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ARTICLE *in* JOURNAL OF PHYSICS B ATOMIC MOLECULAR AND OPTICAL PHYSICS · NOVEMBER 2008

Impact Factor: 1.92 · DOI: 10.1088/0953-4075/41/23/235502

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2008 J. Phys. B: At. Mol. Opt. Phys. 41 235502

(http://iopscience.iop.org/0953-4075/41/23/235502)

J. Phys. B: At. Mol. Opt. Phys. **41** (2008) 235502 (6pp) doi:10.1088/0953-4075/41/23/235502

Generation of entanglement between two spatially separated atoms via dispersive atom–field interaction

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Received 31 July 2008, in final form 13 October 2008
Published 19 November 2008
Online at stacks.iop.org/JPhysB/41/235502

Abstract
Based on the dispersive atom–field interaction, a scheme is proposed for deterministically generating entanglement between two Λ-type atoms, which are trapped individually in two spatially separated cavities coupled by an optical fibre. In the present scheme, it is found that the atomic spontaneous decay and photon leakage out of the fibre can be efficiently suppressed via choosing appropriately the frequency detuning of atom–field and the coupling intensity of cavity–fibre, respectively. The influence of photon leakage out of the cavities is also discussed, and the strictly numerical simulation shows that our proposal is good enough to demonstrate the generation of entanglement between two distant atoms with high fidelity.

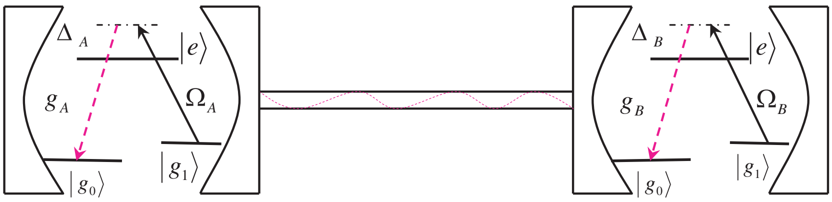
(Some figures in this article are in colour only in the electronic version)

1. Introduction

Entanglement is known to play an essential role in many quantum information processes, such as quantum teleportation, quantum cryptography, quantum computers, etc [1–9]. Over the past few years, a large number of schemes have been proposed for generating entangled states in various quantum systems [10–17], including semiconductor quantum dots [13], cavity quantum electrodynamics (CQED) [14], trapped ions [16] and so on. Among these quantum systems, the CQED system is always favoured because of its low decoherence rate [14] and has many advantages in entanglement engineering [18]. Based on the theory of CQED, researchers have already realized experimentally the entangled sate of two Rydberg atoms crossing a nonresonant cavity [19].

In recent years, more and more theory investigations have been devoted to the generation of entanglement between atoms trapped in distant optical cavities, through the detection of leaking photons [20–22] or through the direct linking of the cavities [23–30]. For example, Feng *et al* [20] proposed a scheme for generating entangled state of two distant atoms via the interference effects of polarized photons. However, it is a probabilistic scheme as it depends on the detection of the photons decaying from two leaking cavities and thus high efficient photon detectors are required. Subsequently, based on the cavity–fibre–cavity system, Serafini *et al* [25] proposed a scheme for realizing highly reliable swap and entangling gates via employing two two-level atoms trapped individually in the spatially separated cavities. Using a similar set-up, two schemes were proposed for deterministically generating entanglement between two spatially separated atoms, through the photon emission–absorption process [26] and adiabatic passage [28], respectively. Multiatom and resonant interaction schemes for generating entanglement between two remote cavities connected by an optical fibre also have been proposed in the paper [29]. Summing up the previous schemes, it is noticed that most of them are based on the resonant atom–field interaction, and then the atomic spontaneous emission usually has a strong influence on the fidelity of realizing entanglement [26, 29].

In this paper, based on the dispersive atom–field interaction, we propose an alternative scheme for deterministically generating entangled state of two Λ-type atoms, which are trapped individually in two spatially separated cavities coupled by an optical fibre. In the present



(A) (B)

Figure 1. Two Λ-type atoms are trapped in two spatially separated cavities A and B, respectively. The cavities are linked by an optical fibre.

scheme, the influence of atomic spontaneous decay and photon leakage from the fibre can be suppressed effectively via choosing large atom–field detuning and appropriate fibre–field coupling strength. As a result, the highly reliable entangled state of two spatially separated atoms can be realized, based on our scheme.

The remainder of this paper is organized as follows. In section 2, we first describe the model under consideration and then derive the effective Hamiltonian of the system. In section 3, the generation of atomic entangled states is provided and discussed. In section 4, we numerically simulated effects of atomic spontaneous decay, photon leakage out of the cavities and fibre. Finally, we conclude with a brief summary in section 5.

2. Model and Hamiltonian

As shown in figure 1, we consider a cavity–fibre–cavity system, consisting of two single-mode cavities (cavities A and B) connected by an optical fibre. Two Λ-type atoms are individually trapped in the cavities A and B. In the cavity i (i = A, B), the atomic transition $|g_1\rangle_i \leftrightarrow |e\rangle_i$ (with resonant frequency $\omega_{eg_1}^i$) is dispersively driven by a classical field with center frequencies ω_i. The cavity mode with frequency ν_i dispersively interacts with the atomic transition $|g_0\rangle_i \leftrightarrow |e\rangle_i$ (with resonant frequency $\omega_{eg_0}^i$). The frequency detuning of cavity field from the atomic transition $|g_0\rangle_i \leftrightarrow |e\rangle_i$ and classical field from the atomic transition $|g_1\rangle_i \leftrightarrow |e\rangle_i$ are denoted as Δ_{iq} $\left(\Delta_{iq} = \nu_i - \omega_{eg_0}^i\right)$ and Δ_{ic} $\left(\Delta_{ic} = \omega_i - \omega_{eg_1}^i\right)$, respectively. We have assumed that the corresponding frequency detunings satisfy the two-photon resonance condition $\Delta_{iq} = \Delta_{ic} = \Delta_i$ in the following computation.

Then, in the interaction picture, under the dipole and rotating wave approximation, the interaction Hamiltonian of the atom–cavity system can be written as ($\hbar = 1$) [34–36]

$$H_I^{\mathrm{ac}} = \sum_{i=\mathrm{A,B}} [-\Delta_i |e\rangle_i\langle e| + (\Omega_i |e\rangle_i\langle g_1| + g_i a_i |e\rangle_i\langle g_0| + \mathrm{h.c.})], \tag{1}$$

where the symbol h.c. means the Hermitian conjugate and we have taken the ground state $|g_0\rangle$ as the energy origin for the sake of simplicity. $a_i^\dagger$ and a_i are the creation and annihilation operators associated with the corresponding quantized cavity modes. Ω_i and g_i denote the one-half Rabi frequency and atom–field coupling constant, respectively. They are assumed to be real in this paper, without loss of generality.

By applying standard quantum optical techniques [31], under the large-detuning condition, $|\Delta_i| \gg |\Omega_i|, |g_i|$, the excited state $|e\rangle_i$ of an atom is only virtually excited in the process of atom–field interaction. So, we can adiabatically eliminate the excited state $|e\rangle_i$ and obtain the effective Hamiltonian [24, 32, 33]

$$H_{\mathrm{eff1}}^{\mathrm{ac}} = \sum_{i=\mathrm{A,B}} \left[\frac{g_i^2}{\Delta_i} a_i^\dagger a_i |g_0\rangle_i\langle g_0| + \frac{\Omega_i^2}{\Delta_i} |g_1\rangle_i\langle g_1| + \left(\frac{g_i\Omega_i}{\Delta_i} a_i |g_1\rangle_i\langle g_0| + \mathrm{h.c.}\right)\right], \tag{2}$$

where the first two terms represent cavity- and laser-induced atomic level shifts, respectively. The last two terms correspond to the effective Raman coupling rates. According to [32], the terms of cavity- and laser-induced atomic level shifts can be compensated for quite straightforwardly via using two second lasers which couple the corresponding atomic levels $|g_0\rangle_i$ and $|g_1\rangle_i$ nonresonantly with two additional levels farther up in the atomic level scheme. Then, the effective Hamiltonian can be further reduced as

$$H_{\mathrm{eff2}}^{\mathrm{ac}} = \Omega_{eA} a_A |g_1\rangle_A\langle g_0| + \Omega_{eB} a_B |g_1\rangle_B\langle g_0| + \mathrm{h.c.}, \tag{3}$$

where $\Omega_{ei} = \frac{g_i\Omega_i}{\Delta_i}$ is the effective Rabi frequency for the corresponding Raman transition $|g_0\rangle_i \to |e\rangle_i \to |g_1\rangle_i$.

In the short fibre limit $(2L\bar{\nu})/(2\pi c) \ll 1$, where L is the length of the fibre and $\bar{\nu}$ is the decay rate of the cavity field into a continuum of fibre modes, the interaction Hamiltonian for the cavity fields coupled to the fibre modes reads [25]

$$H_I^{\mathrm{cf}} = \eta\left[b\left(a_A^\dagger + a_B^\dagger\right) + \mathrm{h.c.}\right], \tag{4}$$

where b is the resonant mode of the fibre, when it interacts with the cavity modes a_i. η denotes the corresponding coupling strength between the cavity and the fibre. The relative phase φ between two cavity modes, which is due to the propagation of the field through the fibre of length L, has been absorbed into the cavity mode $a_B^\dagger$ [26].

Lastly, in the interaction picture, the total Hamiltonian of this cavity–fibre–cavity system can be written as

$$\begin{aligned} H_I &= H_{\mathrm{eff2}}^{\mathrm{ac}} + H_I^{\mathrm{cf}} \\ &= \Omega_{eA} a_A |g_1\rangle_A\langle g_0| + \Omega_{eB} a_B |g_1\rangle_B\langle g_0| \\ &\quad + \eta b\left(a_A^\dagger + a_B^\dagger\right) + \mathrm{h.c.} \end{aligned} \tag{5}$$

3. Entanglement of two atoms

In this section, we begin to study the generation of entanglement between two spatially separated Λ-type atoms.

First, we will show that an extremely entangled state of the atoms $|\Psi_e\rangle = (|g_1\rangle_A|g_0\rangle_B + |g_0\rangle_A|g_1\rangle_B)/\sqrt{2}$ can be deterministically generated in an ideal situation. Introduce three normal bosonic modes c and $c_\pm$ by the canonical transformations [25]

$$c = (a_A - a_B)/\sqrt{2}, c_\pm = (a_A + a_B \pm \sqrt{2}b)/\sqrt{2}. \quad (6)$$

Then, based on the new bosonic operators, the Hamilton (5) can be re-expressed as

$$H_I = \tfrac{1}{2}[\Omega_{eA}(\mathrm{e}^{-\mathrm{i}\sqrt{2}\eta t}c_+ + \mathrm{e}^{\mathrm{i}\sqrt{2}\eta t}c_- + \sqrt{2}c)|g_1\rangle_A\langle g_0| + \Omega_{eB}(\mathrm{e}^{-\mathrm{i}\sqrt{2}\eta t}c_+ + \mathrm{e}^{\mathrm{i}\sqrt{2}\eta t}c_- - \sqrt{2}c)|g_1\rangle_B\langle g_0| + \mathrm{h.c.}]. \quad (7)$$

It can be easily seen from equation (7) that the normal mode c resonantly interacts with the atomic transitions $|g_0\rangle_A \leftrightarrow |g_1\rangle_A$ and $|g_0\rangle_B \leftrightarrow |g_1\rangle_B$; however, the normal modes $c_\pm$ are off-resonant from the corresponding atomic transitions with frequency detunings $\pm\sqrt{2}\eta$. So, the off-resonant modes can be safely neglected under the limit $|\eta| \gg |\Omega_{eA}|, |\Omega_{eB}|$, and the Hamiltonian (7) becomes

$$H_I = \tfrac{1}{\sqrt{2}}(\Omega_{eA}c|g_1\rangle_A\langle g_0| - \Omega_{eB}c|g_1\rangle_B\langle g_0| + \mathrm{h.c.}). \quad (8)$$

Now, the time evolution of our system will be governed by the Hamiltonian (8) and the Schrödinger equation ($\hbar = 1$)

$$\mathrm{i}\frac{\partial}{\partial t}|\psi(t)\rangle = H_I|\psi(t)\rangle. \quad (9)$$

Consider that at the initial time the system is in the state $|\psi(0)\rangle = |g_1\rangle_A|g_0\rangle_B|0\rangle_c$, where $|0\rangle_c$ denotes the vacuum state of the normal mode c. Then, the state of the system at time t, $|\psi(t)\rangle$ can be expressed as

$$\begin{aligned}|\psi(t)\rangle &= C_1|\phi_1\rangle + C_2|\phi_2\rangle + C_3|\phi_3\rangle \\ &= C_1|g_1\rangle_A|g_0\rangle_B|0\rangle_c + C_2|g_0\rangle_A|g_1\rangle_B|0\rangle_c \\ &\quad + C_3|g_0\rangle_A|g_0\rangle_B|1\rangle_c,\end{aligned} \quad (10)$$

where $|\phi_1\rangle, |\phi_2\rangle$ and $|\phi_3\rangle$ compose the subspace of system evolvement. Associating with equations (9) and (10), we can get the expressions of coefficients C_j ($j = 1,2,3$),

$$C_1 = \frac{\mathrm{Cos}\left(\sqrt{(\Omega_{eA}^2 + \Omega_{eB}^2)/2}t\right)\Omega_{eA}^2}{\Omega_{eA}^2 + \Omega_{eB}^2} + \frac{\Omega_{eB}^2}{\Omega_{eA}^2 + \Omega_{eB}^2}, \quad (11a)$$

$$C_2 = -\frac{\mathrm{Cos}\left(\sqrt{(\Omega_{eA}^2 + \Omega_{eB}^2)/2}t\right)\Omega_{eA}\Omega_{eB}}{\Omega_{eA}^2 + \Omega_{eB}^2} + \frac{\Omega_{eA}\Omega_{eB}}{\Omega_{eA}^2 + \Omega_{eB}^2}, \quad (11b)$$

$$C_3 = \frac{\mathrm{i\,Sin}\left(\sqrt{(\Omega_{eA}^2 + \Omega_{eB}^2)/2}t\right)\Omega_{eA}}{\sqrt{\Omega_{eA}^2 + \Omega_{eB}^2}}. \quad (11c)$$

From the above equations, we note that the state of system will evolve into the state $|\Psi_e\rangle|0\rangle_c$ (corresponding to $C_1 = C_2 = 1/\sqrt{2}, C_3 = 0$) under the condition $\Omega_{eB} = (1+\sqrt{2})\Omega_{eA}$, when $t = \frac{\sqrt{2}m\pi}{\sqrt{\Omega_{eA}^2+\Omega_{eB}^2}} (m = 1, 3, 5, \ldots)$. The state $|\Psi_e\rangle|0\rangle_c = \frac{1}{\sqrt{2}}(|g_1\rangle_A|g_0\rangle_B + |g_0\rangle_A|g_1\rangle_B) \otimes |0\rangle_c$ is a product state of the two-atom entangled state $|\Psi_e\rangle$ and the vacuum state $|0\rangle_c$, and hence we get an extremely entangled state ($|\Psi_e\rangle$) of two spatially separated atoms, which is completely separated from the cavity fields and fibre modes.

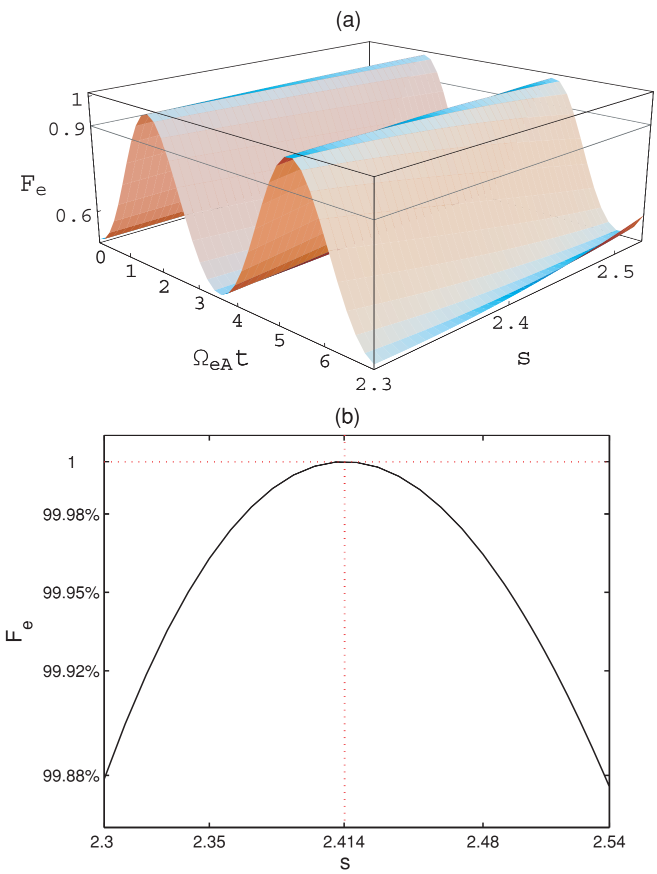


Figure 2. The fidelity F_e for realizing the extremely entangled state $|\Psi_e\rangle$ versus time Ω_{eA} t and ratio coefficient s (a); versus deviation s when $\Omega_{eA}t = 1.7$ (b).

Summing up the above discussions, it is noted that the extremely entangled state, $|\Psi_e\rangle$, can be deterministically generated in an ideal situation, which includes $\Omega_{eB} = (1+\sqrt{2})\Omega_{eA}$. But there exists usually some deviation of the ratio coefficient between two effective Rabi frequencies from the value $1+\sqrt{2}$ in practical situations. In order to study the influence of this deviation on the fidelity F_e of realizing atomic entangled state, we present the three-dimensional figure of the dependence of F_e on $\Omega_{eA}t$ and ratio coefficient s, as shown in figure 2(a). The fidelity F_e is defined as $F_e = |_c\langle 0|\langle\Psi_e|\psi(t)\rangle|^2$ and the ratio coefficient s satisfies the equation $\Omega_{eB} = s\Omega_{eA}$. It is clearly shown from figure 2(a) that F_e will reach periodically the maximal value, and it is highly stable to the deviation of the ratio coefficient s from the condition $s = 1+\sqrt{2} \approx 2.4$, with which the atomic entangled state can be realized determinately. To see this high stability of F_e more clearly, we also plot the two-dimensional curve of F_e versus s in figure 2(b), when $\Omega_{eA}t = 1.7$. From figure 2(b), it is noted that there is just a little decrease ($\sim$0.12%) of the fidelity F_e compared to $F_e = 1$, when the ratio coefficient s changes from $s = 2.3$ to 2.54. As a result, based on our scheme, the atomic entangled state $|\Psi_e\rangle$ can also be realized with high fidelity ($\geqslant$99.88%), even that

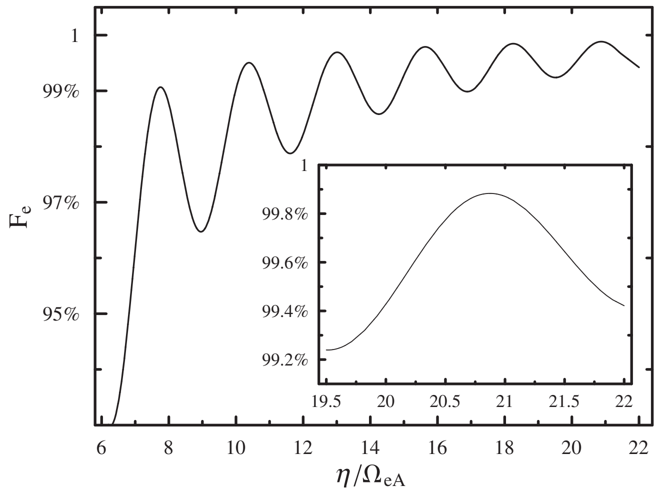


Figure 3. The fidelity of realizing the extremely entangled state $|\Psi_e\rangle$ versus coupling strength η when $\Omega_{eA}t = 1.7$, $\Omega_{eB} = (1+\sqrt{2})\Omega_{eA}$.

the ideal condition $\Omega_{eB} = (1+\sqrt{2})\Omega_{eA}$ could not be satisfied accurately in practical situations.

Up to now, the above discussions are all based on the approximation of neglecting the nonresonant normal modes $c_\pm$ under the condition $|\eta| \gg |\Omega_{eA}|, |\Omega_{eB}|$. It has been shown that the entangled state $|\Psi_e\rangle$ can be realized deterministically ($F_e = 1$) in an ideal situation, i.e., $\Omega_{eB} = (1+\sqrt{2})\Omega_{eA}$, $\sqrt{\Omega_{eA}^2 + \Omega_{eB}^2}t = \sqrt{2}m\pi (m = 1, 3, 5, \ldots)$, under this approximation. In order to check the validity of this approximation, we will directly solve the Schrödinger equation (9) with the help of Hamiltonian (5). Consider that at the initial time the system is in the state $|\psi(0)\rangle = |g_1\rangle_A|g_0\rangle_B|0\rangle_A|0\rangle_f|0\rangle_B$, where $|0\rangle_A, |0\rangle_f, |0\rangle_B$ denote the vacuum state of cavity A, cavity B and fibre mode, respectively. Then, the whole system state at t time $|\psi(t)\rangle$ will be restricted to the subspace with one excitation number spanned by the basis vectors

$$|\phi_1\rangle = |g_1\rangle_A|g_0\rangle_B|0\rangle_A|0\rangle_f|0\rangle_B, \tag{12a}$$

$$|\phi_2\rangle = |g_0\rangle_A|g_0\rangle_B|1\rangle_A|0\rangle_f|0\rangle_B, \tag{12b}$$

$$|\phi_3\rangle = |g_0\rangle_A|g_0\rangle_B|0\rangle_A|1\rangle_f|0\rangle_B, \tag{12c}$$

$$|\phi_4\rangle = |g_0\rangle_A|g_0\rangle_B|0\rangle_A|0\rangle_f|1\rangle_B, \tag{12d}$$

$$|\phi_5\rangle = |g_0\rangle_A|g_1\rangle_B|0\rangle_A|0\rangle_f|0\rangle_B. \tag{12e}$$

In this subspace, by solving numerically equation (9) with the help of Hamiltonian (5), we give the effects of coupling strength η on the fidelity of realizing the extremely entangled state $|\Psi_e\rangle$, as shown in figure 3. The fidelity is defined as $F_e = |{}_A\langle 0|_f\langle 0|_B\langle 0|\langle\Psi_e|\psi(t)\rangle|^2$ and the corresponding parameters are chosen as $\Omega_{eA}t = 1.7$, $\Omega_{eB} = (1+\sqrt{2})\Omega_{eA}$. It is shown from figure 3 that the fidelity F_e rapidly approaches $F_e = 1$, which corresponding the result under the approximation of neglecting nonresonant modes $c_\pm$ due to $|\eta| \gg |\Omega_{eA}|, |\Omega_{eB}|$, with fluctuant way, and becomes more and more stable to the coupling strength η along with its increase. More specifically, the maximum value of F_e can almost reach 99.9% and the minimum value also larger than 99.3%, when the coupling strength $|\eta| \geqslant 20|\Omega_{eA}|\,(\geqslant 10\Omega_{eB})$. As a result, the nonresonant normal modes $c_\pm$ can be safely neglected in our scheme, when $|\eta| \geqslant 20|\Omega_{eA}|\,(\geqslant 10\Omega_{eB})$.

Before ending this section, it should be pointed out that the propagation effects of field in the fibre have been neglected in this paper. First, the relative phase φ between two cavity modes, which is due to the propagation of the field through the fibre of length L, can be absorbed into the annihilation and creation operators of the mode of cavity B [26]. Moreover, similar to the descriptions of [25], under the conditions of short fibre limit $(2L\bar{\nu})/(2\pi c) \ll 1$ and $|\eta| \gg |\Omega_{ei}|$ $(i = \text{A, B})$, the cavity–fibre–cavity system considered here can be reduced to two atoms resonantly coupled through a single harmonic oscillator, which has no contribution from the fibre mode b, as shown in Hamiltonian (8). So, the system gets in this instance insensitive to the influence of the fibre. So, the propagation effects of field in the fibre can be effectively neglected in this paper.

4. Effects of atomic spontaneous decay and photon leakage

In this section, we will study the influence of atomic spontaneous decay and photon leakage out of the cavities and fibre on the generation of atomic entangled state $|\Phi_e\rangle$. Using the density-matrix formalism, the master equation for the density matrix of the whole system can be expressed as

$$\begin{aligned}\dot{\rho} = &-\mathrm{i}\left[H_I^{\mathrm{ac}} + H_I^{\mathrm{cf}}, \rho\right] - \frac{\gamma_f}{2}(b^\dagger b\rho - 2b\rho b^\dagger + \rho b^\dagger b)\\ &- \sum_{i=A,B}\frac{\kappa_i}{2}\left(a_i^\dagger a_i\rho - 2a_i\rho a_i^\dagger + \rho a_i^\dagger a_i\right)\\ &- \sum_{j=g_0,g_1}\left[\frac{\gamma_{Aa}^{ej}}{2}\left(\sigma_{ee}^A\rho - 2\sigma_{je}^A\rho\sigma_{ej}^A + \rho\sigma_{ee}^A\right)\right.\\ &\left.- \frac{\gamma_{Ba}^{ej}}{2}\left(\sigma_{ee}^B\rho - 2\sigma_{je}^B\rho\sigma_{ej}^B + \rho\sigma_{ee}^B\right)\right],\end{aligned} \tag{13}$$

where H_I^{ac} and H_I^{cf} are given by equations (1) and (4), respectively; γ_{Aa}^{ej} and γ_{Ba}^{ej} denote the spontaneous decay rates of atoms from level $|e\rangle_i$ to $|j\rangle_i$ $(i = \text{A, B})$, κ_i and γ_f denote the decay rates of cavity fields and fibre mode, respectively; $\sigma_{mn}^i = |m\rangle_i\langle n|$ denotes the usual Pauli matrix. By solving numerically equation (13) in the subspace spanned by the basis vectors (12) and $|\phi_6\rangle = |e\rangle_A|g_0\rangle_B|0\rangle_A|0\rangle_f|0\rangle_B$, $|\phi_7\rangle = |g_0\rangle_A|e\rangle_B|0\rangle_A|0\rangle_f|0\rangle_B$, we present the effects of the decay rates γ_a $\left(\gamma_a = \gamma_a^{eg_0} + \gamma_a^{eg_1}\right)$, κ and γ_f on the fidelity F_e of generating atomic entangled state $|\Psi_e\rangle$, as shown in figure 4. In the calculation, we have chosen all parameters reduced to dimensionless units by scaling γ and $\gamma_{ia}^{eg_0} = \gamma_{ia}^{eg_1} = \gamma_a/2$ $(i = \text{A, B})$, $\kappa_A = \kappa_B = \kappa$ for simplicity, without loss of generality. From figure 4, it is easily noted that the influence of decay rates γ_a, κ and γ_f on the fidelity F_e is very little when $\kappa = \gamma_a = \gamma_f \leqslant 0.1\gamma$. More specifically, the fidelity F_e still can be

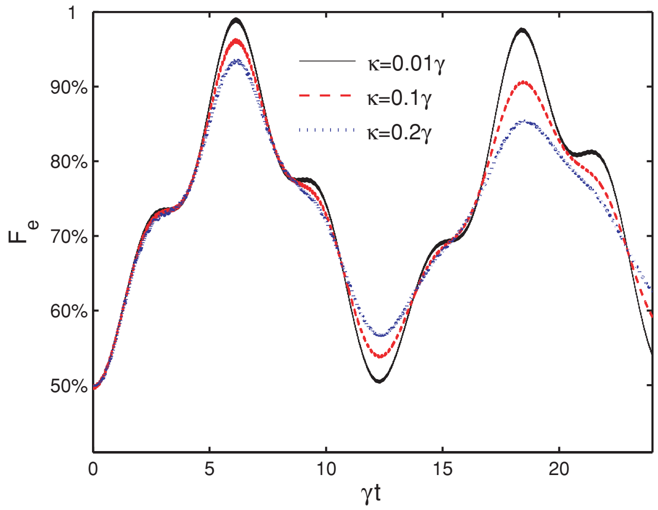


Figure 4. Fidelity of realizing the extremely entangled state $|\Psi_e\rangle$ versus time γt for different decay rates κ ($\gamma_a = \gamma_f = \kappa$). The corresponding system parameters are chosen as $g_A = 10\gamma$, $g_B = 23\gamma$, $\Omega_A = \Omega_B = 10\gamma$, $\eta = 25\gamma$, $\Delta_A = \Delta_B = -200\gamma$.

larger than 96%, when $\kappa = \gamma_a = \gamma_f = 0.1\gamma$. In order to see the influence of decay rates κ, γ_a and γ_f on F_e respectively, we also plot the function curves for F_e versus κ, γ_a and γ_f in figure 5. Comparing the main parts and inserted parts of figures 5(a) and (b), we note that the influence of atomic spontaneous decay rate γ_a and fibre decay rate γ_f on F_e is much smaller than that of the cavity field decay rate κ, and hence they can be neglected safely in our scheme. These numerical results are very consistent with our discussions in above sections and can be qualitatively explained as follows. Under the conditions $|\Delta_i| \gg |\Omega_i|, |g_i|$ and $|\eta| \gg |\Omega_{ei}|$ $\left(\Omega_{ei} = \frac{g_i\Omega_i}{\Delta_i}; i = \mathrm{A, B}\right)$, the atomic excited state $|e\rangle_i$ and fibre mode b are only virtually excited in the whole interaction process, and hence the effects of atomic decay rate γ_a and fibre decay rate γ_f are suppressed strongly in figure 5, which satisfies the above conditions. It is also shown from figure 5 that the low proper decay rate ($\kappa \leqslant 0.04\gamma$) of the cavity field is still required for getting atomic entangled state with high fidelity ($F_e \geqslant 98\%$) in our scheme.

Before ending this section, let us briefly discuss the experimental feasibility of our scheme. First, the Λ-type energy level configurations for atoms A and B can be realized by choosing the hyperfine-split levels for the D lines of cold alkali-metal atoms [37, 38]. For instance, in the case of the cesium atom with nuclear spin $I = 7/2$, the levels of atoms and polarizations of cavity fields and classical fields can be chosen as shown in figure 6 according to the selective rule of photon absorption and emission. Second, based on the recent experiments [39], the parameter conditions $g_A/2\pi \approx 750$ MHz, $\gamma_a/2\pi = \gamma_f/2\pi \approx 7.5$ MHz, $\kappa/2\pi \approx 3$ MHz (corresponding to the cavity quality factor $Q \sim 10^7$) have been achieved in a toroidal microcavity system with the cavity mode wavelength about 852 nm. In addition, the corresponding transition wavelenght $\left(\lambda_{eg_0}\right)$ of cesium atom considered here is also about 852 nm [37], so the above system parameters are

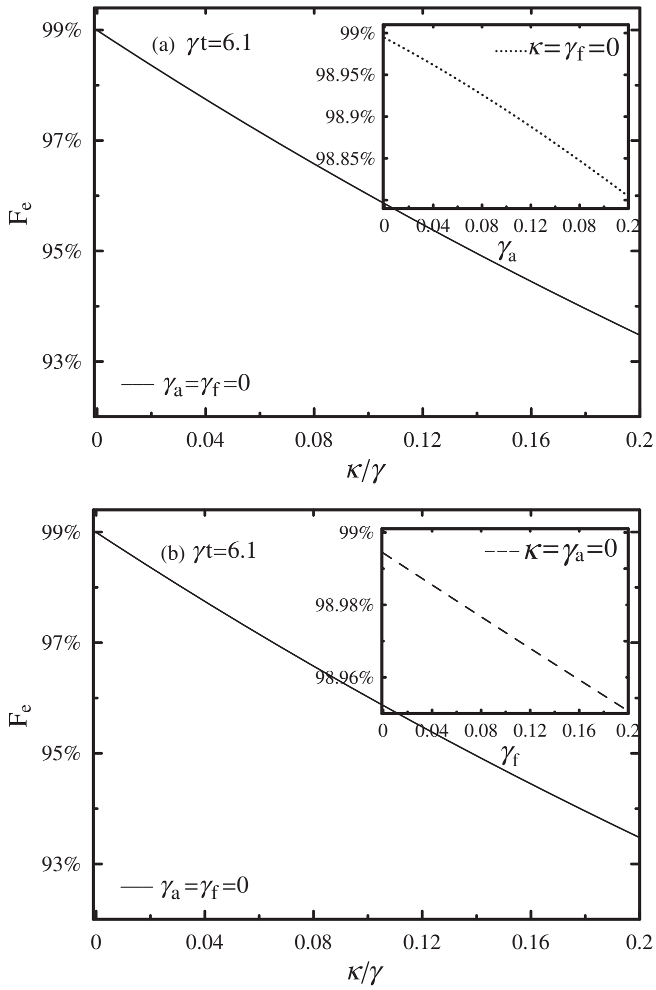


Figure 5. Fidelity of realizing the extremely entangled state $|\Psi_e\rangle$ versus κ and γ_a (a); versus κ and γ_f (b); when $\gamma t = 6.1$. The other system parameters are same as in figure 4.

suited for our scheme. Based on those experiment parameters, the other system parameters of our scheme can be chosen as $\Delta_A/2\pi = \Delta_B/2\pi \approx 15$ GHz, $g_B = 1.7$ GHz, $\Omega_A = \Omega_B = 750$ MHz, $\eta = 800$ MHz. Then, the conditions $\kappa \leqslant 0.04\gamma$ and $|\eta| \geqslant 20|\Omega_{eA}|$ ($\geqslant 10|\Omega_{eB}|$), which are required for realizing atomic entangled sate with high fidelity, can be satisfied with the above system parameters. Here, it should be pointed out that the condition $|\eta| \geqslant 20|\Omega_{eA}|$ ($\geqslant 10|\Omega_{eB}|$) dose not imply that the value of $|\eta|$ is very large, which is different from [26, 29]. In the present scheme, the effective Rabi frequency $\Omega_{ei} = \frac{g_i\Omega_i}{\Delta_i}$ ($i = \mathrm{A, B}$) and $|\Delta_i| \gg |\Omega_i|$ and $|g_i|$, and hence $|\Omega_{ei}|$ actually is a very small value in this situation. So, the coupling strength between the cavity and the fibre, $|\eta|$, is not required to be much large actually in our scheme. However, we also should emphasize that the fidelity will be certainly reduced in a real situation because the coupling between the cavity and the fibre is not ideal in the current experiments [40]. Lastly, along with the progress of fibre–cavity coupling techniques, we believe that the atomic entangled state $|\Psi_e\rangle$ with oppositely high fidelity can be realized based on our scheme.

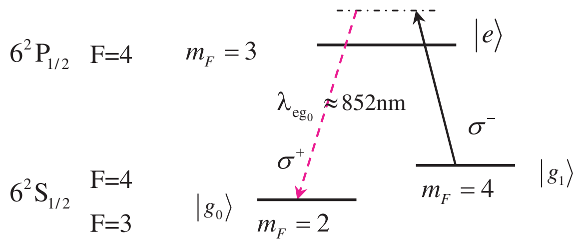


Figure 6. The Λ-type energy level configuration of the cesium atom. σ^- and σ^+ denote left and right circular polarization of light, respectively.

5. Conclusion

In conclusion, based on the dispersive atom–field interaction, we have proposed a scheme for deterministically generating entanglement between two spatially separated Λ-type atoms. In the present scheme, the atomic spontaneous decay and photon leakage out of the fibre can be efficiently suppressed because the excited sates of atoms and fibre mode are only virtually excited in the whole interaction process. Moreover, we also show that this scheme is highly stable to the deviation of the ratio coefficient between two effective Rabi frequencies from that in the ideal situation. Lastly, the experimental feasibility of our scheme is discussed and as a result, it is considered as a promising scheme for realizing entanglement with high fidelity.

Acknowledgments

The research is supported in part by the Natural Science Foundation of China (Grant Nos 10575040, 90503010, 10634060, 10874050) and by National Basic Research Program of China under Contract No 2005CB724508. We would like to thank Professor Ying Wu for helpful discussion and his encouragement.

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