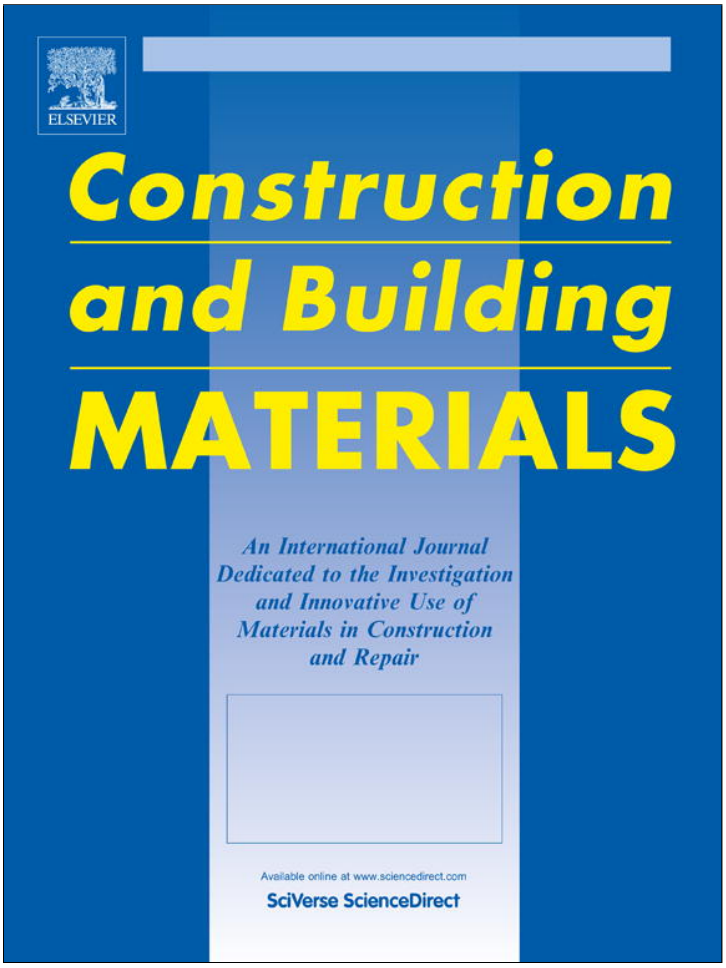
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A modified micro-mechanics model for estimating effective elastic modulus of concrete

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HIGHLIGHTS

▶ A new equivalent micro-mechanics model is obtained for the estimation of elastic modulus of the composite/concrete.
▶ Numerical analyses are carried out to obtain the bulk/shear modulus for various volume fractions of aggregates.
▶ The equations obtained can account for the evolution of interfacial debonding damage between the aggregate and cement.

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ABSTRACT

A modified equivalent micro-mechanics model, which is considered based on an initial model of an infinite matrix with an inclusion, has been developed for estimation of the elastic modulus of concrete. It is assumed that aggregate or cement is the inclusion of equivalent model, and elastic modulus of the model is considered according to the volume fraction of each component. The equations of effective elastic modulus are obtained based on this equivalent model, which contains volume fractions of aggregates and voids as its variables. The equations obtained can account for the evolution of interfacial debonding damage between aggregate and cement. The results show that the concrete modulus increases with the increase of aggregate volume fraction. However, the Poisson's ratio decreases with the increase of aggregate volume fraction.

1. Introduction

The effective elastic modulus of composites that plays an important role in the analysis and design of composite materials, has been studied extensively. Many researches have attempted to estimate the effective elastic modulus of composites using both analytical and numerical methods. The concrete is supposed as a two-phase composite, which has a cement matrix contained aggregates. This two-phase model is the simplest micro-mechanics model for concretes, and it can provide a reasonable estimation of elastic modulus for concretes [1]. In order to investigate the interface effect on mechanical properties of concretes, several researchers have developed a three-phase model for this kind of particulate-reinforced composite. The three-phase model assumes that concrete is composed of aggregate, cement matrix, and interfacial transition zone [2].

The difference between the real value and the estimated value by two-phase model arose from the fact that the model did not consider interfacial transition zone in concretes. Composite material models were applied to estimate the elastic modulus of slag concrete [3]. A numerical concrete model that adopts three-phase model and finite element with material discontinuity was proposed to analyze concrete with complex interface in three dimensions [4]. Interfacial transition zone was modeled as thin shell surrounding aggregate and the model gave reliable results [5–9]. However, it is difficult to estimate elastic modulus of the concrete practically using the three-phase model as its interfacial transition zone is too thin to be modeled precisely using conventional numerical methods [10].

In this paper, the effective bulk and shear moduli of the concrete have been developed by establishing a link between two strains: the strain of cement/aggregate and the strain of an equivalent material, of which the modulus is known as the average value of elastic moduli of the cement and aggregate. In the present study, the modified micro-mechanics model, which is based on Eshelby's equivalent inclusion method and self-consistent method, can account for the interfacial debonding damage between aggregates and cement. Numerical analyses were carried out to obtain the bulk/shear modulus for various volume fractions of aggregates.

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The estimation equations obtained can describe the change rule of the effective elastic modulus due to debonding damage.

2. Model development

The two key mechanical properties of concrete are presented in Table 1. Fig. 1a shows the concrete model, which is experiencing interfacial debonding damage between aggregates and cement. The concrete contained intact, partially debonded and fully debonded aggregates. As shown in Fig. 1b, a damage model is developed based on micro-mechanics. The volume fractions of intact aggregates and voids are indicated as f_a and f_v, respectively. Here, f_a^I is the initial aggregate volume fraction, and the incremental deformation process is denoted by df.

2.1. An equivalent model

The model of an infinite cement-matrix with one single aggregate has been chosen as an original model. The elastic modulus and strains of cement and aggregates are E_c, ε^c and E_a, ε^a, respectively as shown in Fig. 2a. According to self-consistent method, the aggregate is displaced by an equivalent strain $\bar{\varepsilon}$, and with an elastic modulus, E_c, which is the same as that of the cement-matrix as shown in Fig. 2b. In the present work, it was supposed that the cement is in a same situation as the particles, that is, matrix and particles are both inclusions of the composite. Thus, a modified model has been developed as shown in Fig. 2c. For this model, an equivalent matrix is employed, and its elastic modulus is E_e, which equals to the average value of the elastic moduli of cement and aggregates, and is written as $E_e = (3K_e, 2\ G_e)$. The equivalent strain due to this equivalent matrix is $\bar{\varepsilon}^e$. Here, the Ω-area in Fig. 2c denotes cement or aggregates. The model modified can be used to obtain the relations between the strains of cement and aggregate and the equivalent strain, separately. The equivalent strain $\bar{\varepsilon}^e$ is written as $\bar{\varepsilon}^e_{mn}$, and the equations can be written as follows:

$$d\varepsilon^c_{mn} = A_c \cdot d\bar{\varepsilon}^e_{mn} \tag{1}$$

$$d\varepsilon^a_{mn} = A_a \cdot d\bar{\varepsilon}^e_{mn} \tag{2}$$

where $d\bar{\varepsilon}^e_{mn}$ is the link between $d\varepsilon^c_{mn}$ and $d\varepsilon^a_{mn}$. There is no need to study the exact value of $d\bar{\varepsilon}^e_{mn}$. A_c, A_a is Eshelby tensor of the inclusion, and is given as follows:

$$A_c = \left(\frac{K}{K_c}\rho^1_c, \frac{G}{G_c}\rho^2_c\right), \quad A_a = \left(\frac{K}{K_a}\rho^1_a, \frac{G}{G_a}\rho^2_a\right) \tag{3}$$

where K and G are the average bulk and shear modulus of the equivalent composite, respectively, and given as follows:

$$K = f^I_c \cdot K_c + f^I_a \cdot K_a, \quad G = f^I_c \cdot G_c + f^I_a \cdot G_a \tag{4}$$

where K_c and K_a are bulk modulus of cement and aggregate, respectively. G_c and G_a are shear modulus of cement and aggregate, respectively. f^I_c and f^I_a are the initial volume fraction of cement and aggregate, respectively, and:

Table 1
Mechanical properties of concrete.

Parameter	Cement (matrix)	Aggregates (particles)
Young's modulus (GPa)	28.60	76.80
Poisson's ratio	0.21	0.16

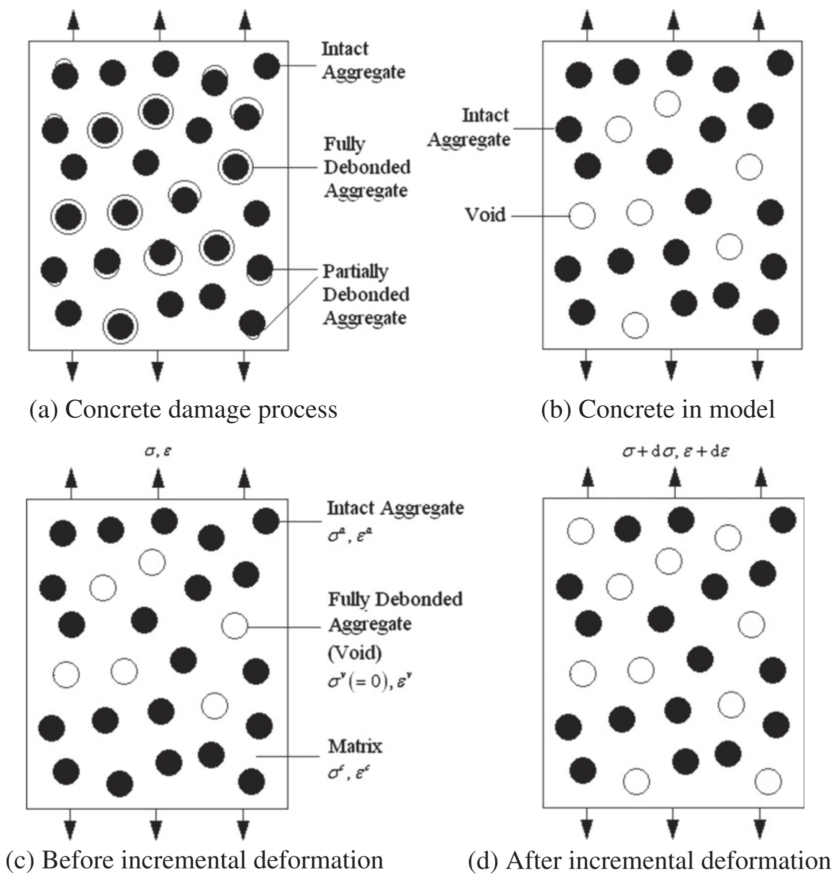
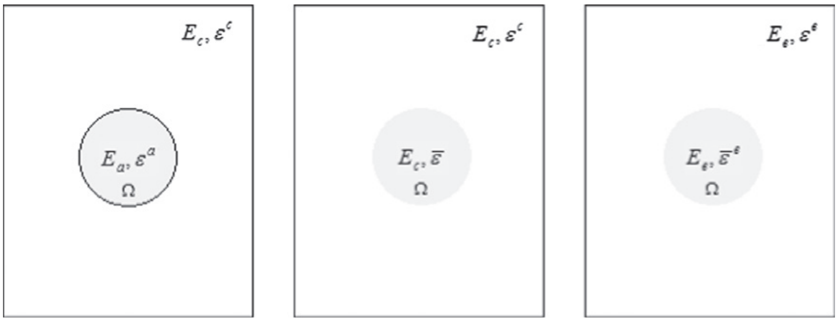


Fig. 1. Schematic illustrations of a concrete in debonding damage and a concrete in micro-mechanics model.



(a) Original model (b) Self-consistent model (c) A modified model

Fig. 2. Different micro-mechanics models of a concrete unit.

$$\rho_c^1=\frac{K}{K(1-\alpha)+\alpha K_c},\quad \rho_c^2=\frac{G}{G(1-\beta)+\beta G_c},\quad \rho_a^1=\frac{K}{K(1-\alpha)+\alpha K_a},\quad \rho_a^2=\frac{G}{G(1-\beta)+\beta G_a} \tag{5}$$

$$\alpha=\frac{3K}{3K+4G},\quad \beta=\frac{6(K+2G)}{5(3K+4G)} \tag{6}$$

The equation for $d\varepsilon_{mn}^c$ and $d\varepsilon_{mn}^a$ can be obtained by Eqs. (1) and (2) as:

$$d\varepsilon_{mn}^a=\frac{A_a}{A_c}\cdot d\varepsilon_{mn}^c \tag{7}$$

where $d\varepsilon_{mn}^a=(d\varepsilon_{kk}^a, d\varepsilon_{mn}'^a), d\varepsilon_{mn}^c=(d\varepsilon_{kk}^c, d\varepsilon_{mn}'^c)$. $d\varepsilon_{kk}^c$ and $d\varepsilon_{kk}^a$ are hydrostatic strains of cement and aggregates, respectively. $d\varepsilon_{mn}'^c$ and $d\varepsilon_{mn}'^a$ are deviatric strains of cement and aggregates, respectively. Substituting Eqs. (3), (5) and (6) into Eq. (7), we got:

$$d\varepsilon_{kk}^a=\frac{K(1-\alpha)+\alpha K_c}{K(1-\alpha)+\alpha K_a}d\varepsilon_{kk}^c,\quad d\varepsilon_{mn}'^a=\frac{G(1-\beta)+\beta G_c}{G(1-\beta)+\beta G_a}d\varepsilon_{mn}'^c \tag{8}$$

Supposing that

$$\frac{d\varepsilon_{mn}^a}{d\varepsilon_{mn}^c}=\frac{A_a}{A_c}=\bar{A},\quad \bar{A}=(\bar{A}_1,\bar{A}_2) \tag{9}$$

then

$$\bar{A}_1=\frac{K(1-\alpha)+\alpha K_c}{K(1-\alpha)+\alpha K_a},\quad \bar{A}_2=\frac{G(1-\beta)+\beta G_c}{G(1-\beta)+\beta G_a} \tag{10}$$

2.2. Incremental constitutive relation under progressive debonding damage

The effective elastic modulus of aggregates can be obtained by the following equation:

$$\bar{L}_a\cdot d\varepsilon_{mn}^a=(1-f_a-f_v)\cdot E_c\cdot d\varepsilon_{mn}^c+f_a\cdot E_a\cdot d\varepsilon_{mn}^a \tag{11}$$

where $\bar{L}_a=(3\bar{K}_a, 2\bar{G}_a)$ is the effective elastic modulus of aggregates, $E_c=(3K_c, 2G_c)$ and $E_a=(3K_a, 2G_a)$ are the elastic moduli of cement and aggregates, respectively. Term $(1-f_a-f_v)$ denotes the cement volume fraction. Substituting Eq. (7) into Eq. (11), the following equation can be obtained:

$$\bar{L}_a=(1-f_a-f_v)\cdot E_c\cdot\bar{A}+f_a\cdot E_a \tag{12}$$

Substituting Eqs. (9) and (10) into Eq. (12), the bulk and shear moduli of aggregate can be obtained:

$$\bar{K}_a=[K(1-\alpha)+\alpha K_a]\cdot\left[\frac{1-f_a-f_v}{K(1-\alpha)/K_c+\alpha}+\frac{f_a}{K(1-\alpha)/K_a+\alpha}\right] \tag{13}$$

$$\bar{G}_a=[G(1-\beta)+\beta G_a]\cdot\left[\frac{1-f_a-f_v}{G(1-\beta)/G_c+\beta}+\frac{f_a}{G(1-\beta)/G_a+\beta}\right] \tag{14}$$

Based on Hook's Law, the stress–strain relation of cement and aggregates can be written as follows:

$$E_c\cdot d\varepsilon_{mn}^c=d\sigma_{mn}^c \tag{15}$$

$$E_a\cdot d\varepsilon_{mn}^a=d\sigma_{mn}^a \tag{16}$$

where $E_c=(3K_c, 2G_c)$ and $E_a=(3K_a, 2G_a)$ are the elastic modulus of cement and aggregates, respectively, $d\varepsilon_{mn}^c=(d\varepsilon_{kk}^c, d\varepsilon_{mn}'^c)$ and $d\sigma_{mn}^c=(d\sigma_{kk}^c, d\sigma_{mn}'^c)$ are the strain and stress of cement, and $d\varepsilon_{mn}^a=(d\varepsilon_{kk}^a, d\varepsilon_{mn}'^a)$ and $d\sigma_{mn}^a=(d\sigma_{kk}^a, d\sigma_{mn}'^a)$ are the strain and stress of aggregates, respectively.

The debonding stress of concretes consists of two parts: the current stress $d\sigma_{mn}$ and the stress $\sigma_{mn}^a df_a$ released by aggregates, which are in the process of debonding. According to Hook's Law, the relation between the aggregate stress and the overall stress of the concrete is written as follow:

$$\bar{L}_a\cdot d\varepsilon_{mn}^a=d\sigma_{mn}+\sigma_{mn}^a df_a \tag{17}$$

where $\bar{L}_a=(3\bar{K}_a, 2\bar{G}_a)$ is the effective elastic modulus of aggregates, $d\sigma_{mn}=(d\sigma_{kk}, d\sigma_{mn}')$ is the overall stress of concrete, and $\sigma_{mn}^a=(\sigma_{kk}^a, \sigma_{mn}'^a)$ is aggregate stress.

By Eqs. (9) and (15), (16), (17), the relations between the cement stress and the overall stress of concrete are obtained as follows:

$$d\sigma_{mn}^c=\frac{E_c}{\bar{L}_a\bar{A}}(d\sigma_{mn}+\sigma_{mn}^a df_a) \tag{18}$$

Substituting Eq. (10) into Eq. (18) and turning Eq. (18) into hydrostatic and deviatric forms:

$$d\sigma_{kk}^c=\frac{K_c}{\bar{K}_a\bar{A}_1}(d\sigma_{kk}+\sigma_{kk}^a df_a),\quad d\sigma_{mn}'^c=\frac{G_c}{\bar{G}_a\bar{A}_2}(d\sigma_{mn}'+\sigma_{mn}'^a df_a) \tag{19}$$

where $\bar{K}_a$ and $\bar{G}_a$ are defined in Eqs. (13) and (14), respectively. Substituting Eq. (19) into Eq. (15), the hydrostatic and deviatric parts of cement strain can be obtained.

$$d\varepsilon_{kk}^c=\frac{1}{3\bar{K}_a\bar{A}_1}(d\sigma_{kk}+\sigma_{kk}^a df_a),$$

$$d\varepsilon_{mn}'^c=\frac{1}{2\bar{G}_a\bar{A}_2}(d\sigma_{mn}'+\sigma_{mn}'^a df_a) \tag{20}$$

By Eqs. (16) and (17), the relations between the aggregate stress and the overall stress of concrete are obtained as:

$$d\sigma_{kk}^a=\frac{K_a}{\bar{K}_a}(d\sigma_{kk}+\sigma_{kk}^a df_a),\quad d\sigma_{mn}'^a=\frac{G_a}{\bar{G}_a}(d\sigma_{mn}'+\sigma_{mn}'^a df_a) \tag{21}$$

Substituting Eq. (21) into Eq. (16), the relations between the aggregate strain and the overall stress of concrete are obtained:

$$d\varepsilon_{kk}^a=\frac{1}{3\bar{K}_a}(d\sigma_{kk}+\sigma_{kk}^a df_a),\quad d\varepsilon_{mn}'^a=\frac{1}{2\bar{G}_a}(d\sigma_{mn}'+\sigma_{mn}'^a df_a) \tag{22}$$

With a volume fraction of aggregates debonded completely, a set of voids arise, then Eq. (2) can be defined as following:

$$d\varepsilon^{v}_{mn} = A_v \cdot d\bar{\varepsilon}_{mn} \tag{23}$$

where $d\varepsilon^{v}_{mn}$ is the void strain, and

$$A_v = \left(\frac{1}{1-\alpha}, \frac{1}{1-\beta}\right) \tag{24}$$

From Eqs. (1), (2), (8), (23) and (24), the relations between the strains of cement and voids are as follows:

$$d\varepsilon^{v}_{kk} = \frac{K(1-\alpha)+\alpha K_c}{K(1-\alpha)} d\varepsilon^{c}_{kk}, \quad d\varepsilon'^{v}_{mn} = \frac{G(1-\beta)+\beta G_c}{G(1-\beta)} d\varepsilon'^{c}_{mn} \tag{25}$$

By Eqs. (20) and (25), the relations between the void strain and the overall stress of concrete can be defined as follows:

$$\begin{aligned} d\varepsilon^{v}_{kk} &= \frac{K(1-\alpha)+\alpha K_c}{3\overline{K}_a K(1-\alpha)} (d\sigma_{kk} + \sigma^{a}_{kk} df_a), \\ d\varepsilon'^{v}_{mn} &= \frac{G(1-\beta)+\beta G_c}{2\overline{G}_a G(1-\beta)} (d\sigma'_{mn} + \sigma'^{a}_{mn} df_a) \end{aligned} \tag{26}$$

2.3. Effective bulk and shear moduli of concrete

The overall stress–strain equation of concrete is given as follows:

$$d\varepsilon_{mn} = \frac{1}{\overline{L}_t} d\sigma_{mn} + \frac{1}{\overline{L}_v} \sigma^{a}_{mn} df_a \tag{27}$$

where $d\sigma_{mn}$ and $d\varepsilon_{mn}$ are the overall stress and strain of concrete, $\overline{L}_t = (3\overline{K}_t, 2\overline{G}_t)$ is the overall effective elastic modulus of concrete, and $\overline{L}_v = (3\overline{K}_v, 2\overline{G}_v)$ is the effective elastic modulus of voids.

Based on Reuss theory, the overall effective modulus $\overline{L}_t$ can be written as follows:

$$\frac{1}{\overline{L}_t} = (1-f_a-f_v) \cdot \frac{1}{\overline{L}_c} + f_a \cdot \frac{1}{\overline{L}_a} + f_v \cdot \frac{1}{\overline{L}_v} \tag{28}$$

where $\overline{L}_c = (3\overline{K}_c, 2\overline{G}_c)$ and $\overline{L}_a = (3\overline{K}_a, 2\overline{G}_a)$ are the equivalent elastic modulus of cement and aggregates, respectively. From Eqs. (20) and (26) the following equations are obtained:

$$\overline{K}_c = \overline{K}_a \overline{A}_1, \quad \overline{G}_c = \overline{G}_a \overline{A}_2 \tag{29}$$

$$\overline{K}_v = \frac{3\overline{K}_a K(1-\alpha)}{K(1-\alpha)+\alpha K_a}, \quad \overline{G}_v = \frac{2\overline{G}_a G(1-\beta)}{G(1-\beta)+\beta G_a} \tag{30}$$

By Eqs. (13), (14), and (28), (29), (30), the overall effective bulk modulus and the overall effective shear modulus of concrete are obtained:

$$\overline{K}_t = \frac{\dfrac{(1-f_a-f_v)\cdot K_c}{K(1-\alpha)+\alpha K_c} + \dfrac{f_a \cdot K_a}{K(1-\alpha)+\alpha K_a}}{\dfrac{1-f_a-f_v}{K(1-\alpha)+\alpha K_c} + \dfrac{f_a}{K(1-\alpha)+\alpha K_a} + \dfrac{f_v}{K(1-\alpha)}} \tag{31}$$

$$\overline{G}_t = \frac{\dfrac{(1-f_a-f_v)\cdot G_c}{G(1-\beta)+\beta G_c} + \dfrac{f_a \cdot G_a}{G(1-\beta)+\beta G_a}}{\dfrac{1-f_a-f_v}{G(1-\beta)+\beta G_c} + \dfrac{f_a}{G(1-\beta)+\beta G_a} + \dfrac{f_v}{G(1-\beta)}} \tag{32}$$

2.4. Effective Young's modulus of concrete

Using Eqs. (31) and (32), the effective Young's modulus of concrete can be obtained:

$$\overline{E}_t = \frac{9\overline{K}_t\overline{G}_t}{3\overline{K}_t+\overline{G}_t} \tag{33}$$

where $\overline{K}_t$ and $\overline{G}_t$ are the effective elastic and the shear moduli which are defined in Eqs. (31) and (32), respectively. According to Eqs. (31)–(33), the effective Young's modulus can be written as a function $\overline{E}_t = \overline{E}_t(f_a, f_v)$ of which the variables are the aggregate volume fraction f_a and the void volume fraction f_v. As the debonding damage occurs, the void volume fraction f_v is increased while the aggregate volume fraction f_a decreases. The algebraic increment of function $(f_a + f_v)$ remains constant, which equals the initial volume fraction f^I_a of aggregate, in any stage of debonding damage. The effective Young's modulus $\overline{E}_t = \overline{E}_t(f_a, f_v)$ turns to $\overline{E}_t = \overline{E}_t(f^I_a - f_v, f_v) = \overline{E}_t(f_v)$ with a variable of void volume fraction f_v $(0 \leqslant f_v \leqslant f^I_a)$. Therefore, the effective Young's modulus affected by the interfacial debonding damage can be obtained.

3. Results and discussion

In the work of Togho et al. [11], to describe the deformation and the damage of composite in the incremental process, the Eshelby's equivalent inclusion method and Mori–Tanaka's mean field concept were applied for the heterogeneous body containing intact particles with $f_p - df$, voids with f_v, and particles to be debonded with df. For this incremental deformation, the strain–stress relation of composite is given as follows:

$$d\varepsilon_{kk} = \frac{1}{3K_t} d\sigma_{kk} + \frac{1}{3K_v} \sigma^{p}_{kk} df \tag{34}$$

$$d\varepsilon'_{mn} = \frac{1}{2G_t} d\sigma'_{mn} + \frac{1}{2G_v} \sigma'^{p}_{mn} df \tag{35}$$

where

$$K_t = \frac{K_0(1-\alpha)A_h}{[(1-\alpha)(1-f_p-f_v)+f_v][K_0+(K_p-K_0)\alpha]+K_0(1-\alpha)f_p} \tag{36}$$

$$G_t = \frac{G_0(1-\beta)A_v}{[(1-\beta)(1-f_p-f_v)+f_v][G_0+(G_p-G_0)\beta]+G_0(1-\beta)f_p} \tag{37}$$

$$K_v = \frac{K_0(1-\alpha)A_h}{K_0+(K_p-K_0)\alpha} \tag{38}$$

$$G_v = \frac{G_0(1-\beta)A_v}{G_0+(G_p-G_0)\beta} \tag{39}$$

and

$$A_h = (1-f_p-f_v)[K_0+(K_p-K_0)\alpha]+K_pf_p \tag{40}$$

$$A_v = (1-f_p-f_v)[G_0+(G_p-G_0)\beta]+G_pf_p \tag{41}$$

$$\alpha = \frac{1}{3}\frac{1+v_0}{1-v_0} \tag{42}$$

$$\beta = \frac{2}{15}\frac{4-5v_0}{1-v_0} \tag{43}$$

where K_t and G_t are the overall effective bulk and shear moduli, respectively, K_v and G_v are the equivalent bulk and shear moduli of void, respectively, K_p and G_p are the bulk and shear moduli of particle, K_0 and G_0 are the bulk and shear moduli of matrix, respectively, f_p and f_v are the volume fractions of particle and void, respectively, and v_0 is the Poisson's ratio of matrix.

Table 2 presents the effective elastic modulus of concretes with different aggregate volume fractions. It can be found from the columns 2, 3, and 5 in Table 2 that the concrete modulus, in all cases, increases with the increase of aggregate volume fraction. However, the Poisson's ratio decreases with the increase of aggregate volume fraction, as shown in column 4 of Table 2. In practical, there are many parameters which affect the modulus of elasticity. It is impossible to include all of the parameters in one model. This could be the main reason why the values of the modulus of elasticity in Table 2 seem to be somewhat higher.

Fig. 3 shows a comparison between the effective elastic modulus obtained from the present work and the work of Togho et al.

Table 2
Calculated Effective Elastic Modulus and Poisson's Ratio of Concrete.

Aggregate volume fraction f_a	Bulk modulus $\bar{K}_t$ (GPa)	Shear modulus $\bar{G}_t$ (GPa)	Poisson's ratio $\bar{\nu}_t$
5	17.09	12.41	0.20
10	17.80	13.04	0.20
15	18.56	13.72	0.20
20	19.36	14.46	0.19
25	20.21	15.24	0.19
30	21.09	16.08	0.18
35	22.02	16.96	0.18
40	23.00	17.89	0.18
45	24.01	18.88	0.18
50	25.06	19.91	0.17
55	26.16	21.00	0.17
60	27.29	22.14	0.17
65	28.46	23.33	0.17
70	29.67	24.57	0.17
75	30.91	25.86	0.17
80	32.19	27.20	0.16

[11]. For Fig. 3a and b, the aggregate volume fractions are 0.01, 0.03, 0.05, 0.07, and 0.09, respectively, and for Fig. 3c and d, they are 0.1, 0.2, 0.3, 0.4, and 0.5, respectively. The solid curves in Fig. 3 are obtained from the present modified model. Fig. 3a and c show the effective bulk modulus calculated by Eq. (31), and Fig. 3b and d present the effective shear modulus calculated by Eq. (32). The dash lines in Fig. 3 are obtained from the work of Togho et al. [11]. As shown in Fig. 3, the results obtained from the present modified model are higher than those obtained from the work of Togho et al. [11] in most cases. Note that every single curve describes the debonding damage effect on concrete effective elastic modulus. It is shown in Fig. 3 that the difference between the results from these two works is decreased with the decrease of the effective elastic modulus.

Fig. 4 shows a comparison between the effective Young's modulus obtained from the Eq. (33) and the work of Togho et al. [11]. Results obtained from the present modified model are higher than

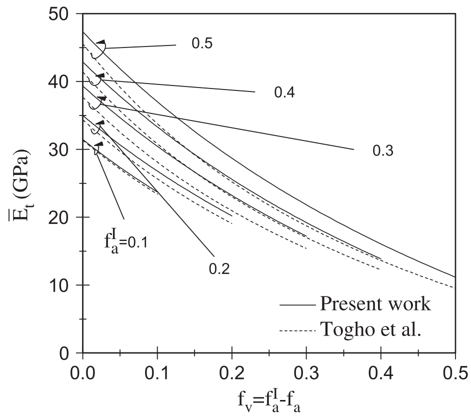


Fig. 4. Relation between effective Young's modulus and initial aggregate volume fractions.

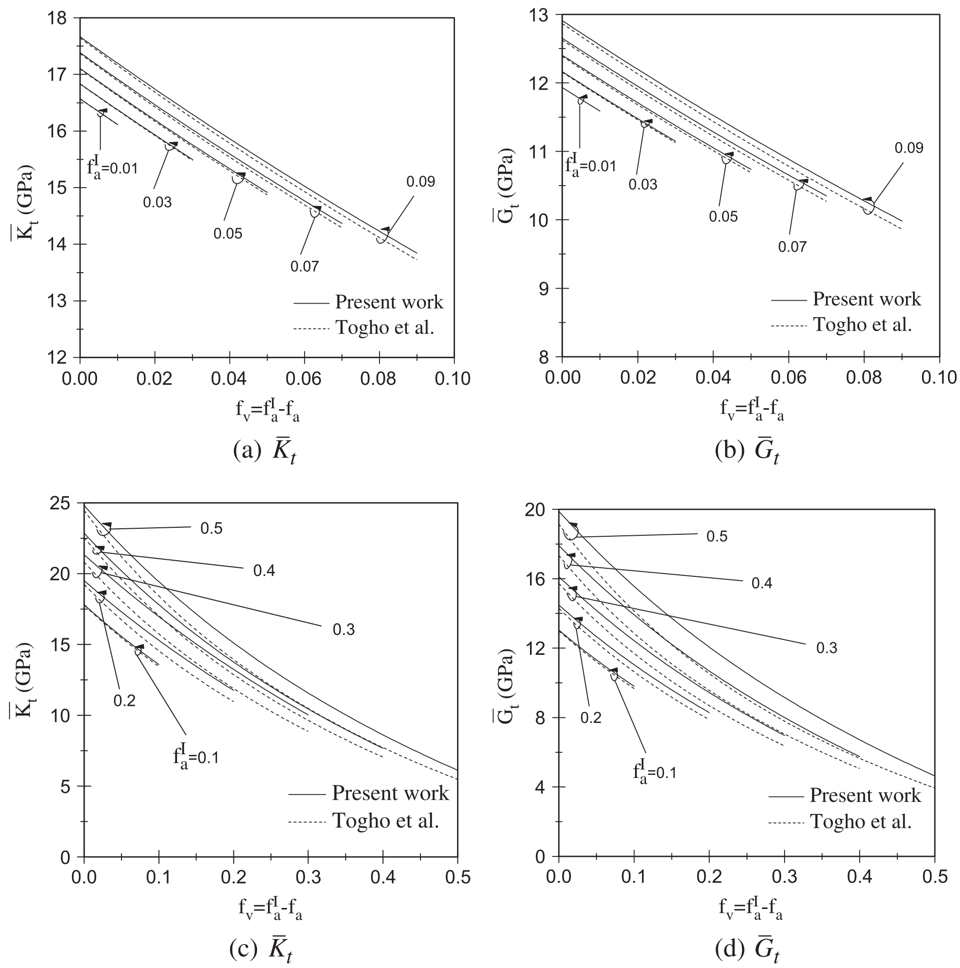


Fig. 3. Relations between effective elastic modulus and aggregate volume fractions.

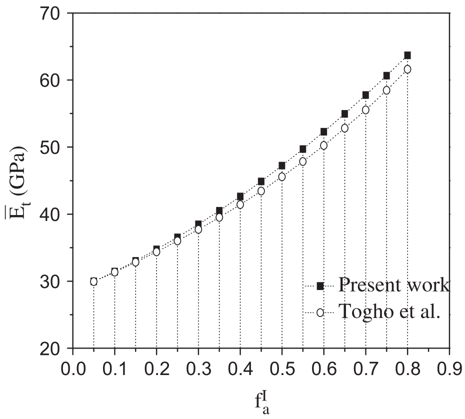


Fig. 5. Relation between effective Young's modulus and initial aggregate volume fractions $f_a^I = 0.05 * c$ (c = 1,2,3,...,16).

those obtained from the work of Togho et al. [11] in most cases. Note that every single curve describes the debonding damage effect on concrete effective elastic modulus. It is shown in Fig. 4 that the difference between the results from these two works is decreased with the decrease of the effective elastic modulus.

Fig. 5 presents the relation between the effective Young's modulus defined by Eq. (33) and the initial aggregate volume fractions. The initial volume fractions f_a^I are 0.05, 0.1, 0.15, 0.2, 0.25, ..., 0.75, 0.8, which can be written as a function $f_a^I = 0.05 * c$ ($c = 1, 2, 3, \ldots, 16$). It can be seen from Fig. 5 that the results obtained from the present modified model are higher than those obtained from the work of Togho et al. [11] in all the cases. However, they are closer, and the difference between these two works is decreased with the decrease of aggregate volume fractions.

4. Conclusions

A modified micro-mechanics model was used to estimate the effective elastic modulus of concrete in this paper. The estimating equations for the effective bulk and shear moduli of concrete were obtained using the equivalent model which was proposed on the basis of self-consistent model. These equations can calculate the effective elastic modulus of concrete with various initial aggregate volume fractions, and can also account for the micro-structure damage evolution of concrete by changing a volume fraction of intact aggregates instead of a volume fraction of voids. Moreover, the effective elastic modulus in any stage of the damage process can be obtained using this new model. It was found that the concrete modulus, in all cases, increases with the increase of aggregate volume fraction. However, the Poisson's ratio decreases with the increase of aggregate volume fraction.

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