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A Maximally Permissive Deadlock Prevention Policy for FMS Based on Petri Net Siphon Control and the Theory of Regions

Zhiwu Li, MengChu Zhou, and MuDer Jeng

***Abstract*—This paper addresses the deadlock problems in flexible manufacturing systems (FMS) by using a Petri net siphon control method and the theory of regions. The proposed policy consists of two stages. The first one, called siphons control, is to add, for every siphon that we identify, a monitor to the original net model such that it is optimally invariant controlled. In the second stage, the theory of regions is utilized to derive the net supervisors such that deadlocks can be prevented. The first-stage work significantly lowers the computational cost compared with the approach where the theory of regions is used alone. An FMS example is presented to illustrate the technique. By varying the markings of given net structures, this paper shows its computational advantages.**

***Note to Practitioners*—Deadlock is a constant problem in flexible manufacturing systems (FMS) with shared resources, which often offsets the advantages of these systems since deadlock can cause unnecessary cost, such as long downtime and low use of some critical and expensive resources, and may lead to catastrophic results in highly automated FMS. Behavior permissiveness has been an important criterion in designing the liveness–enforcing supervisor for an uncontrolled system. The theory of region is an effective method to derive a maximally permissive supervisor from a plant net model. However, it is rather inefficient. In this particular research, we develop a hybrid approach that combines siphon control and the theory of regions to derive a maximally permissive liveness–enforcing Petri net supervisor for a large class of FMS.**

***Index Terms*—Deadlock prevention, flexible manufacturing system, Petri net, siphon, theory of regions.**

I. INTRODUCTION

Petri nets [14] as well as digraphs and automata are major mathematical tools to characterize, analyze, and control deadlocks in various resource allocation systems including flexible manufacturing systems (FMS) [5], [10], [18]. Many researchers use Petri nets as a formalism to describe the behavior of FMS and to develop appropriate deadlock resolution methods [15], [16], [4]. Deadlock prevention is one of the strategies to cope with deadlocks in FMS. It is achieved by either effective system design or using an offline mechanism to control the requests for resources to ensure that deadlocks never occur. Monitors or control places and related arcs in the context of Petri nets are often used to achieve such purposes [4], [6], [7], [11], [19].

The theory of regions [1] that can derive Petri nets from automation-based models is an important method for supervisory control of discrete event systems (DESs) [19]. Ghaffari *et al.* [6] explore the conditions on

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the existence of a monitor-based liveness–enforcing net supervisor that is maximally permissive, and develops a methodology to synthesize such a supervisor. The most attractive advantage of the approach is that a maximally permissive net supervisor can be always obtained by adding monitors that are used to separate events from unsafe states, when such a supervisor exists. This paper tries to explore a way to alleviate the computational overhead of the methodology in [6] for a class of nets, Systems of Simple Sequential Processes with Resources (S^3PR) [4].

The rest of this paper is organized as follows. Section II briefly reviews preliminaries used throughout the paper. A method of identifying siphons in an S^3PR is developed in Section III. Section IV presents the deadlock control policy and an FMS example to illustrate it. Experimental results are given in Section V. Section VI concludes this paper.

II. Preliminaries

A. Petri Nets [14]

A (Petri) net N is a 4-tuple (P, T, F, W) where P and T are finite, nonempty, and disjoint sets. P is the set of places and T is the set of transitions. $F \subseteq (P \times T) \cup (T \times P)$ is called the set of directed arcs. $W : F \rightarrow \mathbf{N}^+$ is a mapping that assigns a weight to an arc, where $\mathbf{N}^+ = \{1, 2, \cdots\}$. N is said to be ordinary, denoted as (P, T, F), if $\forall f \in F, W(f) = 1$. A marking is a mapping $M : P \rightarrow \mathbf{N}$, where $\mathbf{N}$=$\mathbf{N}^+ \cup \{0\}$. A self-loop free net can be represented by its incidence matrix $[N] = [N]^+ - [N]^-$, where $[N]^+(p, t) = W(f(t, p))$ and $[N]^-(p, t) = W(f(p, t))$. A transition $t \in T$ in an ordinary net is enabled at marking M iff $\forall p \in {}^\bullet t$, $M(p) \geq 1$. This fact is denoted as $M[t\rangle$. When fired in a usual way, this gives a new marking M'. It is denoted as $M[t\rangle M'$. M' is reachable from M iff there exists a firable transition sequence $\sigma = t_1 t_2 \cdots t_n$ such that $M[t_1\rangle M_1[t_2\rangle M_2 \cdots M_{n-1}[t_n\rangle M'$ holds, which is denoted as $M[\sigma\rangle M'$ and satisfies the state equation $M' = M + [N]\overrightarrow{\sigma}$. Here, $\overrightarrow{\sigma} : T \rightarrow \mathbf{N}$ is a vector of non-negative integers, called counting vector, and $\overrightarrow{\sigma}(t)$ indicates the algebraic sum of all occurrences of t in σ. The set of markings reachable from M in net N is denoted as $R(N, M)$. The behavior of a net can be described by its reachability graph $RG(N, M)$ in which nodes correspond to reachable markings, and there is an arc labelled t from node M to M' if $M[t\rangle M'$.

A string $x_1 \cdots x_n$ $(x_i \in P \cup T)$ is called a path of N iff $\forall i \in \{1, 2, \cdots, n-1\}, x_{i+1} \in x_i^\bullet$. A simple path from x_1 to x_n is a path whose nodes are all different (possibly, except for x_1 and x_n). It is called a simple circuit iff it is a simple path with $x_1 = x_n$. A circuit containing x is denoted as $C(x)$.

A nonempty set $S \subseteq P$ is a siphon iff ${}^\bullet S \subseteq S^\bullet$. S is a trap iff $S^\bullet \subseteq {}^\bullet S$. A siphon is minimal iff there is no siphon contained in it as a proper subset. A siphon is said to be strict minimal iff it is minimal and does not contain a marked trap. A $P(T)$-vector is a column vector $I(J) : P(T) \rightarrow Z$ indexed by $P(T)$, where Z is the set of integers. P-vector I is a P-inv (place invariant) of net $N = (P, T, F)$ iff $I \neq 0$ and $I^T[N] = \mathbf{0}^T$ hold. P-inv I is called a P-semiflow if $\forall p \in P, I(p) \neq 0$ implies $I(p) > 0$. Siphon S is inv-controlled by P-inv I under M_0 iff $I^T M_0 > 0$ and $\{p \in P \,|\, I(p) > 0\} \subseteq S$ [8]. $\|I\| = \{p | I(p) \neq 0\}$ is called the support of P-inv I. For economy of space, we use $\sum_{p \in \|I\|} I(p)p$ $(\sum_{t \in \|J\|} J(t)t)$ to denote a $P(T)$-vector $I(J)$ and $\sum_{p \in P} M(p)p$ to denote a marking M. $[N](p, \cdot)$ denotes the incidence vector of place p.

B. Supervisory Control Problem [6], [17]

We are concerned with the forbidden state problem for liveness requirement. Let M_F be the set of markings for which specifications do not hold in (N, M_0). The markings in M_F are also called unsafe ones [9]. The objective is to determine a convenient set of monitors that, once added to a given plant net model, prevent the whole system from reaching these states. All transitions of the plant model are assumed to be controllable in this paper.

Definition 1: The set M_L of legal or admissible markings is the maximal set of reachable markings such that 1) $M_L \cap M_F = \emptyset$, and 2) it is possible to reach initial marking M_0 from any legal marking without leaving M_L. Let R_c be the reachability graph containing all legal markings.

Clearly, $M_L = R(N, M_0) - M_F$, which represents the set of legal markings, such that whatever the marking in M_L, the system cannot be led outside M_L. A marking in M_L is called a dangerous one if an unsafe marking (in M_F) can be possibly reached depending on supervisory control. To solve the control problem, one has to identify the set of state/event separation instances (or marking/transition separation instances (MTSI) in net terminology) from an admissible marking to a nonadmissible one. The additional monitors are used to prevent these transitions from occurring in order to keep the state space of the controlled system in the set of legal markings. Formally, the set of MTSI that the supervisor has to disable is $\Omega = \{(M, t) | M[t > M' \wedge M \in M_L \wedge M' \notin M_L\}$, where M is a dangerous marking. Let M_D be the set of dangerous markings. Clearly, we have $M_D = \{M | M \in M_L \wedge \exists t \in T, M[t\rangle M' \wedge M' \in M_F\}$.

A maximally permissive (optimal) supervisor is the one that ensures the reachability of all markings in M_L and forbids all MTSI in Ω. An algorithm is proposed in [6], which can give legal behavior R_c, M_L, Ω, M_D, and the set of transitions leading outside R_c.

C. Theory of Regions in Synthesis of Net Supervisors [6]

The theory of regions is proposed for the synthesis of pure nets from given finite transition systems [1], which can be adopted to synthesize the liveness–enforcing net supervisor (LENS) for a plant model [6], [19].

Given a plant model (N, M_0) of a system to control and R_c, the theory of regions can be used to design monitors $\{p_c\}$ to add. Consider a new monitor p_c. Every marking M in R_c must still be reachable after the addition of p_c, which implies that p_c has to satisfy reachability condition, i.e.,

$$M(p_c) = M_0(p_c) + [N](p_c, \cdot)\overrightarrow{\Gamma}_M \geq 0, \forall M \in R_c \tag{1}$$

where Γ_M is any nonoriented path in R_c from M_0 to M, $\overrightarrow{\Gamma}_M$ is a T-vector, and $\overrightarrow{\Gamma}_M(t)$ indicates the algebraic sum of all occurrences of t in Γ_M. $\overrightarrow{\Gamma}$ is called the counting vector of Γ. Similarly, each monitor p_c should satisfy cycle equations for each cycle in R_c, i.e.,

$$\Sigma_{t \in T}[N](p_c, t) \cdot \overrightarrow{\gamma}(t) = 0, \forall \gamma \in S_c \tag{2}$$

where γ is any nonoriented cycle of R_c, $\overrightarrow{\gamma}$ is a T-vector, $\overrightarrow{\gamma}(t)$ denotes the algebraic sum of all occurrences of t in γ, and S_c is the set of nonoriented cycles of graph R_c.

For the supervisory problem under consideration, only transitions leading the system from R_c to outside R_c need to be considered. Hence, the set of MTSI is Ω. Each additional monitor p_c must solve at least one MTSI (M, t) in Ω, i.e.,

$$M_0(p_c) + [N](p_c, \cdot)\overrightarrow{\Gamma}_M + [N](p_c, t) \leq -1. \tag{3}$$

Relations (1)–(3) determine the additional monitor p_c. An algorithm, denoted as $A1$, is developed in [6] to compute monitors added to the plant net model, which can lead to an LENS.

Consider a net system from [6] and [21] and its reachability graph in Fig. 1(a) and (b), respectively. It is not live since in $M_7 = (0, 1, 2, 0, 0, 0)^T$, no transition can fire. Let us apply Algorithm A1 to synthesize the set of monitors that avoid reaching M_7. The

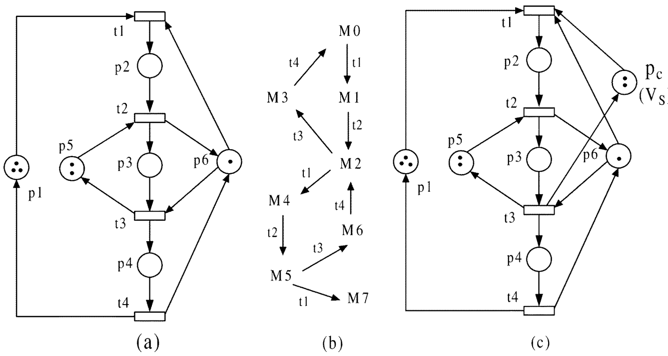

Fig. 1. (a) Net model (N, M_0). (b) Its reachability graph. (c) Its supervisor.

legal behavior is a reachability graph derived from the one in Fig. 1(b) by removing M_7. We have $M_L = \{M_i | i \in \{1, 2, \cdots, 6\}\}$ and $\Omega = \{(M_5, t_1)\}$. The single marking/transition separation condition to solve is $M_7(p_c) = M_0(p_c) + 3[N](p_c, t_1) + 2[N](p_c, t_2) \leq -1$.

The reachability graph contains two cycles that have the same equation $\sum_{i=1}^{4}[N](p_c, t_i) = 0$.

Whereas the reachability conditions are as follows:

$$
\begin{aligned}
&M_0(p_c) \geq 0; M_1(p_c) = M_0(p_c) + [N](p_c, t_1) \geq 0;\\
&M_2(p_c) = M_0(p_c) + [N](p_c, t_1) + [N](p_c, t_2) \geq 0;\\
&M_3(p_c) = M_0(p_c) + [N](p_c, t_1)\\
&\qquad + [N](p_c, t_2) + [N](p_c, t_3) \geq 0;\\
&M_4(p_c) = M_0(p_c) + 2[N](p_c, t_1) + [N](p_c, t_2) \geq 0;\\
&M_5(p_c) = M_0(p_c) + 2[N](p_c, t_1) + 2[N](p_c, t_2) \geq 0; \text{and}\\
&M_6(p_c) = M_0(p_c) + 2[N](p_c, t_1)\\
&\qquad + 2[N](p_c, t_2) + [N](p_c, t_3) \geq 0.
\end{aligned}
$$

The above linear system can be solved by taking $[N](p_c, \cdot) = (-1, 0, 1, 0)$, and $M_0(p_c) = 2$. The LENS is shown in Fig. 1(c).

D. S^3Pr [4]

Our deadlock prevention method targets S^3PR. In what follows, $\mathbf{N}_m$ denotes set $\{1, 2, \cdots, m\}$.

Definition 2: An S^3PR is defined as the union of a set of nets $N_i = (P_i \cup \{p_i^0\} \cup P_{R_i}, T_i, F_i)$ sharing common places, where the following statements are true.

1) p_i^0 is called the process idle place of N_i. Places in P_i and P_{R_i} are called operation and resource ones, respectively.
2) $P_{R_i} \neq \emptyset; P_i \neq \emptyset; p_i^0 \notin P_i; (P_i \cup \{p_i^0\}) \cap P_{R_i} = \emptyset; \forall p \in P_i, \forall t \in {}^\bullet p, \forall t' \in p^\bullet, \exists r_p \in P_{R_i}, {}^\bullet t \cap P_{R_i} = t'^\bullet \cap P_{R_i} = \{r_p\}$; $\forall r \in P_{R_i}, {}^{\bullet\bullet} r \cap P_i = r^{\bullet\bullet} \cap P_i \neq \emptyset$; $\forall r \in P_{R_i}, {}^\bullet r \cap r^\bullet = \emptyset$; ${}^{\bullet\bullet}(p_i^0) \cap P_{R_i} = (p_i^0)^{\bullet\bullet} \cap P_{R_i} = \emptyset$.
3) For $r \in P_{R_i}, H(r) = ({}^{\bullet\bullet} r) \cap P_i$, the operation places that use r is called the set of holders of r.
4) $\forall p \in P_i, \exists$ unique resource $r \in P_{R_i}$ such that $p \in H(r)$.
5) N_i' is a strong connected state machine, where $N_i' = (P_i \cup \{p_i^0\}, T, F)$ is the resultant net after the places in P_{R_i} and related arcs are removed from N_i.
6) Every circuit of N_i' contains place p_i^0.
7) Any two N_i's are composable when they share a set of resource places. Every shared place must be a resource one.

Definition 3: Let $N = \bigcirc_{i=1}^{k} N_i = (P \cup P_0 \cup P_R, T, F)$ be an S^3PR and S be a minimal siphon that is not a trap in N, where $S = S_P \cup S_R, S_R = S \cap P_R$, and $S_P = S \backslash S_R$. $[S] = (\cup_{r \in S_R} H(r)) \backslash S$ is called the complementary set of S.

If S is strict minimal in an S^3PR, $|S \cap P_R| > 1$ [4]. We also have (1) $\forall i \in \mathbf{N}_k, \forall r \in P_R, P_i \cup \{p_i^0\}$, and $H(r) \cup \{r\}$ are the supports of P-semiflows of an S^3PR. (2) Given a strict minimal siphon S in N, $[S] \cup S$ is the support of a P-semiflow I, where $\forall I(p) \neq 0, I(p) = 1$. A strict minimal siphon in an S^3PR may become unmarked during its evolution.

III. Siphon Identification and Control

A. Siphon Identification

An approach is developed to find some, in general, not all, siphons in an S^3PR based on its structural analysis.

Definition 4: Let $\{r_1, r_2, \cdots, r_m\} \subseteq P_R$ $(m \geq 2)$ be a set of resources in an S^3PR $N = (P \cup P_0 \cup P_R, T, F)$. A simple circuit $C(r_1, t_1, r_2, t_2, \cdots, r_m, t_m)$ is called a resource circuit if 1) $\forall i \in \mathbf{N}_m, r_i \in {}^\bullet t_i$; 2) $\forall i \in \{2, \cdots, m\}, r_i \in t_{i-1}^\bullet$; and 3) $r_1 \in t_m^\bullet$.

We use $C^R = \{r_i | i \in \mathbf{N}_m\}$ to denote the set of resources in a resource circuit C in an S^3PR N.

Theorem 1: $S = C^R \cup \{p | p \in \cup_{r \in C^R} H(r) \wedge (p^{\bullet\bullet} \cap (P \cup P_0)) \nsubseteq \cup_{r \in C^R} H(r)\}$ is a siphon in N. And if S does not contain the support of any P-semiflow, S is strict minimal.

Proof: We first claim that S is a siphon. For this, we have to prove $\forall t \in {}^\bullet S, t \in S^\bullet$. $\forall t \in {}^\bullet S$, either $t \in {}^\bullet C^R$ or $t \in {}^\bullet \{p | p \in \cup_{r \in C^R} H(r) \wedge (p^{\bullet\bullet} \cap (P \cup P_0)) \nsubseteq \cup_{r \in C^R} H(r)\}$ holds. We accordingly have the following two cases.

1) $t \in {}^\bullet C^R$ means $\exists i \in \mathbf{N}_m, t \in {}^\bullet r_i$. By the definition of resource circuits, we have two subcases.

a) There exists $j \in \mathbf{N}_m (i \neq j)$ such that ${}^\bullet r_i \cap r_j^\bullet \neq \emptyset$. If $t \in {}^\bullet r_i \cap r_j^\bullet, t \in C^{R\bullet}$ holds. That is to say, $t \in S^\bullet$ is true, and S is, hence, a siphon.

b) $j \in \mathbf{N}_m$ does not exist such that $t \in {}^\bullet r_i \cap r_j^\bullet$. By the definitions of S^3PR, there necessarily exists an operation place $p_x \in H(r_i)$ such that $t \in p_x^\bullet$. Therefore, one needs to prove $p_x \in S$. Since $S = C^R \cup \{p | p \in \cup_{r \in C^R} H(r) \wedge (p^{\bullet\bullet} \cap (P \cup P_0)) \nsubseteq \cup_{r \in C^R} H(r)\}$, one has to prove $(p_x^{\bullet\bullet} \cap (P \cup P_0)) \nsubseteq \cup_{r \in C^R} H(r)$ is true. By contradiction. Assume $(p_x^{\bullet\bullet} \cap (P \cup P_0)) \subset \cup_{r \in C^R} H(r)$. Hence, we have $\exists j \in \mathbf{N}_m$, $(p_x^{\bullet\bullet} \cap (P \cup P_0)) \subseteq H(r_j)$ and $t \in {}^\bullet r_i \cap r_j^\bullet$ by the definitions of S^3PR. This clearly contradicts the condition that $j \in \mathbf{N}_m$ does not exist such that $t \in {}^\bullet r_i \cap r_j^\bullet$. Consequently, if a transition $t \in {}^\bullet C^R$ exists and does not exist $j \in \mathbf{N}_m$ such that $t \in {}^\bullet r_i \cap r_j^\bullet$, there certainly exists a place $p_x \in H(r_i)$ such that $t \in p_x^\bullet$ and $p_x \in S$ (i.e., $t \in S^\bullet$). Therefore, $\forall t \in {}^\bullet C^R, t \in S^\bullet$ holds.

2) $t \in {}^\bullet \{p | p \in \cup_{r \in C^R} H(r) \wedge (p^{\bullet\bullet} \cap (P \cup P_0)) \nsubseteq \cup_{r \in C^R} H(r)\}$ means $\exists r_i \in C^R, p \in H(r_i)$ such that $t \in {}^\bullet p$. By the definitions of S^3PR, $t \in r_i^\bullet$ holds. Thus, we can conclude that $\forall t \in {}^\bullet \{p | p \in \cup_{r \in C^R} H(r) \wedge (p^{\bullet\bullet} \cap (P \cup P_0)) \nsubseteq \cup_{r \in C^R} H(r)\}$ means $t \in S^\bullet$.

By the above discussions for cases 1) and 2), one can get $\forall t \in {}^\bullet (C^R \cup \{p | p \in \cup_{r \in C^R} H(r) \wedge (p^{\bullet\bullet} \cap (P \cup P_0)) \nsubseteq \cup_{r \in C^R} H(r)\}), t \in S^\bullet$ holds (i.e., ${}^\bullet S \subseteq S^\bullet$). Trivially, S is a siphon.

Suppose that S does not contain the support of any P-semiflow. Next, we prove S is minimal. By contradiction, assume that S is not minimal. That is to say, there exists a siphon S_X such that $S_X \subset S$ holds. Let $S = S_R \cup S_P, S_R = S \cap P_R, S_P = S \backslash S_R, S_X = S_{XR} \cup S_{XP}, S_{XR} = S_X \cap P_R$, and $S_{XP} = S_X \backslash S_{XR}$. If $S_X \subset S$, then one of the following three cases holds: 1) $S_{XR} = S_R, S_{XP} \subset S_P$; 2) $S_{XR} \subset S_R, S_{XP} = S_P$; and 3) $S_{XR} \subset S_R, S_{XP} \subset S_P$. We first deal with case 1). By $S_{XP} \subset S_P$, at least a place exists $p \in H(r)$ such that $r \in S_R$ $(S_{XR}), p \in S_P$ and $p \notin S_{XP}$ are true, as illustrated in Fig. 2. From the definition of S^3PR, we know that transition exists $t \in p^\bullet \cap {}^\bullet r_i$ such that $t \in {}^\bullet S_{XR}$ and $t \in {}^\bullet S_X$, as shown in Fig. 2. Owing to $p \notin S_{XP}$ and the definition of S^3PR, there does not exist a place $p_X \in S_{XP}$ such that both $\exists t \in p^\bullet \cap p_X^\bullet$ and $t \in S_{XP}^\bullet$ hold.

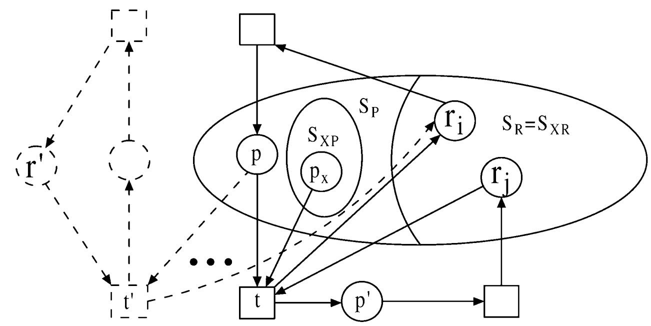

Fig. 2. Case of $S_{XR} = S_R$ and $S_{XP} \subset S_P$.

We can see that $t \notin S_{XP}^{\bullet}$. Next, we prove that $t \notin S_{XR}^{\bullet}$. We have the following two subcases:

i) $(p^{\bullet\bullet} \cap (P \cup P_0))$ contains only one element t, as seen in Fig. 2 without the dashed parts. In this case, there impossibly exists a place $r_j \in S_{XR}(S_R)$ such that $t \in r_j^{\bullet}$. Otherwise, it leads to $(p^{\bullet\bullet} \cap (P \cup P_0)) \subseteq \cup_{r \in C^R} H(r)$ that contradicts $(p^{\bullet\bullet} \cap (P \cup P_0)) \not\subseteq \cup_{r \in C^R} H(r)$ defined in S. Hence, we have $t \notin S_{XR}^{\bullet}$.

ii) $(p^{\bullet\bullet} \cap (P \cup P_0))$ contains at least two elements t and t', as seen in Fig. 2. In this case, if resource place $r' \in S_R$ and S_{XR} (note that $S_R = S_{XR}$), we are led to infer $p \notin S$ that contradicts $p \in S_P$. On the other hand, if $r' \notin S_R(S_{XR})$, then $p \in S_{XP}$ is true since, otherwise, we have $t' \in {}^{\bullet} S_X$ and $t' \notin S_X^{\bullet}$ (S_X is not a siphon), or S contains the support of a P-semiflow. Note that $p \in S_{XP}$ contradicts the fact that $p \in S_P$ but $p \notin S_{XP}$. Therefore, there does not exist a resource place $r_j \in S_{XR}(S_R)$ such that $t \in r_j^{\bullet}$. We have $t \notin S_{XR}^{\bullet}$.

In conclusion, we can say that S_X is not a siphon since a transition t exists such that both $t \in {}^{\bullet} S_X$ and $t \notin S_{XR}^{\bullet} \cup S_{XP}^{\bullet} (t \notin S_X^{\bullet})$ hold, which clearly contradicts the assumption that S_X is a siphon. Hence, S is minimal. Cases b) and c) can be similarly proved. The strictness of S is trivial.

Therefore, $S = C^R \cup \{p | p \in \cup_{r \in C^R} H(r) \wedge (p^{\bullet\bullet} \cap (P \cup P_0)) \not\subseteq \cup_{r \in C^R} H(r)\}$ is a siphon. Note that the support of any P-semiflow is marked under the initial marking in an S^{3}PR. Hence, if S does not contain the support of any P-semiflow, S is strict minimal. ■

To find resource circuits in an S^{3}PR $N = (P \cup P_0 \cup P_R, T, F)$, let us consider the directed graph $G_N = (V, E)$ derived from N: 1) $V = P_R$; and 2) let $r, r' \in P_R$, there is an edge in E from r to r' iff $r^{\bullet} \cap {}^{\bullet} r' \neq \emptyset$. As a result, finding a resource circuit in net N is equivalent to finding a cycle in G_N, and this can be done in $O(|P_R| + |P_R^{\bullet} \cap {}^{\bullet} P_R|)$ time (the number of nodes plus the number of edges in G_N) [3].

In the first stage of our deadlock control policy, we confine ourselves to find, at most, h resource circuits only due to Theorem 1, where $h = min\{|P|, |T|\}$.

Definition 5: In an S^{3}PR resource circuits, C_1 and C_2 are said to be connected if $\exists r \in P_R, r \in C_1^R \cap C_2^R$.

Definition 6: C_1 and C_2 are two connected resource circuits in an S^{3}PR. Let $C^R = C_1^R \cup C_2^R$ and $S = C^R \cup \{p | p \in \cup_{r \in C^R} H(r) \wedge (p^{\bullet\bullet} \cap (P \cup P_0)) \not\subseteq \cup_{r \in C^R} H(r)\}$. S is called a resource circuit composed siphon if it is strict minimal.

The minimality of a siphon can be decided in polynomial time [2]. The strictness of a siphon in an S^{3}PR can be verified by checking if it contains the marked support of a P-semiflow.

We denote by Π the set of siphons generated by Theorem 1 and Definition 6. We need to control them in the first stage. If there is no siphon found due to Theorem 1, the net system is live [4].

If $g (g \leq h)$ siphons are produced by Theorem 1, then they can, at most, generate $\sum_{i=1}^{g-1} i = g(g-1)/2$ siphons due to Definition 6. Therefore, in the first stage, we can have $g + g(g-1)/2 = g(g+1)/2$ siphons to control in the worst case. The siphons identification and control method ensures that our first stage is of polynomial complexity with respect to the size of a plant net model.

B. Siphon Control

For a strict minimal siphon S in an S^{3}PR, we add a monitor V_S such that it is optimally inv-controlled. S is said to be optimally inv-controlled if the markings removed due to the addition of V_S are unsafe ones only. This can be done by a well-established approach in [8] and [20].

For example, $S = \{p_4, p_5, p_6\}$ is the only strict minimal siphon of net (N, M_0) in Fig. 1(a). By Definition 3, $[S] = (H(p_5) \cup H(p_6)) \backslash S = \{p_2, p_3\}$. Adding a monitor V_S to N yields the resultant net denoted as (N_1, M_1). The incidence vector of V_S is $L_{V_S} = (0,0,0,1,1,1) \cdot [N] = (-1,0,1,0)$, and $M_1(V_S) = M_0(S) - 1 = 3 - 1 = 2$. Clearly, $M_1(V_S) = M_1(p_c)$ and $N_1(V_S, \cdot) = N_1(p_c, \cdot)$, as shown in Fig. 1(c).

IV. Deadlock Prevention Policy

A. Deadlock Prevention Policy

A two-stage deadlock prevention policy, LZJ policy for short, is presented as follows.

Input: a plant S^{3}PR net model (N, M_0).

Output: an LENS (N_2, M_2).

Step 1) $\Pi := \{S_i | i \in \mathbf{N}_n\}$, the set of siphons due to Theorem 1 and Definition 6.

Step 2) **if** $\Pi = \emptyset$, **then** $N_2 := N, M_2 := M_0$, go to Step 11).

Step 3) $i := 1$.

Step 4) $\xi_{S_i} := 1$, add monitor V_{S_i} for S_i due to [20], [8], $i := i + 1$

Step 5) **If** $i = n + 1$, **then** go to Step 6)

else go to Step 4)

endif.

/* The new net with monitors is denoted by (N_1, M_1). */

Step 6) Generate the reachability graph $RG(N_1, M_1)$ for (N_1, M_1).

Step 7) Generate the legal behavior R_c from $RG(N_1, M_1)$, find set $\Omega = \{(M, t) | (M, t)$ is an MTSI$\}$.

/* $\mathbf{V}_M$ is used to denote the set of monitors to be added. */

Step 8) $\mathbf{V}_M := \emptyset$.

Step 9) **If** $\Omega = \emptyset$, **then** $N_2 := N_1, M_2 := M_1$, go to Step 11)

else $\forall (M, t) \in \Omega$, design monitor p_m s.t. p_m implements

$$(M, t), \Omega := \Omega - \{(M, t)\}$$

endif.

Step 10) **If** $\nexists p_{m'} \in \mathbf{V}_M$ s.t. $p_{m'}$ implements (M, t), **then** add p_m to (N_1, M_1), $\mathbf{V}_M := \mathbf{V}_M \cup \{p_m\}$, denote the resultant net system as (N_1, M_1), go to Step 9).

else go to Step 9).

endif.

Step 11) Output (N_2, M_2).

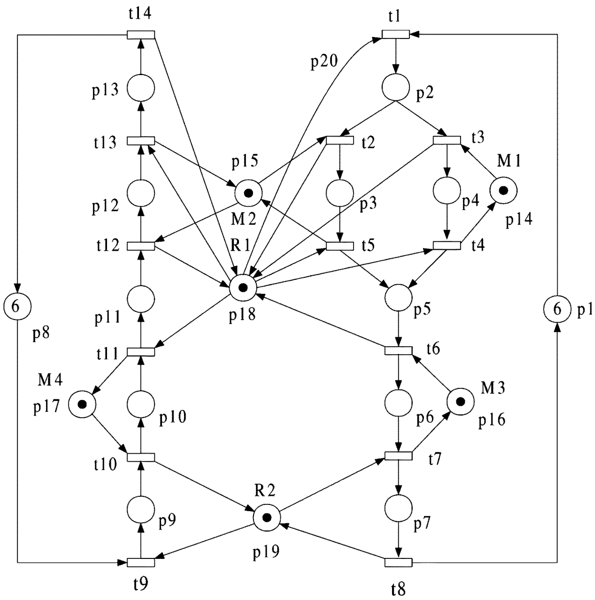

Fig. 3. Plant net model (N, M_0).

In this algorithm, we first find a portion of siphons, in general, and make them optimally inv-controlled. Then, the theory of regions is utilized to design LENS by solving systems of inequalities. The optimal controllability of siphons found in the plant model makes the number of systems of inequalities in the second stage much smaller, which alleviates the computational burden in the second stage. Steps 1)–5) are of polynomial time. Having been extensively discussed in [6], the complexity of Steps 6)–11) remains to be exponential in the worst case since we have to generate the reachability graph. However, the experimental results presented next show that our deadlock control policy is still interesting from the point of computational cost. Furthermore, it is optimal.

Theorem 2: Our deadlock prevention policy is maximally permissive.

Proof: The first stage, siphon control, is optimal since no safe markings can be removed due to the addition of monitors. In the second stage, we use the method proposed in [6] to design monitors such that all of the separation instances can be solved, where only unsafe markings can be removed whenever possible. Due to Theorem 2 of [6], our second stage is maximally permissive as long as such an LENS exists. ■

B. FMS Example

An FMS [19] consists of two robots R1–2, four machine tools M1–4, two loading buffers I1–2, and two unloading buffers O1–2. Two part types are considered in this FMS. As shown in Fig. 3, its net model (N, M_0) is an S^{3}PR if $P_0 = \{p_1, p_8\}$, $P_R = \{p_{14}, \cdots, p_{19}\}$, and $P = \{p_j | j \in \{2, \cdots, 7, 9, \cdots, 13\}\}$. This example has been investigated in [19] using the theory of regions alone.

The net has eight P-semiflows, one of which is $\|I^*\| = \{p_2, p_5, p_{11}, p_{13}, p_{18}\}$. There are three resource circuits $C_1 = C(p_{14}, t_3, p_{18}, t_4)$, $C_2 = C(p_{15}, t_{12}, p_{18}, t_5)$, and $C_3 = C(p_{16}, t_6, p_{18}, t_{11}, p_{17}, t_{10}, p_{19}, t_7)$.

These resource circuits form three siphons according to Theorem 1. They are $S_1 = \{p_2, p_5, p_{11}, p_{13}, p_{14}, p_{18}\}$, $S_2 = \{p_2, p_5, p_{13}, p_{15}, p_{18}\}$, and $S_3 = \{p_2, p_7, p_{11}, p_{13}, p_{16}, p_{17}, p_{18}, p_{19}\}$ due to C_1, C_2, and C_3, respectively. Note that $S_1 = \{p_2, p_5, p_{11}, p_{13}, p_{14}, p_{18}\}$ contains $\|I^*\|$. Therefore, S_1 is not a minimal siphon and cannot be emptied.

Note that $C_1^R \cap C_2^R = \{p_{18}\}$. Due to Definition 6, connected circuits C_1 and C_2 lead to $S_4 = \{p_{14}, p_{15}, p_{18}, p_5, p_{13}\}$, a strict minimal siphon in Fig. 3. Note that $\forall p \in \{p_2, p_3, p_4, p_{11}, p_{12}\}, \exists r \in \{p_{14}, p_{15}, p_{18}\}, p^{\bullet\bullet} \cap (P \cup P_0) \subseteq H(r)$ but not the case for p_5 and p_{13}. By Definition 6, connected circuits C_1 and C_3 lead to a set of places $\{p_7, p_{13}, p_{14}, p_{16}, p_{17}, p_{18}, p_{19}\}$ that are not a siphon. Neither is the set $\{p_7, p_{13}, p_{15}, p_{16}, p_{17}, p_{18}, p_{19}\}$ as generated by C_2 and C_3.

Therefore, siphons $S_2 = \{p_2, p_5, p_{13}, p_{15}, p_{18}\}$, $S_3 = \{p_2, p_7, p_{11}, p_{13}, p_{16}, p_{17}, p_{18}, p_{19}\}$, and $S_4 = \{p_5, p_{13}, p_{14}, p_{15}, p_{18}\}$ are strict minimal. Accordingly, three monitors V_{S_1}, V_{S_2}, and V_{S_3} are added to make S_2, S_4, and S_3 optimally inv-controlled, respectively. The new net system is denoted by (N_1, M_1). It is easy to see that $[N_1](V_{S_1}, \cdot) = -t_2 + t_5 - t_{11} + t_{13}, [N_1](V_{S_2}, \cdot) = -t_1 + t_4 + t_5 - t_{11} + t_{13}, [N_1](V_{S_3}, \cdot) = -t_4 - t_5 + t_7 - t_9 + t_{11}, M_1(V_{S_1}) = 1, M_1(V_{S_2}) = 2$, and $M_1(V_{S_3}) = 3$. We call (N_1, M_1) the partially controlled net model.

Now, the theory of regions is utilized to design monitors to prevent deadlocks for (N_1, M_1). There are 210 reachable markings, denoted by M_1–M_{210}, in $R(N_1, M_1)$ and only one of them is the deadlock marking $M_{57} = 2p_1 + p_3 + p_4 + p_6 + 3p_8 + p_9 + p_{10} + p_{18}$. Also, there are 5 dangerous markings, and 8 MTSI that are (M_{43}, t_9), (M_{44}, t_9), (M_{49}, t_9), (M_{48}, t_9), (M_{47}, t_9), (M_{53}, t_4), (M_{59}, t_1), and (M_{74}, t_2), where

$$\begin{aligned}
M_{43} &= 2p_1 + p_2 + p_4 + p_6 + 4p_8 \\
&\quad + p_{10} + p_{15} + p_{19} + p_{20} + p_{22}, \\
M_{44} &= 2p_1 + p_3 + p_4 + p_6 + 4p_8 \\
&\quad + p_{10} + p_{18} + p_{19} + p_{22}, \\
M_{49} &= 2p_1 + p_2 + p_3 + p_6 + 4p_8 \\
&\quad + p_{10} + p_{14} + p_{19} + p_{22}, \\
M_{48} &= 3p_1 + p_3 + p_6 + 4p_8 \\
&\quad + p_{10} + p_{14} + p_{18} + p_{19} + p_{21} + p_{22}, \\
M_{47} &= 3p_1 + p_3 + p_5 + 4p_8 \\
&\quad + p_{10} + p_{14} + p_{16} + p_{19} + p_{21} + p_{22}, \\
M_{53} &= 3p_1 + p_3 + p_4 + 3p_8 + p_9 \\
&\quad + p_{10} + p_{16} + p_{18} + p_{22}, \\
M_{59} &= 3p_1 + p_4 + p_6 + 3p_8 + p_9 \\
&\quad + p_{10} + p_{15} + p_{18} + p_{20} + p_{21}, \text{and} \\
M_{74} &= 3p_1 + p_2 + p_6 + 3p_8 + p_9 \\
&\quad + p_{10} + p_{14} + p_{15} + p_{20} + p_{21}.
\end{aligned}$$

By solving eight sets of inequalities determined by the above MTSI, one can find that three monitors, namely V_{S_4}, V_{S_5}, and V_{S_6}, have to be added to implement these separation instances, where $M(V_{S_4}) = 3, {}^\bullet V_{S_4} = \{t_6, t_{11}\}, V_{S_4}^\bullet = \{t_2, t_4, t_9\}, M(V_{S_5}) = 3, {}^\bullet V_{S_5} = \{t_5, t_7, t_{11}\}, V_{S_5}^\bullet = \{t_2, t_6, t_9\}, M(V_{S_6}) = 4, {}^\bullet V_{S_6} = \{t_2, t_4, t_7, t_{11}\}$, and $V_{S_6}^\bullet = \{t_1, t_6, t_9\}$. The final monitor-based net supervisor, denoted by (N_2, M_2), is thus obtained, which is live and maximally permissive (the number of markings in the reachability graph of (N_2, M_2) is 205). Hence, by adding only six monitors in total, deadlock prevention is implemented for this example. However, in [19], the theory of regions is used alone, where 59 separation instances (i.e., 59 sets of inequalities) have to be solved, and nine monitors are added.

Just from this small example, we can see that after some siphons are controlled, the number of separation instances is significantly reduced, which alleviates the computational burden at the second stage. The experimental studies in the next section can further show the computational advantage of this approach.

TABLE I
PARAMETERS IN THE PLANT AND PARTIALLY CONTROLLED MODELS WITH VARYING MARKINGS

case	$\lvert\mathcal{S}_C\rvert$	$\lvert R\rvert$	$\lvert M_L\rvert$	$\lvert R_D\rvert$	$\lvert R_D^{[c]}\rvert$	N_{sep}	$N_{sep}^{[c]}$	r_a
1	3	282	205	16	1	59	8	13.6%
2	3	600	484	27	1	95	8	8.4%
3	3	972	870	26	6	103	10	9.7%
4	3	570	421	16	1	107	8	7.5%
5	3	4011	3711	42	9	288	15	5.2%
6	3	27152	26316	84	28	886	48	5.4%
7	3	124110	122235	145	60	2115	105	5.0%
8	3	440850	437190	228	108	4311	192	4.5%

TABLE II
SOLUTION TIME OF THE PARAMETERIZED PROBLEM CASES

case	$\lvert\mathcal{S}_C\rvert$	T_S	$N_{sep}^{[c]}$	$T_{sep}^{[c]}$	N_{sep}	T_{sep}	$T^{[c]}$
1	3	0.23s	8	0.40s	59	2.27s	0.63s
2	3	0.23s	8	0.64s	95	3.66s	0.87s
3	3	0.23s	10	0.82s	103	3.95s	1.05s
4	3	0.23s	8	0.59s	107	3.58s	0.82s
5	3	0.23s	15	1.54s	288	31.72s	1.77s
6	3	0.23s	48	9.51s	886	185.48s	9.74s
7	3	0.23s	105	65.67s	2115	1323.46s	65.90s
8	3	0.23s	192	215.34s	4311	4838.68s	215.57s

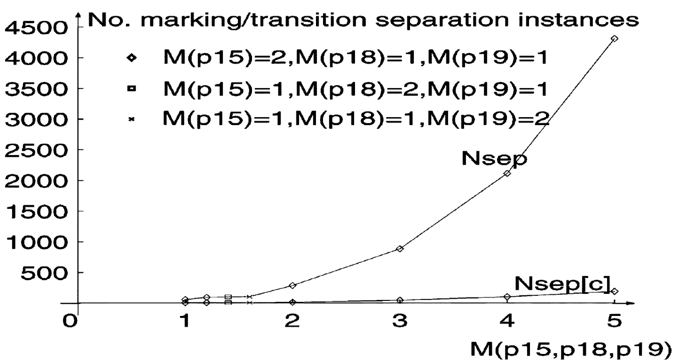

Fig. 4. Case when $M(p_{15}), M(p_{18})$, and $M(p_{19})$ vary.

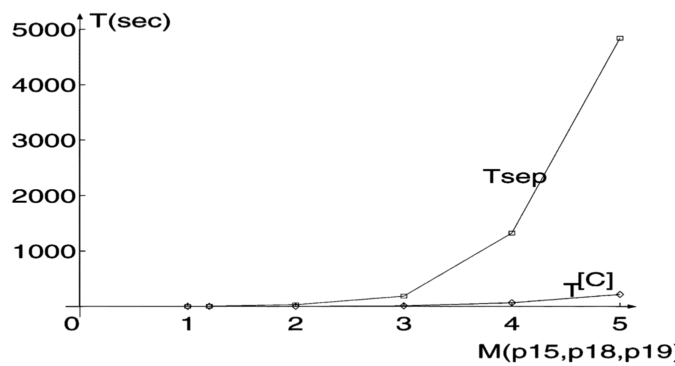

Fig. 5. Solution time when $M(p_{15}), M(p_{18})$, and $M(p_{19})$ vary.

V. EXPERIMENTAL RESULTS

The net structure in Fig. 3 is selected for our experimental studies. We vary the initial markings of places p_{15}, p_{18}, and p_{19} that model shared resources. Note that the initial markings of idle places p_1 and p_8 have to be changed accordingly to ensure no permissive behavior is restricted by them. Table I shows various parameters in the plant and partially controlled net models, where $M(p_{15})$, $M(p_{18})$, and $M(p_{19})$ vary; $|\mathcal{S}_C|$, $|R|$, $|M_L|$, $|R_D|$, and N_{sep} indicate the number of siphons to be controlled in our first stage, the number of reachable states, maximally permissive (legal) states (behavior), dead states, and the MTSI of the plant models, respectively. $|R_D^{[c]}|$ and $N_{\text{sep}}^{[c]}$ represent the number of dead states and the MTSI in the partially controlled models, respectively. The last column is $r_a = N_{\text{sep}}^{[c]}/N_{\text{sep}}$.

There are eight cases that we investigate: the marking vector of p_1, p_8, p_{15}, p_{18}, and p_{19} is (1) $[6, 5, 1, 1,1]^T$; (2) $[7,6, 2, 1, 1]^T$; (3) $[7, 6, 1, 2, 1]^T$; (4) $[7, 6, 1, 1, 2]^T$; (5) $[9, 8, 2, 2, 2]^T$; (6) $[12, 11, 3, 3, 3]^T$; (7) $[15, 14, 4, 4, 4]^T$; and (8) $[18, 17, 5, 5, 5]^T$.

Fig. 4 is the graphical representation of the case in Table I. It is easy to see that the number of MTSIs N_{sep} in the plant model grows quickly when $M(p_{15}), M(p_{18})$, and $M(p_{19})$ become larger. However, $N_{sep}^{[c]}$ increases quite slowly. For instance, when $M(p_{15}) = M(p_{18}) = M(p_{19}) = 5$, we have $N_{sep} = 4311$, which means we have to solve 4311 systems of inequalities when the theory of regions is used alone. Note that $N_{\text{sep}}^{[c]} = 192$, which means that we need to solve only 192 systems of inequalities after siphons S_{2-4} are controlled.

Table II gives the computation time of the parameterized problem cases, where T_S ($T_{\text{sep}}^{[c]}$) is the time to compute monitors in the first (second) stage, and T_{sep} is the time if the theory of regions is used alone. Fig. 5 is the graphical representation of T_{sep} and $T^{[c]}$, where $T^{[c]} = T_S + T_{\text{sep}}^{[c]}$ is the execution time of our two-stage deadlock control method.

VI. CONCLUSION

The major contributions of this paper include a siphon solution approach in the S^{3}PR and synthesis of the net supervisor while lowering the computational cost when using the theory of regions. The deadlock prevention policy in [4] is of exponential complexity since complete siphon enumeration is needed. Moreover, the supervisors obtained in [4], are not maximally permissive in general. Our combined approach can produce the maximally permissive LENS for an S^{3}PR model and is more efficient than using the theory of regions alone.

A naturally arising issue is to derive LENS from other more condensed net representation, such as net unfolding [12] and BDD-based reachable state space representation [13] based on the idea of the theory of regions. Another is to extend the proposed method to other classes of Petri nets.

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Coordinated Logistics Scheduling for In-House Production and Outsourcing

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***Abstract*—In this paper, we address a new scheduling model for a firm with an option of outsourcing. A job can be processed by either in-house production or outsourcing. All outsourced jobs have to be transported back to the firm in batches, and the transportation costs have to be taken into account. We model the situation as a scheduling problem with transportation considerations. We discuss four commonly used objective functions, and solve them by dynamic programming algorithms.**

***Note to Practitioners*—An efficient supply chain management needs the coordination of production and transportation. Such problems exist in many different scenarios. This research considers a particular problem for a firm that has an option of using a subcontractor to fulfill part of its orders. The production schedule has to be coordinated with logistics issues for the transportation from the subcontractor to the firm. The purposes of this paper are twofold. First, we build models and provide optimal solutions for the specific cases discussed in this paper. Second, we hope to raise the issue of coordinated logistics scheduling, and motivate future research on more complicated models.**

***Index Terms*—Logistics, outsourcing, scheduling.**

I. Introduction

With widespread globalization, outsourcing has become prevalent in all areas of industries, both as a strategic tool to reduce operational cost, and as a tactical means to hedge against the capacity shortage when facing a large demand. At the strategic level, the practice of outsourcing is often to transfer an entire section of production or service to some subcontractors. Besides cost reduction, the concerns of outsourcing at the strategic level include union and governmental regulations, the long term goal of the company, and even political issues.

At the tactical level, outsourcing is often used as a complementary source to its own in-house production which has a limited capacity. When the demand level is beyond the in-house production capacity, a manufacturer may have the option to outsource some orders to available subcontractors so that all orders can be completed as early as possible. In doing this, the manufacturer needs to pay the subcontractor for what is outsourced; the outsourced jobs, upon completion, need to be transported back from the subcontractor to the manufacturer, and thus necessary transportation delay and transportation cost may be incurred. The problem is then how to coordinate the in-house production and outsourcing in an efficient way. We will study such an outsourcing problem in this research.

Specifically, we consider an integrated model for in-house production and outsourcing under the context of machine scheduling. For a given set of jobs, the decisions we need to make include the selection of a subset of jobs to be outsourced, the schedule of the jobs, and a transportation plan for the outsourced jobs.

The study of outsourcing under machine scheduling models just started recently. Motivated by their industrial consulting experience, Chung *et al.* [12] study a job shop scheduling model where an operation can be done either on an in-house machine or on an outsourcing machine with an extra cost; but the model neglects the transportation

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