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A mathematical model for simulating spring discharge and estimating sinkhole porosity in a karst watershed

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Abstract Documenting and understanding water balances in a karst watershed in which groundwater and surface water resources are strongly interconnected are important aspects for managing regional water resources. Assessing water balances in karst watersheds can be difficult, however, because karst watersheds are so very strongly affected by groundwater flows through solution conduits that are often connected to one or more sinkholes. In this paper we develop a mathematical model to approximate sinkhole porosity from discharge at a downstream spring. The model represents a combination of a traditional linear reservoir model with turbulent hydrodynamics in the solution conduit connecting the downstream spring with the upstream sinkhole, which allows for the simulation of spring discharges and estimation of sinkhole porosity. Noting that spring discharge is an integral of all aspects of water storage and flow, it is mainly dependent on the behavior of the karst aquifer as a whole and can be adequately simulated using the analytical model described in this paper. The model is advantageous in that it obviates the need for a sophisticated numerical model that is much more costly to calibrate and operate. The model is demonstrated using the St. Marks River Watershed in northwestern Florida.

Keywords Sinkhole porosity · Watershed balance · Karst aquifer · Turbulent flow · Analytical model

The views expressed in this paper are solely those of the authors and do not necessarily reflect the views or policies of the U.S. Environmental Protection Agency.

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Ein mathematisches Modell zur Simulation des Quellabflusses und Berechnung der Dolinendichte in einem Karsteinzugsgebiet

Zusammenfassung Die Bestimmung von Wasserbilanzen in Karsteinzugsgebieten, in welchen Grundwasser und Oberflächenwasser miteinander in Kontakt stehen, ist ein wichtiger Aspekt für die Bewirtschaftung regionaler Grundwasserressourcen. Das Abschätzen der Wasserbilanzen kann allerdings schwierig sein, denn die Grundwasserströmung in Karsteinzugsgebieten wird sehr stark durch hochdurchlässige Karströhren beeinflusst, die oft mit einer oder mehreren Dolinen verbunden sind. In diesem Artikel wird die Entwicklung eines mathematischen Modells zur Abschätzung der Dolinendichte (Anteil von Dolinen an der Einzugsgebietsfläche) aus dem Schüttungsverhalten einer Quelle präsentiert. Das Modell ist eine Kombination aus einem traditionellen linearen Reservoirmodell unter Berücksichtigung turbulenter Verhältnisse der Karströhre, welche die Quelle und die Doline miteinander verbindet. Der Quellabfluss repräsentiert als integrales Gebietssignal alle Aspekte von Strömung und Wasserspeicherung und hängt von der Summe der Systemeigenschaften des Quelleinzugsgebietes ab. Er kann durch das in diesem Artikel beschriebene analytische Modell simuliert werden. Das Modell kann den Einsatz aufwendiger numerischer Modelle überflüssig machen, welche einen höheren Aufwand für Kalibrierung und Anwendung benötigen. Die Anwendung des Modells wird am Beispiel des St. Marks Flusseinzugsgebietes im Nordwesten Floridas demonstriert.

Introduction

Proper management of karst watersheds is of critical importance for maintaining high-quality drinking water and for maintaining the health of the watershed ecosystem for both the developed world and the developing world. Major karst regions such as may be found in China, Europe, and the Americas all have a major stake in managing their respective karst watersheds, but to do so requires considerable effort and resources dedicated to investigating and understanding the complex processes (e.g., groundwater and surface water interactions) that affect karst watersheds.

Understanding the basic water balance in a watershed is one aspect that is important for the management of regional water resources. For a karst watershed, groundwater flow is a controlling factor, because cavernous conduits rapidly transmit tremendous amounts of water underground. For many surface streams in karst terranes, precipitation is well recorded. However, evapotranspiration acquired from micrometeorological stations can vary over a wide range from site to site, thus only providing a reference value for the regional evapotranspiration. Therefore, measurements of river and spring discharges are useful when calculating water balances in karst watersheds.

A spring or river responds to rainfall, typically with an abrupt increase followed by a retarded decrease in discharge. This type of response is often attributed to sinkhole flooding, which is caused by sinkholes receiving excessive amounts of water from nearby sources in a short period of time (Field 2010) and the water table beneath sinkholes rapidly rising. Consequently, the rapidly rising water table beneath and within sinkholes may force the underground water into the surrounding limestone fissures to cause a form of bank storage. Later, the water transiently stored in the limestone matrix is slowly released back into the conduits (Rorabaugh 1964, Atkinson 1977, Palmer 1986, Field 1993, Li 2004, Kovács et al. 2005, Li et al. 2008, Birk & Hergarten 2010). Birk et al. (2006) used MODFLOW to simulate the water head in a fissured matrix and the Darcy-Weisbach equation to model the flow in a karst conduit, with a linear exchange term (Barenblatt et al. 1960) to describe the bank storage and release processes between the matrix and the conduit. The Birk et al. (2006) approach was numerical, and required solving two partial differential equations. A significant aspect of their work was a finding that the breakthrough curve of calcium concentration was strongly asymmetric. However, the modeled spring discharge in their Figure 3 exhibited very limited skewness. If the conduit wall were impermeable, there would be no skewness in spring discharge from their numerical model. A motivation for this paper is to investigate, without the bank storage mechanism (i.e., no water enters the matrix from the conduits), whether the discharge of a spring can respond to precipitation with an abrupt increase followed by slow attenuation. According to Peterson & Wicks (2005), ignoring bank storage is a valid assumption.

In this paper, an analytical flow model is developed that simulates the response of spring discharge to a rainfall event. The model allows for a reasonable estimation of sinkhole porosity, which is an essential element for understanding watershed balances in karst terranes. The St. Marks River watershed is used for evaluating the model. The St. Marks River watershed basin is underlain by the shallow Florida Karst Aquifer in which surface water and groundwater exhibit significant interactions. Understanding how water is transited and balanced in the St. Marks River watershed and how this watershed is different from other watersheds is important from a management perspective. The existence of such common karst features as sinkholes, solution conduits, and springs provide preferential pathways for water flow, which significantly affects water fluxes at the watershed scale.

Mathematical model of spring discharge

A diagenetically mature limestone might be fine-grained, homogeneous, and almost impermeable. However, due to releases along bedding-plane partings and stress in near surface tectonic movements, many openings form, which may become enlarged through dissolution by acidic water and may evolve into large cavernous conduits with diameters of up to tens of meters. Pores in the matrix of a karst aquifer constitutes the primary porosity of the aquifer where flow is slow and laminar, while the fractures represent a secondary porosity and the solution conduits constitute a tertiary porosity in which flow may be fast and is often turbulent. Reynolds numbers for turbulent flows in solution conduits may range from as low as 2000 to as much as 10^6.

Conceptually, the spring base flow consists of the steady seepage from the matrix into the conduits. A critical sinkhole head is here defined as representing the sinkhole water table at some level above the spring head (which has zero head by convention) when there is no flow at the sinkhole (i.e., the sinkhole water table is static). Because sinkhole head is equal to the critical head, a sinkhole maintains a static water table, in which case the spring water issues solely from the matrix discharge/seepage into the conduits. As the sinkhole head increases as a result of precipitation, additional water pressure is exerted in the conduits as well as at the springs resulting in an abrupt increase in discharge. Because the matrix discharge into the conduits is assumed to be constant, the sinkhole water table will decrease accordingly to maintain mass conservation in the phreatic conduits.

A spring watershed generally has many sinkholes connected with the downstream spring. Spring discharge can be conceptualized as consisting of sinkhole drainage that accounts for the response of spring discharge to a rainfall event, and the matrix discharge into the conduits that is responsible for the base flow of the spring.

For constructing our mathematical model, the response of spring discharge Q_{sp}^R is

$$[1] \quad Q_{sp}^R = K(h_S - h_{cr}),$$

where K is the hydraulic conductivity of conduits and h_S and h_{cr} are the head in the sinkholes and the critical head, respectively. When the head in the sinkholes is higher than the critical head, the water table at the sinkholes tends to decrease to its original static state, and vice versa. It will be noted that a higher sinkhole head at any given moment does not necessitate an upward movement of the sinkhole water table at that moment (e.g., the decreasing head of the high sinkhole head following a storm). When a storm abruptly increases the sinkhole head so that it exceeds the critical head, the spring discharge will be increased, and therefore, the sinkhole water will tend to flow downward to maintain water mass conservation in conduits. Equation (1) is correct when $h_S - h_{cr}$ is small. This is because at small ranges any complex function (e.g., the non-linear reservoir model developed by Halihan & Wicks 1998) can be accurately approximated with a linear relation via Taylor expansion. For this reason, application of Equation (1) should be restricted to the scenario where the peak spring discharge is not significantly larger than the spring base flow.

The sinkhole flow Q_{sp}^R is related to the decreased rate of the sinkhole head via

$$[2] \quad Q_{sp}^R = -\phi_S A_S \frac{dh_S}{dt},$$

where ϕ_S and A_S are the porosity of sinkholes and the area of the watershed, respectively. Substituting Equation (2) into Equation (1) yields the governing equation for the sinkhole head,

$$[3] \quad \frac{dh_S}{h_S - h_{cr}} = \frac{-K}{\phi_S A_S} dt.$$

Integrating Equation (3) and using the initial condition, $h_S(t = 0) = h_0$, yields

$$[4] \quad h_S - h_{cr} = (h_0 - h_{cr}) \exp\left[\frac{-K}{\varphi_s A_s} t\right],$$

and substituting (4) into (1), results in

$$[5] \quad Q_{sp}^R = K(h_0 - h_{cr}) \exp\left[\frac{-K}{\phi_S A_S} t\right].$$

The precipitation allocated to the sinkholes P_S satisfies

$$[6] \quad P_S A_S = \phi_S A_S (h_0 - h_{cr}) = \int Q_{sp}^R dt = V_{sp}^R,$$

where V_{sp}^R is the discharge volume of all sinkholes, which as indicated, can be obtained from an integration of the response part of the spring discharge/flux with respect to time.

Finally, the response discharge is

$$[7] \quad Q_{sp}^R = K^* P_S A_S \exp\left[-K^* t\right],$$

where $K^* = K/(\phi_S A_S)$. Equation (7) indicates exponential attenuation of spring discharge with time, and can be rewritten as

$$[8] \quad Q_{sp}^R = Q_{sp}^{Max} \exp\left[\frac{-Q_{sp}^{Max}}{V_{sp}^R} t\right],$$

where Q_{sp}^{Max} is the amplitude above the base flow, which can be read from the measured curve of spring discharge. Also K^* can be determined via

$$[9] \quad K^* = \frac{Q_{sp}^{Max}}{P_S A_S}.$$

The matrix water seeps into the conduits and flows to the spring. This matrix discharge constitutes the base flow of the spring, coming solely from the seepage from the matrix into the conduits. Applying water-mass conservation to the conduits, the spring discharge is the sum of the matrix discharge Q_{sp}^B (into the conduits), and the sinkhole flow Q_{sp}^R. Thus,

$$[10] \quad Q_{sp} = Q_{sp}^R + Q_{sp}^B.$$

The base flow, which comes from the matrix is then

$$[11] \quad Q_{sp}^B = K_M \bar{h},$$

where $\bar{h}$ is the averaged water head in the matrix, above the spring. Because the resistance against the base flow originates mainly from the viscous force in fissures, we approximately have $K_M \propto \frac{1}{\bar{h}}$ for an unconfined aquifer so that $Q_{sp}^B \approx const.$

For this aquifer system, and karst aquifers in general, the meaning of the conduit conductivity K is worth exploring. The spring discharge may be expressed as a function of the averaged hydraulic head in the matrix and in the sinkhole head

$$[12] \quad Q_{sp} = F(\bar{h}, h_S),$$

which is the state equation. Applying a Taylor expansion of the first order to Equation (12) results in

$$[13]\quad Q_{sp} = Q_{sp}(\bar{h}, h_{cr}) + \frac{\partial F(\bar{h}, h_{cr})}{\partial h_S}(h_S - h_{cr}) = Q_{sp}^B + Q_{sp}^R.$$

Comparing Equation (13) with Equation (1) yields

$$[14]\quad K = \left.\frac{\partial F(\bar{h}, h_S)}{\partial h_S}\right|_{h_S = h_{cr}},$$

which is the definition of K from the viewpoint of the state equation. In reality, Q_{sp} may not be a linear function of $h_S - h_{cr}$, if $h_S - h_{cr}$ is large. Nevertheless, when $h_S - h_{cr}$ is small, K can be treated as a constant. Equation (14) states that the extra sinkhole head causes an extra spring discharge response via K. Essentially, K is a coefficient emerging from the linearization of the turbulent flow law at a given discharge, which may be termed turbulent hydraulic conductivity as in Reimann et al. (2012). This coefficient is related to conduit diameter; similar to the way laminar conductivity is associated with pore size.

The model is more or less similar to a linear reservoir model which has been used extensively in the past to represent recessions from springs (e.g. Maillet 1905, Geyer et al. 2008). However, that model used recession coefficient to infer (laminar) hydraulic conductivity, while the sinkhole model presented in this paper enables us to infer sinkhole porosity, as elucidated in the following.

Equation (1) is consistent with turbulent hydrodynamics in conduits. Moreover, combining turbulent hydrodynamics in conduits with the model will yield a formula for sinkhole porosity ϕ_S.

For turbulent flow in a rough pipe, the square law (Schlichting 1968) states that discharge from a spring is proportional to the square root of the pressure gradient at the spring. With the pressure gradient roughly approximated using the sinkhole head, we obtain

$$[15]\quad \frac{h_S}{h_{cr}} \approx \left(\frac{Q_{sp}}{Q_{sp}^B}\right)^2.$$

For the condition of $h_S - h_{cr} \ll h_{cr}$, we obtain

$$[16]\quad \frac{Q_{sp}^R}{Q_{sp}^B} \approx \frac{h_S - h_{cr}}{2h_{cr}}.$$

Comparing Equation (16) with Equation (1) yields

$$[17]\quad K = \frac{Q_{sp}^B}{2h_{cr}}.$$

Equation (1) is consistent with the square law for turbulent conduit flow if $h_S - h_{cr} \ll h_{cr}$. Substituting $K = K^* \phi_S A_S$ into Equation (17) yields

$$[18]\quad \phi_S = \frac{Q_{sp}^B}{2h_{cr} K^* A_S}.$$

Substituting Equation (9) into (18) yields an alternative form

$$[19]\quad \varphi_S = \frac{Q_{sp}^B P_S}{2Q_{sp}^{Max} h_{cr}},$$

which states that sinkhole porosity depends on the ratio of the base-flow discharge to the maximum discharge (above the base flow), and the ratio of the precipitation allocated to sinkholes to the critical head.

The derived model is for a single spring watershed. The model can be generalized to a larger watershed with many springs, if K^* in Equation (7) is assumed to be a constant and each spring responds to a rainfall in a synchronous manner. The model is also applicable to sinking streams.

Sinkhole head attenuates slowly. This is so because the relatively high sinkhole head (i.e., the peak sinkhole head) causes a significant pressure gradient that results in a rapidly decreasing sinkhole drainage. As the sinkhole head decreases, the pressure gradient also decreases, resulting in a lessening drainage rate and a slowing decrease in sinkhole head. Releasing water back into a conduit after accumulating in bank storage may also additionally slow the decreasing rate of the sinkhole head.

Example application

St. Marks River starts from creeks on hills in Thomas County, South Georgia, and flows southward. After flowing across the boundary between Georgia and Florida, the river enters Lake Miccosukee, a large swampy prairie lake in Northern Jefferson County, Florida (Figure 1). At Natural Bridge Sink near the boundary between Leon County and Wakulla County, Florida, St. Marks River disappears down a swallet, becomes a subterranean river for about 800 m, and reemerges at the St. Marks River Rise as a first-magnitude spring with a discharge of 12 $m^3\,s^{-1}$. At the historic Town of St. Marks, the river is joined by its largest tributary, Wakulla River, which contributes an average flow of $9.80 \cdot 10^5\ m^3\,d^{-1}$. Wakulla River originates from Wakulla Springs, which is one of the largest and deepest springs in the world. Finally, the confluence of St. Marks River and Wakulla River discharges, on average, $2.68 \cdot 10^6\ m^3$ of freshwater a day (Lewis et al. 2009) into Apalachee Bay.

St. Marks River has a drainage basin of 3,030 km^2 (Lewis et al. 2009). Immediately adjacent to the coast of the Gulf of Mexico is the St. Marks National Wildlife Refuge (Figure 1), a renowned U.S. conservation wetland. From the wetland to the Cody Scarp (an ancient shoreline) is the

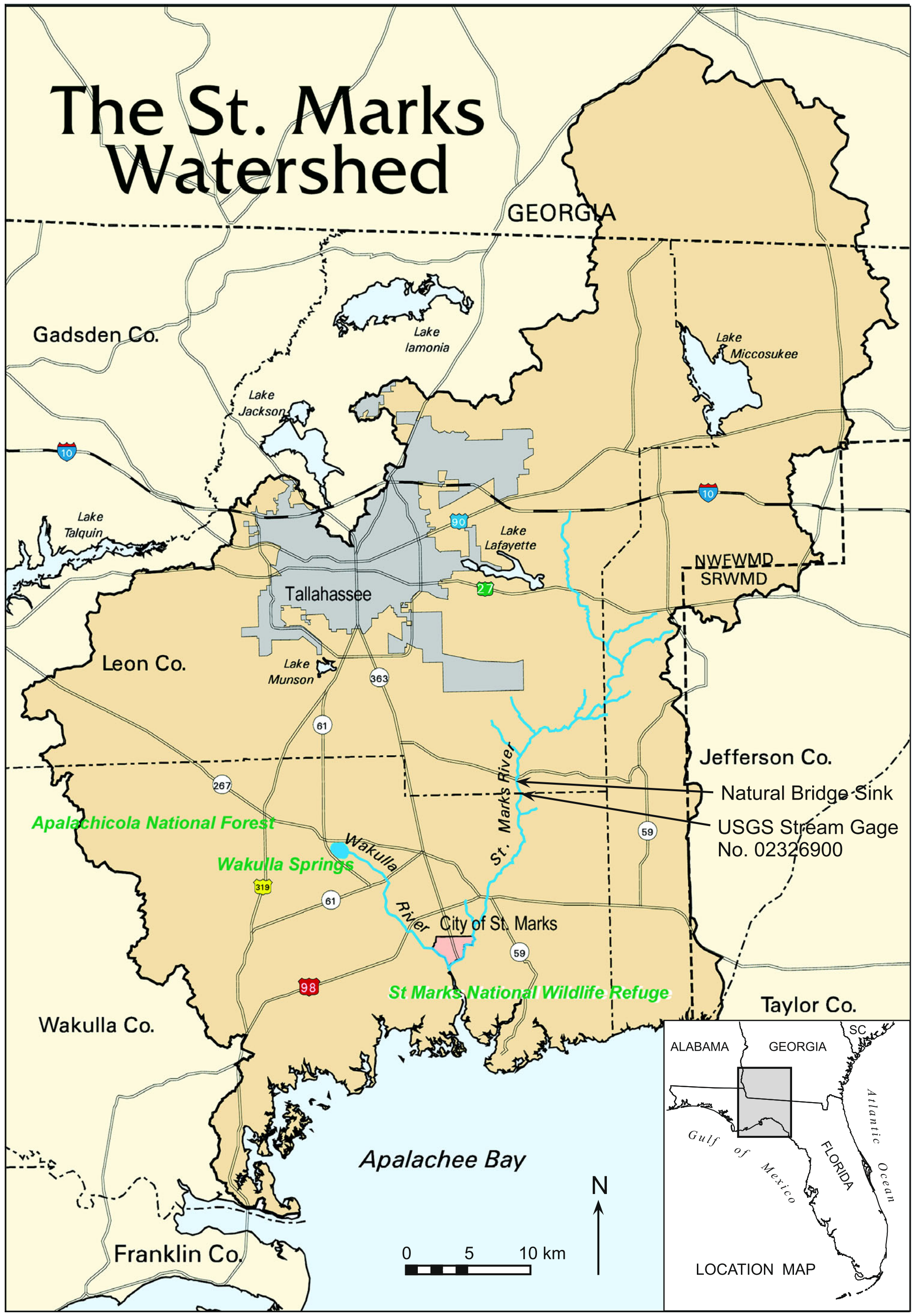


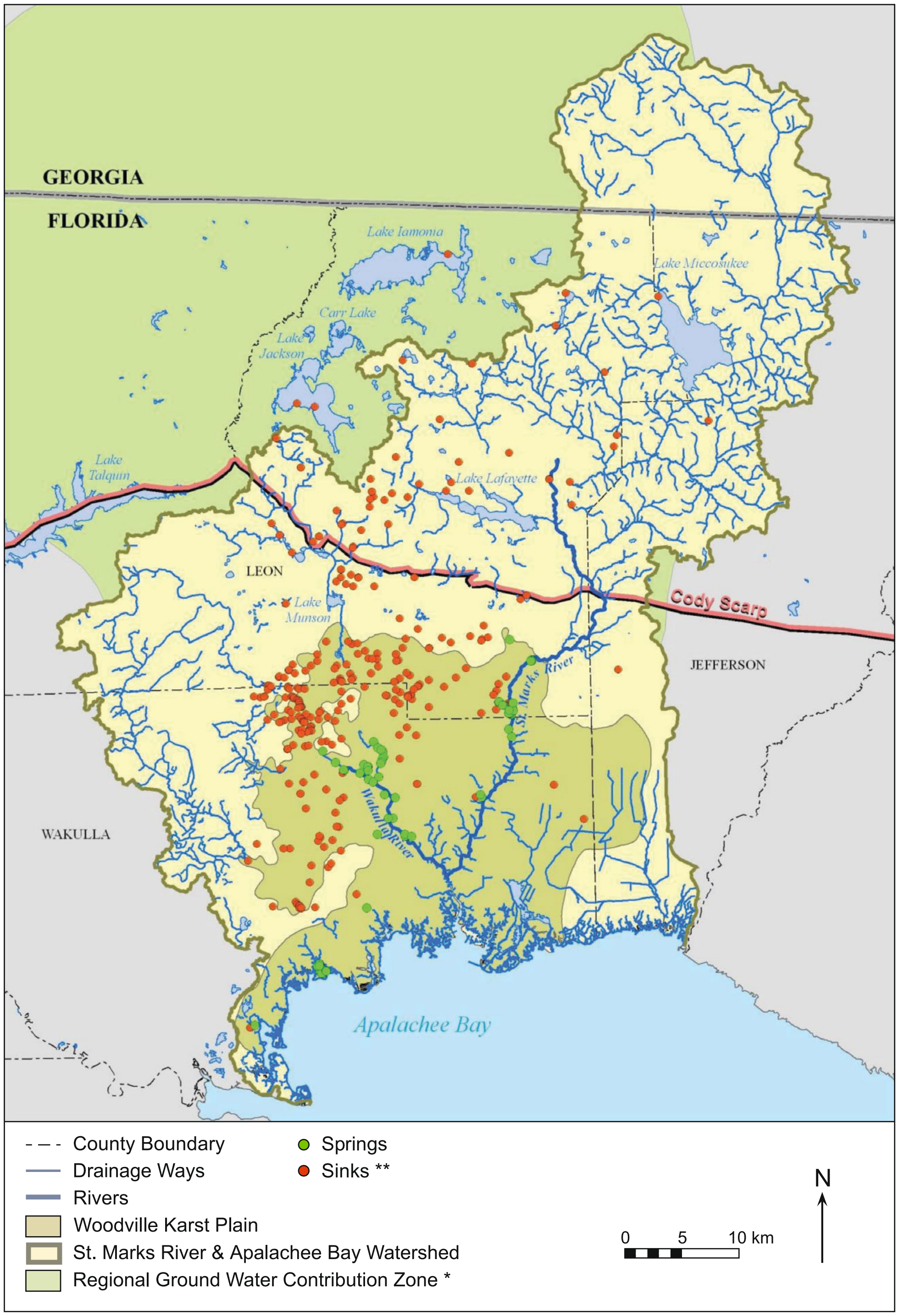
Fig. 1 The St. Marks River and Apalachee Bay watershed. Modified from Eidse (undated)

Woodville Karst Plain with an average elevation of about 8 m above sea level (Figure 2). Further north from the Scarp are a series of hills extending to Georgia.

There are numerous sinks and springs in the St. Marks River watershed (Figure 2). Scuba divers have mapped the Leon Sinks Cave system (the longest underwater cave in the United States) that connects Big Dismal Sink with Wakulla Springs. Dye-tracing experiments have confirmed a rapid water connection from Ames Sink to Indian Spring, Sally Ward Spring, and Wakulla Springs (Kincaid 2012). The connection suggests strong interactions between surface water and groundwater are a prominent feature of the St. Marks River watershed.

There are few publications on modeling water flow and contaminant transport in the St. Marks River watershed at the basin scale. The only significant modeling work was conducted by Hazlett-Kincaid, Inc. (2007). They numerically simulated the regional water head distribution, includ-

Fig. 2 Surface water and identified karst features (e.g., sinkholes and springs) of the St. Marks River watershed. Modified from Lewis et al. (2009); * approximate; ** data for features designated as sinks is from Leon and Wakulla County governments and Florida Department of Environmental Protection. Features may or may not have been verified



ing the head in the major conduits and rivers. A notable finding was that the rivers and conduits tend to push the head contours from the Gulf Coast inland. This is physically reasonable, because the rivers and conduits receive recharge from the neighboring limestone rock and thus should have a lower head than that on the two sides. Hazlett-Kincaid, Inc. has not yet reported on the water fluxes in the conduits and rivers.

Typically, it is unlikely to get an accurate flux in a conduit or river from regional-scale modeling. Taking the coupled continuum pipe flow (CCPF) model in MODFLOW-2005 as an example, the conduit/river flux is the head gradient multiplied by a hydraulic-conductivity coefficient. Although heads can be obtained from regional modeling efforts, the conduit/river flux remains underdetermined because the conductivity coefficient is unknown. An alternative method for assessing the conduit flux is to model the flow between a conduit and the limestone matrix (Birk et al. 2006, Loper & Chicken 2011). However, that method is a far-field approach requiring specifying an unknown exchange coefficient between the conduit and the matrix. As such, the conduit flux obtained from the modeling efforts is very uncertain. For surface rivers, other complications arise, because the contribution from numerous local creeks and streams are generally unknown. In practice, flux in a conduit or river is often acquired from field measurements using flow meters, weirs, etc.

Hydrology of the St. Marks River watershed

The drainage basin has three typical geographic features: hills, karst plain, and wetland. In the hilly area during a drought season, the water table approximates the ground surface, albeit more gently (Fetter 2000). This is because surface water can flow away quickly, whereas groundwater in pores and fissures meets much greater resistance/friction from the wall of rocks to retard flow. Consequently, the aquifer maintains a higher water table and acts like a big reservoir slowly providing water to creeks and surface streams which in turn recharge rivers. During a rainy season, a significant part of precipitation runs off, and directly contributes to creeks and surface streams, while the other portion of precipitation slowly percolates downward into the ground by gravity to the underlying aquifer, replenishing groundwater reserves.

In a karst plain (i.e., Woodville Karst Plain in Figure 2) where sinkholes, sinking streams, cavernous conduits, and springs are prevalent, water flow has a pattern similar to that in a hilly area, except for one major difference. As precipitation falls onto the ground surface, very permeable limestone allows greater infiltration downward into the shallower aquifer than that occurring in a hilly area. The portion of local precipitation that contributes to surface runoffs is significantly less than that occurring in a hilly area (the flatter ground surface is also an important factor).

Very hot and humid weather in the watershed in summer means that evaporation from surface-water bodies cannot be neglected, and rich vegetation implies transpiration by plants may be important. Evapotranspiration in an area with rich vegetation and hot weather, such as wetlands in Florida, plays an important role in the water balance. For instance, Abtew (1996) estimated an average evapotranspiration of 135 $\mathrm{cm\,y^{-1}}$ in South Florida, slightly less than the annual precipitation of 142 cm $\mathrm{y^{-1}}$. Sutula et al. (2001) concluded that in Taylor Slough in the Southern Everglades, Florida, annual precipitation is 138 $\mathrm{cm\,y^{-1}}$, roughly in balance with annual evapotranspiration (154 $\mathrm{cm\,y^{-1}}$). Evapotranspiration is considered to be roughly in balance with the precipitation in that region because it is not easy to discern the actual balance from physics, and water balances do not necessitate that evapotranspiration be accurately measured. Regardless, evapotranspiration in a wetland with hot weather (e.g., the Everglades) should be more important compared to that in other areas (e.g., the hills and karst plain in the St. Marks River watershed) so the ratio of evapotranspiration to precipitation in the wetland should be higher than that in other areas (assuming the same precipitation).

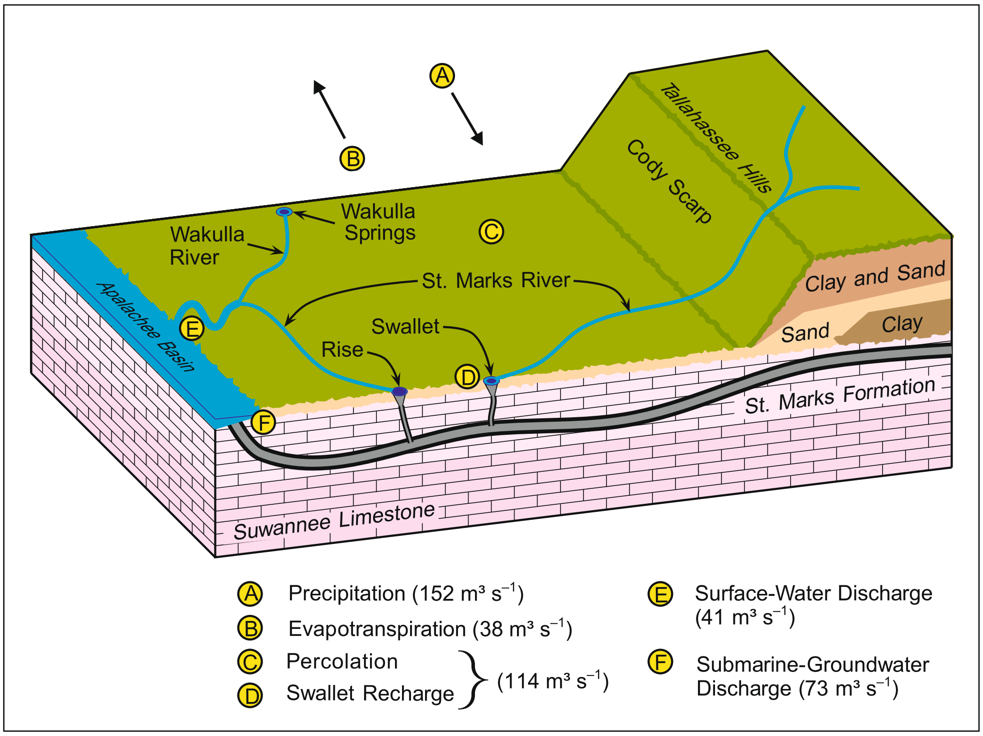
St. Marks River watershed regional water balance

A major assumption for the calculation of water balance in the watershed is that groundwater from land nearby to the catchment basin may be neglected. As Spechler & Schiffer (1995) pointed out, the source of Florida's water is mainly precipitation falling on the land of the State, which is contrary to a popular misconception that a majority of Florida's water comes from other states. For this reason, our assumption is reasonable. The basic idea is that the deficit between regional precipitation and discharge from the St. Marks River watershed into the Gulf of Mexico is equal to the regional evapotranspiration.

The St. Marks River watershed covers an area of 3,030 $\mathrm{km^2}$. The annual average precipitation in the watershed for a 29-year period was 161 $\mathrm{cm\,y^{-1}}$ (Lewis et al. 2009), which translates into an average water input of 152 $\mathrm{m^3\,s^{-1}}$. Discharge from the watershed to the Gulf of Mexico can be calculated from the sum of submarine groundwater discharge and that from surface rivers and streams. The Spring Creek Springs Group has the largest discharge among all springs in Florida, and is the dominant set of submarine springs in the watershed, discharging approximately 57 $\mathrm{m^3}$ of water a second to the Gulf of Mexico (Spechler & Schiffer 1995). Among surface streams discharging to the sea, St. Marks River (the dominant river in the watershed) conveys an average of 32 $\mathrm{m^3}$ of water per second into Apalachee Bay. Therefore, the combined output from these two dominant spring/river discharges is 89 $\mathrm{m^3\,s^{-1}}$, which constitutes a lower limit for the total discharge from the watershed to the sea. Unknown submarine springs and secondary surface streams also contribute freshwater into the Gulf of Mexico. According to Spechler & Schiffer (1995), discharge from all second-magnitude springs in Florida is approximately 28 % of that from all first-magnitude springs. Using this ratio (based upon a similarity assumption), the unknown secondary discharge to the sea is about 28 % of the dominant discharge from Spring Creek Springs Group and St. Marks River. Lower magnitude springs are neglected because their discharge is small, although there are many smaller springs. Thus the total freshwater output is probably 114 $\mathrm{m^3\,s^{-1}}$.

Boning (2007) states that six second-magnitude springs contribute to the St. Marks River. Therefore, the above calculation may ignore the heritage and double-count those

Fig. 3 Schematic diagram of the Florida Karst Aquifer and the annual water balance for the St. Marks River watershed



secondary springs. However, because the second-magnitude springs do not dominate the total flux, the error from this possible double-counting is not significant.

The annual mass of water in the watershed is approximately conserved, such that evapotranspiration ET is equal to the deficit between precipitation P and discharge to the sea Q. That is

$$[20] \quad ET = P - Q.$$

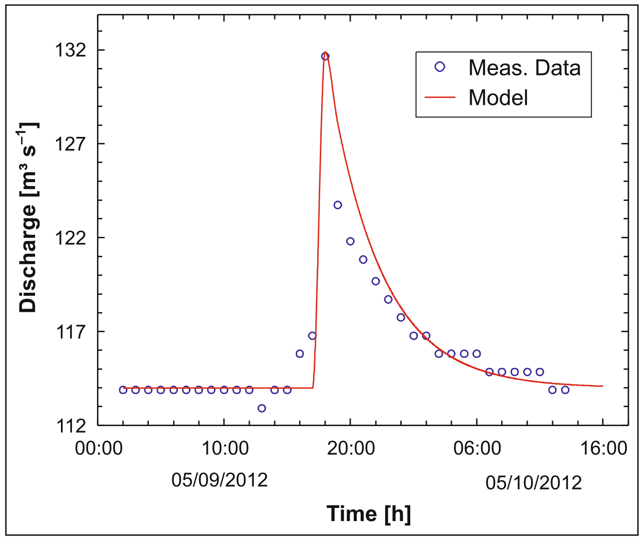
The precipitation to the watershed is 152 $m^3\,s^{-1}$, and the discharge from the watershed to the sea is at least 89 $m^3\,s^{-1}$ and probably 114 $m^3\,s^{-1}$. According to Equation (20), evapotranspiration in the St. Marks River watershed has a probable average value of 38 $m^3\,s^{-1}$ and an upper limit of 63 $m^3\,s^{-1}$. Infiltration recharge and sink recharge are combined to be 114 $m^3\,s^{-1}$, but these two types of recharge are not readily distinguished from each other. Figure 3 depicts schematically the annual water balance in the St. Marks River watershed.

St. Marks River Watershed analysis

The U.S. Geological Survey has a station (02326900) to gage the water flux of St. Marks River near Newport (Figure 1). The measured flux is disturbed by tides. In this paper, we selected the measured discharge data from May 9–11, 2012 (USGS 2012), to test the above model, because during this period, the disturbance from tides is much smaller than the response of discharge to the rainfall event.

St. Marks River is a spring-fed river with a high base flow. The first step is to transform the measured data at that station to the total discharge from the watershed to the sea. The base flow at the station is 16.6 $m^3\,s^{-1}$, while the base discharge from the watershed to the sea can be approximated with the average flux 114 $m^3\,s^{-1}$ in Figure 3. Thus the measured discharge at the station is transformed to the basin-scale discharge by multiplying with a factor of 6.87. By integration of the response part, the sinkholes discharge a volume of $2.73 \cdot 10^5$ m^3. The area of the St. Marks River watershed is 3,030 km^2, which translates into a precipitation allocated to the sinkholes P_S to be $\approx 9.1 \cdot 10^{-5}$ m. This number is so small that it is concluded that most of the precipitation percolated down into the aquifer via infiltration. The maximum response above the base flow (Q_{sp}^{Max}) is 18 $m^3\,s^{-1}$, and thus K^* is $6.6 \cdot 10^{-5}$ s^{-1} from Equation (9). The sinkhole porosity can be calculated via Equation (18). Setting $h_{cr} \approx 1$ m sinkhole porosity is $2.9 \cdot 10^{-4}$, which is the same as that calculated via (19). Using Equation (6), $h_0 - h_{cr} \approx 0.31$ m. An alternative method for estimating $h_0 - h_{cr}$ is direct application of the square law, which yields a value of 0.34 m. The amplitude of the sinkhole head variation is small, which facilitates the Taylor expansion in Equation (13). An advantage of our model and the above analysis is that they are not relevant to the primary porosity (matrix porosity), which is often unknown. This is because the sinkholes are hydraulically decoupled from the fissured matrix in the model. The obtained small sinkhole porosity is for the 2-D plane, which may roughly represent the 3-D conduit porosity. This con-

Fig. 4 Measured discharge and modeled discharge for the St. Marks River watershed into Apalachee Bay for May 9–10, 2012



firms that although the conduits convey a large amount of water, they are only secondary in storage of water.

As shown in Figure 4, the discharge calculated by the model with use of Equation (8), agrees well with that transformed from the stream gage measurement. The observed hydrograph presents a more rapid recession, clearly deviating from the modeled exponential behavior at the early stage. This phenomena is quite typical, e.g. (Birk & Hergarten 2010). The question on this phenomena can be equivalently transformed to why the late recession observed is slower than that from model. This is probably due to the retardation of infiltration recharge in the vadose zone.

The decreased rate of the averaged head in the matrix is

$$[21] \quad \frac{d\bar{h}}{dt} = \frac{-Q_{sp}^{B}}{\phi_M A_S} \approx -1.1\ \mathrm{cm\,d^{-1}},$$

where the matrix porosity, $\phi_M \approx 0.3$. It is evident from Figure 4 that the water table decreases more slowly than it increased.

Summary and conclusions

Discharge from all second-magnitude springs in Florida is approximately 28 % of that from all first-magnitude springs (Spechler & Schiffer 1995). Based on a similarity assumption, unknown secondary discharge from the St. Marks River watershed to the Gulf of Mexico is probably 25 $\mathrm{m^3\,s^{-1}}$, and thus the total freshwater discharge to the sea is probably 114 $\mathrm{m^3\,s^{-1}}$. According to water-balance estimates for the St. Marks River watershed, the average evapotranspiration in this watershed is probably one fourth of the annual average precipitation (equal to 40 $\mathrm{cm\,y^{-1}}$), with an upper limit of 66 $\mathrm{cm\,y^{-1}}$. This suggests that evapotranspiration from surface water bodies and vegetation is moderately important in water-balance estimates at the basin scale.

In terms of the ratio of evapotranspiration to precipitation, the St. Marks River watershed is significantly different from South Florida where evapotranspiration is roughly in balance with precipitation. This is because the St. Marks River watershed is dominated by a karst terrain where a majority of precipitation reaches the underlying Florida Karst Aquifer either through percolation into permeable limestone or by sinkholes. In contrast, South Florida has more wetlands present (thus more area of surface water and more vegetation) and is subject to higher temperatures that facilitates evaporation from surface-water bodies and transpiration by plants.

A flow model was developed to simulate the response of spring discharge to a rainfall event. The reasoning behind this model development was the fact that in a situation where carbonate rock is telogenetic rather than eogenetic, the bank storage and release effect may be neglected. Therefore, our model attributed the response of spring discharge to water table fluctuations in sinkholes.

The model adequately simulated the discharge of St. Marks River measured by the U.S. Geological Survey at a station near Newport from May 9–11, 2012. It also yielded a sinkhole porosity of $2.9 \cdot 10^{-4}$, and for that rainfall event, the sinkhole head dropped from 1.31 m down to 1 m. The model is regarded as reasonable because it essentially describes the main physics of spring discharge in two aspects. First, the base flow of a spring comes from matrix seepage into the conduits, while the steep response is caused by precipitation into the sinkholes via runoff, followed by slow attenuation of the sinkhole head. Second, the model is consistent with the dynamics of turbulent conduit flow.

Our model is novel in that it combined the traditional linear reservoir model with turbulent hydrodynamics in conduits, which enabled us to estimate the sinkhole porosity. Our model also showed that without the bank storage mechanism, the response of spring discharge to a storm can still present a steep increase followed by slower decrease. It is well known that sophisticated numerical models can simulate detailed head/flow distributions in an aquifer. However, because the spring discharge is an integral of all water storage and flow aspects, it mainly depends on the behavior of the aquifer as a whole, rather than the individual parts. For this reason, when simulating spring discharge, a simpler analytical model can be effective, as demonstrated by this paper.

The most important assumptions in this paper are: (1) The bank storage and release process is not a dominant process (thus our sinkhole model is a counterpart of traditional models that focused on bank storage and release); (2) The peak spring discharge is not significantly larger than the spring base flow, leading to the linearization of Equation (15) and the exponential recession of hydrograph.

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