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A new chaotic Hopfield network with piecewise linear activation function*

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This paper presents a new chaotic Hopfield network with a piecewise linear activation function. The dynamic of the network is studied by virtue of the bifurcation diagram, Lyapunov exponents spectrum and power spectrum. Numerical simulations show that the network displays chaotic behaviours for some well selected parameters.

Keywords: Hopfield network, chaos, piecewise linear function

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1. Introduction

There is a lot of interest in studying the chaotic activities in biological and artificial neural networks.[1−28] Guevara *et al.* found that deterministic mathematical models of neural systems gave rise to complex chaotic dynamics.[1] Babloyantz and Lourenco considered a model cortex comprising two interconnected spatiotemporal chaotic networks, which could discriminate among different patterns presented as inputs.[2] Freeman studied the chaotic activity in the olfactory cortex, and found that chaos was essential for brain activity.[3,4] More and more evidences show that chaos plays an important role for information creation and storage in biological neural networks. Therefore, researches of chaotic neural networks are significant for exploring the qualities of the brain in information processing.

The Hopfield neural network is a simple recurrent network which can work as an efficient associative memory, and it can store certain memories in a manner rather similar to the brain.[29,30] In fact, the Hopfield network is a complex nonlinear dynamic system and one can expect it to display chaotic behaviours. Recently, chaotic activities in artificial neural networks, especially in the Hopfield neural network, have been studied in Refs. [5]–[17] to simulate the chaotic behaviour of the brain. However, most of these researches focus on the Hopfield networks with hyperbolic tangent activation function.

This paper proposes a new chaotic Hopfield network with a piecewise linear activation function. The dynamic of the network is discussed by virtue of the bifurcation diagram and Lyapunov exponents spectrum. It is shown that the proposed network exhibits rich dynamics for different parameters, e.g. chaotic attractors and quasi-periodical motions. The proposed network displays some new double-scroll chaotic attractors.

2. Network model

Consider a Hopfield neural network of the form

$$\dot{X} = -X + W\varphi(X), \tag{1}$$

where $X = [x_1, x_2, x_3]^{\mathrm{T}} \in \mathbb{R}^3$ is the neuron state vector, $\varphi(X) = [\mathrm{satlin}(x_1),\ \mathrm{satlin}(x_2),\ \mathrm{satlin}(x_3)]^{\mathrm{T}}$, the piecewise linear activation function satlin(·) is defined as

$$\mathrm{satlin}(x) = \begin{cases} -1, & \text{if} \quad x \le -1, \\ x, & \text{if} \quad -1 < x \le 1, \\ 1, & \text{if} \quad x \ge 1, \end{cases}$$

and synaptic weights matrix is defined as

$$\boldsymbol{W} = \begin{bmatrix} 1.68 & 3.81 & -2.23 \\ -5.4 & 1.8 & -4.4 \\ -4.38 & -2.2 & a \end{bmatrix}.$$

The parameter a varies from -0.6 to 0.2. Figure 1 illustrates the connection topology of the network (1), which shows a full-connected structure. In simulations, we find that network (1) exhibits chaotic

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behaviours for some well selected parameters a. It should be mentioned that some chaotic Hopfield networks with a piecewise linear activation function are proposed in Refs.[14]–[17]. The connection topology or coupling strength of network (1) is apparently different from that of the networks in Refs. [14]–[17]. In the following, the dynamic of network (1) is studied by virtue of the bifurcation diagram, Lyapunov exponents spectrum and power spectrum.

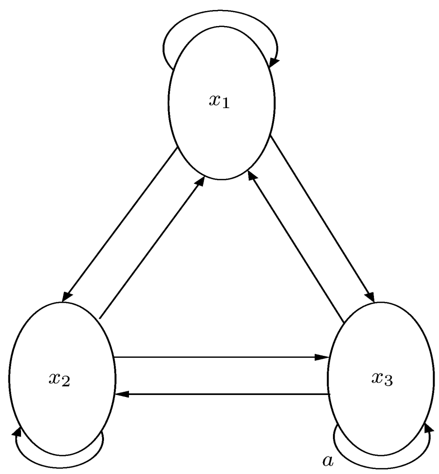

Fig. 1. Connection topology of network (1).

3. Bifurcation and chaotic behaviour

To explore the dynamics of network (1) further, the corresponding bifurcation diagram of state x_2 versus the parameter a is obtained as shown in Fig. 2, the cross-section is defined as the plane $x_1 = 0$. It can be observed from Fig. 2 that, the network response enters into chaos from quasi-periodical motion

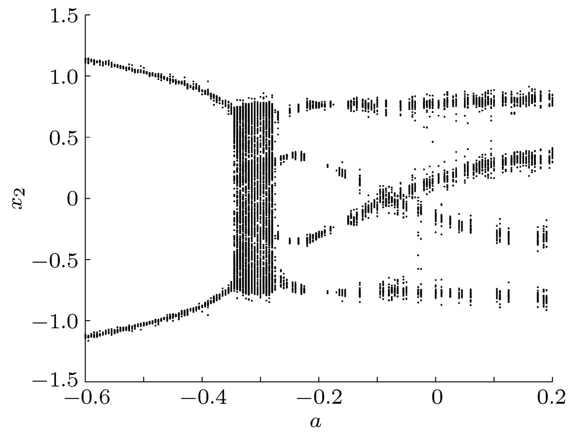

Fig. 2. Bifurcation diagram of state x_2 versus the parameter a.

nearly at $a = -0.35$, and the network response enters into quasi-periodical motion from chaos nearly at $a = -0.27$. Figure 3 shows the Lyapunov exponents spectrum as a varying from -0.6 to 0.2 with step 0.01, following the method listed in Ref. [8]. Usually, a positive maximum Lyapunov exponent is taken as an indication that the system is chaotic.

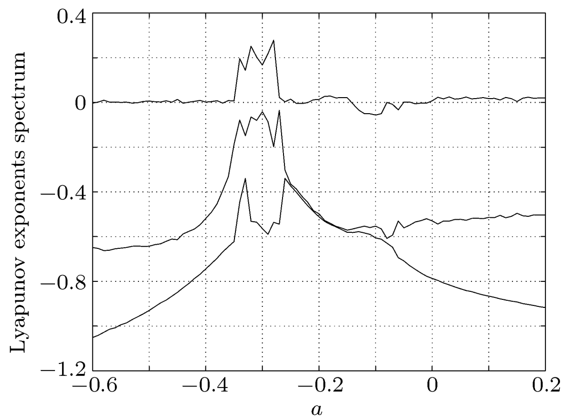

Fig. 3. Lyapunov exponents spectrum of network (1).

As shown in Figs. 2 and 3, the bifurcation diagram coincides well with the corresponding Lyapunov exponents spectrum. Moreover, it can be observed from Figs. 2 and 3 that network (1) displays a robust chaos, because there is no periodic window in the chaotic region. Furthermore, the robust chaos in Hopfield networks with hyperbolic tangent activation function, which displays chaotic behaviours in a large parameter space, has been observed in Ref. [13]. The chaotic region of network (1) is much smaller than that of the network in Ref. [13].

For $a = -0.3$, it can be verified that network (1) has three equilibria, namely $X^1 = [0, 0, 0]^{\mathrm{T}}$, $X^2 = [-0.71, 0.71, 1.24]^{\mathrm{T}}$, $X^3 = [0.71, -0.71, -1.24]^{\mathrm{T}}$.

The Jacobian matrix of network (1) evaluated at X^1 is

$$\boldsymbol{J}_1 = \begin{bmatrix} 0.68 & 3.81 & -2.23 \\ -5.4 & 0.8 & -4.4 \\ -4.38 & -2.2 & -1.3 \end{bmatrix}.$$

It can be calculated that the eigenvalues of $\boldsymbol{J}_1$ are

$$\lambda_1 = 1.83,\ \lambda_2 = -0.83 + 1.45\ \mathrm{i},\ \lambda_3 = -0.83 - 1.45\ \mathrm{i},$$

where λ_1 is a positive real number, λ_2 and λ_3 become a pair of complex conjugate eigenvalues with negative real parts, which implies that the equilibrium point X^1 is a saddle point. Similarly, it can be calculated that X^2 and X^3 are also saddle points.

Figure 4 shows the phrase portrait of network (1) with the initial state $[-0.66, -0.059, 0.648]^{\mathrm{T}}$ for $a =$

−0.3. It can be calculated that the Lyapunov exponents are 0.17, −0.04, −0.56, which implies that the network is chaotic. The calculated Lyapunov exponent is not a rigorous method, so that it is necessary to study the dynamic of network (1) by other methods, such as the Poincare section and power spectrum.

To plot the Poincare section of network (1) with $a = -0.3$, we define the cross-section plane as $x_1 = 0$. Figure 5 shows the Poincare section and power spectrum of network (1). As illustrated in Fig. 5(a), the Poincare section is composed of a large number of thick dots, and the chaotic attractor is almost symmetric with respect to the origin. The power spectrum has a broad-band nature. This confirms that the motion displayed in Fig. 4 is indeed a chaotic attractor.

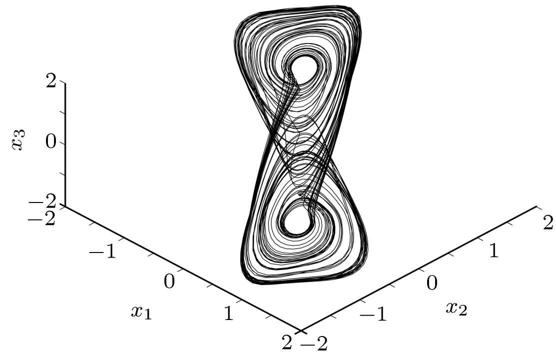

Fig. 4. Phrase portrait of the network with $a = -0.3$.

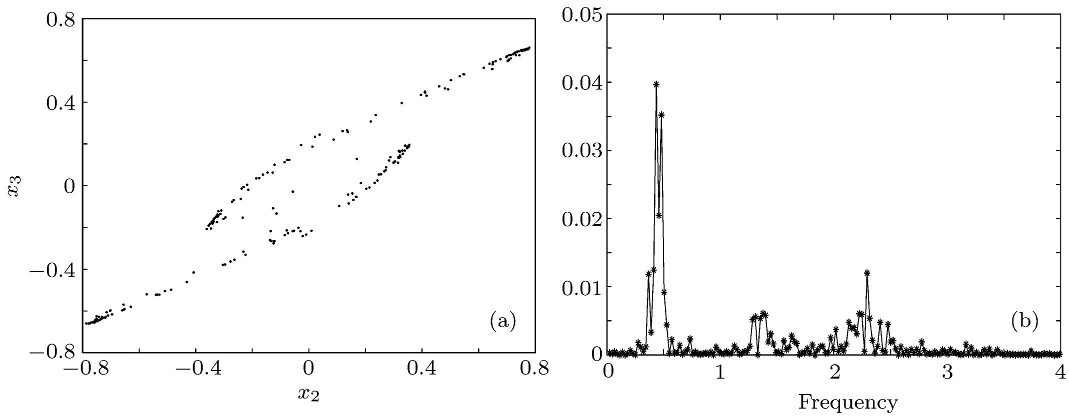

Fig. 5. The Poincare section and power spectrum of the network with $a = -0.3$. (a) Poincare section, (b) Power spectrum of x_2.

For $a = -0.4$, Fig. 6(a) shows the phrase portrait of network (1) with the initial state $[0.745, 0.1927, -0.4336]^{\mathrm{T}}$. Figure 6(b) plots the power spectrum of x_2. The power spectrum possesses several frequencies which are very close to each other. Thus, the network is quasi-periodic, but the motion is very close to a periodic orbit.

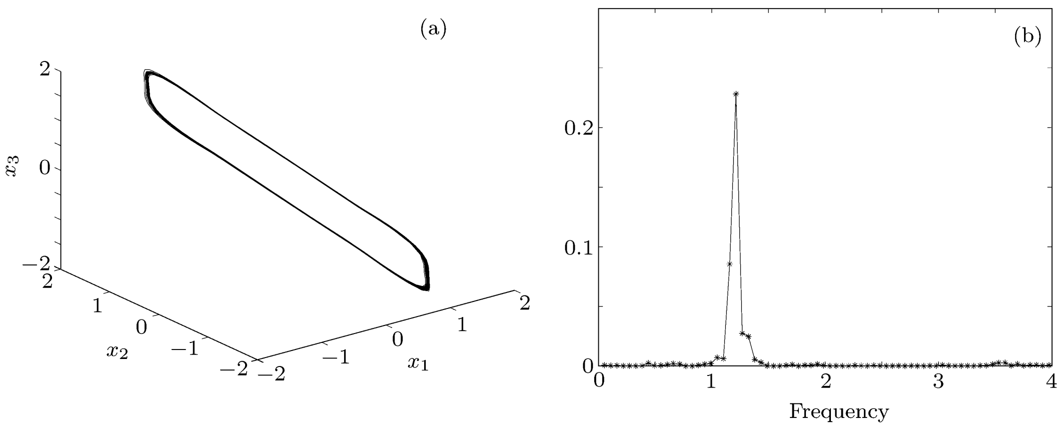

Fig. 6. The phrase portrait and power spectrum of the network with $a = -0.4$. (a) Phrase portrait, (b) Power spectrum of x_2.

For $a = -0.2$, Fig. 7 plots the phrase portrait and the power spectrum of network (1) with the initial state $[0.666, 0.228, -0.606]^{\mathrm{T}}$. As shown in Fig. 7(b), the power spectrum possesses several distinctly different frequencies. Thus, the network is quasi-periodic when $a = -0.2$.

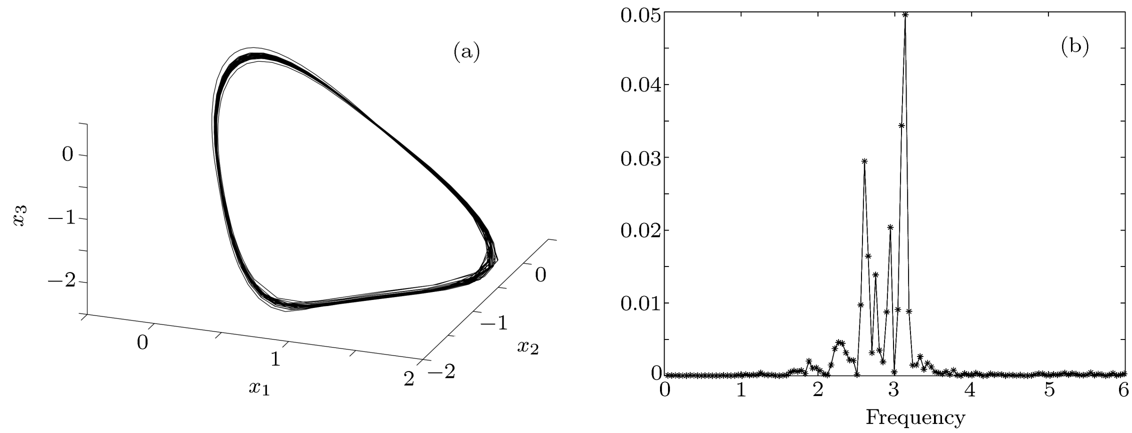

Fig. 7. The phrase portrait and power spectrum of the network with $a = -0.2$. (a) Phrase portrait, (b) Power spectrum of x_2.

In numerical simulations, we find that the network dynamic exhibits sensitive dependence on the connection strength. To illustrate this, Fig. 8 plots the phrase portrait of network (1) with $a = -0.28$. Figures 4 and 8 have the same initial state, angle of view, initial time and final time. Although the parameter a is only slightly changed, but the network dynamic changes apparently. Compared with the network in Ref. [13], we find that the dynamic of network (1) shows a more sensitive dependence on the connection strength.

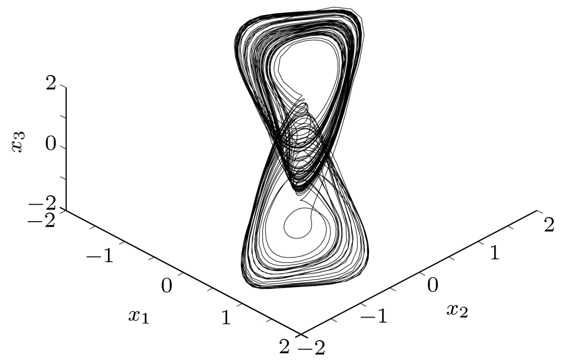

Fig. 8. Phrase portrait of the network with $a = -0.28$.

4. Conclusion

In this paper, a new chaotic Hopfield network with piecewise linear activation function is presented. It is shown that the network displays rich dynamics for different parameters, e.g. chaotic attractors and quasi-periodical motions. The network dynamic exhibits sensitive dependence on the connection strength, and it exhibits chaotic behaviours for some well selected parameters.

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