A new theorem relating quantum tomogram to the Fresnel operator*

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According to Fan–Hu's formalism (Fan Hong-Yi and Hu Li-Yun 2009 *Opt. Commun.* **282** 3734) that the tomogram of quantum states can be considered as the module-square of the state wave function in the intermediate coordinate–momentum representation which is just the eigenvector of the Fresnel quadrature phase, we derive a new theorem for calculating quantum tomogram of density operator, i.e., the tomogram of a density operator ρ is equal to the marginal integration of the classical Weyl correspondence function of $F^{\dagger}\rho F$, where F is the Fresnel operator. Applications of this theorem to evaluating the tomogram of optical chaotic field and squeezed chaotic optical field are presented.

Keywords: quantum tomogram, Fresnel operator, Weyl correspondence

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1. Introduction

It is well known that X-ray or optical tomographic imaging technique (classical tomography) derives two-dimensional data from a three-dimensional object to obtain a slice image of the internal structure and thus it has an ability to peer into the object. In the context of phase space theory of quantum statistical physics Vogel and Risken[1] pointed out the probability distribution for the rotated quadrature phase can be expressed in terms of Wigner function and the reverse is also true (named as Vogel–Risken relation), i.e., one can obtain the Wigner distribution by tomographic inversion of a set of measured probability distributions of the quadrature amplitude. In quantum optics theory, $\sigma X + \tau P$, where σ and τ are real numbers, $X = \left(a^{\dagger} + a\right)/\sqrt{2}, P = \mathrm{i}\left(a^{\dagger} - a\right)/\sqrt{2}$, representing all possible linear combinations of quadratures X and P of the oscillator field mode a and $a^{\dagger}$ can be measured by the homodyne measurement just by varying the phase of the local oscillator.[2,3] The average of the measurements, at a given local oscillator phase, is related to the marginal distribution of Wigner function, thus the homodyne measurement of the light field allows the reconstruction of the Wigner function[4–12] of a quantum system by varying the phase of the local oscillator. Thus a tomographic approach of quantum theories offers a description of tomographic probability. Recently, a tomogram approach,[1,13–20] which is related to a standard positive probability distribution function describing the quantum state in quantum statistics and quantum optics, has aroused much interest of physicists. A challenge thus arises: how is the mixed state tomogram calculated in a concise way? In particular, how is the mixed state tomogram effectively derived? As Dirac pointed out: " When one has a particular problem to work out in quantum mechanics, one can minimize the labour by using a representation in which the representatives of the more important abstract quantities occurring in that problem are as simple as possible."[21] In Ref. [22] a new formalism of quantum tomogram has been established. It has been shown that for tomographic approach there exists the quantum mechanical representations $|x\rangle_{s,r}$ (named the intermediate coordinate–momentum representation) and that the Radon transform of the Wigner operator is just the pure-state density matrices $|x\rangle_{s,rs,r}\langle x|$. As a result, the tomogram of quantum states can be considered as the module-square of the state wave function in the intermediate coordinate-momentum representation. In Ref. [22] Fan and Hu further found that $|x\rangle_{s,r} = F|x\rangle$, F is the Fresnel operator proposed as the quantum image of the classical Fresnel transformation,[23–26] thus the probability distribution for the Fresnel quadrature phase is just the core of tomography. In this way the Radon transform

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of Wigner function can be formulated much simply and more elegantly, which enables us to evaluate the tomogram of quantum mixed state in a more direct way. In this way, as one can see shortly later, we find a new theorem, i.e., the tomogram of a density operator ρ is equal to the marginal integration of the classical Weyl correspondence function of $F^{\dagger}\rho F$, where F is the Fresnel operator. The rest of the present paper is arranged as follows. In Section 2 we briefly review the relationship between Fresnel transformation and quantum tomography. In Section 3 we derive the new theorem. As an application of this theorem, in Section 4 we calculate the tomogram of the optical chaotic field. In Section. 5, we calculate the tomogram of the squeezed optical chaotic field. Throughout the paper we take full advantage of the technique of integration within Weyl ordered product (IWWOP) of operators and Weyl ordering invariance under similar transformations.

2. Brief review of the relationship between Fresnel transformation and quantum tomography

In Ref. [22] Fan and Hu have proved that the Wigner operator's Radon transform is

$$\iint_{-\infty}^{\infty} \mathrm{d}x' \mathrm{d}p' \delta\left[x-(Dx'-Bp')\right] \Delta\left(x',p'\right) = \left|x\right\rangle_{s,r\,s,r}\left\langle x\right|, \tag{1}$$

or

$$\iint_{-\infty}^{\infty} \mathrm{d}x' \mathrm{d}p' \delta\left[\,p-(Ap'-Cx')\right] \Delta\left(x',p'\right) = \left|p\right\rangle_{s,r\,s,r}\left\langle\, p\,\right|, \tag{2}$$

where $\Delta(x,p)$ is the Wigner operator and its normal ordering form is[27]

$$\Delta(x,p)=\frac{1}{\pi} : \mathrm{e}^{-(x-X)^2-(p-P)^2} : ; \tag{3}$$

s and r are two complex parameters, related to $(A;B;C;D)$ through the following relations:

$$A=\frac{1}{2}\left(s^{*}-r^{*}+s-r\right),$$
$$B=\frac{1}{2\mathrm{i}}\left(s^{*}-s+r^{*}-r\right),$$
$$C=\frac{1}{2\mathrm{i}}\left(s-r-s^{*}+r^{*}\right),$$
$$D=\frac{1}{2}\left(s+s^{*}+r+r^{*}\right) \tag{4}$$

with the constraint $|s|^2-|r|^2=1$, or $AD-BC=1$; the Radon projection in Eq. (1) is in the (B,D) direction, while in Eq. (2) is in the (A,C) direction; state vector $\left|x\right\rangle_{s,r}$ is named the tomography representation, while $\left|p\right\rangle_{s,r}$ is just its conjugate,

$$\int_{-\infty}^{+\infty} \mathrm{d}x \left|x\right\rangle_{s,r\,s,r}\left\langle x\right| = 1,$$
$$\int_{-\infty}^{+\infty} \mathrm{d}p \left|p\right\rangle_{s,r\,s,r}\left\langle p\right| = 1. \tag{5}$$

It has been revealed in Ref. [22] that

$$\left|x\right\rangle_{s,r}=F\left|x\right\rangle, \quad \left|p\right\rangle_{s,r}=F\left|p\right\rangle, \tag{6}$$

where

$$\left|x\right\rangle=\pi^{-1/4}\exp\left[-\frac{x^2}{2}+\sqrt{2}xa^{\dagger}-\frac{1}{2}a^{\dagger 2}\right]\left|0\right\rangle, \tag{7}$$

and

$$\left|p\right\rangle=\pi^{-1/4}\exp\left[-\frac{p^2}{2}+\sqrt{2}\mathrm{i}pa^{\dagger}+\frac{1}{2}a^{\dagger 2}\right]\left|0\right\rangle \tag{8}$$

is the momentum representation, $P\left|p\right\rangle=p\left|p\right\rangle$ and F is the so-called Fresnel operator[23,24]

$$F\left(s,r\right)=\exp\left(-\frac{r}{2s^{*}}a^{\dagger 2}\right)\exp\left\{\left(a^{\dagger}a+\frac{1}{2}\right)\ln\frac{1}{s^{*}}\right\} \times \exp\left(\frac{r^{*}}{2s^{*}}a^{2}\right), \tag{9}$$

generating the kernel of classical optical Fresnel diffraction

$$\left\langle x'\right| F\left(s,r\right)\left|x\right\rangle = \frac{1}{\sqrt{2\pi\mathrm{i}B}}\exp\left[\frac{\mathrm{i}}{2B}\left(Ax^2-2x'x+Dx'^2\right)\right], \tag{10}$$

such that an output light field $g\left(x\right)$ of an $ABCD$ optical instrument [$(A;B;C;D)$ are elements of a ray transfer matrix] is related to its input light field $f\left(x\right)$ by the Fresnel integration[28]

$$g\left(x'\right) = \frac{1}{\sqrt{2\pi\mathrm{i}B}}\int_{-\infty}^{\infty}\exp\left[\frac{\mathrm{i}}{2B}\left(Ax^2-2x'x+Dx'^2\right)\right] \times f\left(x\right)\mathrm{d}x, \tag{11}$$

or $F\left(s,r\right)\left|f\right\rangle=\left|g\right\rangle$. Then

$$FXF^{\dagger}=X_F, \tag{12}$$

can be named the Fresnel transformed quadrature phase ($FPF^{\dagger} = P_F$ is its conjugate operator). The Wigner operator in x-representation is expressed as[4]

$$\Delta(p,x) = \int_{-\infty}^{+\infty} \frac{\mathrm{d}v}{2\pi}\, \mathrm{e}^{\mathrm{i}pv} \left| x + \frac{v}{2} \right\rangle \left\langle x - \frac{v}{2} \right|. \tag{13}$$

Hence the probability distribution for the Fresnel quadrature phase is just the tomography.

3. A new theorem relating quantum tomogram to the Fresnel operator

Multiplying both sides of Eq. (1) by a density matrix ρ and then performing the trace, noting the Wigner function $W(p,x) = \mathrm{Tr}\left[\rho\Delta(p,x)\right]$, we see

$$\begin{aligned} &\mathrm{Tr}\left[\iint_{-\infty}^{\infty} \mathrm{d}x'\mathrm{d}p'\delta\left[x-(Dx'-Bp')\right]\Delta\left(x',p'\right)\rho\right] \\ &= \mathrm{Tr}\left(|x\rangle_{s,r\,s,r}\langle x|\,\rho\right) \\ &= {}_{s,r}\langle x|\,\rho\,|x\rangle_{s,r} \\ &= \langle x|\,F^{\dagger}\rho F\,|x\rangle \\ &= \iint_{-\infty}^{\infty} \mathrm{d}x'\mathrm{d}p'\delta\left[x-(Dx'-Bp')\right]W(p',x'). \end{aligned} \tag{14}$$

The right-hand side of Eq. (14) is commonly defined as the tomogram of quantum states in the (B,D) direction, so in our view the calculation of tomogram in (B,D) direction is reduced to calculating

$$\langle x|\,F^{\dagger}\rho F\,|x\rangle \equiv \Xi. \tag{15}$$

This is a concise and neat formula. Similarly, the tomogram in (A,C) direction is reduced to calculating $\langle p|\,F^{\dagger}\rho F\,|p\rangle$.

According to the Weyl correspondence rule[29]

$$H\left(X,P\right) = \iint_{-\infty}^{\infty} \mathrm{d}p\mathrm{d}xh(p,x)\Delta(p,x), \tag{16}$$

and the Weyl ordering form of $\Delta(p,x)$[30,31]

$$\Delta(p,x) = \vdots\,\delta\left(x-X\right)\delta\left(p-P\right)\vdots \tag{17}$$

where the symbol $\vdots\ \vdots$ denotes Weyl ordering, the classical correspondence of a Weyl ordered operator $\vdots\, h(X,P)\,\vdots$ is obtained just by replacing $X \to x, P \to p$ in h, i.e.,

$$\vdots\, h(X,P)\,\vdots = \iint_{-\infty}^{\infty} \mathrm{d}p\mathrm{d}xh(p,x)\Delta(p,x). \tag{18}$$

Let the classical Weyl correspondence of $F^{\dagger}\rho F$ be $h(p,x)$

$$F^{\dagger}\rho F = \iint_{-\infty}^{\infty} \mathrm{d}p\mathrm{d}xh(p,x)\Delta(p,x), \tag{19}$$

then using Eqs. (15) and (19) we have

$$\begin{aligned} \Xi &= \langle x|\,F^{\dagger}\rho F\,|x\rangle \\ &= \langle x|\iint_{-\infty}^{\infty} \mathrm{d}p\mathrm{d}x'h(p,x')\Delta\left(p,x'\right)|x\rangle \\ &= \iint_{-\infty}^{\infty} \mathrm{d}p\mathrm{d}x'h(p,x')\int_{-\infty}^{+\infty}\frac{\mathrm{d}v}{2\pi}\,\mathrm{e}^{\mathrm{i}pv} \\ &\quad\times \left\langle x\middle| x'+\frac{v}{2}\right\rangle\left\langle x'-\frac{v}{2}\middle| x\right\rangle \\ &= \iint_{-\infty}^{\infty} \mathrm{d}p\mathrm{d}x'h(p,x')\int_{-\infty}^{+\infty}\frac{\mathrm{d}v}{2\pi}\,\mathrm{e}^{\mathrm{i}pv}\delta \\ &\quad\times \left(x-x'+\frac{v}{2}\right)\delta\left(x-x'-\frac{v}{2}\right) \\ &= \frac{1}{\pi}\iint_{-\infty}^{\infty} \mathrm{d}p\mathrm{d}x'h(p,x')\,\mathrm{e}^{\mathrm{i}2p\left(x-x'\right)}\delta\left(2x'-2x\right) \\ &= \int_{-\infty}^{\infty}\frac{\mathrm{d}p}{2\pi}h(p,x). \end{aligned} \tag{20}$$

Thus we reach a new theorem as follows: the tomogram of a density operator ρ is equal to the marginal integration of the classical Weyl correspondence $h(p,x)$ of $F^{\dagger}\rho F$, where F is the Fresnel operator, expressed as

$$\mathrm{Tr}\left[\rho\,|x\rangle_{s,r\,s,r}\langle x|\right] = \int_{-\infty}^{\infty}\frac{\mathrm{d}p}{2\pi}h(p,x), \tag{21}$$

or

$$\mathrm{Tr}\left[\rho\,|p\rangle_{s,r\,s,r}\langle p|\right] = \int_{-\infty}^{\infty}\frac{\mathrm{d}x}{2\pi}h(p,x). \tag{22}$$

In this way the relationship between tomogram of a density operator ρ and the Fresnel transformed ρ's classical Weyl function is established.

4. Tomogram of optical chaotic field

As an application of the above theorem, we now consider the density operator of chaotic field, which is expressed as

$$\rho_{\mathrm{c}} = \left(1-\mathrm{e}^{\lambda}\right)\mathrm{e}^{\lambda a^{\dagger}a}, \tag{23}$$

where $\lambda = -\omega\hbar/kT$, k is the Boltzmann constant, ω is frequency and T is temperature of the chaotic field. ρ_{c} is qualified to be a density operator, since ($\mathrm{tr}\rho_{\mathrm{c}} = 1$). In order to evaluate the classical Weyl correspondence

$h_c(p,x)$ of $F^\dagger\rho_c F$, we first present the Weyl ordered form of $\left(1-\mathrm{e}^{\lambda}\right)\mathrm{e}^{\lambda a^\dagger a}$ as follows:[29]

$$\begin{aligned}&\left(1-\mathrm{e}^{\lambda}\right)\mathrm{e}^{\lambda a^\dagger a}\\&=\frac{2\left(1-\mathrm{e}^{\lambda}\right)}{\mathrm{e}^{\lambda}+1}\vdots\exp\left[\frac{2\left(\mathrm{e}^{\lambda}-1\right)}{\mathrm{e}^{\lambda}+1}a^\dagger a\right]\vdots\,,\end{aligned}\tag{24}$$

then using operators' Weyl ordering invariance under similar transformations,[29] and the property under the Fresnel transformation,

$$F^\dagger a^\dagger F=sa-ra^\dagger,\ F^\dagger aF=-r^*a+s^*a^\dagger\,,\tag{25}$$

and letting $\beta\equiv(1-\mathrm{e}^{\lambda})/(\mathrm{e}^{\lambda}+1)$, we can convert $F^\dagger\rho_c F$ into its Weyl ordering form

$$\begin{aligned}&F^\dagger\rho_c F\\&=2\beta F^\dagger\vdots\exp\left[-2\beta a^\dagger a\right]\vdots F\\&=2\beta\vdots\exp\left[-2\beta\left(s^*a^\dagger-r^*a\right)\left(sa-ra^\dagger\right)\right]\vdots.\end{aligned}\tag{26}$$

Due to

$$a=\frac{Q+\mathrm{i}P}{\sqrt{2}},\quad a^\dagger=\frac{Q-\mathrm{i}P}{\sqrt{2}},\tag{27}$$

equation (26) is equal to

$$\begin{aligned}&F^\dagger\rho_c F\\&=2\beta\vdots\exp\bigg\{-2\beta\Big[(P^2-X^2)\,\mathrm{Re}(r^*s)\\&\quad+2\,\mathrm{Im}(r^*s)QP+(r^*r+s^*s)\frac{Q^2+P^2}{2}\Big]\bigg\}\vdots,\end{aligned}\tag{28}$$

so according to Eq. (18) the classical Weyl correspondence function $h_c(p,x)$ of $F^\dagger\rho_c F$ is

$$\begin{aligned}&2\beta\exp\bigg\{-2\beta\Big[(p^2-x^2)\,\mathrm{Re}(r^*s)+2xp\,\mathrm{Im}(r^*s)\\&\quad+(r^*r+s^*s)\frac{x^2+p^2}{2}\Big]\bigg\}\equiv h_c(p,x).\end{aligned}\tag{29}$$

Then substituting Eq. (29) into the theorem (Eq. (21)) we derive the tomogram of the chaotic field

$$\mathrm{Tr}[\rho_c|x\rangle_{s,rs,r}\langle x|]=\int_{-\infty}^{\infty}\frac{\mathrm{d}p}{2\pi}h_c(p,x)=\frac{\sqrt{\beta}}{\sqrt{\pi\tau}}\exp\left\{\beta\frac{2(\mathrm{Im}(r^*s))^2-4\,\mathrm{Re}(r^*s)+2(r^*r+s^*s)}{\tau}x^2\right\},\tag{30}$$

where

$$2\,\mathrm{Re}\,(r^*s)+(r^*r+s^*s)\equiv\tau,\tag{31}$$

by noticing

$$\mathrm{Re}(r^*s)=\frac{1}{4}\left[-A^2+D^2+B^2-C^2\right],\quad \mathrm{Im}\,r^*s=\frac{1}{4}\left(BA+CD\right),\quad r^*r+s^*s=\frac{1}{2}\left(A^2+D^2+B^2+C^2\right),\tag{32}$$

and $AD-BC=1$, we can rewrite Eq. (30) as

$$\mathrm{Tr}\left[\rho_c\left|x\right\rangle_{s,rs,r}\left\langle x\right|\right]=\int_{-\infty}^{\infty}\frac{\mathrm{d}p}{2\pi}h_c(p,x)=\frac{\sqrt{\beta}}{\sqrt{\pi\left(D^2+B^2\right)}}\exp\left\{\frac{-\beta x^2}{D^2+B^2}\right\},\tag{33}$$

a neat result. This is the tomogram of ρ_c in the (B,D) direction. It is a normal distribution and

$$\int_{-\infty}^{\infty}\mathrm{d}x\frac{\sqrt{\beta}}{\sqrt{\pi\left(D^2+B^2\right)}}\exp\left\{\frac{-\beta x^2}{D^2+B^2}\right\}=1.\tag{34}$$

Similarly, we can derive the tomogram of ρ_c in the (A,C) direction

$$\mathrm{Tr}\left[\rho_c\left|p\right\rangle_{s,rs,r}\left\langle p\right|\right]=\int_{-\infty}^{\infty}\frac{\mathrm{d}x}{2\pi}h_c(p,x)=\frac{\sqrt{\beta}}{\sqrt{\pi\left(A^2+C^2\right)}}\exp\left\{\frac{-\beta p^2}{A^2+C^2}\right\},\tag{35}$$

which is also a normal distribution, with

$$\int_{-\infty}^{\infty}\mathrm{d}p\frac{\sqrt{\beta}}{\sqrt{\pi\left(A^2+C^2\right)}}\exp\left\{\frac{-\beta p^2}{A^2+C^2}\right\}=1.\tag{36}$$

By choosing $\beta=1$, and $(A,B,C,D)=(1/2,2,1/2,4)$, which satisfies the relation $AD-BC=1$. Figure 1 shows the curves representing the tomograms of ρ_c in the (B,D) direction (dot curve) and in the (A,C) direction (solid curve), which exhibits not only normal distributions in the two directions, but also their difference. Note that the two curves are complemented to each other, while dot-curve shape is tall and slim, solid-curve shape is short and fat, since $AD - BC = 1$.

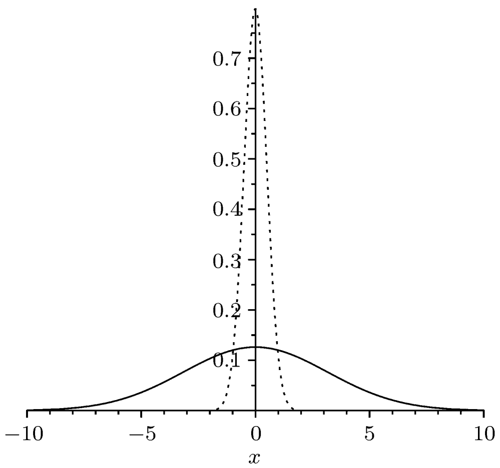

Fig. 1. Tomogram of the optical chaotic field.

5. Tomogram of squeezed optical chaotic field

Furthermore, the density operator of squeezed[32−35] chaotic field is expressed as

$$\rho_{\rm s} = S\left(\gamma\right)\rho_{\rm c} S^{\dagger}\left(\gamma\right), \tag{37}$$

where $S\left(\gamma\right) = \exp\left[{\rm i}\gamma\left(QP+PQ\right)/2\right]$ is the single-mode squeezing operator and $[X,P] = {\rm i}$, $\hbar = 1$, γ is the squeezing parameter. Using the properties of the single-mode squeezing operator $S\left(\gamma\right)$,

$$SPS^{-1} = {\rm e}^{\gamma}P,\ SXS^{-1} = {\rm e}^{-\gamma}X, \tag{38}$$

we have

$$\begin{aligned} &\rho_{\rm s} \\ &= \left(1-{\rm e}^{\lambda}\right)S\left(\gamma\right){\rm e}^{\lambda a^{\dagger}a}S\left(\gamma\right)^{-1} \\ &= \frac{2\left(1-{\rm e}^{\lambda}\right)}{{\rm e}^{\lambda}+1}\vdots\exp\left\{\frac{{\rm e}^{\lambda}-1}{{\rm e}^{\lambda}+1}\left[{\rm e}^{2\gamma}P^2+{\rm e}^{-2\gamma}X^2\right]\right\}\vdots\,. \end{aligned} \tag{39}$$

Using the same method as that used in Section 3, we have

$$\begin{aligned} F^{\dagger}\rho_{\rm s}F &= \vdots\, h_{\rm s}(P,X)\vdots \\ &= 2\beta\vdots\exp\{-2\beta[{\rm e}^{2r}(DP+CX)^2 \\ &\quad + {\rm e}^{-2r}(BP+AX)^2]\}\vdots \end{aligned} \tag{40}$$

and the classical Weyl correspondence $h_{\rm s}(p,x)$ of $F^{\dagger}\rho_{\rm s}F$ as

$$\begin{aligned} h_{\rm s}\left(p,x\right) = 2\beta\exp\big\{-2\beta\big[&{\rm e}^{2r}\left(Dp+Cx\right)^2 \\ &+ {\rm e}^{-2r}\left(Bp+Ax\right)^2\big]\big\}. \end{aligned} \tag{41}$$

We then obtain the tomogram of the density operator of squeezed chaotic field in the (B,D) direction as

$$\begin{aligned} &{\rm Tr}\left[\rho_{\rm s}\left|x\right\rangle_{s,rs,r}\left\langle x\right|\right] \\ &= \int_{-\infty}^{\infty}\frac{{\rm d}p}{2\pi}h_{\rm s}(p,x) \\ &= \frac{\sqrt{\beta}}{\sqrt{\pi\left({\rm e}^{-2r}B^2+{\rm e}^{2r}D^2\right)}} \\ &\times\ \exp\left\{\frac{-\beta}{\left({\rm e}^{-2r}B^2+{\rm e}^{2r}D^2\right)}x^2\right\}. \end{aligned} \tag{42}$$

Similarly, we have the tomogram of $\rho_{\rm s}$ in (A,C) direction as

$$\begin{aligned} &{\rm Tr}\left[\rho_{\gamma}\left|p\right\rangle_{s,rs,r}\left\langle p\right|\right] \\ &= \int_{-\infty}^{\infty}\frac{{\rm d}x}{2\pi}h_{\rm s}(p,x) \\ &= \frac{\sqrt{\beta}}{\sqrt{\pi\left({\rm e}^{-2\gamma}A^2+{\rm e}^{2\gamma}C^2\right)}} \\ &\times\ \exp\left\{\frac{-\beta}{\left({\rm e}^{-2\gamma}A^2+{\rm e}^{2\gamma}C^2\right)}p^2\right\}. \end{aligned} \tag{43}$$

Choosing ${\rm e}^{2\gamma} = 2, \beta = 1$, and (A,B,C,D) as $(1/2,2,1/2,4)$ which satisfies $AD-BC=1$, we plot their figures as shown in Fig. 2, where dot curve describes the (B,D) direction and solid curve describes the (A,C) direction. By comparing Fig. 2 with Fig. 1 we see the squeezing of the optical chaotic field.

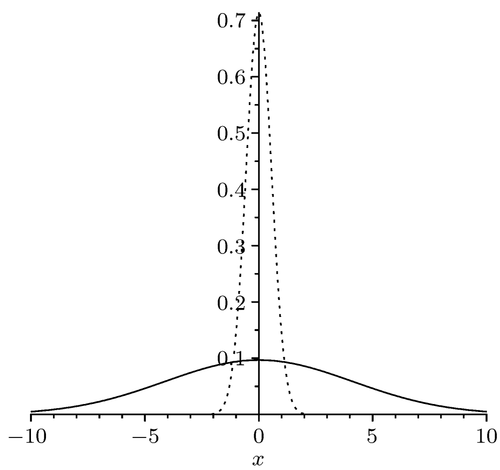

Fig. 2. Tomogram of the squeezed optical chaotic field.

In summary, we have found a new theorem in calculating density operator tomogram, which not only brings much convenience for the mixed state case, but also reveals the relationship between tomogram of density operator ρ and Fresnel transformed ρ classical Weyl correspondence function. The theorem may be applied to calculating the tomogram of more complicated quantum state in a neat and concise way. There is operational advantage of using the new theorem, since the steps are as follows: 1) derive the Weyl ordering form of the given density operator ρ; 2) deduce the Weyl ordering form of $F^{\dagger}\rho F$ according to the fact that Weyl ordering is invariant under Fresnel transformations; 3) write down the Weyl ordered operator classical correspondence just by replacing $X \to x$, $P \to p$, thereby directly obtaining classical correspondence $h\,(p, x)$ of $F^{\dagger}\rho F$; 4) perform integration $\int_{-\infty}^{\infty}(\mathrm{d}p/2\pi)h\,(p,x)$ to obtain the tomogram of ρ. In our approach we make fully use of the properties of Weyl ordering of operators, which is the origin of the operational advantage.

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