Downscaling 1-km Topographic Index Distributions to a Finer Resolution for the TOPMODEL-Based GCM Hydrological Modeling

Feifei Pan[1] and Anthony W. King[2]

Abstract: TOPMODEL predictions of surface runoff and subsurface flow are fundamentally developed on the basis of the topographic index distribution (TID). The scale dependency of the TID [i.e., dependency on the resolution of the digital elevation model (DEM) data used to compute the topographic indexes] determines that downscaling of the TID computed from a coarser resolution DEM to a finer resolution is needed before the TOPMODEL concepts can be applied to simulate hydrological processes at some larger scales than the scale of hillslopes. It was found that adjusting only the mean values cannot achieve an accurate downscaling of the TID because the difference between 2-m TIDs and the downscaled TIDs from a coarser resolution to 2 m through adjusting only mean values resulted in overestimation of the fraction of the saturation area and surface runoff under wet conditions and underestimation under dry conditions. It was found that downscaling by correcting for scale-dependencies in the first three moments of TIDs produced better predictions. A series of empirical relationships among mean, standard deviation, and coefficient of skewness of TIDs of nine catchments in eastern Tennessee at resolutions of 2, 10, and 100 m and 205 watersheds across the contiguous United States at resolutions of 10 m and 1 km were developed for downscaling TIDs from 1 km to 2 m through approximating TIDs by a 3-parameter gamma distribution function. The errors in the downscaled TIDs from 1 km to 10 m over 205 watersheds across the contiguous United States decreased with increasing watershed size and approached a minimum (approximately 6%) as the watershed drainage area was larger than approximately 500 km^2. With the constructed empirical relationships, topographic indexes computed from 1-km DEM can be scaled down to 2 m for reducing errors and uncertainties in the TOPMODEL-based general circulation model (GCM) hydrological simulations. **DOI:** **10.1061/(ASCE)HE.1943-5584.0000438.**

CE Database subject headings: Hydrologic models; Base flow; Runoff; Water table.

Author keywords: TOPMODEL; Topographic index distribution (TID); Digital elevation model (DEM); 3-parameter gamma distribution; Saturation area; Base flow; Surface runoff; Depth to water table.

Introduction

Although the original TOPMODEL concept was proposed to represent hydrological processes at the scale of hillslopes (Beven and Kirkby 1979), it has been applied at much larger spatial scales. For example, TOPMODEL was implemented in the Land Surface Model (LSM) (Bonan 1998), the Common Land Model (CLM) (Dai et al. 2003), and embedded in general circulation models (GCM) (e.g., Koster et al. 2000; Ducharne et al. 2000; Yang and Niu 2003; Niu and Yang 2003). TOPMODEL predictions of surface runoff and subsurface flow are fundamentally based on the topographic index distribution (TID). The TID is the spatial probability distribution of the topographic index of a basin, watershed, or catchment, and the topographic index is the natural logarithm of the ratio of the specific flow accumulation area a to the ground surface slope $\tan\beta$, calculated for each cell of a computational grid superimposed on a basin, watershed, or catchment. Many studies have shown that TIDs are scale dependent; i.e., the distribution is strongly affected by the resolution of the digital elevation model (DEM) used to produce the topographic index (e.g., Wolock and Price 1994; Zhang and Montgomery 1994; Quinn et al. 1995). DEMs used in GCMs are generally much coarser (e.g., 1-km resolution) than those used to define hillslopes in more traditional TOPMODEL applications (normally down to 10-m resolution). Therefore, a spatial rescaling of TIDs between GCM resolutions and hillslope scales is needed before the TIDs can be used to predict saturation area and runoff in GCMs.

Generally, GCMs are run at spatial resolutions of 2.5, 5, or even 10° latitude and longitude. With the continued growth in high performance computing, a future with GCM runs at 1-km resolution is no longer "science fiction." Indeed, numerical tests at 1-km resolution had already been conducted on the Japanese Earth Simulator (JES) (Habata et al. 2003). With the advent of 1-km GCMs, the DEMs supporting them will likely have resolutions of tens of meters. However, for the near future, GCM climate simulations will continue with spatial resolutions on the order of one to several degrees latitude/longitude, and the DEMs defining topography in GCMs will likely have spatial resolutions no finer than 1 km. Thus, there exists an immediate need for downscaling 1-km TIDs to a finer resolution for the TOPMODEL-based GCM hydrological simulations.

Although several studies have focused on this issue, how to best achieve this downscaling for GCM simulations is uncertain. Wolock and McCabe (2000) developed an empirical relationship between TIDs at 1-km and 100-m resolutions which can be used for downscaling TIDs from 1 km to 100 m. Kumar et al. (2000)

[1]Assistant Professor, Dept. of Geography, Univ. of North Texas, Denton, TX 76203 (corresponding author). E-mail: feifei.pan@unt.edu

[2]Scientist, Environmental Science Division, MS 6335, Oak Ridge National Laboratory, Oak Ridge, TN 37831-6335.

Note. This manuscript was submitted on November 12, 2010; approved on May 19, 2011; published online on May 21, 2011. Discussion period open until July 1, 2012; separate discussions must be submitted for individual papers. This paper is part of the ***Journal of Hydrologic Engineering***, Vol. 17, No. 2, February 1, 2012. ©ASCE, ISSN 1084-0699/2012/2-243–251/$25.00.

constructed an empirical downscaling similar to Wolock and McCabe's (2000), finding that the mean, standard deviation, and the coefficient of skewness (hereafter skewness) of TIDs scaled linearly between 90 m and 1 km. Chen and Kumar (2001) used this relationship to scale 1-km topographic indexes down to 100 m in a simulation for North America. Yang and Niu (2003) used a 1-km DEM to compute TIDs for two catchments and applied these results to define a constant topographic index for every grid cell of a global climate simulation. Ibbitt and Woods (2004) assumed that the TIDs computed from different resolution DEM data have similar shapes and thus downscaling a TID can be achieved through "shifting" the distribution, i.e., adjusting the mean value of the TID. Pradhan et al. (2006) proposed a method for downscaling TIDs computed from 1-km DEMs to 50 m through adjusting the specific flow accumulation area by a DEM resolution factor and scaling the slope using a fractal method. Sørensen and Seibert (2007) applied 5-m DEM derived from light detection and ranging (LIDAR) data to study the effects of DEM resolution on the calculation of the topographic indexes. Their results showed that there are considerable differences between topographic indexes computed from DEMs of different grid resolution, and the computed specific flow accumulation area seemed to be more affected by the DEM resolution than the computed slope. Yong et al. (2009) conducted statistical analyses on the TIDs computed from 90-m and 1-km DEMs in China and found that a series of linear relationships existed between mean values of 90-m and 1-km TIDs in three sampled blocks with different sizes (i.e., $0.1° \times 0.1°$, $0.5° \times 0.5°$, and $1° \times 1°$).

Although all the noted preceding studies focused on the scale dependency of TIDs and downscaling of TIDs computed from a coarser DEM resolution to a finer resolution, there are still two unresolved issues: (1) What is the fine resolution DEM needed for computing the topographic index to give accurate TOPMODEL-based hydrologic predictions? To what fine resolution should the TIDs computed from 1-km DEM be downscaled? According to Zhang and Montgomery (1994), 10-m or even finer resolution is needed for TOPMODEL-based hydrological modeling. Therefore, downscaling of 1-km TIDs to 10-m or even finer resolutions may be required before TOPMODEL concepts can be accurately incorporated into GCM climate simulations. However, it is computational expensive to directly compute topographic indexes for global land surface using 10-m DEMs and actually the 10-m resolution DEM data are now available only for the contiguous United States and limited parts of Alaska and Hawaii. (2) Can an accurate downscaling of the TIDs computed from 1-km DEMs be achieved through "shifting" the mean values (e.g., Ibbitt and Woods 2004)? Are the shapes of the TIDs scale independent? To answer these questions, a series of statistical analyses were conducted of TIDs computed from 2-, 10-, and 100-m resolution DEMs in eastern Tennessee and 10-m and 1-km resolution DEMs in 205 watersheds across the contiguous United States. The main objective of this paper is to improve the understanding of the effects of DEM resolution on TIDs and establish a series of empirical relationships among watershed statistical moments of TIDs computed from 2-, 10-, and 100-m and 1-km resolution DEMs. With the constructed empirical relationships, 1-km TIDs can be downscaled to 2 m for reducing errors and uncertainties in the TOPMODEL-based GCM hydrological simulations.

The second objective of this study is to identify the minimum size of watershed suitable for applying the TOPMODEL concept with a TID downscaled from 1-km resolution. The TOPMODEL assumes that the recharge rate is spatially uniform (Beven and Kirkby 1979). In reality, the recharge rate is a function of soil hydraulic properties, and the spatial heterogeneity in soil hydraulic properties dictates that the larger a watershed (encompassing greater heterogeneity), the larger the error in assuming a spatially uniform recharge rate for the watershed. Therefore, to limit the error, TOPMODEL should be applied to small, homogeneous, watersheds. However, the watershed cannot be too small relative to the resolution of the DEM because of significant errors in the downscaled TID arising from too few DEM grid cells (i.e., sampling points) from which to form the TID. Therefore, there may be an optimal range of watershed size for applying the TOPMODEL with a downscaled TID computed from a 1-km resolution DEM. This threshold scale of watershed size is similar to the representative elementary area (REA) first suggested by Wood et al. (1988). Wood et al. argued that, at the REA, the spatial pattern of topography, soil, vegetation, and rainfall is not important for hydrological modeling because the hydrological processes can be captured by their statistical characteristics (Wood et al. 1988; Wood 1995). In this study, only the topographic aspects of the REA are considered.

Topographic Index Calculation

Calculating TOPMODEL's topographic index requires quantification of ground surface slope, $\tan\beta$, and the specific flow accumulation area, a. The surface slope can be evaluated from DEM data. The specific flow accumulation area is the total flow accumulation area (or upslope area), A, through a unit contour length, L, i.e., $a = A/L$. The total flow accumulation area is computed by starting at the DEM cell of interest and tracking flow directions upslope to the upstream divide of the watershed; A is calculated as the total area of upslope cells contributing to the drainage area of the original downstream cell.

Generally speaking, three types of algorithm are commonly used to compute the topographic indexes (Pan et al. 2004): single flow direction (SFD) (O'Callaghan and Mark 1984), biflow direction (BFD) (Tarboton 1997), and multiple flow direction (MFD) (Quinn et al. 1995; Pan et al. 2004). Among these algorithms, the MFD method is the most accurate followed by the BFD method and the SFD method (Pan et al. 2004). Accordingly, the MFD algorithm is adopted in this study.

TOPMODEL Predictions of Surface Runoff and Base Flow

The water table depth (or depth to water table) is a critical lower boundary condition for solving the Richards equation (Richards 1931) and thus influences modeled runoff. On the basis of the TOPMODEL concepts and assumptions, Sivapalan et al. (1987) related the local water table depth z_i (i.e., depth from ground surface to the water table level) to the watershed averaged water table depth, $\bar{z}$, and the average topographic index, λ, of a catchment or watershed:

$$z_i = \bar{z} - \frac{1}{f}\left[\ln\left(\frac{aT_e}{T_o \tan\beta}\right) - \lambda\right] \tag{1}$$

In Eq. (1), $\ln(a/\tan\beta)$ = topographic index; λ = areal average of $\ln(a/\tan\beta)$; T_o = local soil transmissivity; T_e = areal average of T_o; and f = empirical parameter describing the exponential decay of soil transmissivity with soil depth. The total saturation area, A_c, = summation of all pixels that satisfy the following expression:

$$z_i = \bar{z} - \frac{1}{f}\left[\ln\left(\frac{aT_e}{T_o \tan\beta}\right) - \lambda\right] \le 0, \quad \text{or}$$
$$\ln\left(\frac{aT_e}{T_o \tan\beta}\right) \ge f\bar{z} + \lambda \tag{2}$$

If the spatial variation of transmissivity is neglected, expression (2) becomes:

$$\ln\left(\frac{a}{\tan\beta}\right) \geq f\bar{z} + \lambda \tag{3}$$

where $f\bar{z} + \lambda$ can be considered as a threshold, and the fraction of the saturation area A_c/A (A = total catchment area) can be estimated from the complementary distribution (i.e., one minus the cumulative distribution) of the topographic index distribution (TID), i.e., the interception between the A_c/A curve and the threshold $f\bar{z} + \lambda$, as shown in Fig. 1. If mean water table depth is small (i.e., water table is close to the surface), the threshold $f\bar{z} + \lambda$ is small, and the fraction of the saturation area is large, and vice versa. As the fraction of the saturation area increases, so does surface runoff from the watershed. The mean value of the TID, λ, also influences the TOPMODEL's predictions of base flow, because, according to Sivapalan, et al. (1987), base flow Q_b can be estimated as follows:

$$Q_b = Q_o \exp(-f\bar{z}) \quad \text{and} \quad Q_o = AT_e \exp(-\lambda) \tag{4}$$

where Q_o = maximum base flow when $\bar{z}$ is zero.

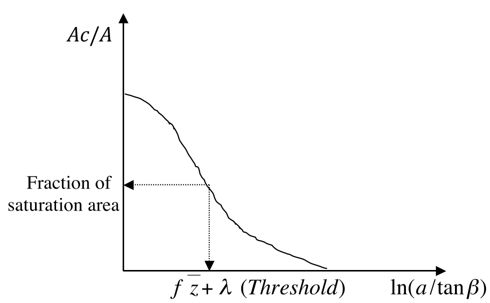

Fig. 1. Schematic plot of the fraction of the saturation area (A_c/A) versus the topographic index $\ln(a/\tan\beta)$, which is also the complementary distribution of the topographic index; the saturation area can be determined by the interception of the threshold $f\bar{z} + \lambda$ and the curve of A_c/A versus $\ln(a/\tan\beta)$

Effects of DEM Resolution on TOPMODEL Surface Runoff and Base Flow Predictions

To illustrate the influence of DEM resolution on the TID, a 2-m resolution topographic dataset for the Oak Ridge Reservation (ORR) in eastern Tennessee was used to compute the topographic indexes. The 2-m DEM data set for the ORR was developed from aerial photography taken during leaf-off in April of 1993. The aerial photographs were flown at 2,195 m above ground level, resulting in a nominal scale of 1:14,400. The data were compiled at 1:2,400 scale (see Fig. 2) yielding a root mean square error (RMSE) of 2.32 m. Two coarser DEM resolution data, i.e., 10 and 100 m, were also downloaded from the U.S. Geology Survey (USGS) National Elevation Dataset (NDE). The overall RMSE of USGS DEMs is 2.44 m according to the *NED Accuracy Document* (http://ned.usgs.gov/Ned/accuracy.asp) and slightly larger than that of 2-m ORR DEM (i.e., 2.32 m).

Since 2-m DEM used in this study was not from the same data source as the 10- or 100-m DEM, to compare the systemic errors and biases among 2-, 10-, and 100-m DEMs, the authors aggregated 2-m ORR DEM to 10 m, and 10-m USGS DEM to 100 m using the mean method (e.g., Shaw et al. 2005). The maximum, minimum, mean, standard deviation, and skewness of each DEM are listed in Table 1. The difference between the aggregated 10-m DEM from 2-m ORR DEM and the original 10-m USGS DEM is comparable to that of between the aggregated 100-m DEM from 10-m USGS DEM and the original 100-m USGS DEM (see Table 1) and implies that the difference in the computed TIDs between 2 and 10 or 100 m is not attributable to different DEM data sources.

Nine catchments were delineated across the ORR (see Fig. 2). The MFD algorithm (Quinn et al. 1995; Pan et al. 2004) was used to compute the topographic index at each DEM cell inside each catchment. For each catchment, three TIDs were obtained, i.e., one for each of three DEM resolutions, i.e., 2, 10, and 100 m.

The first three moments (i.e., mean, standard deviation, and skewness) of each TID were computed and listed in Table 2. The mean values λ of the TIDs decreased with deceasing DEM grid cell size (i.e., increasing DEM data resolution) (Table 2).

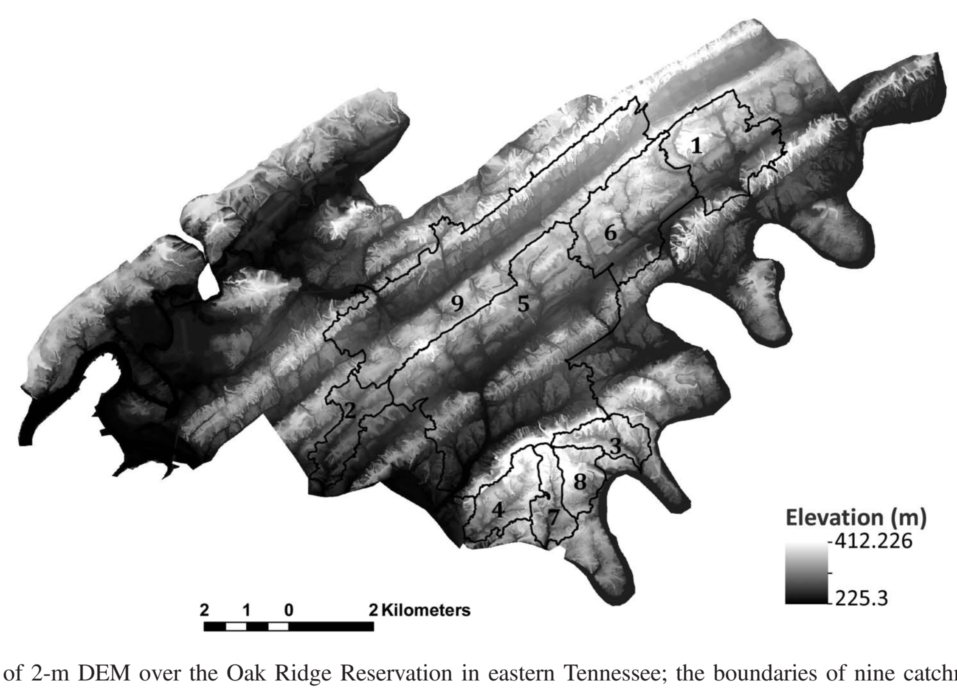

Fig. 2. Depiction of 2-m DEM over the Oak Ridge Reservation in eastern Tennessee; the boundaries of nine catchments are indicated

Table 1. Statistics of the Aggregated and Original DEMs

DEM	Maximum (m)	Minimum (m)	Mean (m)	Standard deviation (m)	Skewness[a]
Aggregated 10-m	411.01	225.30	276.93	33.06	0.50
Original 10-m	413.07	225.26	274.16	31.76	0.59
Difference	−2.06	0.04	2.77	1.30	−0.09
Aggregated 100-m	413.81	225.78	274.88	31.07	0.55
Original 100-m	410.00	226.00	272.68	29.90	0.50
Difference	3.81	−0.22	2.20	1.17	0.05

[a]Coefficient of skewness.

Table 2. Statistics of the Topographic Index Distributions of Nine Catchments over Oak Ridge Reservation at 2-, 10-, and 100-m Resolutions

Number	2-m			10-m			100-m		
	m_1	m_2	m_3	m_1	m_2	m_3	m_1	m_2	m_3
1	5.06	3.01	1.13	6.51	2.15	2.18	9.07	2.19	0.84
2	5.39	2.80	1.09	6.49	1.89	2.59	10.11	1.83	0.28
3	5.04	3.06	1.01	6.16	1.79	2.61	7.99	1.58	1.04
4	4.94	2.86	1.01	6.27	1.94	2.54	8.69	1.85	0.84
5	5.14	3.06	1.12	6.65	2.16	1.87	9.36	2.36	1.20
6	4.94	2.75	1.16	6.07	1.71	2.73	8.96	1.87	1.07
7	4.93	2.99	0.91	6.11	1.73	3.04	8.55	2.05	0.78
8	5.12	2.95	1.12	6.02	1.44	2.78	7.44	0.97	0.25
9	5.24	3.10	1.04	6.65	2.12	2.03	8.91	2.11	1.59

Note: m_1: mean; m_2: standard deviation; m_3: coefficient of skewness.

The λ value derived from 100-m resolution DEM data is about twice that from 2-m data for each catchment. These results agree with those of Wolock and Price (1994), Zhang and Montgomery (1994), and Quinn et al. (1995). In general, a coarser resolution DEM generates a larger mean value of the TID, and a finer resolution produces a smaller mean value of the TID.

To study the effects of the scale dependency exhibited by λ on the TOPMODEL predictions of surface runoff and base flow, the complementary topographic index distributions were plotted (i.e., A_c/A versus $\ln(a/\tan\beta)$ curves) for No.9 Oak Ridge Reservation catchment at 2-, 10-, and 100-m resolutions in Fig. 3(a). Without any scale adjustment, for a given mean depth to water table, the fractions of the saturation area determined from the TIDs among 2, 10, and 100 m are different because of the differences in the mean values and the shapes of the TIDs [see Fig. 3(a)]. Thus, the predicted surface runoff varies with different resolution TIDs. Second, according to Eq. (4), a factor of 2 in λ between 100- and 2-m resolution implies a factor of e^{-2} ($\approx$0.13) in Q_o between 100 and 2 m, and thus the predicted base flow on the basis of a 100-m TID is about 0.13 times that for a 2-m TID. These scale dependencies are generally known (Wolock and Price 1994; Zhang and Montgomery 1994; Quinn et al. 1995). The question is how to correct for them.

Downscaling the Topographic Index Distributions

Downscaling Methodology

If DEM resolution affects the calculation of the topographic indexes, and through the TID, the TOPMODEL predictions of

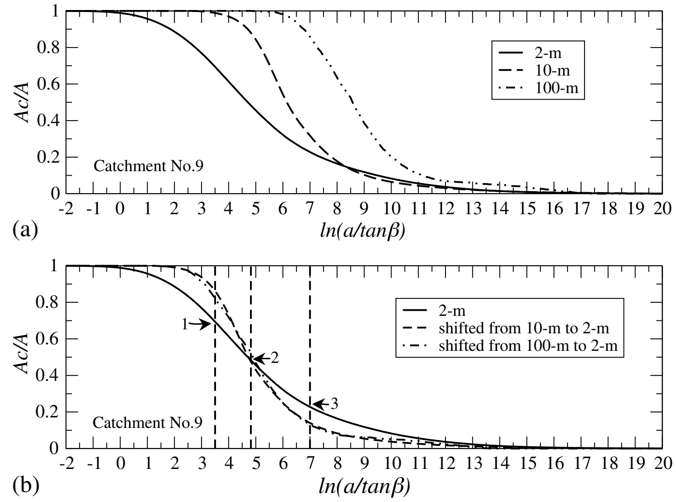

Fig. 3. Complementary topographic index distributions for No. 9 Oak Ridge Reservation catchment at 2-, 10-, and 100-m resolutions and derived by the multiple flow direction algorithm: (a) distributions without any adjustment; (b) adjusted distributions by shifting 10- and 100-m distributions (i.e., adjusting mean values) except the distribution at 2-m [thresholds corresponding to wet (dashed vertical line 1), normal (dashed vertical line 2) and dry (dashed vertical line 3) conditions are marked]

catchment hydrology, how, then, does one rescale a TID to correct for the scale dependency? Two issues need to be resolved in addressing this question. First, is it possible to scale the TID derived from a coarse resolution DEM to the TID computed from a finer resolution DEM, by just "shifting" the distribution and correcting for the difference in the means of the distributions (e.g., Ibbitt and Woods 2004)? Or, in addition to shifting the mean (i.e., the first moment), must the shape of the distribution also be changed by adjusting second and higher moments? Second, how fine must the resolution of DEM data be? Is downscaling to 10 m sufficient. or is it necessary to scale down to a resolution of 2 m or less?

To answer these questions, the scale dependency shown in Fig. 3(a) was first corrected by just matching mean values of the TIDs, i.e., simply adding or subtracting the difference between the mean of a TID and the mean of the reference TID. The TID calculated at 2-m resolution was chosen as the benchmark, because 2-m DEM resolution is the finest available for the catchment. After adjustment, the difference between the 10- and 100-m A_c/A curves was reduced significantly [Fig. 3(b)]. However, the adjustment did not eliminate the shape differences between 2 and 10 m, and between 2 and 100 m [Fig. 3(b)], which indicates that (1) in addition to shifting the mean, the shape of the distributions must be changed by adjusting second and higher moments; and (2) downscaling to 10 m is not enough.

To assess the significance of the remaining differences in the TIDs on the TOPMODEL runoff predictions, three thresholds were chosen: (1) $f\bar{z}_1 + \lambda$, (2) $f\bar{z}_2 + \lambda$, and (3) $f\bar{z}_3 + \lambda$, corresponding to wet, normal, and dry conditions, respectively (i.e., $\bar{z}_1 < \bar{z}_2 < \bar{z}_3$; a small $\bar{z}$ corresponds to a shallow water table and wet conditions, and vice versa). After adjusting the means of the TIDs, λ is the same and f is independent of DEM data resolution because f is a soil parameter rather than a topographic parameter. Under wet conditions [i.e., dashed vertical line 1 in Fig. 3(b)], the TOPMODEL prediction of surface runoff at 2 m is less than that based on adjusted 10- or 100-m TID because the saturation area predicted by the 2-m TID is less than that by either adjusted 10- or 100-m TID. Under dry conditions [i.e., dashed vertical line 3 in Fig. 3(b)], the saturation area predicted by the 2-m TID is greater than that by either adjusted 10- or 100-m TID. This demonstrates that compared with the predictions using the benchmark TID (i.e., 2-m TID), surface runoff predictions could be overestimated under wet conditions and underestimated under dry conditions if a TID computed from a coarser DEM is used and only the mean of the TID is adjusted.

Adjusting for the scale dependencies in the shape of the TIDs and not just correcting for differences between the means first requires a quantification of that shape. A 3-parameter gamma probability density function (e.g., Sivapalan et al. 1987) is commonly used to approximate the distribution:

$$f_x(x) = \frac{|c_2|[c_2(x-c_3)]^{c_1-1}\exp[-c_2(x-c_3)]}{\Gamma(c_1)} \tag{5}$$

where $\Gamma(c_1)$ = gamma function. According to the method of moments (Benjamin and Cornell 1970), there are the following relationships among mean, standard deviation (sd), the coefficient of skewness (γ), and three parameters in Eq. (5), i.e., c_1, c_2, and c_3:

$$\text{for } c_1 > 0, \quad c_2 > 0, \quad \text{and} \quad x > c_3: \ \text{mean} = c_3 + c_1/c_2, \quad sd = c_1/c_2^2, \quad \gamma = c/\sqrt{c_1} \tag{6}$$

Inversely, parameters c_1, c_2, and c_3 can be estimated from the mean, standard deviation, and skewness of the TID:

$$c_1 = 4/\gamma^2, \qquad c_2 = \sqrt{4/(\gamma^2 sd)}, \qquad c_3 = \text{mean} - 2/(\gamma\sqrt{sd}) \tag{7}$$

With Eq. (7), a TID from a coarser resolution can be downscaled to a finer resolution by relating mean, standard deviation, and skewness at the finer resolution (e.g., mean_f, sd_f, γ_f) to those at the coarser resolution (e.g., mean_c, sd_c, γ_c):

$$\begin{cases} \text{mean}_f = f_1(\text{mean}_c, sd_{c,}\gamma_c) \\ sd_f = f_2(\text{mean}_c, sd_c, \gamma_c) \\ \gamma_f = f_3(\text{mean}_c, sd_c, \gamma_c) \end{cases} \tag{8}$$

The relationships between statistical moments of the TIDs computed from a coarser resolution and a finer resolution can be obtained from a series of multiple linear regressions. The procedure for downscaling of the TIDs from 1 km to 2 m is illustrated in the following section.

Empirical Relationships among 2-m, 10-m, and 1-km TIDs

A two-step and hierarchical procedure can be used to scale the TIDs from 1 km to 2 m. This two-step procedure is necessary because there are only 2-m DEM data for nine ORR catchments, which are too small to be adequately resolved at 1-km resolution. First, local relationships between TIDs computed from 10- and 2-m resolution DEMs over the nine ORR catchments were estimated (see Fig. 2). Next, global relationships to scale from 1-km to 10-m based on 205 watersheds across the contiguous United Staes were estimated.

Aset of multiple linear regressions were applied to the statistics of the TIDs of the nine catchments over ORR (Table 2) to obtain the following relationships between 10- and 2-m TIDs:

$$\begin{cases} \text{mean}_{2-\text{m}} = -3.826 + 1.402 \times \text{mean}_{10-\text{m}} - 0.434 \times sd_{10-\text{m}} + 0.328 \times \gamma_{10-\text{m}} \\ sd_{2-\text{m}} = 3.655 - 0.209 \times \text{mean}_{10-\text{m}} + 0.440 \times sd_{10-\text{m}} - 0.091 \times \gamma_{10-\text{m}} \\ \gamma_{2-\text{m}} = 2.266 - 0.023 \times \text{mean}_{10-\text{m}} - 0.245 \times sd_{10-\text{m}} - 0.240 \times \gamma_{10-\text{m}} \end{cases} \tag{9}$$

The RMSE of the estimated mean, standard deviation, and skewness on the basis of Eq. (9) are 0.09, 0.10, and 0.06, respectively, and the corresponding correlation coefficients between the estimated and observed mean, standard deviation, and skewness are 0.86, 0.60, and 0.59, respectively. Fig. 4 illustrates scatter plots of the estimated and the observed mean, standard deviation, and skewness.

The 2-m TID statistics can be estimated by applying these empirical relationships to the 10-m TIDs of each of the nine ORR catchments. These three parameters were estimated by using the downscaling relationships in Eq. (9). The downscaled (10- to 2-m) TIDs, given by Eq. (5), for nine ORR catchments show good agreement with the observed TIDs at 2 m (Fig. 5).

To construct relationships to scale from 1-km to 10-m TIDs, 18 major watersheds with a drainage area magnitude of 1,000–2,000 km^2 in 18 states were delineated (one watershed per state, see Fig. 6) using 10-m DEM data. To investigate the effect of the drainage area on errors in the downscaled TIDs, again using 10-m DEM data, about 10 subwatersheds with different drainage areas (approximately evenly distributed between 50 and 2,000 km^2) inside each of the 18 major watersheds were also delineated and resulted in a total of 205 watersheds. As an example, Fig. 7 depicts the major watershed (i.e., watershed No. 1) and subwatersheds (watersheds Nos. 2-8) in Utah. Each 10-m watershed or subwatershed boundary was projected to 1 km before constructing the 1-km TID. Data used were 1-km DEM data from the USGS GTOPO30. The resulting 1-km to 10-m scaling functions are:

$$\begin{cases} \text{mean}_{10-\text{m}} = 1.136 + 0.657 \times \text{mean}_{1-\text{km}} - 0.640 \times sd_{1-\text{km}} + 0.053 \times \gamma_{1-\text{km}} \\ sd_{10-\text{m}} = -0.128 + 0.187 \times \text{mean}_{1-\text{km}} + 0.168 \times sd_{1-\text{km}} - 0.261 \times \gamma_{1-\text{km}} \\ \gamma_{10-\text{m}} = 3.768 - 0.246 \times \text{mean}_{1-\text{krm}} + 0.317 \times sd_{1-\text{km}} + 0.222 \times \gamma_{1-\text{km}} \end{cases} \tag{10}$$

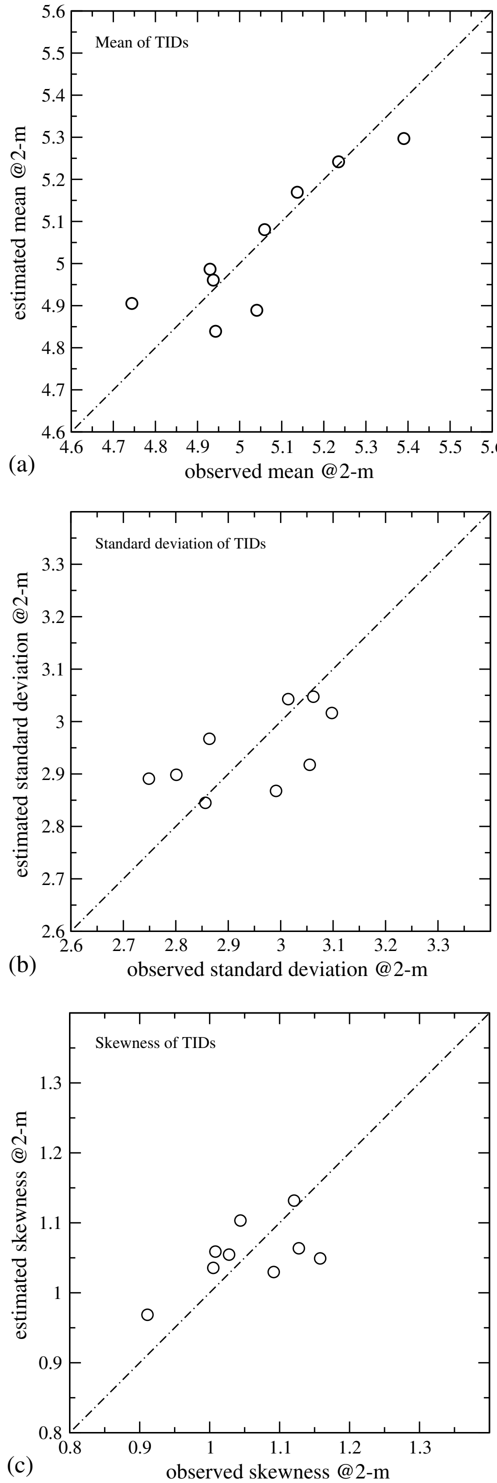

Fig. 4. Scatterplots of the estimated and the observed: (a) mean; (b) standard deviation; (c) coefficient of skewness of TIDs of nine catchments over Oak Ridge Reservation; the observed moments of TIDs are directly computed on the basis of 2-m TIDs, and the estimated moments are computed from downscaled TIDs from 10 to 2 m

The RMSEs of the estimated mean, standard deviation, and skewness at 10-m resolution from Eq. (10) are 0.70, 0.25, and 0.44, respectively. The corresponding correlation coefficients between the estimated and observed mean, standard deviation, and skewness are 0.85, 0.78, and 0.73, respectively. TID statistics scaled from

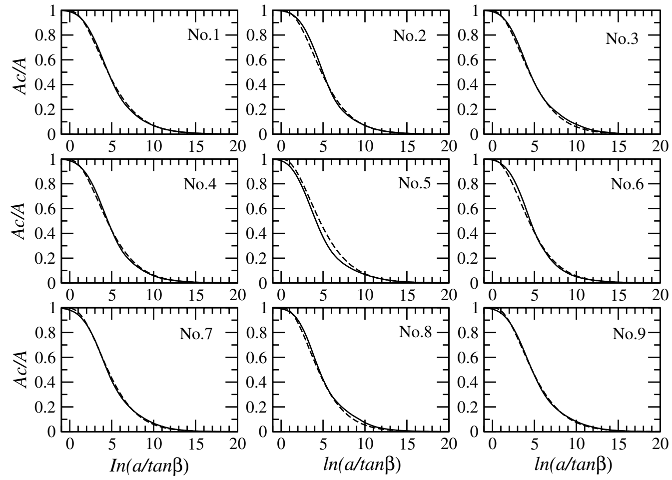

Fig. 5. Plots of TIDs of the nine Oak Ridge Reservation catchments observed at 2 m (solid) and downscaled from 10 to 2 m through correcting three moments (dashed)

1 km to 10 m with the actual 10-m TID statistics can be compared visually (Fig. 8).

Following the same procedure used in downscaling 10-m TIDs to 2 m, the empirical relationships shown in Eq. (10) was used to estimate the mean, standard deviation, and skewness of 10-m TIDs from the 1-km TIDs. The three parameters of the gamma function were determined from Eq. (7), and the downscaled TIDs were obtained using Eq. (5). Fig. 9 shows the computed TIDs at 10 m (solid curves) and the downscaled TIDs (dashed curves) from 1 km of nine major watersheds in nine states.

Errors in the Downscaled TIDs

Using Eqs. (7), (9), and (10), any TID can be downscaled from 1 km to 2 m. Although the errors in the estimated mean, standard deviation, and skewness quantify the accuracy of downscaling to some extent, ultimately, the desire is to quantify the errors in modeled watershed hydrology associated with the downscaled TIDs. For example, the error in the fraction of the saturation area or A_c/A can be expressed as

$$\text{error} = \frac{\int_{x_1}^{x_2} |(\frac{A_c}{A})_o - (\frac{A_c}{A})_e| dx}{x_2 - x_1} \times 100\% \tag{11}$$

where $x = \ln(a/\tan\beta)$; dx = interval for discretizing the A_c/A curve; x_1 and x_2 = minimum and maximum $\ln(a/\tan\beta)$ of an observed TID; and subscripts o and e stand for "observed" and "estimated." "Observed" means the TID is directly derived from DEM data at a finer resolution, whereas "estimated" indicates the TID is downscaled from the TID computed at a coarser resolution DEM to the finer resolution.

The error in the fraction of the saturation area estimated from TIDs downscaled from 1 km to 10 m were compared with those estimated with observed TIDs at 10 m. The error in estimated saturation area fraction decreased with increasing watershed area and reached a lower asymptote (about 6%) for watersheds with area greater than 500 km^2 (Fig. 10).

Summary

This research has shown that not only the mean value (first moment) of the TIDs but also the shape of the TID (second and higher

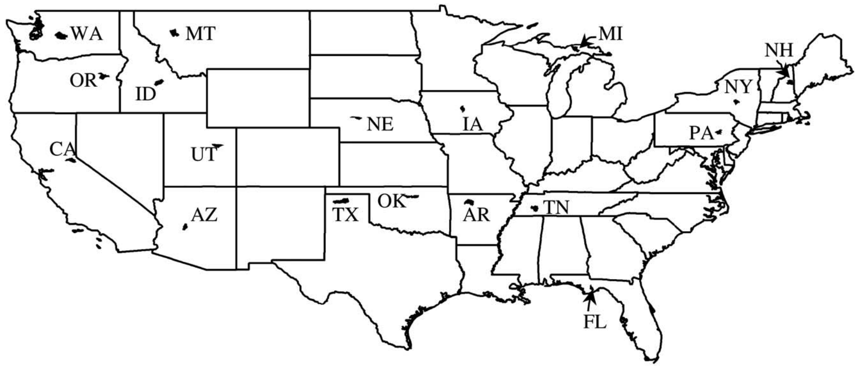

Fig. 6. Eighteen major watersheds chosen from 18 states (shaded areas)

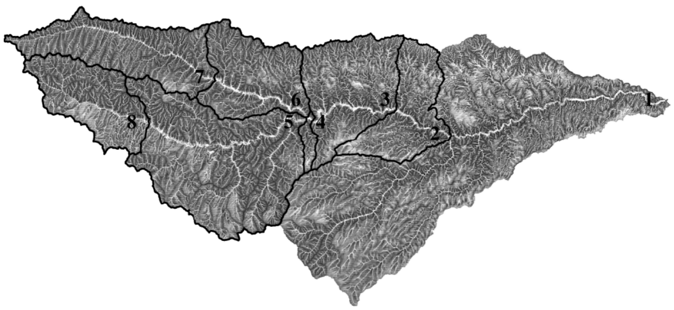

Fig. 7. Topographic index maps of one major watershed (marked as 1 near the outlet) and 7 subwatersheds (marked as 2–8 near each outlet) in Utah

moments) influence TOPMODEL predictions of surface runoff and base flow. The scale dependency of TIDs (i.e., dependency on the resolution of the DEM data used to define them) determines that downscaling of the TID computed from a coarser resolution DEM to a finer resolution is needed before the TOPMODEL concepts can be applied to simulate hydrological processes at some larger scales than the scale of hillslopes. This is particularly true for GCMs utilizing coarse 1-km DEMs.

Next, methods were explored for downscaling TID's. In the first attempt, it was found that downscaling of TIDs could not be achieved simply by shifting the mean value (i.e., correcting for scale dependency in the first moment of the TIDs), because the difference between 2-m TIDs and the downscaled TIDs from a coarser resolution to 2 m through only adjusting mean values resulted in overestimation of the fraction of the saturation area and surface runoff under wet conditions and underestimation under dry conditions.

It was found that downscaling by correcting for scale dependencies in the first three moments of the TID produced better predictions. The authors developed a hierarchical method of applying statistical relationships among moments estimated from different DEM resolutions and parameters of a 3-parameter gamma distribution function that can be used to estimate the TID at the desired resolution. This technique was illustrated by scaling from 10 to 2 m for nine catchments over ORR and from 1 km to 10 m for 205 watersheds across the contiguous United States.

It was found that the errors in the downscaled TIDs of 205 watersheds decreased with an increase in watershed size and reached a minimum for watershed areas greater than 500 km^2. The assumption of the spatially uniform recharge rate in the TOPMDODEL requires that the modeled watershed be as small

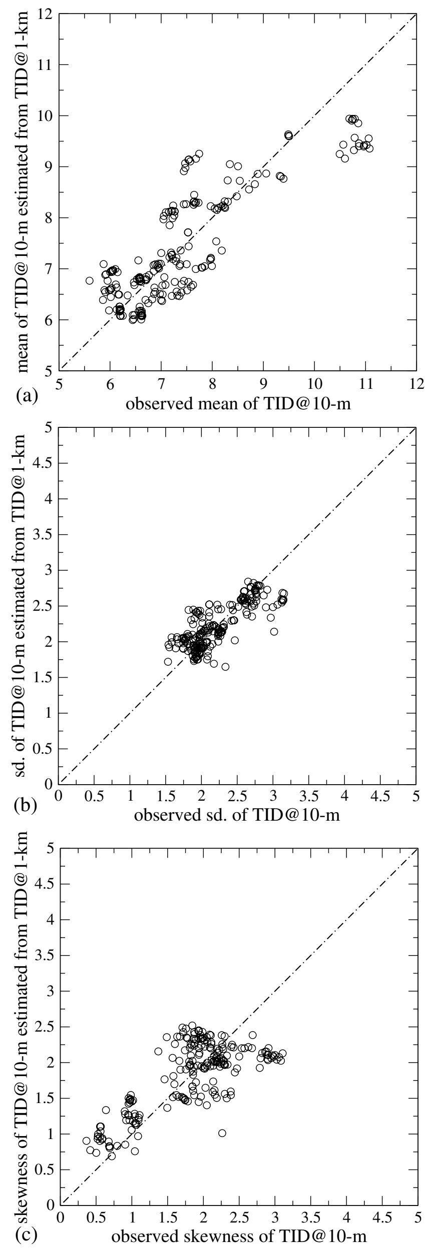

Fig. 8. Scatter-plots of the estimated and the observed: (a) mean; (b) standard deviation; (c) skewness of TIDs of 205 watersheds across the contiguous United States; the observed moments of TIDs are computed on the basis of 10-m TIDs, and the moments are estimated from downscaled TIDs from 1 km to 10 m

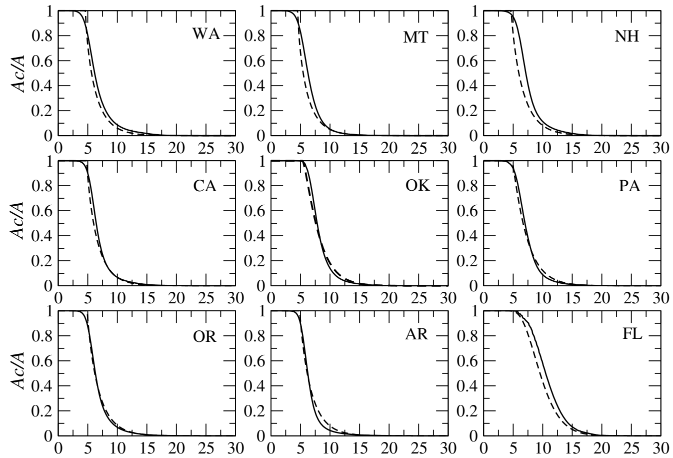

Fig. 9. Plots of TIDs of nine major watersheds in nine states observed at 10 m (solid) and downscaled from 1 km to 10 m through correcting three moments (dashed)

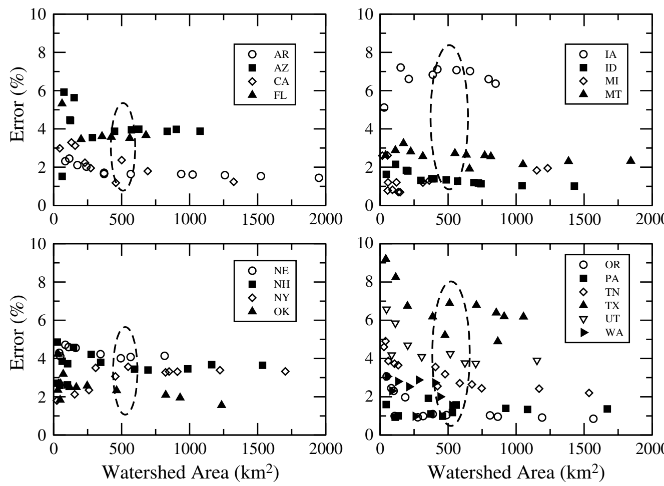

Fig. 10. Scatter plots of errors in downscaled TIDs versus watershed area; ellipses indicate where errors appear to reach a minimum

as possible to minimize the impact of soil heterogeneity on recharge rate. The trade-off between the errors in the downscaled TIDs attributable to DEM resolution and the errors attributable to spatial variation of recharge rate implies that the suitable watershed size for applying the TOPMODEL concepts with TIDs computed from 1-km DEM is about 500 km^2. However, the TIDs for these watersheds must be downscaled from 1-km to 2-m resolution before they can be used for TOPMODEL-based hydrological modeling.

One issue has not been touched in this study, i.e., what is the optimal resolution of DEM for computing the topographic indexes? As DEM resolution becomes finer and finer, more and more small-scale topographic features can be captured. However, there might be a scale limit under which all small-scale topographic variations play a negligible role in controlling hydrological processes. To identify such optimal resolution, more efforts and more finer resolution DEMs are needed to conduct a series of statistical analysis of TIDs computed from different finer resolution DEMs and examine the impacts of geomorphologic characteristics (e.g., channel width and river shape) on TIDs. When finer resolution DEM data are used for computing the topographic indexes, some artificial features such as buildings, parking lots, roads, and bridges and other canopy structures could add "noise" to the terrain information and produce errors in the calculated topographic indexes. Therefore, a preprocessing step is necessary for removing artificial features and canopy structures from finer resolution DEMs before they can be used for computing the topographic indexes.

Acknowledgments

The authors thank H.I. Jager at Oak Ridge National Laboratory for her insightful reviews of the manuscript and three anonymous referees for their valuable comments and suggestions.

References

Benjamin, J. R., and Cornell, C. A. (1970). *Probability, statistics, and decision for civil engineers*, McGraw-Hill, New York, 684.

Beven, K., and Kirkby, M. J. (1979). "A physical based, variable contributing area model of basin hydrology." *Hydrol. Sci. Bull.*, 24(1), 43–69.

Bonan, G. B. (1998). "The land surface climatology of the NCAR land surface model coupled to the NCAR community climate model." *J. Clim.*, 11(6), 1307–1326.

Chen, J., and Kumar, P. (2001). "Topographic influence on the seasonal and interannual variation of water and energy balance of basins in North America." *J. Clim.*, 14(9), 1989–2014.

Dai, Y. et al (2003). "The common land model." *Bull. Am. Meteorol. Soc.*, 84(8), 1013–1023.

Ducharne, A., Koster, R. D., Suarez, M. J., Stieglitz, M., and Kumar, P. (2000). "A catchment-based approach to modeling land surface processes in a general circulation model. 2. Parameter estimation and model demonstration." *J. Geophys. Res.*, 105(D20), 24823–24838.

Habata, S., Yokokawa, M., and Kitawaki, S. (2003). "The earth simulator system." *NEC Res. Develop.*, 44(1), 21–26.

Ibbitt, R., and Woods, R. (2004). "Re-scaling the topographic index to improve the representation of physical processes in catchment models." *J. Hydrol. (Amsterdam)*, 293(1–4), 205–218.

Koster, R. D., Suarez, M. J., Ducharne, A., Stieglitz, M., and Kumar, P. (2000). "A catchment-based approach to modeling land surface processes in a general circulation model. 1. Model structure." *J. Geophys. Res.*, 105(D20), 24809–24822.

Kumar, P., Verdin, K. L., and Greenlee, S. K. (2000). "Basin level statistical properties of topographic index for North America." *Adv. Water Resour.*, 23(6), 571–578.

Niu, G.-Y., and Yang, Z.-L. (2003). "The versatile integrator of surface atmospheric processes—Part 2: Evaluation of three topography-based runoff schemes." *Glob. Planet. Change*, 38(1–2), 191–208.

O'Callaghan, J. F., and Mark, D. M. (1984). "The extraction of drainage networks from digital elevating data." *Comput. Vision Graphics Image Proc.*, 28(3), 323–344.

Pan, F., Peters-Lidard, C. D., Sale, M. J., and King, A. W. (2004). "A comparison of GIS-based algorithms for computing the TOPMODEL topographic index." *Water Resour. Res.*, 40(6), W06303.

Pradhan, N. R., Tachikawa, Y., and Takara, K. (2006). "A downscaling method of topographic index distribution for matching the scales of model application and parameter identification." *Hydrol. Processes*, 20(6), 1385–1405.

Quinn, P. F., Beven, K. J., and Lamb, R. (1995). "The $\ln(a/\tan\text{-beta})$ index—How to calculate it and how to use it within the TOPMODEL framework." *Hydrol. Processes*, 9(2), 161–182.

Richards, L. A. (1931). "Capillary conduction of liquids in porous mediums." *Physics*, 1(5), 318–333.

Shaw, D., Martz, L. W., and Pietroniro, A. (2005). "Flow routing in large-scale models using vector addition." *J. Hydrol. (Amsterdam)*, 307(1–4), 38–47.

Sivapalan, M., Beven, K. J., and Wood, E. F. (1987). "On hydrologic similarity, 2. A scaled model of storm runoff production." *Water Resour. Res.*, 23(12), 2266–2278.

Sørensen, R., and Seibert, J. (2007). "Effects of DEM resolution on the calculation of topographical indices: TWI and its components." *J. Hydrol. (Amsterdam)*, 347(1–2), 79–89.

Tarboton, D. G. (1997). "A new method for the determination of flow directions and upslope areas in grid digital elevation models." *Water Resour. Res.*, 33(2), 309–319.

Wolock, D. M., and McCabe, G. M. (2000). "Differences in topographic characteristics computed from 100-and 1000-m resolution digital elevation model data." *Hydrol. Processes*, 14(6), 987–1002.

Wolock, D. M., and Price, C. V. (1994). "Effects of digital elevation model map scale and data resolution on a topography-based watershed model." *Water Resour. Res.*, 30(11), 3041–3052.

Wood, E. F. (1995). "Heterogeneity and scaling land-atmospheric water and energy fluxes in climate systems." *Space and time scale variability and interdependencies in hydrological processes*, R. A. Feddes, ed., International Hydrology Series, Cambridge University Press, Cambridge, UK, 3–19.

Wood, E. F., Beven, K., Sivapalan, M., and Band, L. (1988). "Effects of spatial variability and scale with implication to hydrology modeling." *J. Hydrol. (Amsterdam)*, 102(1–4), 29–47.

Yang, Z.-L., and Niu, G.-Y. (2003). "The versatile integrator of surface and atmosphere processes Part 1. Model description." *Glob. Planet. Change*, 38(1–2), 175–189.

Yong, B., Zhang, W.-C., Niu, G.-Y., Ren, L.-L., and Qin, C.-Z. (2009). "Spatial statistical properties and scale transform analyses on the topographic index derived from DEMs in China." *Comput. Geosci.*, 35(3), 592–602.

Zhang, W., and Montgomery, D. R. (1994). "Digital elevation model grid size, landscape representation, and hydrologic simulations." *Water Resour. Res.*, 30(4), 1019–1028.