Nutation feature extraction of ballistic missile warhead

S. Huixia and L. Zheng

To elucidate nutation signatures, the model of nutation for a ballistic missile warhead is established and formulas of micro-Doppler induced by a target with nutation are derived. Two micro-Doppler features, which are micro-motion period and micro-Doppler bandwidth, are extracted based on high-resolution time-frequency transform. The validity of the features extracted in this reported work is verified by computer simulations even with low signal-to-noise ratio.

Introduction: Neutralising the threat of an incoming ballistic missile is a difficult task, which makes ballistic missile defence (BMD) more and more important. The midcourse phase which is the longest phase of flight, has a good chance of recognising and intercepting [1]. However, warheads flying outside the aerosphere in the midcourse phase are usually accompanied by decoys and debris moving with the same velocity, which presents a great challenge for ballistic missile warhead detection and recognition [2]. Since warheads are usually spin-stabilised, nutation, which is a special signature of a spinning conic warhead, will occur if there is a latitudinal disturbance, which is generally unavoidable. Nutation may induce additional frequency modulations on the returned radar signal, which generate sidebands about the warhead's Doppler frequency, called micro-Doppler frequency. Micro-Doppler can be regarded as a unique signature and provides additional information that is complementary to existing methods [3]. In [4], an algorithm for micro-Doppler feature extraction of a ballistic missile warhead is proposed, however it only deals with spinning-precession without nutation. Reference [5] establishes a mathematical model of nutation for a ballistic missile target and analyses its micro-Doppler by applying time-frequency transform, but does not give an algorithm for feature extraction.

In this Letter, we present a method for micro-Doppler feature extraction of a ballistic missile warhead under the case of nutation motion. Based on high-resolution time-frequency distribution, both the micro-motion period and the micro-Doppler bandwidth are extracted without velocity estimation, which avoids a high computational requirement.

Problem formulation: Without loss of generality, the geometry of radar and a conic-shaped target is depicted in Fig. 1. The origin of the co-ordinate system is the centroid of the conic-shaped target. The target has a coning motion along the axis oz with the unit vector $\mathbf{w_c} = [0,0,1]^{\mathrm{T}}$, which is also the axis of the coning. The axis oy which is vertical to the axis oz, lies on the plane where the axis oz and radar line of sight (LOS) oo_1 stay. The right hand co-ordinate system is satisfied between the axis ox and the plane yoz. The target spins around its axis of symmetry oo_2 with an angular velocity Ω_s while does the coning motion along the axis oz with an angular velocity Ω_c and nutation angle θ, and the axis of symmetry also oscillates up and down with an angular velocity Ω_w and amplitude θ_w simultaneously. The angle of LOS is γ along the unit vector $\mathbf{n} = [0, -\sin\gamma, \cos\gamma]^T$ while the initial nutation angle is θ_0 along the unit vector $\mathbf{w_s} = [0, \sin\theta_0, \cos\theta_0]^{\mathrm{T}}$. The angle between LOS and the axis of symmetry oo_2 is $\varphi (0 \le \varphi \le \pi)$, which is also called the attitude angle.

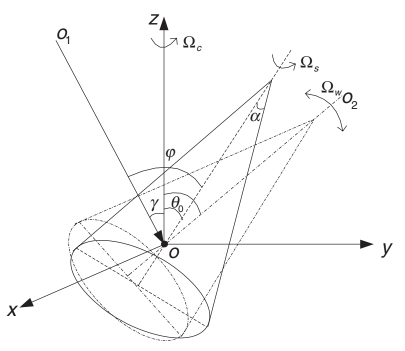

Fig. 1 *Geometry of target with nutation motion*

According to the Rodrigues's formula, at time t the wobbling rotation matrix is:

$$\mathbf{R_w} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_w \sin(\Omega_w t)) & -\sin T(\theta_w \sin(\Omega_w t)) \\ 0 & \sin(\theta_w \sin(\Omega_w t)) & \cos(\theta_w \sin(\Omega_w t)) \end{bmatrix} \tag{1}$$

At time t the coning rotation matrix can be written as follows:

$$\mathbf{R_c} = \mathbf{I} + \widehat{\mathbf{W}}_\mathbf{c} \sin(\Omega_c t) + \widehat{\mathbf{W}}_\mathbf{c}^2 [1 - \cos(\Omega_c t)] \tag{2}$$

where the skew symmetric matrix is defined by:

$$\widehat{\mathbf{W}}_\mathbf{c} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3}$$

At time t the unit vector of the axis of symmetry oo_2 becomes

$$\begin{aligned} \mathbf{r_1} = \mathbf{R_c} \cdot \mathbf{R_w} \cdot \mathbf{w_s} &= [-\sin(\Omega_c t)\sin(\theta_0 - \theta_w \sin(\Omega_w t)), \cos \\ &\times (\Omega_c t)\sin(\theta_0 - \theta_w \sin(\Omega_w t)), \cos(\theta_0 - \theta_w \sin(\Omega_w t))]^{\mathrm{T}} \end{aligned} \tag{4}$$

We can write

$$\begin{aligned} \cos\varphi = \|\mathbf{r}_1 \cdot \mathbf{n}\| / (\|\mathbf{r}_1\| \cdot \|\mathbf{n}\|) &= \cos\gamma\cos(\theta_0 - \theta_w \sin(\Omega_w t)) \\ &- \sin\gamma\cos(\Omega_c t)\sin(\theta_0 - \theta_w \sin(\Omega_w t)) \end{aligned} \tag{5}$$

Therefore, the attitude angle can be expressed by

$$\begin{aligned} \varphi = a\cos(&\cos\gamma\cos(\theta_0 - \theta_w \sin(\Omega_w t)) \\ &- \sin\gamma\cos(\Omega_c t)\sin(\theta_0 - \theta_w \sin(\Omega_w t))) \end{aligned} \tag{6}$$

The backscattered RCS of a cone can be expressed as [6]

$$\sigma(\varphi) = \begin{cases} \lambda L \tan\alpha \tan^2(\varphi - \alpha)/(8\pi \sin\varphi), & \varphi \in (0, \pi), \varphi \ne \pi/2 - \alpha \\ 8\pi L^3 \sin\alpha / (9\lambda \cos^4\alpha), & \varphi \ne \pi/2 - \alpha \end{cases} \tag{7}$$

where λ is the wavelength of the electromagnetic wave. By substituting (6) into the above equation, one can obtain

$$\sigma(t) = \begin{cases} \dfrac{\lambda L \tan\alpha \tan^2(\mathrm{acos}(\cos\gamma\cos(\theta_0 - \theta_w \sin(\Omega_w t)) - \sin\gamma\cos(\Omega_c t)\sin(\theta_0 - \theta_w \sin(\Omega_w t))) - \alpha)}{8\pi \sin(\mathrm{acos})\cos\gamma\cos(\theta_0 - \theta_w \sin(\Omega_w t)) - \sin\gamma\cos(\Omega_c t)\sin(\theta_0 - \theta_w \sin(\Omega_w t))))}, \\ \quad \mathrm{acos}(\cos\gamma\cos(\theta_0 - \theta_w \sin(\Omega_w t)) - \sin\gamma\cos(\Omega_c t) \sin(\theta_0 - \theta_w \sin(\Omega_w t))) \ne 0, \dfrac{\pi}{2} - \alpha \\ \dfrac{8\pi L^3}{9\lambda} \dfrac{\sin\alpha}{\cos^4\alpha}, \\ \quad \mathrm{acos}(\cos\gamma\cos(\theta_0 - \theta_w \sin(\Omega_w t)) - \sin\gamma\cos(\Omega_c t)\sin(\theta_0 - \theta_w \sin(\Omega_w t))) = \dfrac{\pi}{2} - \alpha \end{cases} \tag{8}$$

Assume the initial position of a scatterer P is $\mathbf{r_0} = (x_0, y_0, z_0)^{\mathrm{T}}$. Then at time t, the location of the scatterer P is at $\mathbf{r_t} = \mathbf{R_c} \cdot \mathbf{R_w} \cdot \mathbf{r_0}$. If the radar transmits a sinusoidal waveform with a carrier frequency f_0, the base-band of the signal returned from the particle P is

$$s(t) = \sqrt{\sigma(t)} \exp[j4\pi f_0 r(t)/c] = \sqrt{\sigma(t)} \exp[\mathrm{j}\Phi(t)] \tag{9}$$

where $\sqrt{\sigma(t)}$ is the amplitude of the returned radar signal, $r(t) = \|\mathbf{R}_0 + \mathbf{v}t + \mathbf{r_t}\|$, $\mathbf{R}_0$ is the range vector of the origin o from radar at initial time, $\mathbf{v}$ is the velocity vector of bulk motion, c is the speed of the electromagnetic wave. The phase of the baseband signal is $\Phi(t) = 4\pi f_0 r(t)/c$.

By taking the time derivative of the phase and subtracting the Doppler shift caused by translation, the micro-Doppler frequency shift induced by nutation is obtained

$$\begin{aligned} f_{mD}(t) &= \frac{2f_0}{c}\left[\frac{d}{dt}(\mathbf{R_c} \cdot \mathbf{R_w}) \cdot (x_0, y_0, z_0)^T\right]^{\mathrm{T}} \cdot \mathbf{n} \\ &= \frac{2f_0}{c}\left[\frac{d\mathbf{R_c}}{dt} \cdot \mathbf{R_w} \cdot (x_0, y_0, z_0)^{\mathrm{T}} \right. \\ &\left. + \mathbf{R_c} \cdot \frac{d\mathbf{R_w}}{dt} \cdot (x_0, y_0, z_0)^{\mathrm{T}}\right]^{\mathrm{T}} \cdot \mathbf{n} \end{aligned} \tag{10}$$

where

$$\frac{d\mathbf{R_c}}{dt} = \Omega_c[\widehat{\mathbf{W}}_\mathbf{c}\cos(\Omega_c t) + \widehat{\mathbf{W}}_\mathbf{c}^2\sin(\Omega_c t)]$$

$$\frac{d\mathbf{R_w}}{dt} = \Omega_w\theta_w\cos(\Omega_w t) \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin(\theta_w\sin(\Omega_w t)) & -\cos(\theta_w\sin(\Omega_w t)) \\ 0 & \cos(\theta_w\sin(\Omega_w t)) & -\sin(\theta_w\sin(\Omega_w t)) \end{bmatrix}$$

Feature extraction: From (10), we can conclude that the nutation's micro-Doppler is a time-varying periodic curve. The period and bandwidth of the micro-Doppler can be used as features to recognise a ballistic missile warhead, where the nutation period can be estimated via the autocorrelation method, and the micro-Doppler bandwidth which is twice that of the maximum micro-Doppler frequency can be computed as follows:

1. Compute the smooth pseudo Wigner-Ville distribution of the returned radar signal, written as **SPWVD(M,N)**, where M and N are time samples and frequency samples;
2. Calculate the maximum greyscales of the frequencies on the time-frequency plane

$$g_{\max}(f) = \max\{\text{abs}[(\mathbf{SPWVD}(\mathbf{M}, \mathbf{N}))']\} \tag{11}$$

where $f = -PRF/2, -PRF/2 + PRF/N, \ldots, PRF/2 - PRF/N$, PRF is pulse repetition frequency;
3. Find the frequencies f_l which satisfy $g_{\max}(f) >$ mean$\{g_{\max}(f)\}$, where $l = 1, 2, \ldots, L$, L is the number of frequencies that satisfy the qualification;
4. Search the position l_0 corresponding to the maximum of $g_{\max}(f)$ and translate it into the centre, which can be realised by the Δl-number cyclic shift of $g_{\max}(f)$, where $\Delta l = \lfloor N/2 - l_0 \rfloor$, $\lfloor\cdot\rfloor$ denotes the round-off operation, $g_{\max}(f)$ can be written as $h_{\max}(f)$ after the cyclic shift;
5. Search from $l = 1$ with sort ascending, until $h_{\max}(f_l) < h_{\max}(f_{l+1}) < h_{\max}(f_{l+2})$, then the maximum negative frequency is $f_{\max-} = f_{l+1}$;
6. Search from $l = L$ with degressive order, until $h_{\max}(f_l) < h_{\max}(f_{l-1}) < h_{\max}(f_{l-2})$, then the maximum positive frequency is $f_{\max+} = f_{l-1}$;
7. The bandwidth of the micro-Doppler can be estimated by the following equation:

$$B_{mD} = f_{\max+} - f_{\max-} \tag{12}$$

A conclusion can be drawn that the proposed algorithm extracts features without bulk velocity estimation, which has the property of requiring less computational burden.

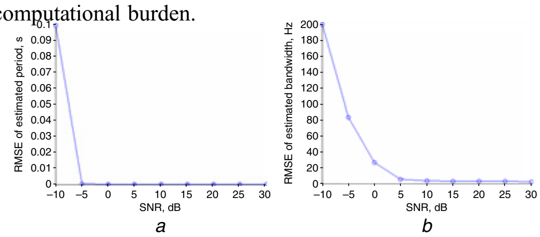

Fig. 2 *Variation of micro-Doppler estimation RMSE with SNR*

a Micro-Doppler period
b Micro-Doppler bandwidth

Simulation results: Assume that radar operates at 5 GHz and transmits a pulse waveform with a PRF of 1000 Hz. The target whose half cone angle is $\alpha = 10°$, is wobbling along axis oo_2 with an initial nutation angle $\theta_{w0} = 10°$, maximum wobbling angle $\theta_{ww} = 10°$ and angular velocity $\Omega_{ww} = 6/\pi$ rad/s, while it is coning with an angular velocity $\Omega_{wn} = 4\pi$ rad/s. For a spinning invariability conic warhead, scatterers substitute a one by one equivalent, which makes no difference for the returned radar signal. Fig. 2*a* is the root mean squared error (RMSE) of micro-Doppler period estimation of the autocorrelation method with SNR. The simulation result shows that the estimated period is nearly identical to the theoretic value when $SNR \geq -5$ dB, since the correlation of the signal and randomicity of the noise. The RMSE of micro-Doppler bandwidth with SNR is shown in Fig. 2*b*, which demonstrates that the estimated error of the proposed method is less than 3% when $SNR > 5$ dB. The number of Monte Carlo trials is 100 in all these simulations.

Conclusion: This study establishes the model of nutation for a ballistic missile warhead, and derived mathematical formulas solving micro-Doppler modulations induced by nutation. Based on high-resolution time-frequency transform, the micro-Doppler period and micro-Doppler bandwidth are extracted without velocity estimation, which is computationally efficient. Simulation results confirm that the presented method is effective even with low SNR.

© The Institution of Engineering and Technology 2011
7 March 2011
doi: 10.1049/el.2011.0580
One or more of the Figures in this Letter are available in colour online.

S. Huixia and L. Zheng (*National Lab for Radar Signal Processing, Xidian University, Xi' an, Shaanxi 710071, People's Republic of China*)

E-mail: hxsun@mail.xidian.edu.cn

References

1 Yongxiang, L., Xiang, L., and Zhaowen, Z.: 'Research on radar target discrimination in the missile defense system', *J. Natl. Univ. Defence Technol.*, 2004, **26**, (4), pp. 6–9
2 Peikang, H.: 'Target recognition in anti-missile system', *J. Strateg. Defence*, 1981, **10**, (5), pp. 1–14
3 Chen, V.C., Li, F., and Ho, S.-S., *et al.*: 'Micro-Doppler effect in radar: phenomenon, model, and simulation study', *IEEE Trans. Aerosp. Electron. Syst.*, 2006, **42**, (1), pp. 2–21
4 Huixia, S., Zheng, L., and Ning, X.: 'Ballistic missile warhead recognition based on micro-Doppler frequency', *Defence Sci. J.*, 2008, **58**, (6), pp. 705–709
5 Zhaoqiang, S., Baozhu, L., and Yaobing, L.: 'A study on nutation property of ballistic target and its micro-Doppler', *Mod. Radar*, 2009, **31**, (4), pp. 24–27
6 Mahafza, B.R.: 'Radar systems analysis and design using matlab' (CRC Press, Boca Raton, FL, 2000), pp. 89–92