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An irreversible heat engine model including three typical thermodynamic cycles and their optimum performance analysis

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Abstract

An irreversible Dual heat engine model, which can include the Otto and Diesel cycles, is established and used to investigate the influence of the multi-irreversibilities mainly resulting from the adiabatic processes, finite time processes and heat leak loss through the cylinder wall on the performance of the cycle. The power output and efficiency of the cycle are derived and optimized with respect to the pressure ratio of the working substance. The maximum power output and efficiency are calculated. The influence of the various design parameters on the performance of the cycle is analyzed. The optimum criteria of some important parameters such as the power output, efficiency and pressure ratio are given. Several special interesting cases are discussed. The results obtained are general, so that the optimal performance of irreversible Otto and Diesel cycles are included in two special cases of the Dual cycle and may be directly derived from that of the Dual heat engine. Moreover, the performance characteristic curves of the three heat engines are presented by using numerical examples.

Keywords: Dual heat engine; Otto cycle; Diesel cycle; Irreversibility; Performance analysis; Optimization criterion

1. Introduction

A study of gas cycles as the models of internal combustion engines is useful for illustrating some of the important parameters influencing engine performance. As we all know, the constant volume heat addition cycle, which is often referred to as the Otto cycle, considers one special case of an internal combustion engine, whose combustion is so rapid that the piston does not move during the combustion process, and thus combustion is assumed to take place at constant volume. The constant pressure heat addition cycle, which is often referred to as the Diesel cycle, models another special case of an internal combustion engine, whose combustion is controlled so that the beginning of the expansion stroke occurs at constant pressure. Modern compression ignition engines resemble neither the constant-volume nor the constant-pressure cycle, but rather the intermediate cycle in which some of the heat is added at constant volume and then the remaining heat is added at constant pressure. The Dual cycle, in which the heat input process of combustion consists of a constant volume followed by a constant pressure, is a gas cycle model that can be used to more accurately model combustion processes that are slower than constant volume, but more rapid than constant pressure [1]. Therefore, the Dual cycle is a better approximation to the modern high speed compression ignition (CI) engine than either the Diesel cycle or the Otto cycle [2].

In recent years, much attention has been paid to the optimization of internal combustion engines for the Dual cycle. Lin et al. [3] considered the effect of heat transfer through a cylinder wall on the work output of the Dual cycle. Şahin et al. [4] made a comparative performance analysis of an endoreversible Dual cycle under the maximum ecological function and maximum power conditions. Chen et al. [5] discussed the effects of cylinder wall heat-transfer and global losses lumped in a friction-like term on the performance of a Dual cycle. Hou [2] analyzed the heat transfer effects on the performance of an air standard Dual cycle. Ust et al. [6] performed an ecological performance analysis for an irreversible Dual cycle by employing the new thermo-ecological criterion as the objective function. Durmayaz et al. [7] reviewed the optimization studies of dual

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Nomenclature

a	proportional coefficient	
C_P	heat capacitance at constant pressure	$\mathrm{J\,K^{-1}}$
C_V	heat capacitance at constant volume	$\mathrm{J\,K^{-1}}$
d_i	parameters defined in Eqs. (16)	
d_i^D	parameters related to the Diesel cycle	
d_i^O	parameters related to the Otto cycle	
K	thermal conductance	$\mathrm{J\,K^{-1}}$
k_i	proportional coefficient	$\mathrm{s\,K^{-1}}$
m_i	parameters defined in Eqs. (43) and (44)	
P	power output	W
P_m	power output at maximum efficiency	W
$P_{\max}$	maximum power output	W
P^*	$= P/\{C_V/[(1+a)k_1]\}$, dimensionless power output	
p_i	pressure of the working substance at state point I	Pa
Q_T	total heat released by combustion per cycle	J
Q_{ij}	heat added to or released by the working substance during the process $i-j$	J
r_p	$= p_4/p_1$, pressure ratio	
r_{pP}	pressure ratio at maximum power output	
$r_{p\eta}$	pressure ratio at maximum efficiency	
T_i	temperature of the working substance at state point I	K
t_{ij}	time of the process $i-j$	s
W	work output	J

Greek symbols

α	constant related to combustion	J
β	constant related to heat transfer	$\mathrm{J\,K^{-1}}$
η	efficiency	
η_c	compression efficiency	
η_e	expansion efficiency	
η_m	efficiency at maximum power output	
$\eta_{\max}$	maximum efficiency	
γ	specific heat ratio	
τ	cycle period	s

Superscripts

D	Diesel cycle
O	Otto cycle
$*$	Dimensionless

cycle and other thermal systems, based on finite-time thermodynamics. Parlak et al. investigated the performance optimization of an irreversible Dual cycle with respect to pressure ratio and temperature ratio and compared with the results of an experimental study of a low heat rejection indirect injection Diesel engine [8,9]. Then they performed a comparative performance analysis and optimization for irreversible Dual and Diesel cycles, based on the maximum power and maximum thermal efficiency criteria [10]. The results obtained will lay a foundation for the further investigation of the Dual heat engine.

In the present paper, a cycle model of the Dual heat engine affected by multi-irreversibilities is established. The power output and efficiency of the cycle are maximized with respect to the pressure ratio, and the main parameters affecting the cycle performance are analyzed and optimized. The optimal criteria of several important parameters are given. The results obtained are presented as some performance characteristic curves through numerical calculation. The studies may be directly extended to the Otto and Diesel cycles as long as some parameters are specially chosen.

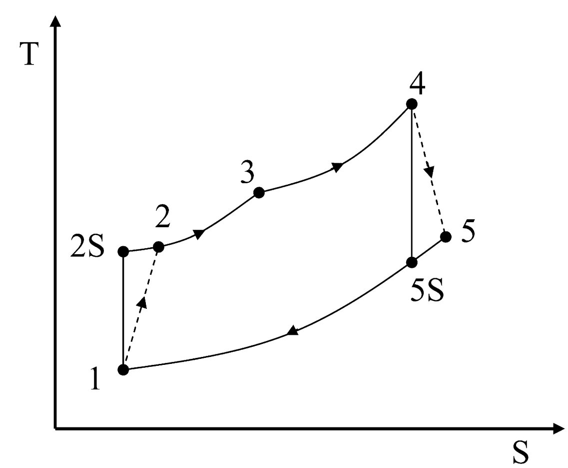

Fig. 1. The temperature-entropy diagram of an irreversible Dual heat engine.

2. An irreversible Dual cycle model

The temperature-entropy ($T \sim S$) diagram of an irreversible Dual heat engine is shown in Fig. 1, where T_1, T_{2S}, T_2, T_3, T_4, T_5 and T_{5S} are the temperatures of the working substance in state points 1, 2S, 2, 3, 4, 5 and 5S. Process 1-2S is an isentropic (reversible adiabatic) compression, while process 1-2 is an irreversible adiabatic process that takes into account the internal irreversibility in the real compression process. The heat addition occurs in two steps: processes 2-3 and 3-4 are heat additions at constant volume and constant pressure, respectively. Process 4-5S is an isentropic (reversible adiabatic) expansion, while process 4-5 takes into account the irreversibility that occurs in the real expansion process. A constant volume heat rejection process 5-1 completes the cycle. Generally, the lowest and the highest temperatures of the working substance, T_1 and T_4, are determined by the ambient temperature and the property of the fuel, so they may be taken as two constants. In the present paper, we consider an irreversible cycle consisting of states 1-2-3-4-5-1, which may include an ideal cycle consisting of 1-2S-3-4-5S-1.

When an ideal gas is used as the working substance, the heats added to and rejected by the working substance are, respectively, given by

$$Q_{in} = Q_{23} + Q_{34} = C_V(T_3 - T_2) + C_P(T_4 - T_3) \tag{1}$$

and

$$Q_{out} = Q_{51} = C_V(T_5 - T_1) \tag{2}$$

where C_V and C_P are the heat capacities at constant volume and constant pressure, respectively. For the two adiabatic processes mentioned above, we may introduce the compression and expansion efficiencies [10–13]

$$\eta_c = (T_{2S} - T_1)/(T_2 - T_1) \tag{3}$$

and

$$\eta_e = (T_5 - T_4)/(T_{5S} - T_4) \tag{4}$$

to describe the irreversibility of the adiabatic processes. Using Eqs. (3) and (4) and the adiabatic equations of an ideal gas, we obtain

$$T_2 = \begin{cases} (1 - 1/\eta_c)T_1 + (1/\eta_c)T_1^{\gamma} T_3^{1-\gamma} r_p^{\gamma-1} & (T_2 < T_3 \leqslant T_4) \\ (1 - 1/\eta_c)T_1 + (1/\eta_c)T_1 r_p^{1-1/\gamma} & (T_3 = T_2) \end{cases} \tag{5}$$

and

$$T_5 = (1 - \eta_e)T_4 + \eta_e T_1^{1-\gamma} T_4^{\gamma} r_p^{1-\gamma} \tag{6}$$

where $\gamma = C_P/C_V$ is the specific heat ratio and $r_p = p_4/p_1 > 1$ is the ratio of the highest to the lowest pressure of the working substance.

For a real engine cycle, there exist other irreversibilities which should not be neglected besides the irreversibility of the adiabatic processes, for example, the heat transfer irreversibility between the working substance and the cylinder wall and the friction-like term loss. The actual heat transfer to the working fluid occurring within the engine cylinder by combustion is very complicated [14]. It is often assumed that the heat leak loss through the cylinder wall is proportional to the average temperature of both the working substance and cylinder wall and that the wall temperature is a constant. The heat added to the working substance by combustion is given by the following linear relation [14–16]

$$Q_{in} = Q_{23} + Q_{34} = \alpha - \beta(T_2 + T_4) \tag{7}$$

where α and β are two constants related to combustion and heat transfer. Eq. (7) implies the fact that heat transfer between the working substance and the cylinder wall obeys a Newtonian law [17,18] and that the heat leak loss through the cylinder wall is proportional to temperature difference between the working substance and cylinder wall. Thus, Eq. (7) may be rewritten as

$$\begin{aligned} Q_{in} &= Q_T - K\big[(T_2 + T_4)/2 - T_0\big] \\ &= Q_T - \beta(T_2 + T_4 - 2T_0) \end{aligned} \tag{8}$$

where Q_T is the total heat released by combustion, $K = 2\beta$ is the thermal conductance between the working substance and the cylinder wall, and T_0 is the average temperature of the cylinder wall.

In order to optimize the power output of the heat engine, it is necessary to calculate the cycle period. It is often assumed the time spent on a process is proportional to the temperature difference of the process [19–22]. Thus, time spent on the heat exchange processes can be, respectively, calculated by

$$t_{23} = k_1(T_3 - T_2) \tag{9}$$

$$t_{34} = k_2(T_4 - T_3) \tag{10}$$

and

$$t_{51} = k_3(T_5 - T_1) \tag{11}$$

where k_1, k_2 and k_3 are proportional coefficients. Furthermore, it can be assumed that the time spent on the two adiabatic processes, which should also be taken into account, are proportional to those spent on the two heat exchange processes and may be expressed as

$$t_{12} + t_{45} = a(t_{23} + t_{34} + t_{51}) \tag{12}$$

where a is a proportional coefficient. Consequently, the cycle period is given by

$$\begin{aligned} \tau &= t_{12} + t_{23} + t_{34} + t_{45} + t_{51} = (1 + a)(t_{23} + t_{34} + t_{51}) \\ &= (1 + a)k_1\left[(T_3 - T_2) + \frac{k_2}{k_1}(T_4 - T_3) + \frac{k_3}{k_1}(T_5 - T_1)\right] \end{aligned} \tag{13}$$

Using the above equations, we can calculate the power output and efficiency of the cycle as

$$\begin{aligned} P &= \frac{W}{\tau} = \frac{C_V}{(1 + a)k_1} \cdot \frac{(T_3 - T_2) + \gamma(T_4 - T_3) - (T_5 - T_1)}{(T_3 - T_2) + \frac{k_2}{k_1}(T_4 - T_3) + \frac{k_3}{k_1}(T_5 - T_1)} \\ &= \begin{cases} \frac{C_V}{(1+a)k_1} \cdot \frac{d_1 + d_2 r_p^{\gamma-1} + d_3 r_p^{2(\gamma-1)}}{d_4 + d_5 r_p^{\gamma-1} + d_3 r_p^{2(\gamma-1)}} & (T_2 < T_3 \leqslant T_4) \\ \frac{C_V}{(1+a)k_1} \cdot \frac{d_1^D + d_2^D r_p^{1-1/\gamma} + d_1 r_p^{1-\gamma}}{d_4^D + d_5^D r_p^{1-1/\gamma} + d_4 r_p^{1-\gamma}} & (T_3 = T_2) \end{cases} \end{aligned} \tag{14}$$

and

$$\begin{aligned} \eta &= \frac{W}{Q_T} = \frac{(T_3 - T_2) + \gamma(T_4 - T_3) - (T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3) + (\beta/C_V)(T_2 + T_4 - 2T_0)} \\ &= \begin{cases} \frac{d_1 + d_2 r_p^{\gamma-1} + d_3 r_p^{2(\gamma-1)}}{d_6 r_p^{\gamma-1} + d_7 r_p^{2(\gamma-1)}} & (T_2 < T_3 \leqslant T_4) \\ \frac{d_1^D + d_2^D r_p^{1-1/\gamma} + d_1 r_p^{1-\gamma}}{d_6^D + d_7^D r_p^{1-1/\gamma}} & (T_3 = T_2) \end{cases} \end{aligned} \tag{15}$$

where

$$d_1 = -\eta_e T_1^{1-\gamma} T_4^{\gamma} \tag{16a}$$

$$d_2 = (1/\eta_c)T_1 + (1 - \gamma)T_3 + (\gamma - 1 + \eta_e)T_4 \tag{16b}$$

$$d_3 = -(1/\eta_c)T_1^{\gamma} T_3^{1-\gamma} \tag{16c}$$

$$d_4 = \frac{k_3}{k_1}\eta_e T_1^{1-\gamma} T_4^{\gamma} \tag{16d}$$

$$\begin{aligned} d_5 &= \left(1/\eta_c - 1 - \frac{k_3}{k_1}\right)T_1 + \left(1 - \frac{k_2}{k_1}\right)T_3 \\ &\quad + \left[\frac{k_2}{k_1} + \frac{k_3}{k_1}(1 - \eta_e)\right]T_4 \end{aligned} \tag{16e}$$

$$\begin{aligned} d_6 &= (1 - 1/\eta_c)(\beta/C_V - 1)T_1 + (1 - \gamma)T_3 \\ &\quad + (\beta/C_V + \gamma)T_4 - 2(\beta/C_V)T_0 \end{aligned} \tag{16f}$$

$$d_7 = (1/\eta_c)(\beta/C_V - 1)T_1^{\gamma} T_3^{1-\gamma} \tag{16g}$$

$$d_1^D = \left[1-\gamma(1-1/\eta_c)\right]T_1 + (\gamma - 1 + \eta_e)T_4 \tag{16h}$$

$$d_2^D = -(1/\eta_c)\gamma T_1 \tag{16i}$$

$$d_4^D = \left(1/\eta_c - 1 - \frac{k_3}{k_1}\right)T_1 + \left[1 + \frac{k_3}{k_1}(1-\eta_e)\right]T_4 \tag{16j}$$

$$d_5^D = -(1/\eta_c)T_1 \tag{16k}$$

$$d_6^D = (1-1/\eta_c)(\beta/C_V - \gamma)T_1 + (\beta/C_V + \gamma)T_4 - 2(\beta/C_V)T_0 \tag{16l}$$

and

$$d_7^D = (1/\eta_c)(\beta/C_V - \gamma)T_1 \tag{16m}$$

It should be noted that the temperature T_3 of the working substance in the Dual heat engine is a controllable parameter and can be theoretically varied from T_2 to T_4. For different temperature T_3, the power output and efficiency of the Dual cycle have different expressions. When $T_3 = T_2$, the Dual cycle will reduce the Diesel cycle. When $T_3 = T_4$, the Dual cycle will reduce the Otto cycle. Thus, one can discuss the performance of the Dual, Diesel and Otto cycles as long as the different values of T_3 are chosen.

Moreover, it is clearly seen from Eq. (15) that when the heat leak loss is considered, the efficiency of the cycle is closely dependent on the heat leak loss. It is important to note that the efficiency defined here is quite different from that defined in some references [2,3,5,23–25], in which the efficiency is independent of the heat leak loss although the heat leak loss was considered.

3. Performance characteristics and parametric optimum criteria

Using Eqs. (14) and (15), one can plot the curves of the power output and efficiency varying with the pressure ratio, as shown in Figs. 2 and 3, where the parameters $T_1 = 350$ K, $T_4 = 2450$ K, $T_3 = 2300$ K, $T_0 = T_1$, $\gamma = 1.4$, $k_1 = k_2 = k_3$ are chosen, $P^* = P/\{C_V/[(1+a)k_1]\}$ is the dimensionless power output, and r_{pP} and $r_{p\eta}$ are, respectively, the pressure ratios at the maximum dimensionless power output $P^*_{\max}$ and the maximum efficiency $\eta_{\max}$. In order to simplify the output plots, the adiabatic efficiencies of the compression and expansion processes of the irreversible cycle have been taken to be equal, i.e., $\eta_c = \eta_e$. The curves in Figs. 2 and 3 show clearly that for the general cases of $\eta_c < 1$, $\eta_e < 1$ and $\beta/C_V > 0$, there exist a maximum power output and a maximum efficiency. From Eqs. (14) and (15) and their extremal conditions, we can prove that when $T_2 < T_3 \leqslant T_4$ and the power output and efficiency attain their extrema, the corresponding pressure ratios r_{pP} and $r_{p\eta}$ are, respectively, given by

$$r_{pP} = \left(\frac{d_1 - d_4 - \sqrt{d_8}}{d_5 - d_2}\right)^{1/(\gamma-1)} \tag{17}$$

and

$$r_{p\eta} = \left(\frac{-d_1 d_7 + \sqrt{d_9}}{d_2 d_7 - d_3 d_6}\right)^{1/(\gamma-1)} \tag{18}$$

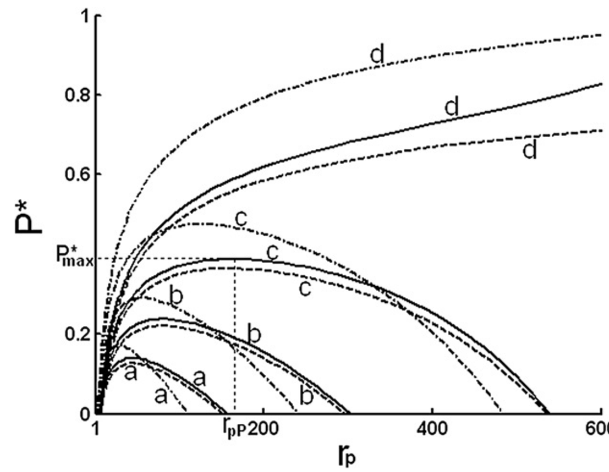

Fig. 2. The $P^* \sim r_p$ curves of the irreversible Dual, Otto and Diesel heat engines for $T_3 = 2300$ K, 2450 K and T_2, respectively, where the parameters $T_1 = 350$ K, $T_4 = 2450$ K, $\gamma = 1.4$, $k_1 = k_2 = k_3$ and $T_0 = T_1$, $P^* = P/\{C_V/[(1+a)k_1]\}$ is the dimensionless power output and r_{pP} is the pressure ratio at the maximum dimensionless power output $P^*_{\max}$. The solid, dashed and dash-dot curves correspond to the Dual, Otto and Diesel cycles, respectively. Curves a, b, c and d correspond to the cases of $\eta_c = \eta_e = 0.7$, 0.8, 0.9 and 1, respectively.

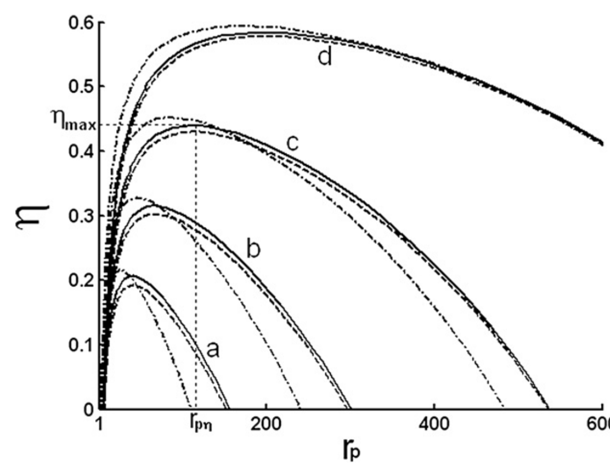

Fig. 3. The $\eta \sim r_p$ curves of the irreversible Dual, Otto and Diesel heat engines for the parameter $\beta/C_V = 0.1$. The values of other parameters are the same as those used in Fig. 2. $r_{p\eta}$ is the pressure ratio at the maximum efficiency $\eta_{\max}$.

where

$$d_8 = (d_1 - d_4)^2 - (d_2 d_4 - d_1 d_5)(d_5 - d_2)/d_3 \tag{19a}$$

and

$$d_9 = d_1^2 d_7^2 - d_1 d_2 d_6 d_7 + d_1 d_3 d_6^2 \tag{19b}$$

Substituting Eqs. (17) and (18) into Eqs. (14) and (15), respectively, one can obtain the maximum power output

$$P_{\max} = \frac{C_V}{(1+a)k_1} \times \left[d_1(d_5-d_2)^2 + d_2(d_5-d_2)(d_1-d_4-\sqrt{d_8}) + d_3(d_1-d_4-\sqrt{d_8})^2\right] \times \left[d_4(d_5-d_2)^2 + d_5(d_5-d_2)(d_1-d_4-\sqrt{d_8}) + d_3(d_1-d_4-\sqrt{d_8})^2\right]^{-1} \tag{20}$$

with the corresponding efficiency

$$\eta_m = \left[d_1(d_5-d_2)^2 + d_2(d_5-d_2)(d_1-d_4-\sqrt{d_8}) + d_3(d_1-d_4-\sqrt{d_8})^2\right] \times \left[d_6(d_5-d_2)(d_1-d_4-\sqrt{d_8}) + d_7(d_1-d_4-\sqrt{d_8})^2\right]^{-1} \tag{21}$$

and the maximum efficiency

$$\eta_{\max} = \left[d_1(d_2d_7-d_3d_6)^2 + d_2(d_2d_7-d_3d_6)(-d_1d_7+\sqrt{d_9}) + d_3(-d_1d_7+\sqrt{d_9})^2\right] \times \left[d_6(d_2d_7-d_3d_6)(-d_1d_7+\sqrt{d_9}) + d_7(-d_1d_7+\sqrt{d_9})^2\right]^{-1} \tag{22}$$

with the corresponding power output

$$P_m = \frac{C_V}{(1+a)k_1} \times \left[d_1(d_2d_7-d_3d_6)^2 + d_2(d_2d_7-d_3d_6)(-d_1d_7+\sqrt{d_9}) + d_3(-d_1d_7+\sqrt{d_9})^2\right] \times \left[d_4(d_2d_7-d_3d_6)^2 + d_5(d_2d_7-d_3d_6)(-d_1d_7+\sqrt{d_9}) + d_3(-d_1d_7+\sqrt{d_9})^2\right]^{-1} \tag{23}$$

When $T_3 = T_2$ and the power output and efficiency attain their extrema, the corresponding pressure ratios r_{pP} and $r_{p\eta}$ are, respectively, determined by

$$\left(d_1^D d_5^D - d_2^D d_4^D\right) r_{pP}^{1/\gamma-\gamma} + \left(d_1 d_5^D - d_4 d_2^D\right)(1+\gamma) r_{pP}^{1/\gamma-1} + \left(d_1 d_4^D - d_4 d_1^D\right)\gamma = 0 \tag{24}$$

and

$$d_2^D d_6^D r_{p\eta}^{1/\gamma-\gamma} + d_2^D d_7^D (1+\gamma) r_{p\eta}^{1/\gamma-1} + \left(d_1^D d_7^D - d_1 d_6^D\right)\gamma = 0 \tag{25}$$

Substituting the solutions of Eqs. (24) and (25) into Eqs. (14) and (15), one can obtain the maximum power output and efficiency and other corresponding parameters through numerical calculation.

The curves in Figs. 2 and 3 also show that the influence of irreversibility of the adiabatic compression and expansion processes on the power output and efficiency of the Dual heat engine is very obvious. Along with the increase of the compression and expansion efficiencies, both the power output and efficiency increase quickly.

From Eqs. (14) and (15), we can further obtain the characteristic curves of the dimensionless power output versus the efficiency for an irreversible Dual heat engine including a Diesel heat engine and an Otto heat engine, as shown in Fig. 4, where P_m^* is the dimensionless power output at the maximum efficiency. Fig. 4 shows clearly that when the heat engine is operated in those parts of the $P \sim \eta$ curve which has a positive slope, the power output will decrease as the efficiency is decreased. These regions are not the optimally operating regions of the heat engine. Obviously, the optimally operating region of the heat engine should be situated in the part of the $P \sim \eta$ curve

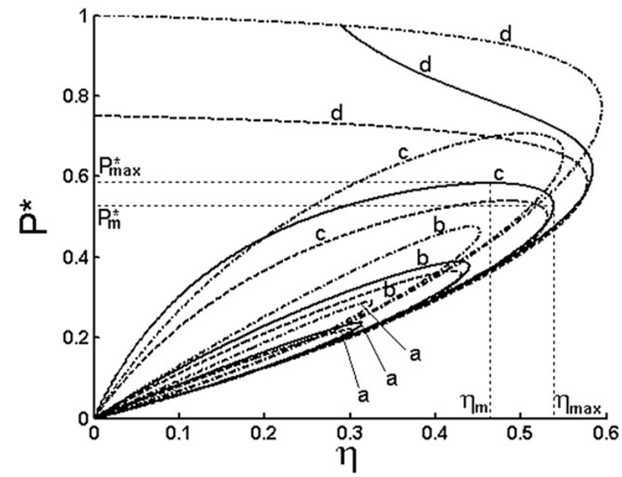

Fig. 4. The $P^* \sim \eta$ curves of the irreversible Dual, Otto and Diesel heat engines. Curves a, b, c and d correspond to the cases of $\eta_c = \eta_e = 0.8, 0.9, 0.97$ and 1, respectively. The values of the relevant parameters are the same as those used in Fig. 3.

which has a negative slope. When the heat engine is operated in the region, the power output will increase as the efficiency is decreased, and vice versa. Thus, the optimal ranges of the power output and efficiency should be

$$P_m \leqslant P \leqslant P_{\max} \tag{26}$$

and

$$\eta_m \leqslant \eta \leqslant \eta_{\max} \tag{27}$$

The above results show that $P_{\max}^*$, $\eta_{\max}$, P_m^* and η_m are four important parameters of the Dual heat engine. $P_{\max}^*$ and $\eta_{\max}$ determine the upper bounds of the dimensionless power output and efficiency, while P_m^* and η_m determine the allowable values of the lower bounds of the dimensionless optimum power output and optimum efficiency. It is clearly seen from Fig. 4 that not only the maximum power output and maximum efficiency but also the optimal ranges of the optimum power output and efficiency decrease with the decrease of the compression and expansion efficiencies.

According to Eqs. (26) and (27), one can further determine the optimal region of the pressure ratio for the irreversible Dual heat engine as

$$r_{pP} \geqslant r_p \geqslant r_{p\eta} \tag{28}$$

This conclusion can be further expounded by Figs. 2 and 3. In the region of $r_p > r_{pP}$, both the power output and efficiency will decrease as the pressure ratio is increased, while in the region of $r_p < r_{p\eta}$, both the power output and efficiency will decrease as the pressure ratio is decreased. It is obvious that the regions of $r_p > r_{pP}$ and $r_p < r_{p\eta}$ are not optimal although the Dual heat engine can be operated in these regions. It is thus clear that both r_{pP} and $r_{p\eta}$ are two important parameters of the Dual heat engine, which determine, respectively, the upper and lower bounds of the optimized pressure ratio. It should be pointed out that the parameters r_{pP} and $r_{p\eta}$ depend not only on the compression and expansion efficiencies η_c and η_e but also on the temperatures of the working substance T_1, T_3 and

T_4 and other parameters. In the design of the Dual heat engine, engineers should choose a rational pressure ratio according to Eq. (28), so that the Dual heat engine can be operated in the optimal region.

It is also seen from Figs. 2–4 that there are the following interesting relations:

$$P_{\max}^{O} \leqslant P_{\max} \leqslant P_{\max}^{D} \tag{29}$$

$$\eta_m \leqslant \eta_m^{O} \leqslant \eta_m^{D} \tag{30}$$

$$r_{pP}^{D} \leqslant r_{pP}^{O} \leqslant r_{pP} \tag{31}$$

$$\eta_{\max}^{O} \leqslant \eta_{\max} \leqslant \eta_{\max}^{D} \tag{32}$$

$$P_m^{O} \leqslant P_m \leqslant P_m^{D} \tag{33}$$

and

$$r_{p\eta}^{D} \leqslant r_{p\eta} \leqslant r_{p\eta}^{O} \tag{34}$$

where η_m^O and r_{pP}^O are the efficiency and pressure ratio of the Otto heat engine at the maximum power output $P_{\max}^O$, P_m^O and $r_{p\eta}^O$ are the power output and pressure ratio of the Otto heat engine at the maximum efficiency $\eta_{\max}^O$, η_m^D and r_{pP}^D are the efficiency and pressure ratio of the Diesel heat engine at the maximum power output $P_{\max}^D$, and P_m^D and $r_{p\eta}^D$ are the power output and pressure ratio of the Diesel heat engine at the maximum efficiency $\eta_{\max}^D$.

It is very significant to note that these parameters of the Otto and Diesel cycles can be directly derived from the above results. For example, the parameters $P_{\max}^O$, $\eta_{\max}^O$, P_m^O, η_m^O, r_{pP}^O and $r_{p\eta}^O$ of the Otto cycle can be deduced from Eqs. (17), (18) and (20)–(23) and are, respectively, given by

$$\begin{aligned}
P_{\max}^{O} = {} & \frac{C_V}{(1+a)k_1} \\
& \times \left[d_1\left(d_5^O - d_2^O\right)^2 + d_2^O\left(d_5^O - d_2^O\right)\left(d_1 - d_4 - \sqrt{d_8^O}\right)\right. \\
& \left. + d_3^O\left(d_1 - d_4 - \sqrt{d_8^O}\right)^2\right] \\
& \times \left[d_4\left(d_5^O - d_2^O\right)^2 + d_5^O\left(d_5^O - d_2^O\right)\left(d_1 - d_4 - \sqrt{d_8^O}\right)\right. \\
& \left. + d_3^O\left(d_1 - d_4 - \sqrt{d_8^O}\right)^2\right]^{-1}
\end{aligned} \tag{35}$$

$$\begin{aligned}
\eta_{\max}^{O} = {} & \left[d_1\left(d_2^O d_7^O - d_3^O d_6^O\right)^2\right. \\
& + d_2^O\left(d_2^O d_7^O - d_3^O d_6^O\right)\left(-d_1 d_7^O + \sqrt{d_9^O}\right) \\
& \left. + d_3^O\left(-d_1 d_7^O + \sqrt{d_9^O}\right)^2\right] \\
& \times\left[d_6^O\left(d_2^O d_7^O - d_3^O d_6^O\right)\left(-d_1 d_7^O + \sqrt{d_9^O}\right)\right. \\
& \left. + d_7^O\left(-d_1 d_7^O + \sqrt{d_9^O}\right)^2\right]^{-1}
\end{aligned} \tag{36}$$

$$\begin{aligned}
P_m^{O} = {} & \frac{C_V}{(1+a)k_1} \\
& \times\left[d_1\left(d_2^O d_7^O - d_3^O d_6^O\right)^2\right. \\
& + d_2^O\left(d_2^O d_7^O - d_3^O d_6^O\right)\left(-d_1 d_7^O + \sqrt{d_9^O}\right) \\
& \left. + d_3^O\left(-d_1 d_7^O + \sqrt{d_9^O}\right)^2\right] \\
& \times\left[d_4\left(d_2^O d_7^O - d_3^O d_6^O\right)^2\right. \\
& + d_5^O\left(d_2^O d_7^O - d_3^O d_6^O\right)\left(-d_1 d_7^O + \sqrt{d_9^O}\right) \\
& \left. + d_3^O\left(-d_1 d_7^O + \sqrt{d_9^O}\right)^2\right]^{-1}
\end{aligned} \tag{37}$$

$$\begin{aligned}
\eta_m^{O} = {} & \left[d_1\left(d_5^O - d_2^O\right)^2 + d_2^O\left(d_5^O - d_2^O\right)\left(d_1 - d_4 - \sqrt{d_8^O}\right)\right. \\
& \left. + d_3^O\left(d_1 - d_4 - \sqrt{d_8^O}\right)^2\right] \\
& \times\left[d_6^O\left(d_5^O - d_2^O\right)\left(d_1 - d_4 - \sqrt{d_8^O}\right)\right. \\
& \left. + d_7^O\left(d_1 - d_4 - \sqrt{d_8^O}\right)^2\right]^{-1}
\end{aligned} \tag{38}$$

$$r_{pP}^{O} = \left(\frac{d_1 - d_4 - \sqrt{d_8^O}}{d_5^O - d_2^O}\right)^{1/(\gamma-1)} \tag{39}$$

and

$$r_{p\eta}^{O} = \left(\frac{-d_1 d_7^O + \sqrt{d_9^O}}{d_2^O d_7^O - d_3^O d_6^O}\right)^{1/(\gamma-1)} \tag{40}$$

where

$$d_2^O = (1/\eta_c)T_1 + \eta_e T_4$$

$$d_3^O = -(1/\eta_c)T_1^{\gamma} T_4^{1-\gamma}$$

$$d_5^O = \left(1/\eta_c - 1 - \frac{k_3}{k_1}\right)T_1 + \left[1 + \frac{k_3}{k_1}(1-\eta_e)\right]T_4$$

$$\begin{aligned}
d_6^O = {} & (1 - 1/\eta_c)(\beta/C_V - 1)T_1 + (\beta/C_V + 1)T_4 \\
& - 2(\beta/C_V)T_0
\end{aligned}$$

$$d_7^O = (1/\eta_c)(\beta/C_V - 1)T_1^{\gamma} T_4^{1-\gamma}$$

$$d_8^O = (d_1 - d_4)^2 - \left(d_2^O d_4 - d_1 d_5^O\right)\left(d_5^O - d_2^O\right)/d_3^O$$

$$d_9^O = d_1^2\left(d_7^O\right)^2 - d_1 d_2^O d_6^O d_7^O + d_1 d_3^O\left(d_6^O\right)^2$$

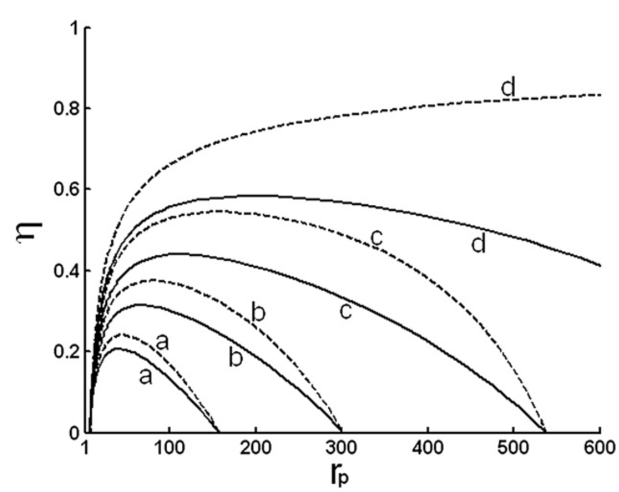

Fig. 5. The $\eta \sim r_p$ curves of the irreversible Dual heat engine for the parameter $\beta/C_V = 0.1$ (solid curves) and 0 (dashed curves) and $T_3 = 2300$ K. The values of the relevant parameters are the same as those used in Fig. 2.

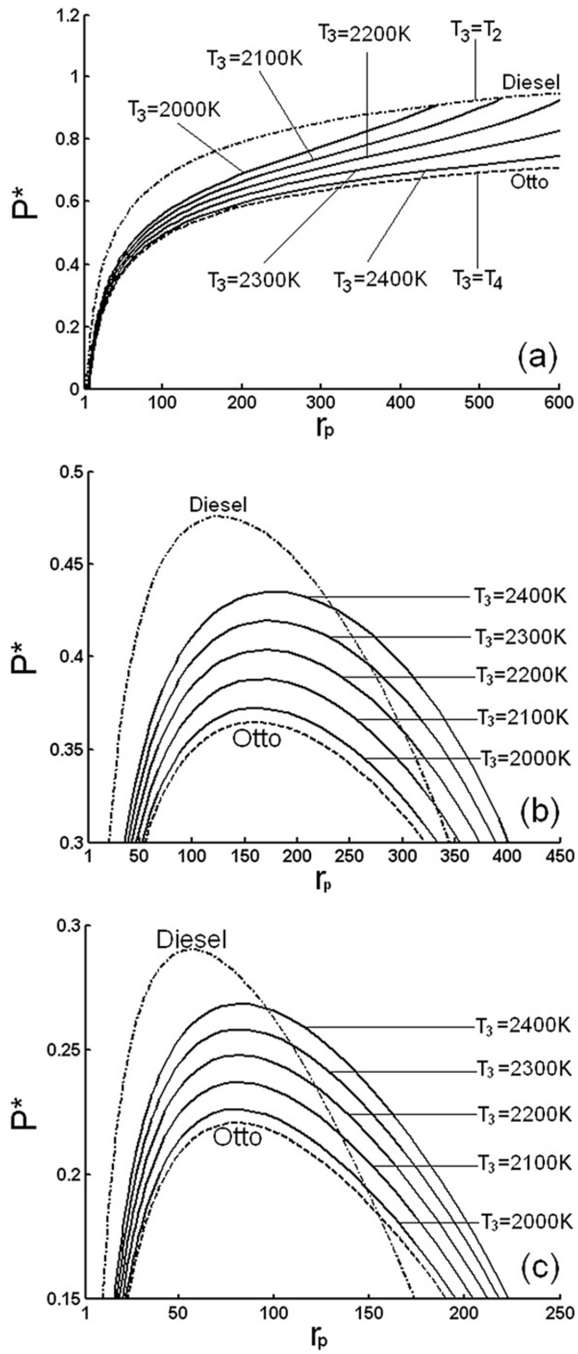

Fig. 6. The influence of T_3 on the $P^* \sim r_p$ curves of the irreversible Dual heat engine under the condition of (a) $\eta_c = \eta_e = 1$, (b) $\eta_c = \eta_e = 0.9$ and (c) $\eta_c = \eta_e = 0.8$. The solid, dashed and dash-dot curves correspond to the Dual, Otto and Diesel cycles, respectively. The values of the relevant parameters are the same as those used in Fig. 2.

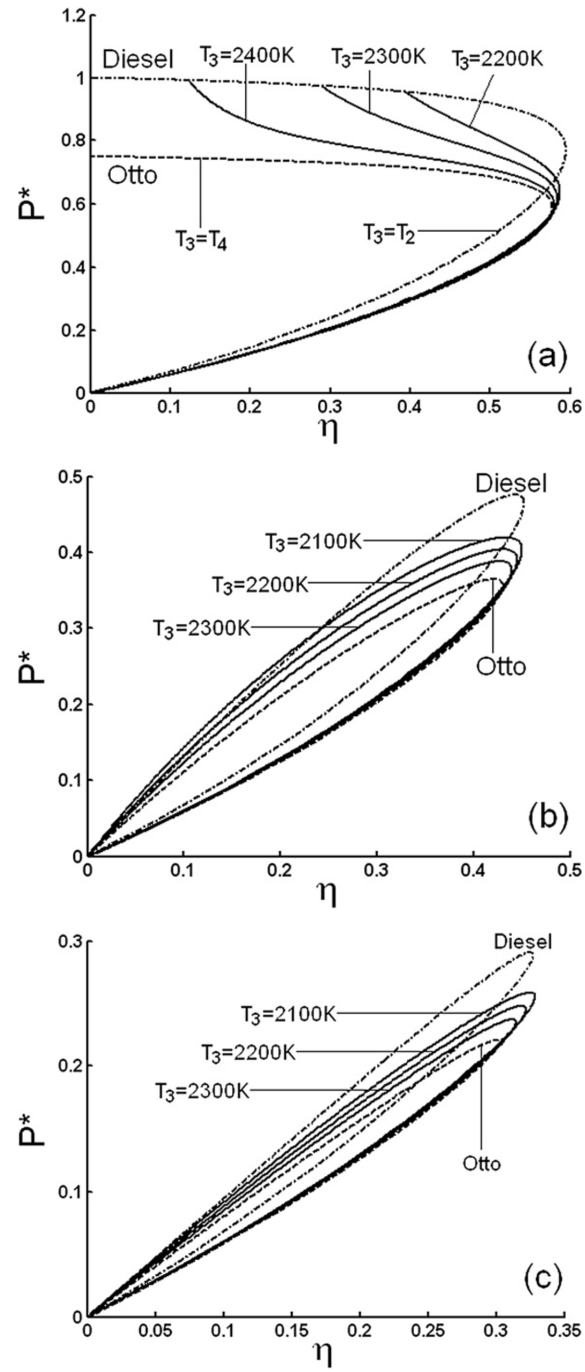

Fig. 7. The influence of T_3 on the $P^* \sim \eta$ curves of the irreversible Dual heat engine under the condition of (a) $\eta_c = \eta_e = 1$, (b) $\eta_c = \eta_e = 0.9$ and (c) $\eta_c = \eta_e = 0.8$. The solid, dashed and dash-dot curves correspond to the Dual, Otto and Diesel cycles, respectively. The values of the relevant parameters are the same as those used in Fig. 3.

As for the Diesel heat engine, the pressure ratios r_{pP}^D and $r_{p\eta}^D$ at the maximum power output and maximum efficiency are, respectively, determined by Eqs. (24) and (25), and the parameters $P_{\max}^D$, $\eta_{\max}^D$, P_m^D, η_m^D, r_{pP}^D and $r_{p\eta}^D$ of the Diesel cycle can be further obtained from Eqs. (14), (15), (24) and (25) through numerical calculation.

It is worthwhile to point out that like the Otto [26] and Diesel cycles, the influence of the heat leak loss always decreases the efficiency of the Dual cycle, which can be clearly seen from Fig. 5.

In order to further expound the influence of temperature T_3 on the performance of the Dual heat engine, Eqs. (14) and (15) can be used to generate the $P \sim r_p$ and $P \sim \eta$ curves of the cycle for some given values of T_3, as shown in Figs. 6 and 7, respectively. It is clearly seen from Figs. 6 and 7 that when T_3 varies from T_2 to T_4, the cycle will vary from the Diesel to the Dual cycle and then to the Otto cycle. Figs. 6 and 7 show once again that the influence of the compression and expansion efficiencies on the power output and efficiency of the Dual heat engine is very obvious.

4. Several interesting cases

(1) When $T_3 = T_2$, processes 2-3 and 3-4 become an isobaric process, so the Dual cycle reduces the Diesel cycle. In such a case, $k_1 = k_2$ and the power output and efficiency of the Diesel cycle are still given by Eqs. (14) and (15). The characteristic curves of the Diesel cycle are presented by the dash–dot curves in Figs. 2–4, 6 and 7.

(2) When $T_3 = T_4$, processes 2-3 and 3-4 become an isochoric process, so the Dual cycle reduces the Otto cycle. In such a case, $k_1 = k_2$ and Eqs. (14) and (15) may be written as

$$P = \frac{C_V}{(1+a)k_1} \cdot \frac{d_1 + d_2^O r_p^{\gamma-1} + d_3^O r_p^{2(\gamma-1)}}{d_4 + d_5^O r_p^{\gamma-1} + d_3^O r_p^{2(\gamma-1)}} \tag{41}$$

and

$$\eta = \frac{d_1 + d_2^O r_p^{\gamma-1} + d_3^O r_p^{2(\gamma-1)}}{d_6^O r_p^{\gamma-1} + d_7^O r_p^{2(\gamma-1)}} \tag{42}$$

The maximum power output and maximum efficiency and the corresponding parameters are, respectively, given by Eqs. (35)-(40). The characteristic curves of the Otto cycle are presented by the dashed curves in Figs. 2–4, 6 and 7.

(3) When the irreversibility of the adiabatic processes in the cycle is negligible, i.e., $\eta_c = \eta_e \to 1$, we can obtain a simple cycle model, and consequently, the results obtained above can be simplified. For example, Eqs. (14) and (15) can be, respectively, written as

$$P = \frac{C_V}{(1+a)k_1} \cdot \frac{m_1 + m_2 r_p^{\gamma-1} + m_3 r_p^{2(\gamma-1)}}{m_4 + m_5 r_p^{\gamma-1} + m_3 r_p^{2(\gamma-1)}} \tag{43}$$

and

$$\eta = \frac{m_1 + m_2 r_p^{\gamma-1} + m_3 r_p^{2(\gamma-1)}}{m_6 r_p^{\gamma-1} + m_7 r_p^{2(\gamma-1)}} \tag{44}$$

where

$$m_1 = -T_1^{1-\gamma} T_4^{\gamma}$$

$$m_2 = T_1 + (1-\gamma)T_3 + \gamma T_4$$

$$m_3 = -T_1^{\gamma} T_3^{1-\gamma}, \qquad m_4 = \frac{k_3}{k_1} T_1^{1-\gamma} T_4^{\gamma}$$

$$m_5 = -\frac{k_3}{k_1} T_1 + \left(1 - \frac{k_2}{k_1}\right) T_3 + \frac{k_2}{k_1} T_4$$

$$m_6 = (1-\gamma)T_3 + (\beta/C_V + \gamma)T_4 - 2(\beta/C_V)T_0$$

and

$$m_7 = (\beta/C_V - 1)T_1^{\gamma} T_3^{1-\gamma}$$

In such a case, the power output is a monotonically increasing function of the pressure ratio, while there is still a maximum value for the efficiency when $r_p = r_{p\eta}$, as shown by curves d in Figs. 2–4 and Figs. 6(a) and 7(a).

(4) When the heat leak loss through the cylinder wall of the engine is negligible, i.e, $\beta/C_V \to 0$, the power output is still expressed by Eq. (14), while the efficiency may be simplified as

$$\eta = \frac{d_1 + d_2 r_p^{\gamma-1} + d_3 r_p^{2(\gamma-1)}}{m_6' r_p^{\gamma-1} + m_7' r_p^{2(\gamma-1)}} \tag{45}$$

where

$$m_6' = (1/\eta_c - 1)T_1 + (1-\gamma)T_3 + \gamma T_4$$

and

$$m_7' = -1/\eta_c T_1^{\gamma} T_3^{1-\gamma}$$

It is clearly seen by comparing the curves without the heat leak loss (the dashed lines) with the curves with the heat leak loss (the solid lines) in Fig. 5 that the efficiency of the heat engine will decrease quickly as the heat leak loss is increased. This shows once again that the influence of the heat leak loss on the efficiency of the heat engine is quite obvious and has to be considered in the performance analysis of the heat engine.

(5) When both the irreversibility of the adiabatic processes and the heat leak loss through the cylinder wall of the engine are negligible, i.e., $\eta_c = \eta_e \to 1$ and $\beta/C_V \to 0$, the expression of the power output is still given by Eq. (43), while the efficiency may be further simplified as

$$\eta = \frac{m_1 + m_2 r_p^{\gamma-1} + m_3 r_p^{2(\gamma-1)}}{m_6'' r_p^{\gamma-1} + m_7'' r_p^{2(\gamma-1)}} \tag{46}$$

where $m_6'' = (1-\gamma)T_3 + \gamma T_4$ and $m_7'' = -T_1^{\gamma} T_3^{1-\gamma}$. In such a case, the efficiency is a monotonically increasing function of the pressure ratio, as shown by the dashed curve d in Fig. 5. This is just the result of a reversible Dual heat engine.

5. Conclusions

We have established an irreversible cycle model of the Dual heat engine which can include the Otto and Diesel cycles. In the cycle model, the multi-irreversibilities coming from the adiabatic compression and expansion processes, finite time processes and heat leak loss through the cylinder wall are taken into account. The power output and efficiency of the cycle are optimized with respect to the pressure ratio of the working substance. The optimum criteria of some important parameters such as the power output, efficiency and pressure ratio are given. Several special interesting cases of the Dual cycle are discussed. The results obtained are general, so that the optimal performance of the Otto and Diesel cycles are included in two special cases of the Dual cycle. It's found that the power outputs of the three heat engines mentioned above are independent of the heat leak loss, but the efficiencies of the three heat engines are closely dependent on the heat leak loss. The increase of the heat leak loss always reduces the efficiencies of these cycles. The irreversibility in the adiabatic processes always reduces the power output and efficiency. These results may be helpful to further understand the inherent relation and essential distinction of the optimal performance of the Dual, Otto and Diesel heat engines and provide some theoretical instructions for the performance evaluation and improvements of a class of real internal combustion heat engines.

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