v))b_r & if \quad v \ge b_r, \\ -[g_l'(\xi_l(v)) + g_r'(\xi_r(v))]v & if \quad b_l < v < b_r, \\ -g_l'(\xi_l(v))b_l, & if \quad v \le b_l \end{cases}$$

and $|d(v)| \le p^*$, p^* is an unknown positive constant with $p^* = (k_{r1} + k_{l1})\max\{b_r, -b_l\}$.

Remark 2 There are many results for the case of linear dead-zone outside the deadband, but equation (8) is to capture the most realistic situation. As shown in [2], we know that $K^T(t)\Phi(t) \in [\beta_0, k_{l1} + k_{r1}] \subset (0, +\infty)$.

The control objective is to design an adaptive fuzzy controller $v(t)$ for the system (1) such that the output y follows the specified desired trajectory y_d with guaranteeing that the system is bounded in probability.

3. Control design and stability analysis

In this section, we will use backstepping to design an adaptive fuzzy controller. First, we introduce the error variables

$$z_i = x_i - \alpha_{i-1}(z_{[i-1]}, y_d^{[i-1]}), \quad i = 1, 2, \cdots, n \tag{9}$$

where $\alpha_{i-1}(z_{[i-1]}, y_d^{[i-1]})$ to be given in the following steps, $\alpha_0 = y_d$, $\alpha_n = u$, $z_{[i-1]} = [z_1, \cdots, z_{i-1}]^T$, $y_d^{[i-1]} = [y_d, \dot{y}_d, \cdots, y_d^{i-1}]^T$, $z = z_{[n]}$.

Step 1 The derivation of z_1 is

$$dz_1 = dx_1 - \dot{y}_d dt = (z_2 + \alpha(z_1, y_d, \dot{y}_d) - \dot{y}_d)dt. \tag{10}$$

Consider the following Lyapunov function candidate

$$V_1 = \frac{z_1^2}{2}, \tag{11}$$

the time derivative of V_1 is

$$dV_1 = z_1(z_2 + \alpha_1(z_1, y_d, \dot{y}_d) - \dot{y}_d)dt.$$

Select virtual control $\alpha_1(z_1, y_d, \dot{y}_d)$ as

$$\alpha_1(z_1, y_d, \dot{y}_d) = -k_1 z_1 + \dot{y}_d, \tag{12}$$

with design constant $k_1 > 0$, it is easy to get

$$dV_1 \le (z_1 z_2 - k_1 z_1^2)dt. \tag{13}$$

Step k $(2 \le k \le n-1)$ A similar procedure is employed recursively for each step k. The derivation of α_{k-1} is

$$d\alpha_{k-1} = \left(\sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial z_j} dz_j + \sum_{j=0}^{k-1} \frac{\partial \alpha_{k-1}}{\partial y_d^{(j)}} y_d^{(j+1)}\right)dt$$

then the derivation of z_k is

$$dz_k = [z_{k+1} + \alpha_k - \sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial z_j} dz_j - \sum_{j=0}^{k-1} \frac{\partial \alpha_{k-1}}{\partial y_d^{(j)}} y_d^{(j+1)}]dt, \tag{14}$$

choose the Lyapunov candidate functions and the virtual control laws as follows,

$$V_k = V_{k-1} + \frac{1}{2} z_k^2, \tag{15}$$

$$\alpha_k = -k_k z_k - z_{k-1} + \sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial z_j} dz_j + \sum_{j=0}^{k-1} \frac{\partial \alpha_{k-1}}{\partial y_d^{(j)}} y_d^{(j+1)}, \tag{16}$$

and we get

$$dV_k \le (z_k z_{k+1} - \sum_{j=1}^{k} k_j z_j^2)dt. \tag{17}$$

Step n The derivation of z_n is

$$\begin{aligned} dz_n &= [f(x)+g(x)u]dt + h^T(x)d\omega \\ &\quad - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial z_j}dz_j - [\sum_{j=0}^{n-1}\frac{\partial\alpha_{k-1}}{\partial y_d^{(j)}}y_d^{(j+1)}]dt, \end{aligned} \tag{18}$$

choose Lyapunov candidate as

$$V_n = V_{n-1} + \frac{z_n^4}{4}, \tag{19}$$

then

$$\begin{aligned} LV_n &\le -k_1 z_1^2 - \cdots - \frac{k_n}{2}z_n^4 + \frac{3}{2}z_n^2 h^T h + z_n[z_{n-1} + \\ &\quad \frac{k_n}{2}z_n^3 + z_n^2(f(x)+g(x)u - \dot{\alpha}_{n-1})] \end{aligned} \tag{20}$$

Consider the following inequalities:

$$\begin{aligned} \frac{3}{2}z_n^2 h^T(x)h(x) &\le \frac{1}{\zeta} + \frac{9}{16}\zeta\|h(x)\|^4 z_n^4 \quad \forall\zeta>0 \\ z_n z_{n-1} &\le \frac{1}{\xi} + \frac{1}{4}(\frac{3\xi}{4})^3 z_{n-1}^4 z_n^4 \quad \forall\xi>0 \end{aligned} \tag{21}$$

So,

$$\begin{aligned} LV_n &\le \frac{1}{\zeta} + \frac{1}{\xi} - \sum_{j=1}^{n-1}k_j z_j^2 - \frac{k_n}{2}z_n^4 + z_n^3\{f(x)+g(x)u \\ &\quad - \dot{\alpha}_{n-1} + [\frac{9}{16}\zeta\|h(x)\|^4 + \frac{1}{4}(\frac{3\xi}{4})^3 z_{n-1}^4 + \frac{k_n}{2}]z_n\} \end{aligned} \tag{22}$$

Then use the first type fuzzy systems to approximate $\bar{f}(x) = g_0^{-1}(f(x) - \dot{\alpha}_{n-1} + \frac{9}{16}\zeta\|h(x)\|^4 z_n)$,

$$\bar{f}(x) = \theta^{*T}\xi(x) + \varepsilon_f, \tag{23}$$

where ε_f is approximation error, according to Assumption 1, there exist $\varepsilon_M > 0$ such that $|\varepsilon_f| < \varepsilon_M$. Let $\hat{\theta}$ is the estimation of θ^*, and $\tilde{\theta} = \hat{\theta} - \theta^*$.

We define the overall Lyapunov function as

$$V = V_n + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}, \tag{24}$$

and choose the adaptive law as

$$\dot{\hat{\theta}} = \Gamma[g_0\xi(x)z_n^3 - \sigma(\hat{\theta} - \theta^0)], \tag{25}$$

with gain matrix $\Gamma > 0$. So, we get

$$\begin{aligned} LV &\le \frac{1}{\zeta} + \frac{1}{\xi} - 2\mu_1 V + z_n^3\{g_0\hat{\theta}^T\xi(x) + g(x)u \\ &\quad + [\frac{1}{4}(\frac{3\xi}{4})^3 z_{n-1}^4 + \frac{k_n}{2} + \frac{1}{4}z_n^2]z_n\} + \mu_2^0 \end{aligned} \tag{26}$$

where $\mu_1 = \min_{1\le i\le n}\{(1-\varepsilon_i)k_i, \sigma\lambda_{\min}(\Gamma)\}$, $\mu_2^0 = \frac{1}{2}\|\theta^* - \theta^0\|^2 + g_0\varepsilon_M^2$. It is easy to see $\mu_1 > 0$ and $\mu_2^0 > 0$. Choose the control law v as follows:

$$v = -\frac{1}{\beta_0}(-u_r - u_f). \tag{27}$$

where

$$u_f = -\hat{\theta}^T\xi(x)\tanh(\frac{z_n\hat{\theta}^T\xi(x)}{\delta}), \tag{28}$$

$$u_r = -g_0^{-1}[\frac{1}{4}(\frac{3\xi}{4})^3 z_{n-1}^4 + \frac{k_n}{2} + \frac{1}{2}z_n^2 + \frac{g_1^2}{4}z_n^2]z_n, \tag{29}$$

with δ is a positive constant. Then

$$\begin{aligned} & z_n^3(g_0\hat{\theta}^T\xi(x) + g(x)u_f) \\ & \le z_n^2 g_0(|z_n\hat{\theta}^T\xi(x)| - z_n\hat{\theta}^T\xi(x)\tanh(\frac{z_n\hat{\theta}^T\xi(x)}{\delta})) \\ & \le \kappa + \frac{1}{3}z_n^6, \end{aligned}$$

where $\kappa = \frac{2}{3}(0.2785 g_0\delta)^{3/2}$ is a positive constant,

$$z_n^3 g(x)d(v) \le \frac{g_1^2}{4}z_n^6 + p^{*2}$$

From (26), it holds true that

$$LV \le -2\mu_1 + \mu_2 \tag{30}$$

with $\mu_2 = \mu_2^0 + \kappa + p^{*2}$.

The main result on the asymptotic stability of the closed-loop system is summarized in the following theorem.

Theorem For the stochastic uncertain nonlinear systems (1) satisfying Assumptions 1-4, with control law (27) and adaptive law (25), then the tracking error is bounded and the mean square tracking error enters inside the region

$$\Omega = \{y(t) \in R \mid E[(y(t) - y_d(t))^2] \le \frac{2\mu_2}{\mu_1}, \quad \forall t \ge T_1\}$$

wherein it remains for all time thereafter, and the variable T_1 will be given later.

Proof From (30) and Lemma 1, it is easy to get that the system is bounded in probability and the mean value of the Lyapunov function satisfies

$$\frac{d}{dt}[E(V)] \le -2\mu_1 E[V(t)] + \mu_2. \tag{31}$$

So,

$$E[V(t)] \le e^{-2\mu_1 t} V_n(0) + \frac{\mu_2}{2\mu_1}, \qquad \forall t \ge 0 \tag{32}$$

there exists a time T_1

$$T_1 = \max\{0, \frac{1}{2\mu_1}\ln[\frac{2\mu_1 V_n(0)}{\mu_2}]\}. \tag{33}$$

such that

$$E[(y(t) - y_d(t))^2] \le 2E[V(t)] \le \frac{2\mu_2}{\mu_1}. \tag{34}$$

4. Computer simulation

Consider the following nonlinear systems:

$$\begin{cases} dx_1 = x_2 dt \\ dx_2 = [x_1 x_2 + (2 + \cos(x_2))u]dt + \frac{1}{3}(x_2 + \sin x_1)d\omega \\ y = x_1 \end{cases} \tag{35}$$

where $D(v)$ as follows:

$$u = D(v) = \begin{cases} 1.5(v-2) & if \ v \ge 2, \\ 0 \quad if & -1.5 < v < 2.5, \\ v + 0.5 & if \ v \le -0.5. \end{cases} \tag{36}$$

with initial conditions $x_1(0) = 0.5$, $x_2(0) = 0.1$, and the reference signal $y_d = 1.2(\sin t + \sin(0.4t))$.

It is easy to get $g_0 = 1, g_1 = 3, \beta_0 = 1$. In the simulation, the fuzzy membership functions are defined as

$$\mu_{F_j^1}(x) = \exp[\tfrac{-(x+2)^2}{4}], \quad \mu_{F_j^2}(x) = \exp[\tfrac{-(x+1)^2}{4}],$$

$$\mu_{F_j^3}(x) = \exp[\tfrac{-(x)^2}{4}], \quad \mu_{F_j^4}(x) = \exp[\tfrac{-(x-2)^2}{4}],$$

$$\mu_{F_j^5}(x) = \exp[\tfrac{-(x-1)^2}{4}].$$

We choose the virtual control law and the control law as

$$\alpha_1 = -k_1 z_1 + \dot{y}_d, \tag{37}$$

$$v = -\hat{\theta}^T \xi(x) \tanh(\frac{z_2 \hat{\theta}^T \xi(x)}{\delta}) - 10[\frac{1}{4}(\frac{3\xi}{4})^3 + \frac{k_2}{2} + \frac{1}{2}z_2^2 + \frac{g_1^2}{4}z_2^2]z_2, \tag{38}$$

where $z_1 = y - y_d = x_1 - y_d$, $z_2 = x_2 - \alpha_1$, $k_1 = k_2 = 2$, $\delta = 0.2$, $\xi = 2$. The adaptive law

$$\dot{\hat{\theta}} = \Gamma\,[\xi(x) z_2^3 - \sigma(\hat{\theta} - \theta^0)], \tag{39}$$

where $\Gamma = 2I$, $\sigma = 2$, θ^0 is randomly taken in the intervals $[-1,1]$. The simulation results are shown in Figure 1 and Figure 2. From Figure 1, it can be seen that fairly good tracking performance is obtained.

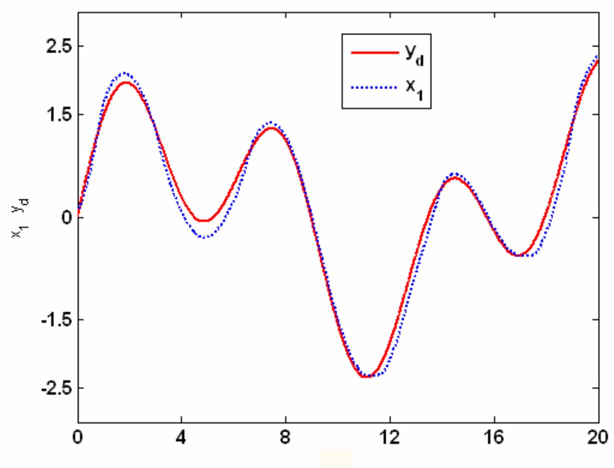

Fig. 1. The response of x_1.

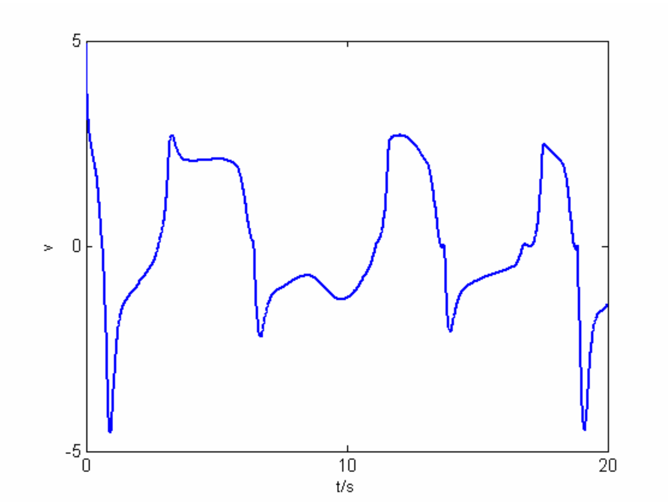

Fig. 2. Control Signal v .

5. Conclusions

In brief, a novel adaptive fuzzy control scheme has been presented for a class of uncertain nonlinear systems with unknown dead-zone input, which is driven by unknown covariance noise inputs. The proposed control scheme ensures that all the error variables are bounded in probability while the mean square tracking error becomes SGUUB in an arbitrarily small area around the origin.

Acknowledgment

This work is partially supported by the Natural Science Foundation of China (60404006,60574006), partially by the Natural Science Foundation of the Jiangsu Higher Education Institutions of China (Grant No.07KJB510125), and partially by the Natural Science Foundation of Yancheng Teachers University.

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