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A New Viewpoint on the Internal Model Principle and Its Application to Periodic Signal Tracking*

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***Abstract*— Periodic signal tracking is certainly easier than general signal tracking. This has been manifested for linear time-invariant systems by applying theories of repetitive control. However, because of the lack of corresponding theories, the difficulties in designing repetitive controllers for both periodic signal tracking and general signal tracking in nonlinear systems are similar or the same. In view of this, this paper proposes a new viewpoint on the internal model principle which is used to explain how the internal models work in the time domain when the desired signals are step signals, sine signals and general periodic signals, respectively. Guided by this viewpoint, the periodic signal tracking problem is considered as a stability problem for nonlinear systems. To demonstrate the effectiveness of this new viewpoint, a new method of designing repetitive controllers is proposed for periodic signal tracking of non-minimum phase nonlinear systems, where the internal dynamics are subject to a periodic disturbance. A simulation example illustrates the effectiveness of the new method.**

***Index Terms*— Internal model principle, Repetitive control, Non-minimum phase nonlinear systems.**

I. INTRODUCTION

The concept of repetitive control (RC) was initially developed for continuous single-input, single-output (SISO) linear time-invariant (LTI) systems by Inoue et al., for high accuracy tracking of a periodic signal with a known period [1]. Later, Hara et a1. extended the RC to multiple-input, multiple-output (MIMO) systems [2]. Since then, RC has begun to receive more attention and applications, and has become a special topic in control theory. In recent years, the development on RC has been uneven. By the use of frequency methods, the theories and applications in LTI systems have developed very well [3],[4]. On the other hand, RC in nonlinear systems has received very limited research effort.

For LTI systems, the design of repetitive controllers mainly depends on transfer functions. By contrast, the leading method of designing repetitive controllers in nonlinear systems is in fact a design method for a special adaptive controller [5]-[8]. The structures of repetitive controllers obtained for the two types of systems are similar or the same, but the ways to obtain these controllers are very different. For LTI systems, we do not need to obtain error dynamics. However, for nonlinear systems, it is often required to derive error dynamics to convert a tracking problem to a disturbance rejection problem or a parameter estimation problem. Then an adaptive control design is adopted to specify certain components of the repetitive controller. In the process, the error dynamics are required. For non-minimum phase nonlinear systems, the ideal internal dynamics are required to obtain the error dynamics. This is difficult and computationally expensive especially when the internal dynamics are subject to an unknown disturbance. As a result, the authors suppose that this is the reason why few RC works on such systems have been reported.

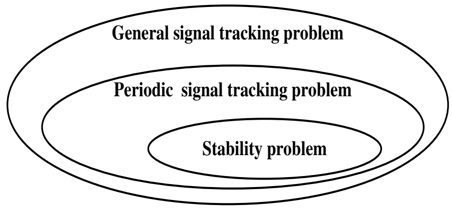

Fig. 1. Relationship between stability and tracking.

As shown in Fig.1, the periodic signal tracking problem is an instance of the general signal tracking problem, and in turn includes the stability problem (means zero signal tracking problem here) as a special case. Consequently, periodic signal tracking should certainly be easier than general signal tracking. Nevertheless, if the repetitive controllers are designed by following existing methods used for general signal tracking problem, then the special feature of periodic signals is in fact under-exploited. Therefore, general methods will not only restrict the development of RC, but also fail to represent the special feature and importance of RC. Since periodic signals are special, we have reason to believe that there should exist another method, different from the general methods, to design repetitive controllers for nonlinear systems. It is expected

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that the new design method will outperform general design methods when dealing with the periodic signal tracking problem.

For LTI systems, the periodic signal tracking problem is usually viewed as a special stability problem. On the other hand, for nonlinear systems, it usually comes down to a pure tracking problem. This is the major difference between dealing with the same problem for LTI systems and nonlinear systems. In our opinion, the periodic signal tracking problem should be a stability problem just as in LTI systems. It is well known that a stability problem is easier than a tracking problem. So, this conversion will greatly reduce the difficulties in periodic signal tracking and moreover conforms to the internal model principle (IMP) [9]. More importantly, when the external signals are periodic, this conversion can help overcome certain weaknesses of existing methods as developed for general signal tracking.

Based on the consideration above, this paper develops a new viewpoint on IMP in the time domain, which relies on the system's behavior. Guided by this viewpoint, the periodic signal tracking problem is viewed as a stabilizing problem for the closed-loop system which incorporates an external signal model. The resulting new method does not require error dynamics. Furthermore, it can unify the repetitive controller design for both LTI systems and nonlinear systems. To demonstrate the effectiveness of the proposed method, we design a repetitive controller to track a periodic signal for a non-minimum phase nonlinear system where a periodic disturbance exists in the internal dynamics. To the authors' knowledge, general methods handle such a case only at highly computational cost [10].

In this paper, C_{PT}^{n} is the space of continuous and periodic functions with periodicity T: $x(t) = x(t-T)$, $x(t) \in \mathbb{R}^n$, $0 \le t < \infty$; $x_\theta(t)$ denotes $x(t-\theta)$. If $x(t)$ is bounded on $[0,\infty)$, we let $\|\cdot\|_a$ denote the quantity $\|x\|_a \triangleq \limsup_{t\to\infty} \|x(t)\|$ [11].

II. A New Viewpoint on IMP

The IMP states that if any exogenous signal can be regarded as the output of an autonomous system, the inclusion of this signal model in a stable closed-loop system can assure perfect tracking or complete rejection of the signal. In other words, the IMP embodies the concept that the tracking problem of a signal can be converted into a stability problem of a closed-loop system into which is incorporated a corresponding model of the signal. This principle plays an important role in forming the basis of RC theories.

For LTI systems, the IMP implies that the internal model is to supply closed-loop transmission zeros which cancel the unstable poles of the disturbances and reference signals. Unfortunately, the transfer function cannot be applied to nonlinear systems. For this reason, a new viewpoint on the IMP is proposed to explain the role of the internal models for step signals, sine signals and generally periodic signals, respectively.

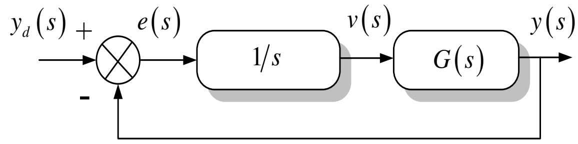

Fig. 2. Step signal tracking.

A. Step Signals

Since the Laplace transformation model of a unit step signal and an integral term are the same, namely $\frac{1}{s}$, the inclusion of the model $\frac{1}{s}$ in a stable closed-loop system can assure perfect tracking or complete rejection of the unit step signal according to the IMP.

Former Viewpoint: As shown in Fig.2, the transfer function from the desired signal to the tracking error is written as follows

$$\begin{aligned} e(s) &= \frac{1}{1+\frac{1}{s}G(s)} y_d(s) = \frac{1}{s+G(s)}\left(s\frac{1}{s}\right) \\ &= \frac{1}{s+G(s)}. \end{aligned} \tag{1}$$

Then, it only requires to verify whether or not the roots of the equation $s+G(s)=0$ are all in the left s-plane, namely whether or not the closed-loop system is stable. If all roots are in the left s-plane, then the tracking error tends to zero as $t\to\infty$. Therefore, the tracking problem has been reduced to a stability problem of the closed-loop system.

New Viewpoint: This new viewpoint will give a new explanation on IMP without using transfer functions. Because of the integral term, the relationship between $v(t)$ and $e(t)$ can be written to be

$$e(t) = \dot{v}(t). \tag{2}$$

If the closed-loop system without external signals is exponentially stable, then, when the system is driven by a unit step signal, it is easy to see that $v(t)$ and $e(t)$ will tend to constants as $t\to\infty$. Consequently, $e(t) = \dot{v}(t) \to 0$ as $t\to\infty$ by (2). Therefore, to confirm that the tracking error tends to zero as $t\to\infty$, it is only required to verify whether or not the closed-loop system without external signals is exponentially stable. This implies that the tracking problem has been reduced to a stability problem.

B. Sine Signals

If the external signal is in the form $a_0\sin(\omega t+\varphi_0)$, where a_0, φ_0 are constants, then perfect tracking or complete rejection can be achieved by incorporating the model $\frac{1}{s^2+\omega^2}$ into the closed-loop system.

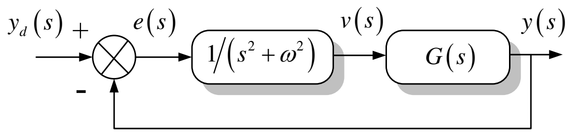

Fig. 3. Sine signal tracking.

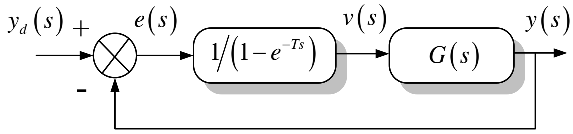

Fig. 4. Periodic signal tracking of a RC system.

Former Viewpoint: As shown in Fig.3, the transfer function from the desired signal to the tracking error is written as follows

$$\begin{aligned} e(s) &= \frac{1}{1+\frac{1}{s^2+\omega^2}G(s)} y_d(s) \\ &= \frac{1}{s^2+\omega^2+G(s)}\left[\left(s^2+\omega^2\right)\frac{b_1 s+b_0}{s^2+\omega^2}\right] \\ &= \frac{b_1 s+b_0}{s^2+\omega^2+G(s)} \end{aligned}$$

where the Laplace transformation model of $a_0 \sin(\omega t+\varphi_0)$ is $\frac{b_1 s+b_0}{s^2+\omega^2}$. Then, it is only required to verify whether or not the roots of the equation $s^2+\omega^2+G(s)=0$ are all in the left s-plane, namely whether or not the closed-loop system is stable. Therefore, the tracking problem has been reduced to a stability problem of the closed-loop system.

New Viewpoint: Because of the term $\frac{1}{s^2+\omega^2}$, the relationship between $v(t)$ and $e(t)$ can be written to be

$$e(t) = \ddot{v}(t) + \omega^2 v(t). \tag{3}$$

If the closed-loop system without external signals is exponentially stable, then, when the system is driven by an external signal in the form of $a_0 \sin(\omega t+\varphi_0)$, it is easy to see that $v(t)$ and $e(t)$ will tend to signals in the form of $a\sin(\omega t+\varphi)$, where a and φ are constants. Consequently,

$$e(t) \to \left(a\sin(\omega t+\varphi)\right)'' + \omega^2\left(a\sin(\omega t+\varphi)\right)$$

as $t\to\infty$ by (3). Therefore, to confirm that the tracking error tends to zero as $t\to\infty$, it only requires to verify whether or not the closed-loop system without external signals is exponentially stable. This implies that the tracking problem has been reduced to a stability problem.

C. Generally Periodic Signal

If the external signal is in the form of $y_d(t) = y_d(t-T)$, which can represent any periodic signal with a period T, then perfect tracking or complete rejection can be achieved by incorporating the model $\frac{1}{1-e^{-sT}}$ into the closed-loop system.

Former Viewpoint: Similarly, the transfer function from the desired signal to the error is written as follows

$$\begin{aligned} e(s) &= \frac{1}{1+\frac{1}{1-e^{-sT}}G(s)} y_d(s) \\ &= \frac{1}{1-e^{-sT}+G(s)}\left[\left(1-e^{-sT}\right)\frac{1}{1-e^{-sT}}\right] \\ &= \frac{1}{1-e^{-sT}+G(s)}. \end{aligned}$$

Then, it is only required to verify whether or not the roots of the equation $1-e^{-sT}+G(s)=0$ are all in the left s-plane. Therefore, the tracking problem has been reduced to a stability problem of the closed-loop system.

New Viewpoint: Because of the term $\frac{1}{1-e^{-sT}}$, the relationship between $v(t)$ and $e(t)$ can be written to be

$$e(t) = v(t) - v(t-T). \tag{4}$$

If the closed-loop system without external signals is exponentially stable, then, when the system is driven by a periodic signal, it can be proved that $v(t)$ and $e(t)$ will both tend to periodic signals with the period T. Consequently, we can conclude that $e(t)\to 0$ as $t\to\infty$ by (4). Therefore, to examine the tracking error tends to zero as $t\to\infty$, it only requires to verify whether or not the closed-loop system without external signals is exponentially stable. This implies that the tracking problem has been reduced to a stability problem.

A controller including the model $\frac{1}{1-e^{-sT}}$ or $\frac{e^{-sT}}{1-e^{-sT}}$ is said to be a repetitive controller and a system with such a controller is called a RC system. How to stabilize the RC system is not a trivial problem due to the inclusion of the time delay element in the positive feedback loop. It was proved in [2] that stability of RC systems could be achieved for continuous-time systems only when the plants are proper but not strictly proper. Moreover, the internal model $\frac{1}{1-e^{-sT}}$ may lead to instability of the system. Therefore, low-pass filters are introduced into repetitive controllers to enhance stability of RC systems, forming modified repetitive controllers which can suppress the high-gain feedback at high frequencies. However, stability is achieved at the sacrifice of performance at high frequencies. With an appropriate filter, the modified repetitive controller can usually achieve a tradeoff between tracking performance and stability, which in turn broadens its application in practice. For example: substituting the model $\frac{q(s)}{1-q(s)e^{-sT}}$ for $\frac{1}{1-e^{-sT}}$ results in the closed-loop system

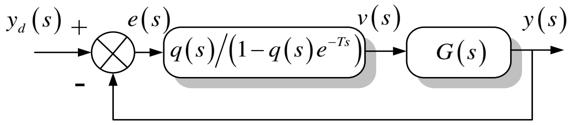

Fig. 5. Periodic signal tracking of a modified RC system.

shown in Fig.5, where $q(s) = \frac{1}{1+\epsilon s}$. Then the relationship between $v(t)$ and $e(t)$ can be written to be

$$e(t) = v(t) - v(t-T) + \epsilon \dot{v}(t). \tag{5}$$

If the closed-loop system without external signals is exponentially stable, then, when the system is driven by a periodic signal, it is easy to see that $v(t)$ and $e(t)$ will both tend to periodic signals as $t \to \infty$. Because of the relationship (5), we can conclude that $e(t) - \epsilon\dot{v}(t) \to 0$. This implies that the tracking error can be adjusted by the filter $q(s)$ or say ϵ. Moreover, if $\dot{v}(t)$ is bounded in t uniformly with respect to (w.r.t) ϵ as $\epsilon \to 0$, then we have $\lim\limits_{t\to\infty,\epsilon\to 0} e(t,\epsilon) = 0$. On the other hand, increasing ϵ can improve stability of the closed-loop system. Therefore, we can achieve a tradeoff between stability and tracking performance by using the modified repetitive controller.

As seen above, the new viewpoint not only explains the IMP in the time domain, but also gives an explanation for the modified repetitive control. For periodic signal tracking, we can conclude that if a periodic signal model with tracking error as the input is incorporated into a closed-loop system all of whose states tend to periodic signals, then perfect tracking is achieved. From the new viewpoint, we do not utilize the transfer function as before. Instead, we only need some tools to verify the system's behavior as $t \to \infty$. This broadens the tools we can choose. For periodic signal tracking, we only need to seek conditions to verify whether or not the system states tend to periodic signals. There exist many conditions on existence of periodic solutions, which usually rely on stability of the closed-loop system. Consequently, stability of the closed-loop system is all that is needed. Therefore, the tracking problem has been reduced to a stability problem of the closed-loop system.

As an application of the new viewpoint, a new method is proposed to design repetitive controllers for periodic signal tracking of non-minimum phase nonlinear systems, where the internal dynamics are subject to a periodic disturbance. To the authors' knowledge, general methods handle such a case only at high computational cost. So the effectiveness is demonstrated.

III. Periodic Signal Tracking of Non-minimum Phase Nonlinear Systems

For clarity, consider a single-input, single-output system in the following normal form

$$\begin{aligned} \dot{\eta} &= \phi(\eta,\xi) + d_\eta \\ \dot{\xi} &= u + d_\xi \\ y &= \xi \end{aligned} \tag{6}$$

where $\eta \in D_\eta \subset \mathbb{R}^{n-1}$, $\xi \in D_\xi \subset \mathbb{R}$, $d_\eta \in C_{PT}^{n-1}$ and $d_\xi \in C_{PT}^1$. The signals d_η and d_ξ are both periodic disturbances. The function $\phi : D_\eta \times D_\xi \to \mathbb{R}^{n-1}$ is locally Lipschitz. In addition, $\phi(0,0) = 0$. The zero dynamics $\dot{\eta} = \phi(\eta, 0)$ in (6) is unstable, so the system (6) is called a non-minimum phase nonlinear system. All the states of (6) are further assumed to be accessible. In this paper, the objective is to design a controller for systems of the form (6) to track a given desired trajectory $y_d \in C_{PT}^1$, while ensuring that the internal state is bounded. Unlike many non-minimum phase nonlinear systems considered in existing literature, an unknown disturbance exists in the internal dynamics of (6). This has brought difficulties for general methods. Now, we take a control law of the form

$$\begin{aligned} \epsilon\dot{v} &= -v + (1-\alpha\epsilon)v_T + [h(y_d) - h(y)] \\ u &= u_{st}(\eta,\xi) + v \end{aligned} \tag{7}$$

where $v_T(t) = v(t-T)$, $\epsilon > 0$, $h : \mathbb{R} \to \mathbb{R}$ denotes a continuous strictly increasing function and $u_{st} : D_\eta \times D_\xi \to \mathbb{R}$ is a state feedback law employed to stabilize the state of the underlying plant. The functions $h(\cdot)$ and $u_{st}(\cdot)$ are both locally Lipschitz. On the other hand, the continuous function v represents a feedforward input which will drive the output y of (6) to track the given desired trajectory $y_d \in C_{PT}^1$. Next, we write the resulting closed-loop system as follows

$$E\dot{x} = F(x) + Bd \tag{8}$$

where

$$x = \begin{bmatrix} v \\ \eta \\ \xi \end{bmatrix}, F(x) = \begin{bmatrix} -v + (1-\alpha\epsilon)v_T - h(\xi) \\ \phi(\eta,\xi) \\ u_{st}(\eta,\xi) + v \end{bmatrix},$$

$$E = \mathrm{diag}(\epsilon, I_n), B = \mathrm{diag}(k, I_n), d = \begin{bmatrix} y_d & d_\eta^T & d_\xi \end{bmatrix}^T.$$

The closed-loop system (8) is a functional differential equation. A definition is needed for developing the following theorem.

Definition 1 [12]. The solutions $x(t_0,\varphi)(t)$ of system (8) with $x(t_0+\theta) = \phi(\theta), \theta \in [-\tau, 0]$ are said to be *uniformly ultimately bounded* with *ultimate bound* B, if for each $A > 0$ there exists $K(A,B) > 0$ such that $[t_0 \in \mathbb{R}, \varphi \in C, \|\varphi\| < A, t \geq t_0 + K(A,B)]$ imply that $\|x(t_0,\varphi)(t)\| < B$.

Theorem 1. Suppose that the solutions of the resulting closed-loop system in (8) are uniformly ultimately bounded.

Then the resulting closed-loop system in (8) has a T-periodic solution. If the solutions of (8) approach the T-periodic solution, then

$$\|h(y_d) - h(y)\|_a \le \epsilon \left(\|\dot{v}\|_a + \alpha \|v\|_a \right).$$

Furthermore, if $\|\dot{v}\|_a$ and $\|v\|_a$ are bounded in t uniformly w.r.t ϵ as $\epsilon \to 0$, then $\lim_{t\to\infty, \epsilon\to 0} \|e(t,\epsilon)\|_a = 0$.

Proof: Define $\dot{x} = f(x,t) := F(x) + Bd$. Since $d \in C_{PT}^{n+1}$, we have $f(x,t) = f(x, t+T)$. Furthermore, $f(x,t)$ is locally Lipschitz. Since the solutions of the resulting closed-loop system (8) are uniformly ultimately bounded, the resulting closed-loop system in (8) has a T-periodic solution according to [12]. By using (7), it follows that

$$h(y_d) - h(y) = \epsilon\dot{v} + v - (1 - \alpha\epsilon) v_T.$$

Taking $\|\cdot\|_a$ on both sides of the equation above yields

$$\begin{aligned} \|h(y_d) - h(y)\|_a &= \|\epsilon(\dot{v} + \alpha v_T) + v - v_T\|_a \\ &\le \|\epsilon(\dot{v} + \alpha v_T)\|_a + \|v - v_T\|_a \\ &\le \epsilon\left(\|\dot{v}\|_a + \alpha\|v\|_a\right) \end{aligned}$$

where the condition that the solutions of (8) approach the T-periodic solution is used. If $\|\dot{v}\|_a$ and $\|v\|_a$ are bounded in t uniformly w.r.t ϵ as $\epsilon \to 0$, then $\epsilon\left(\|\dot{v}\|_a + \alpha\|v\|_a\right) \to 0$ as $\epsilon \to 0$. This implies that $\|h(y_d) - h(y)\|_a \to 0$ as $\epsilon \to 0$. Note that $h(\cdot)$ is a continuous strictly increasing function. Then $\lim_{t\to\infty,\epsilon\to 0} \|e(t,\epsilon)\|_a = 0$. ■

Remark 1: It can be proved that the solutions of (8) will approach the T-periodic solution under suitable conditions [13]. So far, however, the conditions on $F(\cdot)$ are usually conservative. Generally speaking, through many observations from experiments and simulations, we observe that stable systems will always eventually oscillate on being driven by an external periodic signal. Moreover, the oscillation period is the same as that of the external signal. Therefore, the condition that the solutions of (8) approach the T-periodic solution does not need to be verified in practice. In the worst situation, the solutions of the resulting closed-loop system in (8) are uniformly ultimately bounded. So, it is flexible in practice to decide whether or not to adopt this control scheme depending on the tracking performance.

IV. An application

Consider a concrete system in the form of (6) that

$$\begin{aligned} \dot{\eta} &= \sin\eta + \xi + d_\eta \\ \dot{\xi} &= u + d_\xi \\ y &= \xi \end{aligned} \tag{9}$$

where $\eta(0) = 1$ and $\xi(0) = 0$. The desired trajectory $y_d = \sin t$. The disturbances are assumed to be $d_\eta = 0.1\sin t$ and $d_\xi = 0.2\sin t$. Since the zero dynamics are unstable, system (9) is a non-minimum phase nonlinear system. The control is required not only to cause y to track y_d, but also to stabilize the internal dynamics. If the usual method is used to handle this problem, then it may be difficult to obtain the ideal internal dynamics, for the disturbance in the internal dynamics is unknown. Now, we will take a control law of the form (7), and design u_{st} and v to make sure the solutions of the resulting closed-loop system are uniformly ultimately bounded.

The stabilizing controller u_{st} is designed by using back-stepping method. We start with the scalar system

$$\dot{\eta} = \sin\eta + \xi + d_\eta$$

with ξ viewed as the input and proceed to design the feedback control as $\xi = z - q_1\eta - \sin\eta$. Then we obtain $\dot{\eta} = -q_1\eta + z + d_\eta$. To backstep, we use the change of variables $z = \xi + q_1\eta + \sin\eta$ to transform the system into the form

$$\dot{z} = u + (q_1 + \cos\eta)(-q_1\eta + z) + d_\xi + d_\eta(q_1 + \cos\eta).$$

Based on the equation above, the controller is designed to be

$$\begin{aligned} \epsilon\dot{v} &= -v + (1 - \alpha\epsilon) v_T + \rho(y_d - y) \\ u &= -(q_1 + \cos\eta)(-q_1\eta + z) - kz - q_2\eta + v \end{aligned} \tag{10}$$

where the coefficients are specified later to ensure that the solutions of the resulting closed-loop system are uniformly ultimately bounded. Then the closed-loop system becomes

$$\begin{aligned} \epsilon\dot{v} &= -v + (1 - \alpha\epsilon) v_T - \rho(z - q_1\eta - \sin\eta) + \rho y_d \\ \dot{\eta} &= -q_1\eta + z + d_\eta \\ \dot{z} &= -kz - q_2\eta + v + d_\xi + d_\eta(q_1 + \cos\eta). \end{aligned} \tag{11}$$

Design a Lyapunov functional to be

$$V = \frac{1}{2}p_1\eta^2 + \frac{1}{2}p_2 z^2 + \frac{\epsilon}{2}v^2 + \int_{-T}^{0} v_\theta^2 d\theta.$$

Taking the derivative of V along the solutions of (11) results in

$$\begin{aligned} \dot{V} &= p_1\eta\dot{\eta} + p_2 z\dot{z} + \varepsilon v\dot{v} + \frac{1}{2}\left(v^2 - v_T^2\right) \\ &= -p_1 q_1\eta^2 - p_2 k z^2 - \frac{\alpha\varepsilon(2 - \alpha\varepsilon)}{2}v^2 \\ &\quad + (p_1 - p_2 q_2)\eta z + (p_2 - \rho) z v + v(q_1\eta + \sin\eta) \\ &\quad + p_1\eta d_\eta + p_2\left[d_\xi - d_\eta(q_1 + \cos\eta)\right] z + \rho v y_d. \end{aligned}$$

By choosing p_1, q_1 appropriately and $p_2 = \rho$, if k is chosen sufficiently large, then we have

$$\begin{aligned} &-p_1 q_1\eta^2 - p_2 k z^2 - \frac{\alpha\varepsilon(2 - \alpha\varepsilon)}{2}v^2 + (p_1 - p_2 q_2)\eta z \\ &+ (p_2 - \rho) z v + v(q_1\eta + \sin\eta) \le -\theta_1\eta^2 - \theta_2 z^2 - \theta_3 v^2 \end{aligned}$$

where $\theta_1, \theta_2, \theta_3$ are positive numbers. Furthermore, there exists a class $\mathcal{K}$ function $\chi : [0,\infty) \to [0,\infty)$ such that [14]

$$\dot{V} \le -\theta_1'\eta^2 - \theta_2' z^2 - \theta_3' v^2 + \chi\left(\|y_d\|_\infty^2 + \|d_\eta\|_\infty^2 + \|d_\xi\|_\infty^2\right)$$

where $\theta_1', \theta_2', \theta_3'$ are positive numbers. Therefore, the given Lyapunov functional satisfies

$$\gamma_0 \|x(t)\|^2 \leq V \leq \gamma_1 \|x(t)\|^2 + \frac{1}{2}\int_{-T}^{0} \|x_\theta\|^2 d\theta$$
$$\dot{V} \leq -\gamma_2 \|x(t)\|^2 + \chi \left(\|y_d\|_\infty^2 + \|d_\eta\|_\infty^2 + \|d_\xi\|_\infty^2 \right)$$

where $\gamma_0 = \min\left(\frac{1}{2}p_1, \frac{1}{2}p_2, \frac{\varepsilon}{2}\right), \gamma_1 = \max\left(\frac{1}{2}p_1, \frac{1}{2}p_2, \frac{\epsilon}{2}\right)$ and $\gamma_2 = \min\left(\theta_1', \theta_2', \theta_3'\right)$.

According to Theorem 4 in [12], the solutions of (11) are uniformly ultimately bounded. Furthermore, ξ is also uniformly ultimately bounded by using the relationship $\xi = z - q_1\eta - \sin\eta$. By **Theorem 1**, the resulting closed-loop system has a T-periodic solution. If the solutions of (8) approach the T-periodic solution, then the tracking error satisfies $\|e\|_a \leq \epsilon\rho^{-1}\left(\|\dot{v}\|_a + \alpha\|v\|_a\right)$. The controller (10) is chosen to be

$$0.1\dot{v} = -v + 0.99v_T + 5\left(y_d - y\right), v\left(\theta\right) = 0, \theta \in [-T, 0]$$
$$u = -\left(1 + \cos\eta\right)\left(-1 + z\right) - 2z - \eta + v.$$

Then the performance of the proposed controller is shown in Figs 6-7. Fig. 6 shows the response of the closed-loop system from the given initial condition. The output very quickly tracks the desired trajectory actually, in two periods. The internal state is also bounded. In Fig. 7, we present the time history of the stabilizing controller u_{st}, the feedforward input v and finally the controller output $u = u_{st} + v$. It is obvious that they are all bounded.

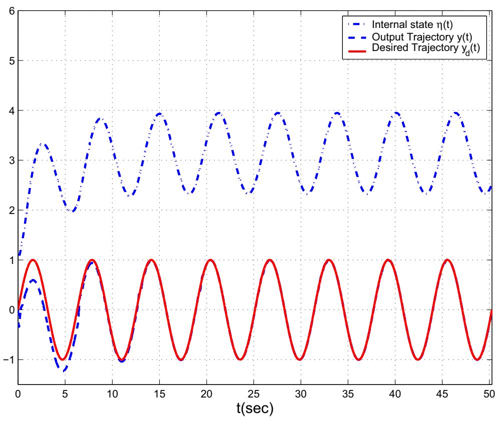

Fig. 6. Time responses of the closed-loop system.

V. Conclusions

A new viewpoint on IMP is given. Guided by this viewpoint, a method is given to design repetitive controllers for periodic signal tracking of non-minimum phase nonlinear systems, where the internal dynamics are subject to a periodic disturbance. A simulation example illustrates the effectiveness of the proposed method. As we expect, the new method really overcomes some weaknesses of existing methods which are applicable to the general signal tracking. This also coincides with the basic idea of the IMP.

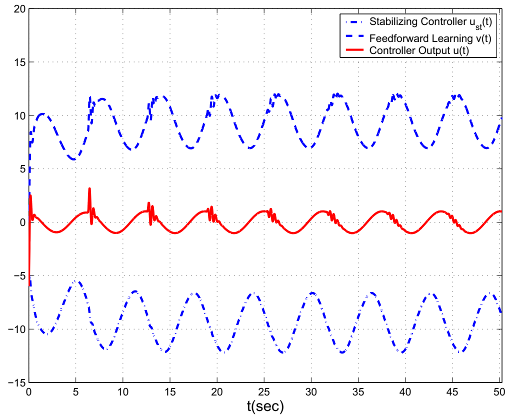

Fig. 7. Time responses of the controller output.

References

[1] T. Inoue, M. Nakano, and S. lwai, "High accuracy control of a proton synchrotron magnet power supply," *Proc. 8thWorld Congr. IFAC*, pp. 216-221, 1981.

[2] S. Hara, Y. Yamamoto, T. Omata, and M. Nakano, "Repetitive control system: a new type servo system for periodic exogenous signals," *IEEE Trans. Autom. Control*, vol. 33, no. 7, pp. 659-668, July 1988.

[3] G. Weiss and M. Hafele, "Repetitive control of MIMO system using H$^\infty$ design," *Automatica*, vol. 35, no. 7, pp. 1185-1199, 1999.

[4] R.W. Longman, "Iterative learning control and repetitive control for engineering practice," *Int. J. Control*, vol. 73, no. 10, pp. 930-954, July 2000.

[5] Y.-H. Kim and I.-J. Ha, "Asymptotic state tracking in a class of nonlinear systems via learning-based inversion," *IEEE Trans. Autom. Control*, vol. 45, no 11, pp. 2011-2027, November 2000.

[6] W.E. Dixon, E. Zergeroglu, and B.T. Costic, "Repetitive learning control: a Lyapunov-based approach," *IEEE Trans. Syst. Man Cybern. Part B-Cybern.*, vol. 32, no. 4, pp. 538-545, August 2002.

[7] M. Sun, S.S. Ge, and I.M.Y. Mareels, "Adaptive repetitive learning control of robotic manipulators without the requirement for initial repositioning," *IEEE Trans. Robot.*, vol. 22, no. 3, pp. 563-568, June 2006.

[8] J.X. Xu and R. Yan, "On repetitive learning control for periodic tracking tasks," *IEEE Trans. Autom. Control*, vol. 51, no. 11, pp. 1842-1848, November 2006.

[9] B.A. Francis and W.M. Wonham, "The internal model principle of control theory," *Automatica*, vol. 12, no. 5, pp. 457-465, 1976.

[10] I.A. Shkolnikov and Y.B. Shtessel, "Tracking in a class of nonminimum-phase systems with nonlinear internal dynamics via sliding mode control using method of system center," *Automatica*, vol. 38, no. 5, pp. 837-842, 2002.

[11] A.R. Teel, "A nonlinear small gain theorem for the analysis of control systems with saturation," *IEEE Trans. Autom. Control*, vol. 41, no. 9, pp. 1256-1270, September 1996.

[12] T.A. Burton and S. Zhang, "Unified boundedness, periodicity, and stability in ordinary and functional differential equations," *Annali di Matematica Pura ed Applicata*, vol. 145, no. 1, pp. 129-158, December 1986.

[13] J. Cao, "On exponential stability and periodic solutions of CNNs with delays," *Phys. Lett. A*, vol. 267, no. 5, pp. 312-318, March 2000.

[14] H.K. Khalil, *Nonlinear Systems,* 3rd Edition, Englewood Cliffs, NJ: Prentice-Hall, 2002.