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# Adaptive actuator failure compensation with unknown control gain signs

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Abstract: This study deals with the problem of adaptive control for a class of non-linear systems with parameterisable timevarying actuator failures. An adaptive output-feedback actuator failure compensation scheme is proposed based on the backstepping technique and the Nussbaum gain approach. The assumption on the control gain signs is removed. The boundedness of all the closed-loop signals and the asymptotic output tracking are guaranteed in spite of the unknown actuator failures. Finally, an example is given to show the effectiveness of the proposed design method.

### 1 Introduction

Most of physical systems suffer from actuator failures. Many efforts have been devoted to providing a theory for the control of such systems. Numerous theoretical results about this topic have been presented (see, e.g. [1, 2], and the references therein). Generally speaking, the control methods can be clarified into the following types: fault detection and diagnosis-based designs [3, 4]; multiple-model designs [5, 6]; linear matrix inequality technique [7-9]; adaptive approach [10, 11]. Among these design methods, adaptive mechanism [12-19] has been extensively employed, and adaptive control has been an effective one for actuator failure compensation. Recent research in this area was reported in [2]. In [20], a novel attempt was made to compensate for the actuator failures in linear time-invariant systems by using adaptive state feedback. Then, in [21], an adaptive output-feedback controller was synthesised. The above two control schemes guarantee the closedloop stability and asymptotic state or output tracking. Furthermore, non-linear systems with actuator failures were investigated. In [22, 23], adaptive state feedback failure compensation schemes were proposed for non-linear systems in the parametric strict-feedback form. In [24, 25], a class of output-feedback non-linear systems were considered. Within the framework of adaptive backstepping technique [14], output-feedback control designs were well developed. In [26-28], the design idea in [24] was further extended to a class of non-linear systems with uncertain non-linearities. However, it is noted that, in the adaptive compensation schemes proposed in [24-28], the signs of control gains  $b_{n^*j}$ ,  $b_{\gamma j}$  or  $b_{m_b j}$  (j = 1, 2, ..., m) were required.

When these signs are unknown, adaptive actuator failure compensation control becomes much more difficult.

Therefore new compensation schemes are needed to cope with completely unknown control gains. Actually, the adaptive control problem with unknown gain signs has been extensively investigated in the existing literature (see, e.g. [29-39]). It is well known that the high-frequency gain has played an important role in the conventional adaptive control for linear systems [12, 13], and some results for relaxing the assumption on the sign of the high-frequency gain have been reported (see, e.g. [29, 30]). It has been shown that if the high-frequency gain sign is unknown, the problem of adaptive control is solvable if we utilise the socalled Nussbaum gain approach. Nussbaum-type function was first proposed in [29], and then it was exploited for adaptive stabilisation of linear or non-linear systems. In the adaptive backstepping control, Nussbaum gain approach is also expensively used. In [33, 34], the parametric strictfeedback systems with unknown constant or time-varying virtual control coefficients were investigated, where two important lemmas were developed to ensure the closed-loop stability and Nussbaum gains were incorporated into the adaptive control design. In [35-37], output-feedback control schemes without requiring the information of control gain signs or control directions were proposed for output-feedback non-linear systems. More recently, Nussbaum gain approach was also applied in adaptive neural control in [38, 39]. However, to the best of authors' knowledge, there is little work to use this method to deal with unknown actuator failures and unknown control gain signs simultaneously.

In this paper, we further address the problem of actuator failure compensation of non-linear systems. The goal is to remove the restrictive assumption on the sign of  $b_{n^*j}$ . We focus on a more general class of non-linear systems than those studied in [24, 25], where some additional parametric uncertainties are considered. Moreover, compared with

[24–27], a parameterisable time-varying actuator failure model is investigated. The proposed controller with Nussbaum gain can guarantee the boundedness of all the signals in the closed-loop system. The output tracking error is proven to tend to zero asymptotically. A simulation example is provided to demonstrate the effectiveness of the proposed control scheme.

The rest of the paper is organised as follows. After introducing the class of the considered system in Section 2, we establish a linearly parameterised model with actuator failure information and present the adaptive controller design in Section 3. Then, we give the stability analysis in Section 4. In Section 5, an example with simulation is presented to illustrate the effectiveness of the design method. Finally, this paper is concluded by Section 6.

### 2 Problem formulation

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Consider a class of non-linear systems in the following form

$$\dot{x}_{i} = x_{i+1} + \varphi_{0,i}(y) + \sum_{j=1}^{q} a_{j}\varphi_{j,i}(y), \quad i = 1, \dots, \rho - 1$$
$$\dot{x}_{\rho} = x_{\rho+1} + \varphi_{0,\rho}(y) + \sum_{j=1}^{q} a_{j}\varphi_{j,\rho}(y) + \sum_{j=1}^{m} b_{n^{*},j}\beta_{j}(y)u_{j}$$
$$\vdots$$

$$\dot{x}_{n-1} = x_n + \varphi_{0,n-1}(y) + \sum_{j=1}^q a_j \varphi_{j,n-1}(y) + \sum_{j=1}^m b_{1,j} \beta_j(y) u_j$$
$$\dot{x}_n = \varphi_{0,n}(y) + \sum_{j=1}^q a_j \varphi_{j,n}(y) + \sum_{j=1}^m b_{0,j} \beta_j(y) u_j$$
$$y = x_1$$
(1)

where  $u_j \in R, j = 1, 2, ..., m$ , are the control inputs whose actuators may fail during operation;  $x = [x_1, x_2, ..., x_n]^T \in R^n$  is the state vector;  $y \in R$  is the system output;  $a_j$ ,  $j = 1, 2, ..., q, b_{r,j}, r = 0, 1, ..., n^* = n - \rho, j = 1, 2, ..., m$ , are unknown constant parameters; the sign of control gain  $b_{n^*,j}$  is unknown, j = 1, 2, ..., m;  $\varphi_{0,i}(y), i = 1, 2, ..., q$ ,  $\beta_{j,i}(y)$ , i = 1, 2, ..., n, j = 1, 2, ..., q,  $\beta_{j}(y)$ , j = 1, 2, ..., m, are known smooth non-linear functions;  $\beta_j(y) \neq 0$  for  $\forall y \in R$ . Only the output y is available for measurement.

A time-varying actuator failure can be modelled as [20]

$$u_j(t) = \bar{u}_j + \bar{d}_j(t), \quad t \ge t_j, \quad j \in \{1, 2, \dots, m\}$$
 (2)

where the failure value  $\bar{u}_j$ , the failure time instant  $t_j$  and the failure index j are unknown;  $\bar{d}_i(t)$  is given by

$$\bar{d}_{j}(t) = \sum_{l=1}^{h} \bar{d}_{jl} f_{jl}(t)$$
(3)

for some unknown scalar constants  $\bar{d}_{jl}$  and known bounded scalar signals  $f_{jl}(t), j = 1, 2, ..., m, l = 1, 2, ..., h, h \ge 1$ .

Suppose that  $p_k$  actuators fail at a time instant  $t_k$ , k = 1, 2,..., q,  $t_0 < t_1 < t_2 < s < t_q < \infty$ . That is, at time  $t \in (t_k, t_{k+1})$ , k = 0, 1, ..., q, with  $t_{q+1} = \infty$ , there are  $p = \sum_{i=1}^{k} p_i$  failed actuators. Thus, the actuator failure mode is

$$u_{j}(t) = \begin{cases} \bar{u}_{j} + \bar{d}_{j}(t), & j = j_{1}, \dots, j_{p} \\ v_{j}(t), & j \neq j_{1}, \dots, j_{p} \end{cases}$$
(4)

where  $v_j(t)$ , j = 1, 2, ..., m, are applied control signals from a feedback control design. For the adaptive compensation control scheme, we use the following actuation scheme

$$v_j(t) = \frac{1}{\beta_j(y)} v_0, \quad j = 1, 2, \dots, m$$
 (5)

where  $\nu_0$  will be determined later.

The control task is that all the closed-loop signals remain bounded, while the plant output y(t) asymptotically tracks a prescribed signal  $y_r(t)$  despite the presence of unknown actuator failures, unknown plant parameters and unknown control gain signs. The reference signal  $y_r(t)$  and its first  $\rho$ derivatives are known and bounded.

*Remark 1:* In the absence of actuator failures, system (1) is in the output-feedback form with multiple inputs (see, [14, pp. 327]). Here, it is worth pointing out that we can extend the proposed design idea to the non-linear systems with unknown virtual control coefficients considered in [33, 34]. Of course, for the actuator failure compensation problem, the control input in [33, 34] should be linearly augmented.

The following assumptions are made for the plant (1) with actuator failures:

Assumption 1: The plant (1) is such that for any up to m-1 actuator failures, the remaining actuators can still achieve a desired control objective, when implemented with the knowledge of the plant parameters and failure parameters.

Assumption 2: The polynomials  $\sum_{j \neq j_1, \dots, j_p} B_j(s)$  are stable,  $\forall \{j_1, \dots, j_p\} \subset \{1, 2, \dots, m\}, \forall p \in \{0, 1, \dots, m-1\},$ where for each  $j = 1, 2, \dots, m, B_j(s)$  is defined as

$$B_{j}(s) = b_{n^{*},j}s^{n^{*}} + b_{n^{*}-1,j}s^{n^{*}-1} + \dots + b_{1,j}s + b_{0,j}$$
(6)

*Remark 2:* Assumption 1 is common (see [20-28]). Assumption 2 can be found in [24-28]. It is noted that the assumption on control gain signs has been made in the existing literature (see (A3) in [24], Assumption 3 in [25], (A4) in [26], (A4) in [27] and Assumption 5 in [28], respectively). However, in this paper, this assumption is not needed. Moreover, the knowledge of sign $[b_{n^*j}]$  does not appear in (5) and Assumption 2.

To deal with the unknown control gain signs, we introduce the knowledge of Nussbaum-type gain. A smooth function  $N(k): R \mapsto R$  is called Nussbaum-type gain if it has the following properties [29]

$$\lim_{s \to +\infty} \sup \frac{1}{s} \int_0^s N(\zeta) d\zeta = +\infty$$
$$\lim_{s \to +\infty} \inf \frac{1}{s} \int_0^s N(\zeta) d\zeta = -\infty$$

For instance,  $k^2 \cos(k)$ ,  $k^2 \sin(k)$  and  $e^{k^2} \cos((\pi/2)k)$  belong to this class of functions. In this paper, an even Nussbaum-type function  $k^2 \cos(k)$  is used.

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# 3 Parameterised model with actuator failure information and adaptive control design

Combining (1)–(5), we represent the plant (1) with p failed actuators as

$$\begin{split} \dot{x}_{i} &= x_{i+1} + \varphi_{0,i}(y) + \sum_{j=1}^{q} a_{j}\varphi_{j,i}(y), \quad i = 1, \dots, \rho - 1 \\ \dot{x}_{\rho} &= x_{\rho+1} + \varphi_{0,\rho}(y) + \sum_{j=1}^{q} a_{j}\varphi_{j,\rho}(y) + \sum_{j=j_{1},\dots,j_{p}} b_{n^{*},j}\bar{u}_{j}\beta_{j}(y) \\ &+ \sum_{j=j_{1},\dots,j_{p}} b_{n^{*},j}\beta_{j}(y) \sum_{l=1}^{h} \bar{d}_{jl}f_{jl}(t) + \sum_{j \neq j_{1},\dots,j_{p}} b_{n^{*},j}\nu_{0} \\ &\vdots \end{split}$$

$$\dot{x}_{n-1} = x_n + \varphi_{0,n-1}(y) + \sum_{j=1}^q a_j \varphi_{j,n-1}(y) + \sum_{j=j_1,\dots,j_p} b_{1,j} \bar{u}_j \beta_j(y) + \sum_{j=j_1,\dots,j_p} b_{1,j} \beta_j(y) \sum_{l=1}^h \bar{d}_{jl} f_{jl}(t) + \sum_{j \neq j_1,\dots,j_p} b_{1,j} \nu_0 \dot{x}_n = \varphi_{0,n}(y) + \sum_{j=1}^q a_j \varphi_{j,n}(y) + \sum_{j=j_1,\dots,j_p} b_{0,j} \bar{u}_j \beta_j(y) + \sum_{j=j_1,\dots,j_p} b_{0,j} \beta_j(y) \sum_{l=1}^h \bar{d}_{jl} f_{jl}(t) + \sum_{j \neq j_1,\dots,j_p} b_{0,j} \nu_0 y = x_1$$
(7)

For a linear parametric model with actuator failures, we define

$$k_{1,r} = \sum_{j \neq j_1, \dots, j_p} b_{r,j}, \quad r = 0, 1, \dots, n^*$$
 (8)

$$k_{2,rj} = \begin{cases} b_{r,j}\bar{u}_j, & r = 0, 1, \dots, n^*, j = j_1, \dots, j_p \\ 0, & r = 0, 1, \dots, n^*, j \neq j_1, \dots, j_p \end{cases}$$
(9)

$$k_{3,rjl} = \begin{cases} b_{r,j}\bar{d}_{jl}, & r = 0, 1, \dots, n^*, j = j_1, \dots, j_p, l = 1, 2, \dots, h \\ 0, & r = 0, 1, \dots, n^*, j \neq j_1, \dots, j_p, l = 1, 2, \dots, h \end{cases}$$
(10)

Applying (8)–(10) to (7), we have

$$\begin{split} \dot{x}_{i} &= x_{i+1} + \varphi_{0,i}(y) + \sum_{j=1}^{q} a_{j}\varphi_{j,i}(y), \quad i = 1, \dots, \rho - 1\\ \dot{x}_{\rho} &= x_{\rho+1} + \varphi_{0,\rho}(y) + \sum_{j=1}^{q} a_{j}\varphi_{j,\rho}(y) + \sum_{j=1}^{m} k_{2,n^{*}j}\beta_{j}(y)\\ &+ \sum_{j=1}^{m} \sum_{l=1}^{h} k_{3,n^{*}jl}\beta_{j}(y)f_{jl}(t) + k_{1,n^{*}}\nu_{0}\\ &\vdots \end{split}$$

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$$\dot{x}_{n-1} = x_n + \varphi_{0,n-1}(y) + \sum_{j=1}^q a_j \varphi_{j,n-1}(y) + \sum_{j=1}^m k_{2,1j} \beta_j(y) + \sum_{j=1}^m \sum_{l=1}^h k_{3,1jl} \beta_j(y) f_{jl}(t) + k_{1,1} \nu_0 \dot{x}_n = \varphi_{0,n}(y) + \sum_{j=1}^q a_j \varphi_{j,n}(y) + \sum_{j=1}^m k_{2,0j} \beta_j(y) + \sum_{j=1}^m \sum_{l=1}^h k_{3,0jl} \beta_j(y) f_{jl}(t) + k_{1,0} \nu_0 y = x_1$$
(11)

Define

$$a = [a_1, a_2, \dots, a_q]^{\mathrm{T}} \in \mathbb{R}^q$$
 (12)

$$k_{2,r} = [k_{2,r1}, k_{2,r2}, \dots, k_{2,rm}]^{\mathrm{T}} \in \mathbb{R}^{m}$$
 (13)

$$k_{3,rj} = [k_{3,rj1}, k_{3,rj2}, \dots, k_{3,rjh}]^{\mathrm{T}} \in \mathbb{R}^{h}$$
 (14)

$$k_{3,r} = [k_{3,r1}^{\mathrm{T}}, k_{3,r2}^{\mathrm{T}}, \dots, k_{3,rm}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{mh}$$
(15)

$$\varphi_i(y) = [\varphi_{1,i}(y), \varphi_{2,i}(y), \dots, \varphi_{q,i}(y)]^{\mathrm{T}}$$
 (16)

$$\boldsymbol{\beta}(\boldsymbol{y}) = \left[\boldsymbol{\beta}_1(\boldsymbol{y}), \, \boldsymbol{\beta}_2(\boldsymbol{y}), \, \dots, \, \boldsymbol{\beta}_m(\boldsymbol{y})\right]^{\mathrm{T}}$$
(17)

$$f_{j}(y, t) = [\beta_{j}(y)f_{j1}(t), \beta_{j}(y)f_{j2}(t), \dots, \beta_{j}(y)f_{jh}(t)]^{\mathrm{T}}$$
(18)

$$f(y, t) = [f_1^{\mathrm{T}}(y, t), f_2^{\mathrm{T}}(y, t), \dots, f_m^{\mathrm{T}}(y, t)]^{\mathrm{T}}$$
(19)

where  $r = 0, 1, ..., n^*, j = 1, 2, ..., m, i = 1, 2, ..., n$ . Then, some terms on the right hand of (11) can be rewritten as

$$\sum_{j=1}^{q} a_{j} \varphi_{j,i}(y) = \varphi_{i}^{\mathrm{T}}(y)a, \quad i = 1, 2, \dots, n$$
 (20)

$$\sum_{j=1}^{m} k_{2,rj} \beta_j(y) = \beta^{\mathrm{T}}(y) k_{2,r}, \quad r = 0, 1, \dots, n^*$$
(21)

$$\sum_{j=1}^{m} \sum_{l=1}^{h} k_{3,rjl} \beta_j(y) f_{jl}(t) = \sum_{j=1}^{m} f_j^{\mathrm{T}}(y, t) k_{3,rj}$$
$$= f^{\mathrm{T}}(y, t) k_{3,r}, \quad r = 0, 1, \dots, n^*$$
(22)

Substituting (20)-(22) into (11), we have

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$$\begin{aligned} \dot{x}_i &= x_{i+1} + \varphi_{0,i}(y) + \varphi_i^{\mathrm{T}}(y)a, \quad i = 1, \dots, \rho - 1 \\ \dot{x}_\rho &= x_{\rho+1} + \varphi_{0,\rho}(y) + \varphi_\rho^{\mathrm{T}}(y)a + \beta^{\mathrm{T}}(y)k_{2,n^*} \\ &+ f^{\mathrm{T}}(y, t)k_{3,n^*} + k_{1,n^*}\nu_0 \end{aligned}$$

$$\dot{x}_{n-1} = x_n + \varphi_{0,n-1}(y) + \varphi_{n-1}^{\mathrm{T}}(y)a + \beta^{\mathrm{T}}(y)k_{2,1} + f^{\mathrm{T}}(y,t)k_{3,1} + k_{1,1}\nu_0 \dot{x}_n = \varphi_{0,n}(y) + \varphi_n^{\mathrm{T}}(y)a + \beta^{\mathrm{T}}(y)k_{2,0} + f^{\mathrm{T}}(y,t)k_{3,0} + k_{1,0}\nu_0 y = x_1$$
(23)

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Furthermore, we can describe (23) as

$$\dot{x} = Ax + \varphi_0(y) + \Phi_a(y)a + \Phi_{k_2}(y)k_2 + \Phi_{k_3}(y, t)k_3 + \begin{bmatrix} 0_{(\rho-1)\times 1} \\ k_1 \end{bmatrix} \nu_0 y = e_1^{\mathrm{T}}x$$
(24)

where  $e_1$  denotes the first coordinate vector in  $\mathbb{R}^n$ 

$$x = [x_1, x_2, \dots, x_n]^{\mathrm{T}}$$
 (25)

$$k_1 = [k_{1,n^*}, k_{1,n^*-1}, \dots, k_{1,0}]^{\mathrm{T}} \in \mathbb{R}^{n^*+1}$$
 (26)

$$k_2 = [k_{2,n^*}^{\mathrm{T}}, k_{2,n^*-1}^{\mathrm{T}}, \dots, k_{2,0}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{(n^*+1)m}$$
 (27)

$$k_3 = [k_{3,n^*}^{\mathrm{T}}, k_{3,n^*-1}^{\mathrm{T}}, \dots, k_{3,0}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{(n^*+1)mh}$$
 (28)

$$\varphi_0(y) = [\varphi_{0,1}(y), \varphi_{0,2}(y), \dots, \varphi_{0,n}(y)]^1$$
(29)

$$\Phi_{a}(y) = [\varphi_{1}(y), \varphi_{2}(y), \dots, \varphi_{n}(y)]^{\mathrm{T}}$$
(30)

$$A = \begin{bmatrix} 0 & & \\ \vdots & I_{n-1} \\ 0 & \dots & 0 \end{bmatrix}$$
(31)

$$\Phi_{k_2}(y) = \begin{bmatrix} 0_{(\rho-1)\times[(n^*+1)m]} \\ \beta^{\mathrm{T}}(y) & 0 & \cdots & 0 \\ 0 & \beta^{\mathrm{T}}(y) & \cdots & 0 \\ & \ddots & \ddots \\ 0 & 0 & \cdots & \beta^{\mathrm{T}}(y) \end{bmatrix}$$
(32)

$$0_{(\rho-1)\times[(n^*+1)mh]}$$

$$\Phi_{k_3}(y,t) = \begin{bmatrix} f^{\mathrm{T}}(y,t) & 0 & \cdots & 0\\ 0 & f^{\mathrm{T}}(y,t) & \cdots & 0\\ & \ddots & \ddots\\ 0 & 0 & \cdots & f^{\mathrm{T}}(y,t) \end{bmatrix}$$
(33)

where  $\Phi_{k_2}(y) \in \mathbb{R}^{n \times [(n^*+1)m]}, \Phi_{k_3}(y, t) \in \mathbb{R}^{n \times [(n^*+1)mh]}$ . By this, we define

$$\theta_1 = [k_2^{\mathrm{T}}, k_3^{\mathrm{T}}, a^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{(n^* + 1)m(h+1)+q}$$
(34)

$$\theta = [k_1^{\mathrm{T}}, \theta_1^{\mathrm{T}}]^{\mathrm{T}} \in R^{(n^*+1)[m(h+1)+1]+q}$$
(35)

$$\Phi(y, t) = [\Phi_{k_2}(y), \Phi_{k_3}(y, t), \Phi_a(y)]$$
(36)

$$F(y, t, v_0)^{\mathrm{T}} = \left[ \begin{bmatrix} 0_{(\rho-1)\times(n^*+1)} \\ I_{n^*+1} \end{bmatrix} \quad v_0, \Phi(y, t) \right]$$
(37)

Combining (24) and (34)–(37), we finally obtain a compact form

$$\dot{x} = Ax + \varphi_0(y) + \Phi(y, t)\theta_1 + \begin{bmatrix} 0_{(\rho-1)\times 1} \\ k_1 \end{bmatrix} \nu_0$$
$$= Ax + \varphi_0(y) + F(y, t, \nu_0)^{\mathrm{T}}\theta$$
$$y = e_1^{\mathrm{T}}x$$
(38)

*Remark 3:* Here, we establish a linear parametric model (38). The underlying idea for introducing this new model is that the unknown plant parameters  $a_j$ , j = 1, 2, ..., q,  $b_{r,j}$ , r = 0,  $1, ..., n^*$ , j = 1, 2, ..., m, and the actuator failure parameters  $\bar{u}_j$ ,  $\bar{d}_{jl}$ , j = 1, 2, ..., m, l = 1, 2, ..., h, are lumped together; that is, they are included in the parameter  $\theta$  (see (35)), which will be estimated online. To achieve this, a new parameter  $k_{3,rjl}$  (see (10)) is defined as a result of introducing the time-varying actuator failure model (see (2)). Moreover, it should be emphasised that (23) or (38) is very useful for the parameter estimations and adaptive control design.

*Remark 4:* It is noted that the parametric model (38) is very similar to (8.3) and (8.7) in [14]. However, unlike Assumption (8.1) in [14], the sign of  $k_{1,n^*}$  is not always known, which implies that the design procedure in [14] is not applicable in this paper. This can be seen from the following explanations. Clearly, the definition of  $k_{1,r}$ ,  $r = 0, 1, \ldots, n^*$ , in this paper (see (8)) is different from that in [24–28] where  $k_{1,r}$  is defined as

$$k_{1,r} = \sum_{j \neq j_1, \dots, j_p} \operatorname{sign}[b_{n^*, j}] b_{r, j}, \quad r = 0, 1, \dots, n^*$$
(39)

If sign $[b_{n^*,j}]$  is known, it follows from the above definition (39) that  $k_{1,n^*} = \sum_{j \neq j_1, \dots, j_p} |b_{n^*,j}| > 0$ , that is, the sign of  $k_{1,n^*}$  is known. Nevertheless, as mentioned in Remark 2, the knowledge of sign $[b_{n^*,j}]$  has not been assumed. On the other hand, it follows from (8) that  $k_{1,n^*} = \sum_{j \neq j_1, \dots, j_p} b_{n^*,j}$ , from which we cannot draw any conclusions on the sign of  $k_{1,n^*}$ . So, following the design steps in [14] is invalid.

*Remark 5:* Fortunately, the adaptive control problem for the system considered in [14] (see, Chapter 8, [14]) which does not require a prior knowledge of the control gain signs has been successfully solved in [36]. Therefore we can follow the same steps as those in [36] to design the controller. Here, we only present the design procedure. For the detailed deduction, please see [36].

We first present the filters design. The following K-filters [14] is employed

$$\dot{\xi} = A_0 \xi + ly + \varphi_0(y)$$

$$\dot{\Xi} = A_0 \Xi + \Phi(y, t)$$

$$\dot{\lambda} = A_0 \lambda + e_n v_0 \qquad (40)$$

$$v_j = A_0^j \lambda, \quad j = 0, 1, \dots, n^*$$

$$\Omega^{\mathrm{T}} = [v_{n^*}, \dots, v_0, \Xi]$$

where  $l = [l_1, l_2, ..., l_n]^T$  is chosen such that  $A_0 = A - le_1^T$  is Hurwitz. Thus,  $\hat{x} = \xi + \Omega^T \theta$  is an observer for x and the estimation error

$$\varepsilon = x - \hat{x} \tag{41}$$

satisfies

$$\dot{\varepsilon} = A_0 \varepsilon \tag{42}$$

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Then, combining (38), (40) and (41) gives

$$\dot{y} = \omega_0 + \omega^{\mathrm{T}} \theta + \varepsilon_2$$
  
=  $k_{1,n^*} v_{n^*,2} + \omega_0 + \bar{\omega}^{\mathrm{T}} \theta + \varepsilon_2$  (43)

where

$$\boldsymbol{\omega} = \left[ v_{n^*,2}, \ \dots, \ v_{0,2}, \ \Phi_{(1)} + \boldsymbol{\Xi}_{(2)} \right]^{\mathrm{T}}$$
(44)

$$\bar{\omega} = [0, v_{n^*-1,2}, \dots, v_{0,2}, \Phi_{(1)} + \Xi_{(2)}]^{\mathrm{T}}$$
 (45)

$$\omega_0 = \varphi_{0,1}(y) + \xi_2 \tag{46}$$

By this, adaptive control design is described as follows.

Step 1:

$$z_1 = y - y_r \tag{47}$$

$$\alpha_1 = N(k)(c_1 z_1 + d_1 z_1 + \omega_0 - \dot{y}_r + \bar{\omega}^{\mathrm{T}} \hat{\theta})$$
(48)

$$\dot{k} = \gamma z_1 (c_1 z_1 + d_1 z_1 + \omega_0 - \dot{y}_r + \bar{\omega}^{\mathrm{T}} \hat{\theta})$$
 (49)

$$\tau_1 = \bar{\omega} z_1 \tag{50}$$

where N(k) is Nussbaum gain;  $\hat{\theta}$  is an estimate of  $\theta$  with the estimation error  $\tilde{\theta} = \theta - \hat{\theta}$ ;  $c_1$ ,  $d_1$  and  $\gamma$  are positive constants.  $c_i$ ,  $d_i$ ,  $i = 2, ..., \rho$ , which will be used later, are positive design parameters.

Step 2:

$$z_2 = v_{n^*,2} - \alpha_1 \tag{51}$$

$$\tau_2 = \tau_1 - \frac{\partial \alpha_1}{\partial y} \omega z_2 + [z_1 z_2, 0, \dots, 0]^{\mathrm{T}}$$
 (52)

$$\begin{aligned} \alpha_{2} &= -\hat{k}_{1,n^{*}}z_{1} - c_{2}z_{2} - d_{2}\left(\frac{\partial\alpha_{1}}{\partial y}\right)^{2}z_{2} + l_{2}v_{n^{*},1} \\ &+ \frac{\partial\alpha_{1}}{\partial y}(\omega_{0} + \omega^{\mathrm{T}}\hat{\theta}) \\ &+ \frac{\partial\alpha_{1}}{\partial\xi}(A_{0}\xi + ly + \varphi_{0}(y)) + \frac{\partial\alpha_{1}}{\partial\Xi}(A_{0}\Xi + \Phi(y, t)) \\ &+ \sum_{j=1}^{2}\frac{\partial\alpha_{1}}{\partial y_{r}^{(j-1)}}y_{r}^{(j)} \\ &+ \sum_{j=1}^{n^{*}+1}\frac{\partial\alpha_{1}}{\partial\lambda_{j}}(-l_{j}\lambda_{1} + \lambda_{j+1}) + \frac{\partial\alpha_{1}}{\partial\hat{\theta}}\Gamma\tau_{2} + \frac{\partial\alpha_{1}}{\partial k}\dot{k} \end{aligned}$$
(53)

Step  $i = 3, \ldots, \rho$ 

$$z_i = v_{n^*,i} - \alpha_{i-1}$$
 (54)

$$\tau_i = \tau_{i-1} - \frac{\partial \alpha_{i-1}}{\partial y} \,\omega z_i \tag{55}$$

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$$\begin{aligned} \alpha_{i} &= -z_{i-1} - c_{i}z_{i} - d_{i} \left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^{2} z_{i} + l_{i}v_{n^{*},1} \\ &+ \frac{\partial \alpha_{i-1}}{\partial y}(\omega_{0} + \omega^{\mathrm{T}}\hat{\theta}) \\ &+ \frac{\partial \alpha_{i-1}}{\partial \xi}(A_{0}\xi + ly + \varphi_{0}(y)) + \frac{\partial \alpha_{i-1}}{\partial \Xi}(A_{0}\Xi + \Phi(y, t)) \\ &+ \sum_{j=1}^{i} \frac{\partial \alpha_{i-1}}{\partial y_{r}^{(j-1)}}y_{r}^{(j)} + \sum_{j=1}^{n^{*}+i-1} \frac{\partial \alpha_{i-1}}{\partial \lambda_{j}}(-l_{j}\lambda_{1} + \lambda_{j+1}) \\ &+ \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\Gamma\tau_{i} + \frac{\partial \alpha_{i-1}}{\partial k}\dot{k} - \sum_{j=2}^{i-1} \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\Gamma\frac{\partial \alpha_{i-1}}{\partial y}\omega z_{j} \end{aligned}$$
(56)

Finally, the actual control signal and parameter adaptive laws are respectively designed as

$$\nu_0 = \alpha_{\rho} - \nu_{n^*, \rho+1} \tag{57}$$

$$\dot{\hat{\theta}} = \Gamma \tau_{\rho} \tag{58}$$

where  $\Gamma = \Gamma^{T} > 0$  is the adaptive gain.

## 4 Stability analysis

To prepare for the stability analysis, a candidate Lyapunov function for the closed-loop system is chosen as

$$V = \frac{1}{2}z^{\mathrm{T}}z + \frac{1}{2}\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta} + \sum_{j=1}^{\rho}\frac{1}{4d_{j}}\varepsilon^{\mathrm{T}}P\varepsilon$$
(59)

where P > 0 satisfies the Lyapunov equation  $PA_0 + A_0^T P = -I$ . Following the same line as that in [36], we can obtain the time derivative of V as

$$\dot{V} \le \gamma^{-1}(k_{1,n^*}N(k) + 1)\dot{k} - \sum_{j=1}^{\rho} c_j z_j^2$$
(60)

$$\leq \gamma^{-1} k_{1,n^*} N(k) \dot{k} + \gamma^{-1} \dot{k}$$
 (61)

We are now at the position to establish the following theorem on the stability of adaptive control system.

*Theorem 1:* The closed-loop adaptive system consisting of the plant (1) under Assumptions 1 and 2, the control law (5), (57), the parameter update laws (49), (58) and the filters (40) has the properties that all the signals are bounded, and the asymptotic output tracking is achieved:  $\lim_{t\to\infty}(y(t) - y_r(t)) = 0$ .

*Proof:* For each time interval  $(t_k, t_{k+1}), k = 0, 1, ..., q$ , we have a Lyapunov function V defined in (59). Starting from the first time interval and integrating (61) on the interval  $(t_0, t), t \in [t_0, t_1)$ , we obtain

$$\int_{t_0}^{t} \dot{V}(\tau) \mathrm{d}\tau \le \gamma^{-1} k_{1,n^*} \int_{t_0}^{t} N(k(\tau)) \dot{k}(\tau) \mathrm{d}\tau + \gamma^{-1} \int_{t_0}^{t} \dot{k}(\tau) \mathrm{d}\tau$$
(62)

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Thus, we have

$$V(t) \le \gamma^{-1} k_{1,n^*} \int_{k(t_0)}^{k(t)} N(u) \mathrm{d}u + \gamma^{-1} k(t) - \gamma^{-1} k(t_0) + V(t_0)$$
(63)

Since k(t) is a continuous function, k(t) is bounded on  $[t_0, t_1]$ . By this, it follows that V(t) is bounded on  $[t_0, t_1]$ , which further implies that z,  $\hat{\theta}$ ,  $\varepsilon$  are all bounded on  $[t_0, t_1]$ . Moreover, from (6) and (8), we have

$$k_{1,n^*}s^{n^*} + k_{1,n^{*-1}}s^{n^*-1} + \dots + k_{1,1}s + k_{1,0}$$
  
=  $\sum_{j \neq j_1,\dots,j_p} (b_{n^*,j}s^{n^*} + b_{n^*-1,j}s^{n^*-1} + \dots + b_{1,j}s + b_{0,j})$   
=  $\sum_{j \neq j_1,\dots,j_p} B_j(s)$  (64)

This together with Assumption 2 implies that the polynomial  $k_{1,n*}s^{n^*} + \cdots + k_{1,1}s + k_{1,0}$  is stable, which corresponds to Assumption (8.2) in [14]. As a result, as discussed in [14], the boundedness of all the closed-loop signals for  $t \in [t_0, t_1)$  can be obtained.

Next, at time  $t = t_1$ ,  $p_1$  actuator failures occur, which results in the abrupt change of  $\theta$ . Since the change of values of these parameters is finite and z,  $\hat{\theta}$ ,  $\varepsilon$  are continuous, it follows from (63) that

$$V(t_{1}^{+}) = V(t_{1}^{-}) + \bar{V}_{1}$$

$$\leq \gamma^{-1} k_{1,n^{*}} \int_{k(t_{0})}^{k(t_{1}^{-})} N(u) du + \gamma^{-1} k(t_{1}^{-})$$

$$- \gamma^{-1} k(t_{0}) + V(t_{0}) + \bar{V}_{1}$$
(65)

with a positive constant  $\overline{V}_1$ , which is similar to (62) in [26]. For all  $t \in (t_1, t_2)$ , integrating (61) on the interval  $(t_1^+, t)$  gives

$$\int_{t_1^+}^t \dot{V}(\tau) \mathrm{d}\tau \le \gamma^{-1} k_{1,n^*} \int_{t_1^+}^t N(k(\tau)) \dot{k}(\tau) \mathrm{d}\tau + \gamma^{-1} \int_{t_1^+}^t \dot{k}(\tau) \mathrm{d}\tau$$
(66)

that is

$$V(t) \le \gamma^{-1} k_{1,n^*} \int_{k(t_1^+)}^{k(t)} N(u) \mathrm{d}u + \gamma^{-1} k(t) - \gamma^{-1} k(t_1^+) + V(t_1^+)$$
(67)

Substituting (65) into (67) and using the continuity of k(t), that is,  $k(t_1^-) = k(t_1^+)$ , we have

$$V(t) \leq \gamma^{-1} k_{1,n^*} \int_{k(t_1^+)}^{k(t)} N(u) du + \gamma^{-1} k(t) - \gamma^{-1} k(t_1^+) + \gamma^{-1} k_{1,n^*} \int_{k(t_0)}^{k(t_1^-)} N(u) du + \gamma^{-1} k(t_1^-) - \gamma^{-1} k(t_0) + V(t_0) + \bar{V}_1$$

$$= \gamma^{-1} k_{1,n^*} \int_{k(t_0)}^{k(t)} N(u) du + \gamma^{-1} k(t) - \gamma^{-1} k(t_0) + V(t_0) + \bar{V}_1$$
(68)

By repeating the same arguments as those on the first time interval, we can prove the boundedness of all the signals for the time interval  $(t_1, t_2)$ .

Continuing in the same manner, we finally obtain

$$V(t) \leq \gamma^{-1} k_{1,n^*} \int_{k(t_0)}^{k(t)} N(u) du + \gamma^{-1} k(t) - \gamma^{-1} k(t_0) + V(t_0) + \sum_{i=1}^{q} \bar{V}_i, \quad t \in (t_q, \infty)$$
(69)

with some positive constants  $\bar{V}_k$ , k = 1, 2, ..., q. On the last time interval  $(t_q, \infty)$ , the boundedness of k(t) can be proved by following the same way as that in [36]. In view of the finite times of actuator failures, it can be obtained that V(t) is bounded for  $\forall t \ge t_0$ , and so are all the closed-loop signals.

To prove the asymptotic output tracking, we consider the last time interval  $(t_q, \infty)$ . From (60) together with the boundedness of k(t), we have  $z_1 \in L_2$ . Furthermore, it follows from the boundedness of all the closed-loop signals that  $\dot{z}_1 \in L_\infty$ . So by using Barbalat's lemma [12], we have  $\lim_{t\to\infty} z_1(t) = \lim_{t\to\infty} (y(t) - y_r(t)) = 0$ . This completes the proof.

### 5 Simulation studies

To verify our results by simulation, we apply the control scheme to a two-axis positioning stage system, which is driven by two linear motors. Actually, this system has been studied in [40, 41], and its mathematical model is described as follows (see [41])

$$\dot{x}_{1} = x_{2} - a_{1}x_{1}$$
  
$$\dot{x}_{2} = q^{T}(y)c + b_{0,1}u_{1} + b_{0,2}u_{2} + b_{0,3}u_{3}$$
  
$$y = x_{1}$$
(70)

where

$$q(y) = [\cos(2\pi y/P), \sin(2\pi y/P), \cos(6\pi y/P), \sin(6\pi y/P)]^{\mathrm{T}}$$
  
$$c = [a_2, a_3, a_4, a_5]^{\mathrm{T}}$$
(71)

 $x_1$  and  $x_2$  are the states; y represents the position of the inertia load of the linear motor;  $u_1$ ,  $u_2$  and  $u_3$  are the voltage signals to the driving motor, which are introduced as redundant actuators for adaptive actuator failure compensation study;  $a_i$ ,  $i = 1, 2, 3, 4, 5, b_{0,i}$ , i = 1, 2, 3, are unknown constants; P is the motor magnet's pitch and P = 60 mm. For the simulation purpose, we consider two actuator failure cases.

Case 1: No failures occur, that is

$$u_{1}(t) = v_{1}(t) = v_{0}, \quad t \in [0, \infty)$$
  

$$u_{2}(t) = v_{2}(t) = v_{0}, \quad t \in [0, \infty)$$
  

$$u_{3}(t) = v_{3}(t) = v_{0}, \quad t \in [0, \infty)$$
  
(72)

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1864 © The Institution of Engineering and Technology 2011 The simulation parameters are as follows

$$a_{1} = 12, a_{2} = -1, a_{3} = 1, a_{4} = -1, a_{5} = 1,$$
  

$$b_{0,1} = 2, b_{0,2} = 3, b_{0,3} = 2$$
  

$$f_{11}(t) = 0.02 \cos(t), f_{21}(t) = 0.02 \sin(t), f_{31}(t) = 0.01 \sin(t)$$
  

$$l = [1, 1]^{T}, c_{1} = d_{1} = 1, c_{2} = d_{2} = 5$$
  

$$\gamma = 20, \Gamma = \text{diag}\{10, 10, 5, 5, 5, 5, 1, 1, 1, 1, 1, 1\}$$
  

$$x(0) = [0.1, -0.1]^{T}, k(0) = 0$$
  

$$\hat{\theta}(0) = \text{diag}\{0, 0, 0, 0, 0, 0, 0, 3, -0.4, 0.4, -0.5, 0.5\}$$
  
(73)

The simulation results are shown in Figs. 1 and 2 for  $y_r = 0.1 \sin(0.5t)$  and Figs. 3 and 4 for  $y_r = 0.1$ , respectively. *Case 2:* We consider the case where  $u_2$ ,  $u_1$  fail at the 20th second and the 40th second, respectively, whereas  $u_3$  does



**Fig. 1** *Case 1: plant output y and reference signal*  $y_r = 0.1 \sin(0.5t)$ 



**Fig. 2** Case 1: control inputs with  $y_r = 0.1 \sin(0.5t)$ 

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**Fig. 3** *Case 1: plant output y and reference signal*  $y_r = 0.1$ 



**Fig. 4** *Case 1: control inputs with*  $y_r = 0.1$ 

not fail; that is

$$u_{1}(t) = \begin{cases} \nu_{1}(t), & t \in [0, 40) \\ \bar{u}_{1} + \bar{d}_{11}f_{11}(t), & t \in [40, \infty) \end{cases}$$

$$u_{2}(t) = \begin{cases} \nu_{2}(t), & t \in [0, 20) \\ \bar{u}_{2} + \bar{d}_{21}f_{21}(t), & t \in [20, \infty) \end{cases}$$

$$u_{3}(t) = \nu_{3}(t), \quad t \in [0, \infty)$$

$$(74)$$

Failure parameters are selected to be

$$\bar{u}_1 = 2, \quad \bar{u}_2 = 1.5, \quad \bar{d}_{11} = 2, \quad \bar{d}_{21} = 2$$
 (75)

and other parameters are same as those in Case 1. The response curves are shown in Figs. 5 and 6 for  $y_r = 0.1\sin(0.5t)$  and Figs. 7 and 8 for  $y_r = 0.1$ , respectively. When we apply the proposed adaptive controller in Theorem 1 to system (70), the above figures show that the closed-loop systems can be stabilised. Then, from this example we conclude that the presented control scheme can work effectively for non-linear systems with or without failures in actuators. Furthermore, from Figs. 5 and 7, we can see that the asymptotical tracking of non-linear system

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**Fig. 5** *Case 2: plant output y and reference signal*  $y_r = 0.1 \sin(0.5t)$ 



**Fig. 6** Case 2: control inputs with  $y_r = 0.1 \sin(0.5t)$ 



**Fig. 7** *Case 2: plant output y and reference signal*  $y_r = 0.1$ 



**Fig. 8** *Case 2: control inputs with*  $y_r = 0.1$ 

(70) with actuator failures (74) is achieved although output tracking errors jump when the actuator failures occur. Figs. 6 and 8 indicate the boundedness of control inputs although they are large in the initial stage.

#### 6 Conclusion

In this paper, an adaptive actuator failure compensation scheme based on the backstepping method has been proposed. A linearly parameterised model with unknown system parameters and actuator failure parameters has been established. The Nussbaum gain approach has been exploited to relax the assumption on the control gain signs. Simulations have been conducted to verify the effectiveness of the proposed control algorithm.

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