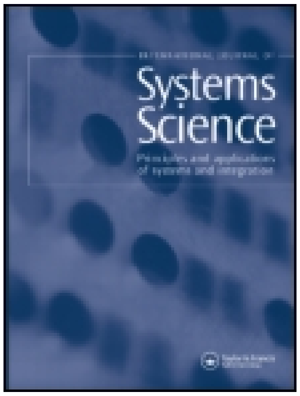


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# Asymptotic stabilisation for a class of feedforward input-delay systems with ratios of odd integers

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This article addresses the stabilisation problem by state-feedback for a class of feedforward input-delay nonlinear systems with ratios of odd integer powers. The designed controller achieves the global asymptotic stability. Based on the appropriate state transformation of time-delay systems and the Lyapunov method, the problem of controller design can be converted into the problem of finding a parameter which can be obtained by appraising the nonlinear terms of the systems. Finally, three simulation examples are given to illustrate the effectiveness of the control algorithm proposed in this article.

**Keywords:** nonlinear time-delay systems; feedforward systems; state-feedback stabilisation; Lyapunov method



## 1. Introduction

It is well known that time delays often appear in many industrial and actual systems such as communication systems, long transmission line systems, electrical systems and chemical engineering systems. The existence of time delay generally makes the control design problems more difficult. For example, it may destroy the closed-loop stability and even may deteriorate the control performance. Hence, the problem of control design of time-delay systems has been attracting more and more researchers, especially engineering researchers. Until now, many interesting results have been obtained, see, Mahmoud (2001), Boukas and Liu (2002), Yue and Han (2005), Zhang and Cheng (2005a, b), Choi and Lim (2006), Mazenc, Modié, and Niculescu (2006), Michiels and Niculescu (2007), Xia, Qiu, Zhang, Gao, and Wang (2008), Ye and Yang (2008), Lian, Zhao, and Dimirovski (2009), Mao, Jiang, and Ding (2009), Yi, Cao, Li, and Guo (2009), Zhang, Li, and Liu (2009), Zhang, Li, Wu, and She (2009), Benzaouia, Hmamed, and Tadeo (2010), Chen, Jiao, Li, and Li (2010), Chen and Zhang (2010, 2011), Feng and Xu (2010), Liu and Sun (2010), Pai (2010), Li and Xi (2011), Rasool and Nguang (2011), Chen, Wu, and Jiao (to appear) and the references therein.

In this article, authors will focus on solving the stabilisation problem of nonlinear input-delay system with the form

$$\begin{cases} \dot{x}_1(t) = x_2^{p_1}(t) + \phi_1(t, x(t), u(t-d_1)), \\ \dot{x}_2(t) = x_3^{p_2}(t) + \phi_2(t, x(t), u(t-d_2)), \\ \qquad \vdots \\ \dot{x}_{n-1}(t) = x_n^{p_{n-1}}(t) + \phi_{n-1}(t, x(t), u(t-d_{n-1})), \\ \dot{x}_n(t) = u(t-d_n), \end{cases} \tag{1}$$

where $x(t)=[x_1(t),\dots,x_n(t)]^T \in R^n$ are the states; $u(\cdot)\in R$ is the control input; $d_i \geq 0$, $i=1,2,\dots,n$ are known constant delays and $d=\max_{1\leq i\leq n}\{d_i\}$. In this article, the arguments of functions will be omitted or simplified whenever no confusion can arise from the context. For example, we may denote $x_i(t)$ and $z_i(t)$ by $x_i$ and $z_i$, respectively. For $i=1,2,\dots,n-1$, $\phi_i(t,x,u)$: $R^+\times R^n\times R\to R$ are continuous functions in $(t,x,u)$, $p_i\in R^* =: \{\ell\in R:\ \ell=h/f\geq 1$ for any positive odd integers $h, f\}$.

When $p_i=1$ for all $i=1,2,\dots,n-1$, system (1) reduces to the well-known feedfoward form which has been the focus of major research interests in the past few years, see Zhang, Gao, and Cheng (2006) and the references therein. The problem of globally uniformly asymptotically and locally exponential stabilisation is solved in Mazenc, Mondié, and Francisco (2004)

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when $d_i = 0$ for $i = 1, 2, \ldots, n-1$. Recently the output feedback stabilisation problem is also investigated in Feng and Xu (2010) for the stochastic nonlinear input-delay systems.

When $p_i > 1$, for the case where $p_i$ is positive odd integer (i.e. $f = 1$), there are some interesting features of system (1). For example, the Jacobian linearisation of the $x$-subsystem is uncontrollable and the $x$-subsystem cannot be linearised by state-feedback. Therefore, the traditional design tools are not applicable to system (1). In order to achieve the global robust stabilisation for system (1) and more general systems, Lin, Qian and others have done much work (see, e.g. Frye, Trevino, and Qian (2007) and the references therein). When $d_i = 0$ for $i = 1, 2, \ldots, n$, the system considered here is exactly studied in Frye et al. (2007). Then, in Zhang, Boukas, Liu, and Baron (2010) the state-feedback controller is designed for system (1) by constructing an appropriate Lyapunov–Krasovskii functional and choosing an appropriate state transformation.

However, for more general feedforward input-delay systems in which systems' powers are ratios of odd integers (i.e. $p_i \in R^*$), to the best of authors' knowledge, there are no reports on the control problem for system (1) with $p_i \in R^*$ at the present stage. This article will focus on this problem. This work is to extend the control method proposed in Zhang et al. (2010) from $p_i$ being positive odd integers to $p_i \in R^*$. It should be pointed out that this extension is not direct and easy since the binomial expansion theorem, as one of key technique used in Zhang et al. (2010), is invalid when $p_i \in R^*$. The main contributions are listed as follows.

(1) As far as generalisation is concerned, system (1) studied in this article is the more general system which includes some systems addressed in Mazenc et al. (2004), Frye et al. (2007), Zhang et al. (2006, 2010) as its special cases.
(2) From the technical viewpoint, the mismatched nonlinear functions are dealt with by the new skilful inequality technique instead of the binomial expansion theorem used in Zhang et al. (2010). Thus, the main obstacle is successfully overcome.

The rest of this article is organised as follows. In Section 2, we present problem statement and preliminaries. Section 3 gives the control design procedure and the main results. In Section 4, three examples are provided to verify the control scheme proposed in this article. In Section 5, we conclude the work of this article.

## 2. Problem statement and preliminaries

The goal of this article can be formulated as follows. Assume the nonlinear functions $\phi_i(t, x, \tilde{u}_i)$ satisfy

$$|\phi_i(t,x,\tilde{u}_i)| \le M\left[\sum_{j=i+2}^{n+1} |x_j|^{p_i p_{i+1} \cdots p_{j-1}} + |\tilde{u}_i|^{p_i p_{i+1} \cdots p_{n-1}}\right], \quad (2)$$

where $\tilde{u}_i = u(t - d_i)$, $i = 1, 2, \ldots, n-1$, $M$ is a known positive constant, $x_{n+1} = 0$ and $p_0 = p_n = 1$. Then, find a state-feedback control law of the form

$$u = K\left(x(t), \int_{t-d_n}^{t} u(s)\mathrm{d}s\right) \quad (3)$$

such that the origin $x = 0$ of system (1) is globally asymptotically stable.

**Remark 1:** Condition (2) is similar to Assumption 1 in Zhang et al. (2010), but the powers are extended from positive integers to the rational numbers. This assumption plays an important role in the following procedure of controller design.

In the remainder of this section, we introduce several key lemmas which will serve as the basis for the development of a state-feedback controller for system (1).

**Lemma 1** (Zhang et al. 2010)**:** *For any continuous function* $\alpha(t)$: $[-d, +\infty) \to R^+$, *and* $p \ge 1$, *the following inequality holds*:

$$\left(\int_{t-d}^{t} \alpha(\tau)\mathrm{d}\tau\right)^p \le d^{p-1}\int_{t-d}^{t} \alpha^p(\tau)\mathrm{d}\tau \quad \forall t \in R^+. \quad (4)$$

**Lemma 2** (Qian and Lin 2001b)**:** *Let* $a \ge 0$, $b \ge 0$ *be real numbers and* $p \ge 1$, $q \ge 1$. *Then*

$$a^{p-1}b^q \le a^p + b^{pq}. \quad (5)$$

**Remark 2:** Lemmas 1 and 2 are the consequence of Young's inequality. Their proofs can be found in Qian and Lin (2001b) and Zhang et al. (2010). Here it should be pointed out that Young's inequality holds where $p > 1$, hence Lemmas 1 and 2 also hold where $p \in R^*$.

**Lemma 3** (Beckenbach and Bellman 1961)**:** *For any* $y_i \ge 0$ $(i = 1, 2, \ldots, k)$ *and* $r_i > 0$ $(i = 1, 2, \ldots, k)$ *satisfying* $\sum_{i=1}^{k} r_i = 1$, *the following inequality holds*:

$$y_1^{r_1} y_2^{r_2} \cdots y_k^{r_k} \le r_1 y_1 + r_2 y_2 + \cdots + r_k y_k. \quad (6)$$

**Lemma 4** (Polendo and Qian 2005)**:** *For* $a \in R$, $b \in R$ *and* $q \ge 1$ *is a constant*, *the following inequalities hold*:

$$|a + b|^q \le 2^{q-1}|a^q + b^q|, \quad (7)$$

$$(|a| + |b|)^{\frac{1}{q}} \le |a|^{\frac{1}{q}} + |b|^{\frac{1}{q}}. \quad (8)$$

*If $q \geq 1$ is odd, then,*

$$|a-b|^q \leq 2^{q-1}|a^q - b^q|, \tag{9}$$

$$|a^{1/q} - b^{1/q}| \leq 2^{(q-1)/q}|a-b|^{1/q}. \tag{10}$$

**Lemma 5** (Ji and Xi 2006)**:** *Suppose n and m are two positive real numbers, and $a \geq 0$, $b \geq 0$ and $\pi \geq 0$ are continuous functions. Then, for any constant $c > 0$,*

$$a^n b^m \pi \leq c a^{n+m} + \frac{m}{n+m}\left[\frac{n}{c(n+m)}\right]^{\frac{n}{m}} b^{n+m} \pi^{\frac{n+m}{m}}. \tag{11}$$

**Lemma 6** (Zhang et al. 2006)**:** *Let $r \in R^*$ and $r > 1$, $x$, $y$ be real-valued functions. For a constant $c > 0$, one has*

$$\begin{aligned} |x^r - y^r| &\leq r|x-y|(x^{r-1} + y^{r-1}) \\ &\leq c|x-y||(x-y)^{r-1} + y^{r-1}|, \end{aligned} \tag{12}$$

*where $c = r$ for $1 < r \leq 2$ and $c = r2^{r-1}$ for $r > 2$.*

**Lemma 7:** *For the nonlinear system described by the equations of the form*

$$\dot{z}_1 = z_2^{p_1}, \quad \dot{z}_2 = z_3^{p_2}, \quad \cdots, \quad \dot{z}_{n-1} = z_n^{p_{n-1}}, \quad \dot{z}_n = v, \tag{13}$$

*where $p_i \in R^*$ $(i = 1, 2, \ldots, n-1)$, there is a positive definite and proper Lyapunov function $\tilde{W}_n(z)$, and a continuous state-feedback controller $v(t)$, such that*

$$\dot{\tilde{W}}_n(z)|_{(13)} \leq -(\xi_1^2 + \xi_2^2 + \cdots + \xi_n^2),$$

*where $\tilde{W}_n(z)$ and $v(t)$ are given by the form:*

$$\tilde{W}_n(z) = \sum_{i=1}^{n} W_i(z_i), \quad v(t) = -\beta_n \xi_n^{\frac{1}{p_1 p_2 \cdots p_{n-1}}},$$

$$W_i(z_i) = \int_{z_i^*}^{z_i} \left(s^{p_0 p_1 \cdots p_{i-1}} - z_i^{*p_0 p_1 \cdots p_{i-1}}\right)^{2 - \frac{1}{p_0 p_1 \cdots p_{i-1}}} \mathrm{d}s, \quad i = 1, 2, \ldots, n,$$

$$\begin{cases} z_1^* = 0, & \xi_1 = z_1 - z_1^*, \\ z_i^{*p_1 \cdots p_{i-1}} = -\beta_{i-1}\xi_{i-1}, & \\ \xi_i = z_i^{p_1 \cdots p_{i-1}} - z_i^{*p_1 \cdots p_{i-1}}, & i = 2, 3, \ldots, n, \end{cases} \tag{14}$$

*with $z = [z_1, z_2, \ldots, z_n]^T$, $\beta_i > 0$, $i = 1, 2, \ldots, n$, being proper real constants.*

**Proof:** Based on Qian and Lin (2001a) and Frye et al. (2007), the results of Lemma 7 can be easily obtained by employing Lemma 2. The proof is omitted here.

## 3. Global stabilisation by state-feedback

In this section, we will develop a constructive control design method for system (1), which makes the closed-loop system globally asymptotically stable. We first make an equivalent transformation:

$$\tilde{x}_n = x_n + \int_{t-d_n}^{t} u(s)\mathrm{d}s$$

and then system (1) can be transformed into

$$\begin{cases} \dot{x}_i(t) = x_{i+1}^{p_i}(t) + \phi_i, \quad i = 1, 2, \ldots, n-2, \\ \dot{x}_{n-1}(t) = \phi_{n-1} + \left(\tilde{x}_n - \int_{t-d_n}^{t} u(s)\mathrm{d}s\right)^{p_{n-1}}, \\ \dot{\tilde{x}}_n(t) = u(t), \end{cases} \tag{15}$$

where $\phi_i = \phi_i\left(t, x_1, \ldots, x_{n-1}, \tilde{x}_n - \int_{t-d_n}^{t} u(s)\mathrm{d}s, \tilde{u}_i\right)$, $i = 1, 2, \ldots, n-1$.

**Remark 3:** The term $\left(\tilde{x}_n - \int_{t-d_n}^{t} u(s)\mathrm{d}s\right)^{p_{n-1}}$ is dealt with by employing the binomial expansion theorem when $p_{n-1}$ is an odd integer in Zhang et al. (2010). However, the binomial expansion theorem is invalid when $p_{n-1} \in R^*$. In this section, this main obstacle is overcome by introducing the new skilful inequality technique.

Noting that if $(x_1, \ldots, x_{n-1}, \tilde{x}_n)$ and $u$ converge to zero asymptotically as $t \to +\infty$, $(x_1, \ldots, x_{n-1}, x_n)$ converges to zero asymptotically as well. Letting

$$\begin{cases} x_i = L^{1 - \sum_{m=1}^{i} \frac{p_0 p_1 \cdots p_{m-1}}{p_0 p_1 \cdots p_{i-1}}} z_i, \quad i = 1, 2, \ldots, n-1, \\ \tilde{x}_n = L^{1 - \sum_{m=1}^{n} \frac{p_0 p_1 \cdots p_{m-1}}{p_0 p_1 \cdots p_{n-1}}} z_n, \\ u = L^{-\sum_{m=1}^{n} \frac{p_0 p_1 \cdots p_{m-1}}{p_0 p_1 \cdots p_{n-1}}} v, \\ \tilde{u}_i = L^{-\sum_{m=1}^{n} \frac{p_0 p_1 \cdots p_{m-1}}{p_0 p_1 \cdots p_{n-1}}} \tilde{v}_i, \end{cases} \tag{16}$$

where $\tilde{v}_i = v(t - d_i)$, and $L > 1$ is an unknown constant, which will be determined later; we can derive from (15) that

$$\begin{cases} \dot{z}_i = L^{-1} z_{i+1}^{p_i} + \varphi_i, \quad i = 1, 2, \ldots, n-1, \\ \dot{z}_n = L^{-1} v(t) + \varphi_n, \end{cases} \tag{17}$$

with

$$\begin{cases} \varphi_i = L^{-1 + \sum_{m=1}^{i} \frac{p_0 p_1 \cdots p_{m-1}}{p_0 p_1 \cdots p_{i-1}}} \phi_i, \quad i = 1, 2, \ldots, n-2, \\ \varphi_{n-1} = L^{-1 + \sum_{m=1}^{n-1} \frac{p_0 p_1 \cdots p_{m-1}}{p_0 p_1 \cdots p_{n-2}}} \\ \quad (\phi_{n-1} + (\tilde{x}_n - \int_{t-d_n}^{t} u(s)\mathrm{d}s)^{p_{n-1}} - \tilde{x}_n^{p_{n-1}}), \\ \varphi_n = 0. \end{cases} \tag{18}$$

Next, we will choose an appropriate constant $L>1$ such that the system (17) with

$$v(t) = -\beta_n \xi_n^{\frac{1}{p_0 \cdots p_{n-1}}}$$

is globally asymptotically stable at $z=0$, where $\beta_n$ and $\xi_n$ are defined as the ones in (14). Taking the derivative of $\tilde{W}_n(z)$ with respect to $t$, we have

$$\begin{aligned}
&\dot{\tilde{W}}_n(z)|_{(15),(17)} \\
&= \sum_{i=1}^{n} \frac{\partial \tilde{W}_n}{\partial z_i} \dot{z}_i \\
&= \sum_{i=1}^{n} \frac{\partial W_i}{\partial z_1} \dot{z}_1 + \cdots + \sum_{i=n-1}^{n} \frac{\partial W_i}{\partial z_{n-1}} \dot{z}_{n-1} + \frac{\partial W_n}{\partial z_n} \dot{z}_n \\
&= \sum_{i=1}^{n} \frac{\partial W_i}{\partial z_1} (L^{-1} z_2^{p_1} + \varphi_1) + \cdots \\
&\quad + \sum_{i=n-1}^{n} \frac{\partial W_i}{\partial z_{n-1}} (L^{-1} z_n^{p_{n-1}} + \varphi_{n-1}) + \frac{\partial W_n}{\partial z_n} (L^{-1} v(t) + \varphi_n) \\
&= L^{-1} \left( \sum_{i=1}^{n} \frac{\partial W_i}{\partial z_1} z_2^{p_1} + \cdots + \sum_{i=n-1}^{n} \frac{\partial W_i}{\partial z_{n-1}} z_n^{p_{n-1}} + \frac{\partial W_n}{\partial z_n} v(t) \right) \\
&\quad + \left( \sum_{i=1}^{n} \frac{\partial W_i}{\partial z_1} \varphi_1 + \cdots + \sum_{i=n-1}^{n} \frac{\partial W_i}{\partial z_{n-1}} \varphi_{n-1} \right), \qquad (19)
\end{aligned}$$

where $\tilde{W}_n(z)$ and $W_i$ are defined as that in Lemma 7. Employing Lemma 7, we can obtain

$$\dot{\tilde{W}}_n(z)|_{(15),(17)} \leq -L^{-1}(\xi_1^2 + \xi_2^2 + \cdots + \xi_n^2) + \sum_{i=1}^{n-1} \left| \frac{\partial \tilde{W}_n}{\partial z_i} \varphi_i \right|. \qquad (20)$$

Some terms on the right-hand side of (20), namely $\left| \frac{\partial \tilde{W}_n}{\partial z_i} \varphi_i \right|$, can be estimated by the following proposition.

**Proposition 1:** *For system* (17), *under condition* (2), *for any* $i = 1, 2, \ldots, n-1$, *there exist positive constants* $\lambda_{i,1}$, $\lambda_{i,2}$, $\lambda_{i,3}$ *and* $\mu_{i,j}$ $(j = 1, 2, \ldots, n)$, *such that*

$$\left| \frac{\partial \tilde{W}_n}{\partial z_i} \varphi_i \right| \leq \frac{1}{L^2} \left( \lambda_{i,1} \xi_n^2 + \lambda_{i,2} \xi_n^2 (t - d_i) + \lambda_{i,3} \int_{t-d_n}^{t} \xi_n^2(s) \mathrm{d}s + \sum_{j=1}^{n} \mu_{i,j} \xi_j^2 \right). \qquad (21)$$

**Proof:** See the appendix.

Hence, from (20) and (21), one has

$$\begin{aligned}
&\dot{\tilde{W}}_n(z)|_{(15),(17)} \\
&\leq -L^{-1}(\xi_1^2 + \xi_2^2 + \cdots + \xi_n^2) + \frac{1}{L^2} \left( \sum_{i=1}^{n-1} \lambda_{i,1} \right) \xi_n^2 \\
&\quad + \frac{1}{L^2} \left( \sum_{i=1}^{n-1} \lambda_{i,2} \xi_n^2 (t - d_i) \right) + \frac{1}{L^2} \left( \sum_{i=1}^{n-1} \lambda_{i,3} \right) \int_{t-d_n}^{t} \xi_n^2(s) \mathrm{d}s \\
&\quad + \frac{1}{L^2} \sum_{j=1}^{n} \left( \sum_{i=1}^{n-1} \mu_{i,j} \right) \xi_j^2. \qquad (22)
\end{aligned}$$

Consider the following functional

$$\begin{aligned}
V = \tilde{W}_n(z) &+ \frac{1}{L^2} \left( \sum_{i=1}^{n-1} \lambda_{i,2} \int_{t-d_i}^{t} \xi_n^2(s) \mathrm{d}s \right) \\
&+ \frac{1}{L^2} \left( \sum_{i=1}^{n-1} \lambda_{i,3} \right) \int_{-d_n}^{0} \int_{\theta+t}^{t} \xi_n^2(s) \mathrm{d}s \, \mathrm{d}\theta. \qquad (23)
\end{aligned}$$

We can conclude that the functional $V$ is a Lypaunov–Krasovskii functional which can be used to study the stability property of the nonlinear input delays system (1), see Zhang et al. (2010).

It follows from (23) that

$$\begin{aligned}
\dot{V}(z) &\leq -L^{-1}(\xi_1^2 + \xi_2^2 + \cdots + \xi_n^2) + \frac{1}{L^2} \left( \sum_{i=1}^{n-1} \lambda_{i,1} \right) \xi_n^2 \\
&\quad + \frac{1}{L^2} \left( \sum_{i=1}^{n-1} \lambda_{i,2} \xi_n^2 (t - d_i) \right) \\
&\quad + \frac{1}{L^2} \left( \sum_{i=1}^{n-1} \lambda_{i,3} \right) \int_{t-d_n}^{t} \xi_n^2(s) \mathrm{d}s + \frac{1}{L^2} \sum_{j=1}^{n} \left( \sum_{i=1}^{n-1} \mu_{i,j} \right) \xi_j^2 \\
&\quad + \frac{1}{L^2} \left( \sum_{i=1}^{n-1} \lambda_{i,2} \xi_n^2 \right) - \frac{1}{L^2} \left( \sum_{i=1}^{n-1} \lambda_{i,2} \xi_n^2 (t - d_i) \right) \\
&\quad + \frac{1}{L^2} \left( \sum_{i=1}^{n-1} d_n \lambda_{i,3} \right) \xi_n^2 - \frac{1}{L^2} \left( \sum_{i=1}^{n-1} \lambda_{i,3} \right) \int_{-d_n}^{0} \xi_n^2(\theta + t) \mathrm{d}\theta \\
&= -L^{-1}(\xi_1^2 + \xi_2^2 + \cdots + \xi_n^2) \\
&\quad + \frac{1}{L^2} \left( \sum_{i=1}^{n-1} (\lambda_{i,1} + \lambda_{i,2} + d_n \lambda_{i,3}) \right) \xi_n^2 + \frac{1}{L^2} \sum_{j=1}^{n} \left( \sum_{i=1}^{n-1} \mu_{i,j} \right) \xi_j^2. \qquad (24)
\end{aligned}$$

Letting $\delta_0 = \sum_{i=1}^{n-1} (\lambda_{i,1} + \lambda_{i,2} + d_n \lambda_{i,3})$ and $\delta_j = \sum_{i=1}^{n-1} \mu_{i,j}$, from (24), one has

$$\dot{V}(z) \leq -L^{-1}(\xi_1^2 + \xi_2^2 + \cdots + \xi_n^2) + \frac{1}{L^2} \delta_0 \xi_n^2 + \frac{1}{L^2} \sum_{j=1}^{n} \delta_j \xi_j^2. \qquad (25)$$

It is concluded that the right hand side of this inequality can be rendered negative definite by choosing a large $L$. Now choosing

$$L > \max_{1 \leq j \leq n} \{\delta_0 + \delta_j\} + \varepsilon, \qquad (26)$$

where $\varepsilon$ is a positive constant, one can get

$$\dot{V}(z) \leq -\frac{\varepsilon}{L^2} (\xi_1^2 + \xi_2^2 + \cdots + \xi_n^2),$$

which indicates that the trivial solution of the closed-loop systems of (15) and (17) are globally asymptotically stable. So the trivial solution of the closed-loop system with

$$\begin{aligned}u(t) &= L^{-1-\sum_{m=1}^{n-1}\frac{1}{p_m\cdots p_{n-1}}}v\\ &= -L^{-1-\sum_{m=1}^{n-1}\frac{1}{p_m\cdots p_{n-1}}}\beta_n\xi_n^{\frac{1}{p_1\cdots p_{n-1}}}\\ &= -L^{-1-\sum_{m=1}^{n-1}\frac{1}{p_m\cdots p_{n-1}}}\beta_n\left[\left(\sum_{i=1}^{n-1}b_i z_i^{p_0\cdots p_{n-1}}\right)+z_n^{p_0\cdots p_{n-1}}\right]^{\frac{1}{p_1\cdots p_{n-1}}}\\ &= -\frac{\beta_n}{L}\left[\left(\sum_{i=1}^{n-1}L^{-\sum_{m=i}^{n-1}p_0\cdots p_{m-1}}b_i x_i^{p_0\cdots p_{i-1}}\right)\right.\\ &\quad\left.+\left(x_n+\int_{t-d_n}^{t}u(s)\mathrm{d}s\right)^{p_1\cdots p_{n-1}}\right]^{\frac{1}{p_1\cdots p_{n-1}}} \end{aligned}\tag{27}$$

is globally asymptotically stable, where $L$ satisfies inequality (26), $b_i=\beta_i\cdots\beta_{n-1}$ are constants, $\beta_i$ are defined as in (14).

Now, we give the main result of this article.

**Theorem 1:** *Under the condition* (2), *there exists a control law* (27) *with design parameters* $\beta_i$, $i=1,\ldots,n$, *and L given in* (14) *and* (26), *respectively*, *such that the closed-loop system with the input delays consisting of system* (1), *the control input* (27), *is globally asymptotically stable*.

**Remark 4:** From (25)–(27), it can be seen that the proposed controller for the system (1) is dependent on the delay $d_n$, and independent of the delays $d_i$, $i=1,2,\ldots,n-1$. Based on (26), it is easily known that the design parameter $L$ will be larger when the delay $d_n$ becomes larger, consequently, the gain of the controller will become very small and the convergence rate of the state of closed-loop system will become very slow as stated in Zhang et al. (2010).

**Remark 5:** The method proposed in this section can be further extended to solve the control problem for the following input-delay nonlinear system with bounded disturbances

$$\begin{cases}\dot{x}_1(t)=x_2^{p_1}(t)+\phi_1(t,x(t),u(t-d_1))\\ \qquad +\Delta_1(t,x(t),u(t-d_1)),\\ \dot{x}_2(t)=x_3^{p_2}(t)+\phi_2(t,x(t),u(t-d_2))\\ \qquad +\Delta_2(t,x(t),u(t-d_2)),\\ \quad\vdots\\ \dot{x}_{n-1}(t)=x_n^{p_{n-1}}(t)+\phi_{n-1}(t,x(t),u(t-d_{n-1}))\\ \qquad +\Delta_{n-1}(t,x(t),u(t-d_{n-1})),\\ \dot{x}_n(t)=u(t-d_n),\end{cases}\tag{28}$$

where $p_i$, $d_i$, $\phi_i(\cdot)$ and $u(\cdot)$ are defined as in system (1); $\phi_i$ satisfies condition (2) and $\Delta_i(\cdot)$ is the bounded disturbance, i.e. $|\Delta_i(\cdot)|\le\tilde{\Delta}_i^{\frac{1}{p_0\cdots p_{i-1}}}$ with the known positive constant $\tilde{\Delta}_i$. Based on the similar proof of Theorem 5.1 in Ho, Li, and Niu (2005), extending the method in this section, we only obtain a controller of the form (27) such that $\dot{V}(x)\le -W(x)+\delta$ with $W(x)$ and $\delta$ being a positive-definite function and a positive constant, respectively, which implies all signals of the closed-loop system are uniformly ultimately bounded. The control performance can be guaranteed by choosing appropriate design parameters.

## 4. Simulation study

In this section, we provide three numerical examples to illustrates the effectiveness of the proposed stabiliser by state-feedback. It should be pointed that the existing results in Zhang et al. (2010) are not applicable to the following examples because the systems are considered here with ratios of odd integers. The binomial expansion theorem, as one of key technique used in Zhang et al. (2010), is invalid when $p_i\in R^*$.

**Example 1:** Consider the following nonlinear time-delay system

$$\begin{cases}\dot{x}_1=x_2^{5/3}+u^{5/3}(t-0.5),\\ \dot{x}_2=u(t-0.2),\end{cases}\tag{29}$$

**Step 1:** We design a controller for the following system:

$$\begin{cases}\dot{z}_1=z_2^{5/3},\\ \dot{z}_2=u.\end{cases}\tag{30}$$

Choosing $V_1=\frac{1}{2}z_1^2$, we get

$$\dot{V}_1|_{(29)}=z_1\cdot z_2^{5/3}=z_1\cdot(z_2^{5/3}-z_2^{*5/3})+z_1z_2^{*5/3},$$

where $z_2^*$ is the virtual controller. Letting

$$z_2^{*5/3}=-\frac{1}{2}z_1,\quad \xi_2=z_2^{5/3}-z_2^{*5/3}=z_2^{5/3}+\frac{1}{2}z_1$$

we obtain

$$\dot{V}_1|_{(29)}=-\frac{1}{2}z_1^2+z_1\xi_2.\tag{31}$$

Next, choosing

$$V_2=V_1+5\int_{z_2^*}^{z_2}(s^{5/3}-z_2^{*5/3})^{7/5}\mathrm{d}s,\quad u=-7\xi_2^{3/5},$$

we have

$$\begin{aligned}\dot{V}_2|_{(29)}&=-\frac{1}{2}z_1^2+z_1\xi_2-35\xi_2^2\\ &\quad+7\int_{z_2^*}^{z_2}(s^{5/3}-z_2^{*5/3})^{2/5}\mathrm{d}s\left(\frac{1}{2}z_2^{5/3}\right)\end{aligned}$$

$$\leq -\frac{1}{2}z_1^2 + z_1\xi_2 - 35\xi_2^2 + \frac{7}{2}|z_2 - z_2^*|\xi_2^{2/5}z_2^{5/3}$$
$$\leq -\frac{1}{2}z_1^2 + z_1\xi_2 - 35\xi_2^2 + \frac{7}{2}\cdot 2^{2/5}|\xi_2||\xi_2 - \frac{1}{2}z_1|$$
$$\leq -0.3z_1^2 - 14.7\xi_2^2. \tag{32}$$

**Step 2:** Let

$$\begin{cases} x_1 = z_1, \\ \tilde{x}_2 = L^{-3/5}z_2, \\ \tilde{x}_2 = x_2 + \int_{t-0.2}^{t} u(s)\mathrm{d}s, \\ u = L^{-1-3/5}v, \\ v = -7\xi_2^{3/5}. \end{cases}$$

System (29) can be transformed into

$$\begin{cases} \dot{z}_1 = \frac{1}{L}z_2^{5/3} + \phi(\cdot), \\ \dot{z}_2 = \frac{1}{L}v, \end{cases} \tag{33}$$

where

$$\phi(\cdot) = \left(\tilde{x}_2 - \int_{t-0.2}^{t} u(s)\mathrm{d}s\right)^{5/3} - \tilde{x}_2^{5/3} + u^{5/3}(t-0.5).$$

Using Lemmas 1, 4–6, we can get

$$|\phi(\cdot)| \leq \left|\left(\tilde{x}_2 - \int_{t-0.2}^{t} u(s)\mathrm{d}s\right)^{5/3} - \tilde{x}_2^{5/3}\right| + |u^{5/3}(t-0.5)|$$
$$\leq \frac{5}{3}\left|\int_{t-0.2}^{t} u(s)\mathrm{d}s\right|\left|\left(\int_{t-0.2}^{t} u(s)\mathrm{d}s\right)^{2/3} + \tilde{x}_2^{2/3}\right| + |u^{5/3}(t-0.5)|$$
$$\leq \frac{5}{3}\left|\int_{t-0.2}^{t} u(s)\mathrm{d}s\right|^{5/3} + \frac{5}{3}\left|\int_{t-0.2}^{t} u(s)\mathrm{d}s\right|\tilde{x}_2^{2/3} + |u^{5/3}(t-0.5)|$$
$$= 45.85L^{-8/3}\left|\int_{t-0.2}^{t} \xi_2^{3/5}(s)\mathrm{d}s\right|^{5/3} + 11.7L^{-2}\left|\int_{t-0.2}^{t} \xi_2^{3/5}(s)\mathrm{d}s\right|z_2^{2/3} + 27.5L^{-8/3}|\xi_2(t-0.5)|. \tag{34}$$

Since

$$\left|\int_{t-0.2}^{t} \xi_2^{3/5}(s)\mathrm{d}s\right|^{5/3} \leq 0.2^{2/3}\left|\int_{t-0.2}^{t} \xi_2(s)\mathrm{d}s\right| \leq 0.35\int_{t-0.2}^{t} |\xi_2(s)|\mathrm{d}s, \tag{35}$$

and

$$\left|\int_{t-0.2}^{t} \xi_2^{3/5}(s)\mathrm{d}s\right|z_2^{2/3} \leq |z_2|^{5/3} + \left|\int_{t-0.2}^{t} \xi_2^{3/5}(s)\mathrm{d}s\right|^{5/3} \leq |z_2|^{5/3} + 0.35\int_{t-0.2}^{t} |\xi_2(s)|\mathrm{d}s, \tag{36}$$

substituting (35) and (36) into (34) results in

$$|\phi(\cdot)| \leq 11.7L^{-2}|z_2|^{5/3} + (15.7L^{-8/3} + 4.1L^{-2}) \times \int_{t-0.2}^{t} |\xi_2(s)|\mathrm{d}s + 27.5L^{-8/3}|\xi_2(t-0.5)|. \tag{37}$$

**Step 3:** From (32) and (37), we can obtain

$$\dot{V}_2|_{(32)} = \frac{1}{L}(-0.3z_1^2 - 14.7\xi_2^2) + (|z_1| + 5|\xi_2|) \times (11.7L^{-2}|z_2|^{5/3} + (15.7L^{-8/3} + 4.1L^{-2}) \times \int_{t-0.2}^{t} |\xi_2(s)|\mathrm{d}s + 27.5L^{-8/3}|\xi_2(t-0.5)|)$$
$$\leq \frac{1}{L^2}\Bigg[(-0.3L + 9.05 + 35.5L^{-2/3})z_1^2 + (-14.7L + 189.15 + 108.05L^{-2/3})\xi_2^2 + (55L^{-2/3} + 14.25)\int_{t-0.2}^{t} \xi_2^2(s)\mathrm{d}s + 96.25L^{-2/3}\xi_2^2(t-0.5)\Bigg]. \tag{38}$$

Choosing

$$V = V_2 + 96.25L^{-8/3}\int_{t-0.5}^{t} \xi_2^2(s)\mathrm{d}s + (55L^{-8/3} + 14.25L^{-2})\int_{-0.2}^{0}\int_{t+\theta}^{t} \xi_2^2(s)\mathrm{d}s\,\mathrm{d}\theta$$

and applying (38), we have

$$\dot{V}|_{(32)} \leq L^{-2}[(-0.3L + 9.05 + 35.5L^{-2/3})z_1^2 + (-14.7L + 192.15 + 215.75L^{-2/3})\xi_2^2]. \tag{39}$$

Now we can find $L > 1$ such that

$$-0.3L + 9.05 + 35.5L^{-2/3} < 0$$

and

$$-14.7L + 192.15 + 215.75L^{-2/3} < 0.$$

Let $L = 45$. From (39), we can obtain $\dot{V}|_{(32)} \leq -0.0006989z_1^2 - 0.2242\xi_2^2$. Hence, we get the state-feedback controller for the system (29)

$$u(t) = -0.1556\left(\left(x_2 + \int_{t-0.2}^{t} u(s)\mathrm{d}s\right)^{5/3} + 0.0112x_1\right)^{3/5}.$$

In simulation, the initial values are set to be $x_1(0) = 3$ and $x_2(0) = 2$. The delays are specified as $d_1 = 0.5$ and $d_2 = 0.2$. Simulation results are shown in Figure 1, from which we can see that the system state indeed converges to zero, which accords with Theorem 1.

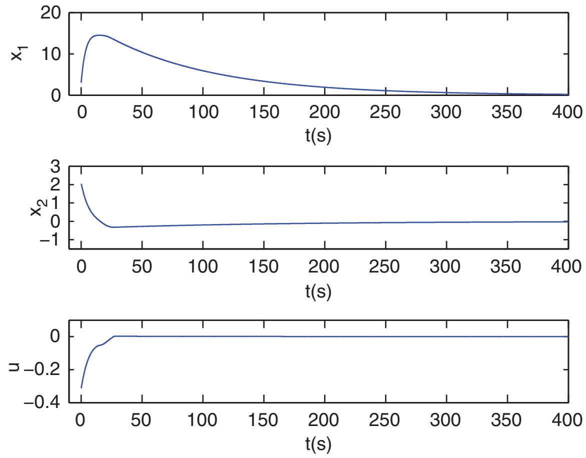


Figure 1. Simulation curves for system (29).

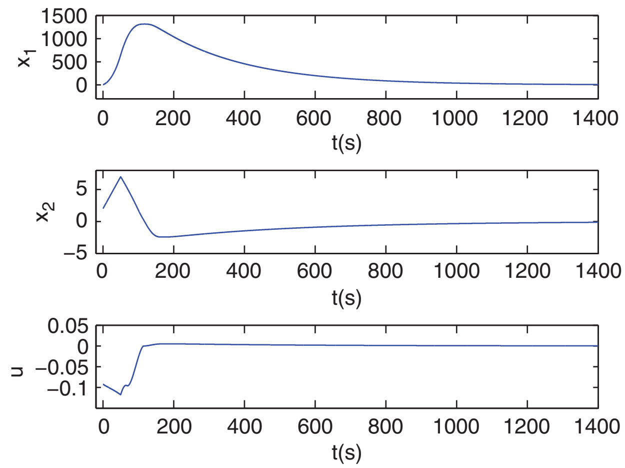


Figure 2. Simulation curves for system (40).



**Example 2:** Now consider another nonlinear time-delay system of the form:

$$\begin{cases} \dot{x}_1 = x_2^{5/3} + u^{5/3}(t-0.5), \\ \dot{x}_2 = u(t-50). \end{cases} \tag{40}$$

Obviously, the system (40) is similar to the system (29). By using the three steps in Example 1, we can obtain $L = 150$. The state-feedback controller can be given as follows:

$$u(t) = -0.046\left(\left(x_2 + \int_{t-50}^{t} u(s)\mathrm{d}s\right)^{5/3} + 0.0033x_1\right)^{3/5}.$$

In simulation, the initial values are set to be $x_1(0) = 3$ and $x_2(0) = 2$. The delays are specified as $d_1 = 0.5$ and $d_2 = 50$. Simulation results are shown in Figure 2.

**Remark 6:** By comparing Example 1 with Example 2, we can see that only the second delay is different. We can obtain controllers with similar construction by the similar design procedure for the two examples. According to the conclusion in Remark 5, the proposed controllers are dependent on the second delays in the systems. The design parameter $L$ will become larger when the last delay becomes larger, consequently, the gain of the controller will become very small and the convergence rate of the state of closed-loop system will become very slow, see Figures 1 and 2.

**Example 3:** Consider the following nonlinear time-delay system with different fractional powers

$$\begin{cases} \dot{x}_1 = x_2^{5/3} + u^{7/3}(t-0.7), \\ \dot{x}_2 = x_3^{7/5}, \\ \dot{x}_3 = u(t-0.1). \end{cases} \tag{41}$$

**Step 1:** We design a controller for the following system:

$$\begin{cases} \dot{z}_1 = z_2^{5/3}, \\ \dot{z}_2 = z_3^{7/5}, \\ \dot{z}_3 = u. \end{cases} \tag{42}$$

Choosing $V_1 = \frac{1}{2}z_1^2$, $z_2^{*5/3} = -\frac{1}{2}z_1, \xi_2 = z_2^{5/3} - z_2^{*5/3} = z_2^{5/3} + \frac{1}{2}z_1$, we get

$$\begin{aligned} \dot{V}_1|_{(42)} = z_1 \cdot z_2^{5/3} &= z_1 \cdot (z_2^{5/3} - z_2^{*5/3}) + z_1 z_2^{*5/3} \\ &= -\frac{1}{2}z_1^2 + z_1\xi_2. \end{aligned} \tag{43}$$

Next, choosing

$$V_2 = V_1 + \frac{1}{2}\int_{z_2^*}^{z_2} (s^{5/3} - z_2^{*5/3})^{7/5}\mathrm{d}s, \quad z_3^{*7/3} = -10.1\xi_2,$$
$$\xi_3 = z_3^{7/3} + 10.1\xi_2,$$

we have

$$\begin{aligned} \dot{V}_2|_{(42)} = &-\frac{1}{2}z_1^2 + z_1\xi_2 + \frac{1}{2}\xi_2^{7/5}z_3^{7/5} \\ &+ \frac{7}{10}\int_{z_2^*}^{z_2}(s^{5/3} - z_2^{*5/3})^{2/5}\mathrm{d}s\left(\frac{1}{2}z_2^{5/3}\right) \\ \le &-\frac{1}{2}z_1^2 + z_1\xi_2 + \frac{7}{20}|z_2 - z_2^*|\xi_2^{2/5}z_2^{5/3} \\ &+ \frac{1}{2}\xi_2^{7/5}(z_3^{7/5} - z_3^{*7/5}) + \frac{1}{2}\xi_2^{7/5}z_3^{*7/5} \\ \le &-0.2z_1^2 - 0.45\xi_2^2 + 0.645\xi_2\xi_3. \end{aligned} \tag{44}$$

Finally, by choosing

$$V_3 = V_2 + \frac{1}{110}\int_{z_3^*}^{z_3}(s^{7/3} - z_3^{*7/3})^{11/7}\mathrm{d}s, u = -750\xi_3^{3/7},$$

then

$$\begin{aligned}\dot{V}_3|_{(42)} &\le -0.2z_1^2 - 0.45\xi_2^2 + 0.645\xi_2\xi_3 + \frac{1}{70}\xi_3^{11/7}u \\ &\quad + \frac{1}{70}\int_{z_3^*}^{z_3}(s^{7/3} - z_3^{*7/3})^{4/7}\mathrm{d}s \\ &\quad \times (16.67z_2^{2/3}z_3^{7/5} + 5.05z_2^{5/3}),\end{aligned} \tag{45}$$

since $z_2^{2/3} = (\xi_2 - \frac{1}{2}\xi_1)^{2/5}$, and $z_3^{7/5} = (\xi_3 - 1001\xi_2)^{3/5}$ and by applying Lemmas 4–6, we obtain

$$\dot{V}_3|_{(42)} \le -0.1425z_1^2 - 0.1063\xi_2^2 - 0.3417\xi_3^2. \tag{46}$$

**Step 2:** Let

$$\begin{cases} x_1 = z_1, \\ x_2 = L^{-3/5}z_2, \\ \tilde{x}_3 = L^{-8/7}z_3, \\ \tilde{x}_3 = x_3 + \int_{t-0.1}^{t} u(s)\mathrm{d}s, \\ u = L^{-1-3/7-5/7}v, \\ v = -750\xi_3^{3/7}. \end{cases}$$

System (41) can be transformed into

$$\begin{cases} \dot{z}_1 = \dfrac{1}{L}z_2^{5/3} + \varphi_1, \\ \dot{z}_2 = \dfrac{1}{L}z_2^{7/5} + \varphi_2, \\ \dot{z}_3 = \dfrac{1}{L}v, \end{cases} \tag{47}$$

where

$$\varphi_1 = u^{7/3}(t-0.7),$$

$$\varphi_2 = L^{3/5}\left(\left(\tilde{x}_3 - \int_{t-0.1}^{t} u(s)\mathrm{d}s\right)^{7/5} - \tilde{x}_3^{7/5}\right).$$

By similar derivation of (34)–(36), we get

$$|\varphi_1| \le 5110538.89L^{-5}|\xi_3(t-0.7)|, \tag{48}$$

and

$$\begin{aligned}|\varphi_2| &\le (5784.667L^{-12/5} + 409.5L^{-2})\int_{t-0.1}^{t}|\xi_3|^{3/5}\mathrm{d}s \\ &\quad + 105L^{-2}|z_3|^{7/5}.\end{aligned} \tag{49}$$

**Step 3:** From (46), (48) and (49), we can obtain

$$\begin{aligned}\dot{V}_2|_{(47)} &\le \frac{1}{L}(-0.145z_1^2 - 0.1063\xi_2^2 - 0.3417\xi_3^2) \\ &\quad + \left(|z_1| + 0.69|\xi_2| + 0.1071|\xi_2 - \frac{1}{2}\xi_1|\right) \\ &\quad \times (5110538.89L^{-5}|\xi_3(t-0.7)|) \\ &\quad + \left(\frac{1}{2}|\xi_2|^{7/5} + 0.3535|\xi_2 - \frac{1}{2}\xi_1|^{2/5}|\xi_3|\right) \\ &\quad \times \left((5784.667L^{-12/5} + 409.5L^{-2})\right. \\ &\quad \left. \times \int_{t-0.1}^{t}|\xi_3|^{3/5}\mathrm{d}s + 105L^{-2}|z_3|^{7/5}\right) \\ &\le \frac{1}{L^2}\Big[(-0.145L + 157.83 + 150.6L^{-2/5} \\ &\quad + 5657365.56L^{-3})z_1^2 + (-0.1063L + 107.73 \\ &\quad + 244.83L^{-2/5} + 9157447L^{-3})\xi_2^2 \\ &\quad + (-0.3417L + 313 + 608.4L^{-2/5})\xi_3^2 \\ &\quad + (5347.85L^{-2/5} + 205)\int_{t-0.1}^{t}\xi_3^2(s)\mathrm{d}s \\ &\quad + 4940612.65L^{-3}\xi_3^2(t-0.7)\Big].\end{aligned} \tag{50}$$

Choosing

$$\begin{aligned}V &= V_3 + 4940612.65L^{-5}\int_{t-0.7}^{t}\xi_3^2(s)\mathrm{d}s \\ &\quad + (5347.85L^{-12/5} + 205L^{-2})\int_{-0.1}^{0}\int_{t+\theta}^{t}\xi_3^2(s)\mathrm{d}s\,\mathrm{d}\theta\end{aligned}$$

and applying (50), we have

$$\begin{aligned}\dot{V}|_{(47)} &\le L^{-2}\big[(-0.145L + 157.83 + 150.6L^{-2/5} \\ &\quad + 5657365.56L^{-3})z_1^2 + (-0.1063L + 107.73 \\ &\quad + 244.83L^{-2/5} + 9157447L^{-3})\xi_2^2 \\ &\quad + (-0.3417L + 333.5 + 1143.185L^{-2/5} \\ &\quad + 4940612.65L^{-3})\xi_3^2\big].\end{aligned} \tag{51}$$

Now we can find a large enough $L > 1$ such that

$$-0.145L + 157.83 + 150.6L^{-2/5} + 5657365.56L^{-3} < 0,$$

$$-0.1063L + 107.73 + 244.83L^{-2/5} + 9157447L^{-3} < 0$$

and

$$-0.3417L + 333.5 + 1143.185L^{-2/5} + 4940612.65L^{-3} < 0.$$

Let $L = 1200$. From (51), we can obtain

$$\begin{aligned}\dot{V}|_{(47)} &\le -0.000005118z_1^2 - 0.0000042062\xi_2^2 \\ &\quad - 0.0000065715\xi_3^2.\end{aligned}$$

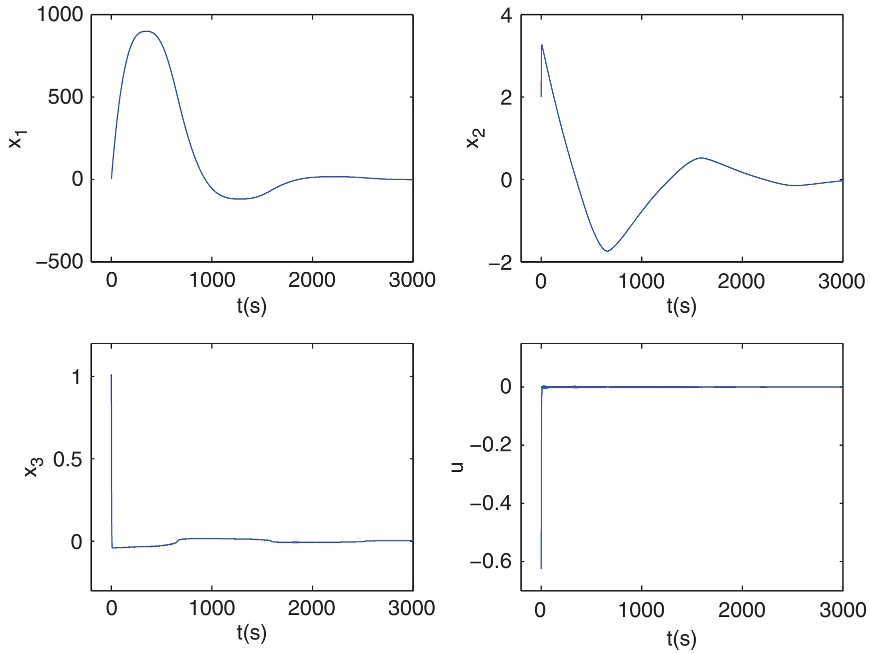


Figure 3. Simulation curves for system (41).

Hence, we get the following state-feedback controller for the system (41)

$$u(t) = -0.625\Bigg(0.0000037x_1 + 0.0007378x_2^{5/3} + \left(x_3 + \int_{t-0.1}^{t} u(s)\mathrm{d}s\right)^{7/3}\Bigg)^{3/7}.$$

In simulation, the initial values are set to be $x_1(0) = 3$, $x_2(0) = 2$ and $x_3(0) = 1$. The delays are specified, $d_1 = 0.7$ and $d_2 = 0.1$. Simulation results are shown in Figure 3, from which we can see that the system state indeed converges to zero.

## 5. Conclusions

This article has dealt with the state-feedback controller design for a class of feedforward input-delay nonlinear systems with a ratio of odd integers. The designed state-feedback controller ensures that the origin of closed-loop system is globally asymptotically stable. One of future works is to generalise the results in this article to the class of stochastic feedforward nonlinear systems with delays in the input.


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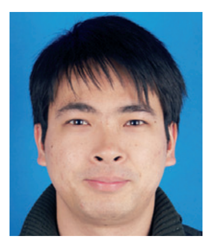

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## Appendix

**Proof of Proposition 1:** $\left|\frac{\partial \tilde{W}_n}{\partial z_i}\varphi_i\right|$ can be estimated by the following three steps.

**Step 1:** We give estimates of $|\varphi_i|$ $(i=1,2,\ldots,n-1)$. Using condition (2), $L>1$, Lemmas 3 and 4, one easily has

$$|\varphi_i| \le \frac{1}{L^2}\left[g_{i,1}\sum_{j=i+2}^{n}|z_j|^{p_i\cdots p_{j-1}} + g_{i,2}\left(\int_{t-d_n}^{t}|v(s)|\mathrm{d}s\right)^{p_i\cdots p_{n-1}} + g_{i,3}|\tilde{v}_i|^{p_i\cdots p_{n-1}}\right] \tag{52}$$

where $g_{i,j}$, $i=1,\ldots,n-2$, $j=1$, 2, 3 are constants. Next we consider the case where $i=n-1$. By Lemmas 5 and 6 and condition (16), we get

$$\begin{aligned}
|\varphi_{n-1}| &\le L^{-1+\sum_{m=1}^{n-1}\frac{p_0p_1\cdots p_{m-1}}{p_0p_1\cdots p_{n-2}}}\left(|\phi_{n-1}| + \left|\left(\tilde{x}_n - \int_{t-d_n}^{t}u(s)\mathrm{d}s\right)^{p_{n-1}} - \tilde{x}_n^{p_{n-1}}\right|\right),\\
&\le \frac{1}{L^2}M|\tilde{v}_{n-1}|^{p_{n-1}} + cL^{-1+\sum_{m=1}^{n-1}\frac{p_0p_1\cdots p_{m-1}}{p_0p_1\cdots p_{n-2}}}\left|\int_{t-d_n}^{t}u(s)\mathrm{d}s\right|\\
&\le \frac{1}{L^2}M|\tilde{v}_{n-1}|^{p_{n-1}} + 2cL^{-1+\sum_{m=1}^{n-1}\frac{p_0p_1\cdots p_{m-1}}{p_0p_1\cdots p_{n-2}}}\left|\int_{t-d_n}^{t}u(s)\mathrm{d}s\right|^{p_{n-1}}\\
&\quad + cL^{-1+\sum_{m=1}^{n-1}\frac{p_0p_1\cdots p_{m-1}}{p_0p_1\cdots p_{n-2}}}|\tilde{x}_n|^{p_{n-1}}\\
&\le \frac{1}{L^2}\left[g_{n-1,1}|z_n|^{p_{n-1}} + g_{n-1,2}\left(\int_{t-d_n}^{t}|v(s)|\mathrm{d}s\right)^{p_{n-1}} + g_{n-1,3}|\tilde{v}_{n-1}|^{p_{n-1}}\right]
\end{aligned} \tag{53}$$

where $g_{n-1,1}$, $g_{n-1,2}$ and $g_{n-1,3}$ are constants. This, together with (52), results in

$$|\varphi_i| \le \begin{cases} \dfrac{1}{L^2}\left[g_{i,1}\displaystyle\sum_{j=i+2}^{n}|z_j|^{p_i\cdots p_{j-1}} + g_{i,2}\left(\int_{t-d_n}^{t}|v(s)|\mathrm{d}s\right)^{p_i\cdots p_{n-1}} + g_{i,3}|\tilde{v}_i|^{p_i\cdots p_{n-1}}\right], \\ \qquad i=1,2\cdots,n-2, \\ \dfrac{1}{L^2}\left[g_{n-1,1}|z_n|^{p_{n-1}} + g_{n-1,2}\left(\displaystyle\int_{t-d_n}^{t}|v(s)|\mathrm{d}s\right)^{p_{n-1}} + g_{n-1,3}|\tilde{v}_{n-1}|^{p_{n-1}}\right], \\ \qquad i=n-1. \end{cases} \tag{54}$$

**Step 2:** We give the estimates of $|\frac{\partial \tilde{W}_n}{\partial z_i}|$, $i=1,2,\ldots n-1$. By using similar derivation of Step 2 in Zhang, Boukas, and Baron (2009), we obtain

$$\left|\frac{\partial \tilde{W}_n}{\partial z_i}\right| \le \sum_{k=1}^{n}\tilde{g}_k\left(z_k^{p_0\cdots p_{k-1}}\right)^{2-\frac{1}{p_0\cdots p_{i-1}}} \tag{55}$$

where $\tilde{g}_k>0$, $k=1,2,\ldots,n$, are constants independent of parameter $L$.

**Step 3:** Based on (54), (55) and Lemma 1, for $i=1$, $2,\ldots,n-1$, we get

$$\begin{aligned}
\left|\frac{\partial \tilde{W}_n}{\partial z_i}\varphi_i\right| &\le \frac{1}{L^2}\left(\lambda_{i,1}\xi_n^2 + \lambda_{i,2}\xi_n^2(t-d_i) + \lambda_{i,3}\int_{t-d_n}^{t}\xi_n^2(s)\mathrm{d}s + \sum_{j=1}^{n}\alpha_{i,j}z_j^{2p_0\cdots p_{j-1}}\right)\\
&\le \frac{1}{L^2}\left(\lambda_{i,1}\xi_n^2 + \lambda_{i,2}\xi_n^2(t-d_i) + \lambda_{i,3}\int_{t-d_n}^{t}\xi_n^2(s)\mathrm{d}s + \sum_{j=1}^{n}\mu_{i,j}\xi_j^2\right),
\end{aligned} \tag{56}$$

where $\lambda_{i,1}$, $\lambda_{i,2}$, $\lambda_{i,3}$, $\alpha_{i,j}$ and $\mu_{i,j}$ are some positive constants, $\lambda_{i,3}$ are constants depending on $d_n$. Thus, the proof of Proposition 1 is complete.