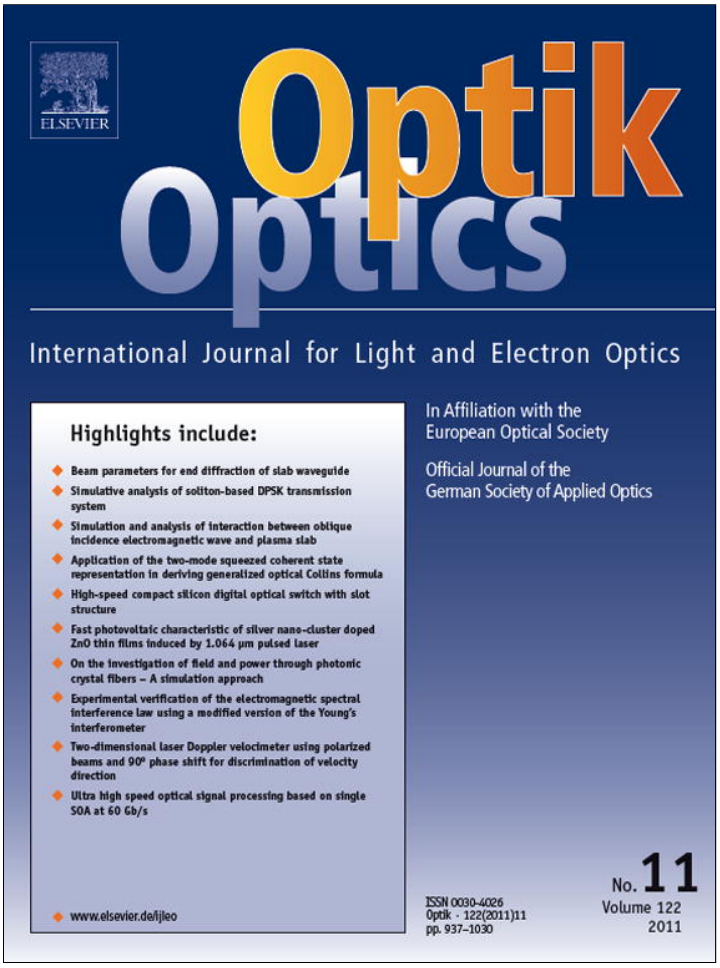
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Application of the two-mode squeezed coherent state representation in deriving generalized optical Collins formula

Chuan-Mei Xie [a,b,*], Hong-Yi Fan [b]

[a] College of Physics & Material Science, Anhui University, Hefei 230039, China
[b] Department of Material Science and Engineering, University of Science and Technology of China, Hefei 230026, China

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ABSTRACT

We propose a new two-mode squeezed coherent state representation $|z_1, z_2\rangle_g$ which is characteristic of the correlation between the squeezing and the displacement. Based on it and using the technique of integration within an ordered product of operators we obtain a generalized two-mode Fresnel operator (GTFO), which is an image of the mapping from (z_1, z_2) to $(sz_1 + rz_2^*, rz_1^* + sz_2)$ in $|z_1, z_2\rangle_g$ representation. The matrix element of GTFO in the coordinate representation leads to a generalized two-dimensional Collins formula (Huygens–Fresnel integration transformation describing optical diffraction) in entangled form.

1. Introduction

In Fourier optical diffraction theory the two-dimensional Collins diffraction integral formula describing the light propagation in an optical system characterized by the $[A, B; C, D]$ ray transfer matrix is [1],

$$U_2(x_2, y_2) = \frac{k\exp(ikz)}{2\pi Bi}\int\int dx_1 dy_1 U_1(x_1, y_1) \times \exp\left\{\frac{ik}{2B}\left[A\left(x_1^2+y_1^2\right) - 2(x_1x_2 + y_1y_2) + D\left(x_2^2+y_2^2\right)\right]\right\}, \tag{1}$$

where $AD - BC = 1$. This formula reflects the Huygens–Fresnel principle of wave propagation [2]. In Ref. [3], based on the coherent state [4,5] $|z\rangle$ representation and using the technique of integration within an ordered product (*IWOP*) [6,7] of operators, the single-mode Fresnel operator $U_1(r, s)$ is derived as an image of the classical transformation from $z \to sz - rz^*$,

$$U_1(r, s) \equiv \sqrt{s}\int \frac{d^2z}{\pi}\left|sz - rz^*\right\rangle\langle z| = \exp\left[\left(-\frac{r}{2s^*}\right)a^{\dagger 2}\right]\exp\left[\left(a^\dagger a + \frac{1}{2}\right)\ln\frac{1}{s^*}\right]\exp\left[\left(\frac{r^*}{2s^*}\right)a^2\right], \tag{2}$$

where $\left[a, a^\dagger\right] = 1$, (s, r), complex number with $|s|^2 - |r|^2 = 1$, are related to a classical ray transfer matrix $\begin{pmatrix} A & B \\ C & D\end{pmatrix}$ by

$$s = \frac{1}{2}[A + D - i(B - C)], \qquad r = -\frac{1}{2}[A - D + i(B + C)], \tag{3}$$

The matrix element in the coordinate $|x\rangle$ representation of U_1 can lead to the transformation kernel of optical Collins formula connecting the input light field $f(x)$ and output light field $g(x')$,

$$g\left(x'\right) = \frac{1}{\sqrt{2piB}}\int_{-\infty}^{\infty}\exp\left[\frac{i}{2B}\left(Ax^2 - 2x'x + Dx'^2\right)\right]f(x)\,dx, \tag{4}$$

* Corresponding author at: College of Physics & Material Science, Anhui University, Hefei 230039, China. Tel.: +86 13856059332.
E-mail address: xiecmei@mail.ustc.edu.cn (C.-M. Xie).

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i.e.,

$$\left\langle x'\right| U_1(r,s)\left|x\right\rangle = \frac{1}{\sqrt{2piB}}\exp\left[\frac{i}{2B}\left(Ax^2-2x'x+Dx'^2\right)\right] \equiv k_1^M\left(x',x\right). \tag{5}$$

M represents the matrix $[A, B; C, D]$, Eq. (5) is the one-dimensional case of Eq. (1).

If we use Dirac's symbol to let $f(x) = \langle x| f\rangle$, then Eq. (4) is expressed as

$$g\left(x'\right) = \int_{-\infty}^{\infty}\left\langle x'\right| U_1(r,s)\left|x\right\rangle\left\langle x\right| f\rangle\, dx = \left\langle x'\right| U_1(r,s)\left|f\right\rangle, \tag{6}$$

which is just the quantum mechanical version of Fresnel transformation. The two-mode case was discussed in Ref. [4]. The obtained Fresnel operator and Collins diffraction integral formula have been widely applied to the relationship between classical optics and quantum optics [8–17].

In this paper we want to derive a new generalized two-dimensional (2D) Collins diffraction integral formula in the context of quantum optics, to be more concrete, we shall firstly construct the two-mode squeezing–displacement related squeezed-coherent state $|z_1,z_2\rangle_g$ representation, which is complete, g is an complex parameter, and then find a generalized two-mode Fresnel operator (GTFO) by mapping from $|z_1,z_2\rangle_g \to \left|sz_1+rz_2^*, rz_1^*+sz_2\right\rangle_g$. It is interesting to demonstrate that the matrix element of GTFO in the coordinate basis just lead to the generalized 2D Collins formula. Our work is arranged as follows: In Section 2, we propose the $|z_1,z_2\rangle_g$ representation, and prove its completeness relation by virtue of the *IWOP* technique. In Section 3, we derive the GTFO. In Section 4, we derive the new generalized 2D Collins diffraction integral formula in entangled form. In this theory, the relationship between quantum optics and classical optics is more clear.

2. The displacement–squeezing related squeezed two-mode coherent state

Since the seventies of last century, increasing attention in the field of quantum optics has been paid to squeezed states of light because of their potential uses in optical communication and interferometers. In this section we construct a new two-mode squeezed coherent state

$$\left|z_1,z_2\right\rangle_g = \exp\left[-\frac{|z_1|^2}{2}-\frac{|z_2|^2}{2}+a^\dagger\left(fz_1+gz_2^*\right)+b^\dagger\left(gz_1^*+fz_2\right)-2fga^\dagger b^\dagger\right]\left|00\right\rangle \tag{7}$$

where $\left|00\right\rangle$ is the vacuum state in Fock space, $\left[b,b^\dagger\right]=1$, fg plays the role of squeezing parameter, with the constraint $ff^*+gg^*=1$, in the expression of $\left|z_1,z_2\right\rangle_g$ one can see that the displacement and squeezing are related. This state is complete, since

$$\frac{1}{\pi^2}\int d^2z_1d^2z_2|z_1,z_2\rangle_{gg}\langle z_1,z_2| = 1, \tag{8}$$

which can be verified with the use of the normal ordering form $\left|00\right\rangle\left\langle 00\right| =: e^{-a^\dagger a - b^\dagger b}:$ and the *IWOP* technique

$$\begin{aligned}
\int\frac{1}{\pi^2}d^2z_1d^2z_2|z_1,z_2\rangle_{gg}\langle z_1,z_2| &= \int\frac{1}{\pi^2}d^2z_1d^2z_2\exp\left[-\frac{|z_1|^2}{2}-\frac{|z_2|^2}{2}+a^\dagger\left(fz_1+gz_2^*\right)+b^\dagger\left(gz_1^*+fz_2\right)-2fga^\dagger b^\dagger\right]\\
&\quad\times\left|00\right\rangle\left\langle 00\right|\exp\left[-\frac{|z_1|^2}{2}-\frac{|z_2|^2}{2}+a\left(f^*z_1^*+g^*z_2\right)+b\left(g^*z_1+f^*z_2^*\right)-2f^*g^*ab\right]\\
&= \int\frac{1}{\pi^2}d^2z_1d^2z_2 :\exp\{-|z_1|^2-|z_2|^2+z_1\left(fa^\dagger+g^*b\right)+z_1^*\left(gb^\dagger+f^*a\right)+z_2\left(fb^\dagger+ag^*\right)\\
&\quad+z_2^*\left(ga^\dagger+f^*b\right)-2fga^\dagger b^\dagger-2f^*g^*ab-a^\dagger a-b^\dagger b\}:\\
&= \exp\left[\left(fa^\dagger+g^*b\right)\left(gb^\dagger+f^*a\right)+\left(fb^\dagger+ag^*\right)\left(ga^\dagger+f^*b\right)-2fga^\dagger b^\dagger-2f^*g^*ab-a^\dagger a-b^\dagger b\right]\\
&= 1.
\end{aligned} \tag{9}$$

Moreover, using the completeness relation of two-mode coherent state

$$\left|\alpha,\beta\right\rangle = \exp\left[-\frac{|\alpha|^2}{2}-\frac{|\beta|^2}{2}+\alpha a^\dagger+\beta b^\dagger\right]\left|00\right\rangle, \int\frac{d^2\alpha d^2\beta}{\pi^2}\left|\alpha,\beta\right\rangle\left\langle\beta,\alpha\right| = 1 \tag{10}$$

and the overlap(11) $\left\langle\alpha,\beta\right| z_1,z_2\rangle_g = \left\langle\alpha\beta\right|\exp\left[-\frac{|z_1|^2}{2}-\frac{|z_2|^2}{2}+\alpha^*\left(fz_1+gz_2^*\right)+\beta^*\left(gz_1^*+fz_2\right)-2fg\alpha^*\beta^*\right]\left|00\right\rangle = \left\langle 00\right|\exp\left[-\frac{|\alpha|^2}{2}-\frac{|\beta|^2}{2}\right.$ $\left.+\alpha^*a+\beta^*b\right]\times\exp\left[-\frac{|z_1|^2}{2}-\frac{|z_2|^2}{2}+\alpha^*\left(fz_1+gz_2^*\right)+\beta^*\left(gz_1^*+fz_2\right)-2fg\alpha^*\beta^*\right]\left|00\right\rangle = \exp\left[-\frac{|\alpha|^2}{2}-\frac{|\beta|^2}{2}-\frac{|z_1|^2}{2}-\frac{|z_2|^2}{2}+\alpha^*\left(fz_1+gz_2^*\right)\right.$ $\left.+\beta^*\left(gz_1^*+fz_2\right)-2fg\alpha^*\beta^*\right]$ we have

$$\langle z_1, z_2|\, z_1, z_2\rangle_g = {}_g\langle z_1 z_2| \int \frac{d^2\alpha d^2\beta}{\pi^2} \left|\alpha\beta\right\rangle \left\langle\beta\alpha\right|\, z_1 z_2\rangle_g = e^{-|z_1|^2-|z_2|^2} \int \frac{d^2\alpha d^2\beta}{\pi^2} \exp[-|\alpha|^2 - |\beta|^2 + \alpha\left(f^* z_1^* + g^* z_2\right) + \beta\left(g^* z_1 + f^* z_2^*\right)$$
$$-2f^* g^* \alpha\beta + \alpha^*\left(f z_1 + g z_2^*\right) + \beta^*\left(g z_1^* + f z_2\right) - 2fg\alpha^*\beta^*] = \frac{1}{1-4\left|fg\right|^2} \tag{12}$$

where we have used the formula

$$\int \frac{d^2 z}{\pi} \exp\{\zeta|z|^2 + \xi z + \eta z^* + f z^2 + g z^{*2}\} = \frac{1}{\sqrt{\zeta^2 - 4fg}} \exp\left\{\frac{-\zeta\xi\eta + \xi^2 g + \eta^2 f}{\zeta^2 - 4fg}\right\}, \tag{13}$$

so the normalization of $\left|z_1, z_2\right\rangle_g$ is

$$\left(1-4\left|fg\right|^2\right)_g \langle z_1, z_2|\, z_1, z_2\rangle_g = 1. \tag{14}$$

Due to

$$\begin{aligned} a|z_1, z_2\rangle_g &= \left[\left(f z_1 + g z_2^*\right) - 2fgb^\dagger\right] |z_1, z_2\rangle_g \\ b|z_1 z_2\rangle_g &= \left[\left(g z_1^* + f z_2\right) - 2fga^\dagger\right] |z_1, z_2\rangle_g, \end{aligned} \tag{15}$$

$|z_1, z_2\rangle_g$ obeys the eigenvector equation

$$\begin{aligned} \frac{a + 2fgb^\dagger}{\sqrt{1-4\left|fg\right|^2}} |z_1, z_2\rangle_g &= \left(f z_1 + g z_2^*\right) |z_1, z_2\rangle_g \\ \frac{b + 2fga^\dagger}{\sqrt{1-4\left|fg\right|^2}} |z_1, z_2\rangle_g &= \left(g z_1^* + f z_2\right) |z_1, z_2\rangle_g. \end{aligned} \tag{16}$$

If we let $a + 2fgb^\dagger/\sqrt{1-4\left|fg\right|^2} \equiv a'$, $b + 2fga^\dagger/\sqrt{1-4\left|fg\right|^2} = b'$, then $\left[a', a'^\dagger\right] = 1$, $\left[b', b'^\dagger\right] = 1$, $|z_1, z_2\rangle_g$ is a two-mode squeezed state indeed, but it differs from the usual one, since its squeezing parameter is related to its displacement parameter.

3. The generalized two-mode Fresnel operator in the $|z_1, z_2\rangle_g$ representation

We now consider that the two-mode squeezed coherent state $|z_1, z_2\rangle_g$ has a movement in (z_1, z_2) space characteristic of $(z_1, z_2) \to \left(s z_1 + r z_2^*, r z_1^* + s z_2\right)$. Corresponding to it, we construct the following ket-bra integration operator,

$$U_{2g}(r, s) = ss^* \int \frac{1}{\pi^2} d^2 z_1 d^2 z_2 \left|s z_1 + r z_2^*, r z_1^* + s z_2\right\rangle_{gg} \langle z_1, z_2|, \tag{17}$$

where s and r are complex, satisfying

$$|s|^2 - |r|^2 = 1, \tag{18}$$

which indicates that the movement is symplectic. Using Eq. (13) and the *IWOP* technique we can perform the integration in Eq. (17) and obtain

$$U_{2g}(r, s) = ss^* \int \frac{1}{\pi^2} d^2 z_1 d^2 z_2 \left|s z_1 + r z_2^*, r z_1^* + s z_2\right\rangle_{gg} \langle z_1, z_2| = ss^* \int \frac{1}{\pi^2} d^2 z_1 d^2 z_2 : \exp\{-|s|^2|z_1|^2 - |s|^2|z_2|^2 - sr^* z_1 z_2 - s^* r z_1^* z_2^*$$
$$+ \left(fsa^\dagger + gr^* a^\dagger + g^* b\right) z_1 + \left(fsb^\dagger + gr^* b^\dagger + g^* a\right) z_2 + \left(frb^\dagger + gs^* b^\dagger + f^* a\right) z_1^* + \left(fra^\dagger + gs^* a^\dagger + f^* b\right) z_2^* - 2fga^\dagger b^\dagger - 2f^* g^* ab$$
$$-a^\dagger a - b^\dagger b\} := \exp\left[\left(\frac{f^2 r}{s^*} + \frac{g^2 r^*}{s}\right) a^\dagger b^\dagger\right] : \exp\left[\left(a^\dagger a + b^\dagger b\right)\left(\frac{|f|^2}{s^*} + \frac{|g|^2}{s} - 1\right)\right] : \times \exp\left[-\left(\frac{f^{*2} r^*}{s^*} + \frac{g^{*2} r}{s}\right) ab\right]. \tag{19}$$

Then with the help of the operator identity

$$\exp\left(l\left(a^\dagger a + b^\dagger b\right)\right) =: \exp\left[\left(e^l - 1\right)\left(a^\dagger a + b^\dagger b\right)\right] : \tag{20}$$

we have

$$U_{2g}(r, s) = \exp\left[\left(\frac{f^2 r}{s^*} + \frac{g^2 r^*}{s}\right) a^\dagger b^\dagger\right] \exp\left[\left(a^\dagger a + b^\dagger b\right) \ln\left(\frac{|f|^2}{s^*} + \frac{|g|^2}{s}\right)\right] \times \exp\left[-\left(\frac{f^{*2} r^*}{s^*} + \frac{g^{*2} r}{s}\right) ab\right]. \tag{21}$$

In particular, when $g = 0$ and $f = 1$ Eq. (21) reduces to

$$U_{2g=0}(r, s) = \sqrt{s^*} U_2(r, s) \tag{22}$$

where

$$U_2(r, s) = s \int \frac{1}{\pi^2} d^2 z_1 d^2 z_2 \left|s z_1 + r z_2^*, r z_1^* + s z_2\right\rangle \langle z_1, z_2| = \exp\left(\frac{r}{s^*} a^\dagger b^\dagger\right) \exp\left[\left(a^\dagger a + b^\dagger b + 1\right) \ln\left(\frac{1}{s^*}\right)\right] \exp\left(-\frac{r^*}{s^*} ab\right) \tag{23}$$

is the two-mode Fresnel operator in Ref. [4]; on the other hand, when $g=1, f=0$,

$$U_{2g=1}(r,s)=\sqrt{s}U_2(r^*,s^*). \tag{24}$$

Thus we name $U_g(r,s)$ the generalized two-mode Fresnel operator (GTFO). Further, using the *IWOP* technique and the formulas Eq. (2) and Eq. (1) we can verify

$$U_{2g}^{\dagger}(r,s)=U_{2g}^{-1}(r,s), \tag{25}$$

so $U_{2g}(r,s)$ is a unitary operator.

Using the normally ordered form of $U_{2g}(r,s)$ in Eq. (19) we take its two-mode coherent state matrix element and immediately have

$$\begin{aligned}\left\langle z_1',z_2'\right| U_{2g}(r,s)\left|z_1,z_2\right\rangle &= \exp\{\left(\frac{f^2r}{s^*}+\frac{g^2r^*}{s}\right)z_1'^*z_2'^*-\left(\frac{f^{*2}r^*}{s^*}+\frac{g^{*2}r}{s}\right)z_1z_2+\left(\frac{|f|^2}{s^*}+\frac{|g|^2}{s}\right)\left(z_1'^*z_1+z_2'^*z_2\right)-\frac{|z_1|^2}{2}-\frac{|z_2|^2}{2}\\&-\frac{|z_1'|^2}{2}-\frac{|z_2'|^2}{2}\}=\exp\left\{Yz_1'^*z_2'^*-Vz_1z_2+K\left(z_1'^*z_1+z_2'^*z_2\right)-\frac{|z_1|^2}{2}-\frac{|z_2|^2}{2}-\frac{|z_1'|^2}{2}-\frac{|z_2'|^2}{2}\right\}\end{aligned} \tag{26}$$

where we have defined

$$K\equiv\frac{|f|^2}{s^*}+\frac{|g|^2}{s},\qquad V\equiv\frac{f^{*2}r^*}{s^*}+\frac{g^{*2}r}{s},\qquad Y\equiv\frac{f^2r}{s^*}+\frac{g^2r^*}{s}, \tag{27}$$

Then, in the entangled state representation [18]

$$\left|\eta\right\rangle=\exp\left(-\frac{|\eta|^2}{2}+\eta a^{\dagger}-\eta^*b^{\dagger}+a^{\dagger}b^{\dagger}\right)\left|00\right\rangle, \tag{28}$$

we can get the entangled state matrix element of $U_{2g}(r,s)$,

$$\begin{aligned}\left\langle \eta'\equiv\sigma\right| U_{2g}(r,s)\left|\eta\right\rangle &= \int\frac{d^2z_1d^2z_2d^2z_1'd^2z_2'}{\pi^4}\left\langle\sigma\right|\left.z_1',z_2'\right\rangle\left\langle z_1',z_2'\right|U_{2g}(r,s)\left|z_1,z_2\right\rangle\left\langle z_1,z_2\right|\left.\eta\right\rangle\\&=\int\frac{d^2z_1d^2z_2d^2z_1'd^2z_2'}{\pi^4}\exp\left[-|z_1|^2-|z_2|^2-|z_1'|^2-|z_2'|^2-\frac{|\sigma|^2}{2}-\frac{|\eta|^2}{2}\right]\\&\quad\times\exp\left[-Vz_1z_2+z_1^*z_2^*+\eta z_1^*+Kz_1'^*z_1+Kz_2'^*z_2+Yz_1'^*z_2'^*+z_1'z_2'+\sigma^*z_1'-\eta^*z_2^*-\sigma z_2'\right]\\&=-\frac{1}{K\left[K-K^{-1}(1+V)(1-Y)\right]}\\&\quad\times\exp\left\{\frac{-|\sigma|^2K\left[K+K^{-1}(1+V)(1+Y)\right]-|\eta|^2K\left[K+K^{-1}(1-Y)(1-V)\right]+2\eta^*\sigma K+2\eta\sigma^*K}{-2K\left[K-K^{-1}(1+V)(1-Y)\right]}\right\}\end{aligned} \tag{29}$$

By introducing

$$\begin{aligned}A&=\frac{1}{2}\left[K+K^{-1}(1-V)(1-Y)\right],\\B&=\frac{i}{2}\left[K-K^{-1}(1+V)(1-Y)\right],\\C&=-\frac{i}{2}\left[K-K^{-1}(1-V)(1+Y)\right],\\D&=\frac{1}{2}\left[K+K^{-1}(1+V)(1+Y)\right],\end{aligned} \tag{30}$$

which just obeys

$$AD-BC=1, \tag{31}$$

then we can rewrite Eq. (29) as

$$\left\langle \eta'\equiv\sigma\right| U_{2g}(r,s)\left|\eta\right\rangle=\frac{1}{K2Bi}\exp\left\{\frac{i}{2B}\left[A|\eta|^2+D|\sigma|^2-(\eta^*\sigma+\eta\sigma^*)\right]\right\}\equiv\frac{\pi}{K}\kappa_2^{(r,s)}\left(\eta',\eta\right)\equiv\frac{\pi}{K}\kappa_2^M\left(\eta',\eta\right) \tag{32}$$

where the superscript M only means that the parameters of κ_2^M are $[A, B; C, D]$ and the subscript 2 means the two-dimensional kernel. Eq. (32) has the similar form as Eq. (5) except for its complex form. Taking $\eta_1=x_1$, $\eta_2=x_2$ and $\sigma_1=x_1'$, $\sigma_2=x_2'$, we have

$$\kappa_2^M\left(\eta',\eta\right)=\kappa_2^M\left(x_1',x_2';x_1,x_2\right)=\kappa_1^M\left(x_1',x_1\right)\otimes\kappa_1^M\left(x_2',x_2\right) \tag{33}$$

This show that $U_{2g}(r,s)$ is really a generalized 2D Fresnel transformation operator.

4. The generalized 2D Collins diffraction integral formula in entangled form.

Now we examine the matrix element of GTFO in coordinate eigenstates. Using Eq. (32) we have

$$\left\langle x_1',x_2'\right| U_{2g}(r,s)\left|x_1,x_2\right\rangle=\int\frac{d^2\sigma d^2\eta}{\pi^2}\left\langle x_1',x_2'\right|\left.\sigma\right\rangle\left\langle\sigma\right|U_{2g}(r,s)\left|\eta\right\rangle\left\langle\eta\right|\left.x_1,x_2\right\rangle, \tag{34}$$

where $|x_1, x_2\rangle$ is the two-mode coordinate eigenstate representation, using the Schmidt-decomposed relation of the bipartite entangled state $\left|\eta\right\rangle$ (or $\left|\eta'\right\rangle$) [4],we can get

$$\begin{aligned}\left\langle \eta = \eta_1 + i\eta_2 \right| |x_1, x_2\rangle = \exp\left(i\eta_1\eta_2\right)\delta\left(\sqrt{2}\eta_1 + x_2 - x_1\right)\cdot\exp\left(-i\sqrt{2}\eta_2 x_1\right),\\ \left\langle x_1', x_2'\right| \left|\sigma = \sigma_1 + i\sigma_2\right\rangle = \exp\left(-i\sigma_2\sigma_1\right)\delta\left(\sqrt{2}\sigma_1 + x_2' - x_1'\right)\cdot\exp\left(i\sqrt{2}\sigma_2 x_1'\right).\end{aligned} \tag{35}$$

On substituting Eqs. (32) and (35) into Eq. (34) we derive

$$\begin{aligned}\left\langle x_1', x_2'\right| U_{2g}(r,s)\left|x_1, x_2\right\rangle &= \frac{1}{4KiB}\int\frac{d\sigma_2 d\eta_2}{\pi^2}\exp\left[i\sigma_2\frac{x_1'+x_2'}{\sqrt{2}} - i\eta_2\frac{x_1+x_2}{\sqrt{2}} + \frac{i}{2B}\left(A\eta_2^2 + D\sigma_2^2 - 2\eta_2\sigma_2\right)\right]\\ &\quad\times\exp\left\{\frac{i}{2B}\left[\frac{A}{2}(x_1-x_2)^2 + \frac{D}{2}\left(x_1'-x_2'\right)^2 - (x_1-x_2)\left(x_1'-x_2'\right)\right]\right\}\\ &= \frac{1}{2K\pi\sqrt{BC}}\exp\left\{\frac{i}{2B}\left[\frac{A}{2}(x_1-x_2)^2 + \frac{D}{2}\left(x_1'-x_2'\right)^2 - (x_1-x_2)\left(x_1'-x_2'\right)\right]\right\}\\ &\quad\times\exp\left\{\frac{-i}{4C}\left[A\left(x_1'+x_2'\right)^2 + D(x_1+x_2)^2 - 2\left(x_1'+x_2'\right)(x_1+x_2)\right]\right\}.\end{aligned} \tag{36}$$

In particular, if let $g=0$ and $f=1$ from Eqs. (27) and (30), we see

$$A = \frac{1}{2}(s - r + s^* - r^*),\qquad B = \frac{i}{2}(s + r - s^* - r^*),\qquad C = -\frac{i}{2}(s - r - s^* + r^*),\qquad D = \frac{1}{2}(s + r + s^* + r^*), \tag{37}$$

in this case Eq. (36) reduces to

$$\left\langle x_1', x_2'\right| U_{2g}(r,s)\left|x_1, x_2\right\rangle = s^*\left\langle x_1', x_2'\right| U_2(r,s)\left|x_1, x_2\right\rangle, \tag{38}$$

where

$$\begin{aligned}\left\langle x_1', x_2'\right| U_2(r,s)\left|x_1, x_2\right\rangle &= \frac{1}{2\pi\sqrt{BC}}\exp\left\{\frac{i}{2B}\left[\frac{A}{2}(x_1-x_2)^2 + \frac{D}{2}\left(x_1'-x_2'\right)^2 - (x_1-x_2)\left(x_1'-x_2'\right)\right]\right\}\\ &\quad\times\exp\left\{\frac{-i}{4C}\left[A\left(x_1'+x_2'\right)^2 + D(x_1+x_2)^2 - 2\left(x_1'+x_2'\right)(x_1+x_2)\right]\right\}.\end{aligned} \tag{39}$$

Eq. (39) is the entangled Fresnel transform in Ref. [4]. So, Eq. (36) is really a generalized two-dimensional Collins diffraction integral formula in entangled form.

In summary, based on the displacement–squeezing related squeezed two-mode coherent state and using the technique of integration within an ordered product of operators we find a generalized two-mode Fresnel operator (GTFO), whose matrix element in the coordinate representation leads to a generalized two-dimensional optical Collins formula. The GTFO corresponds to the variation of $|z_1, z_2\rangle_g$ from $|z_1, z_2\rangle_g \to \left|sz_1 + rz_2^*, rz_1^* + sz_2\right\rangle_g$, this is a new example that there exists formal connection between classical optics and quantum optics. At this point we recall that in the history of quantum mechanics Schrödinger considered that classical dynamics of a point particle should be the "geometrical optics" approximation of a linear wave equation, in the same way as ray optics is a limiting approximation of wave optics. We expect that the generalized two-dimensional optical Collins formula may be realistically used in optical ray diffraction for propagation of light emitted from some complicated objects.

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